An analysis of the risk free rate in the South African capital market

Johann Burger, Honours B.Com. (Risk Management)

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Supervisor: Dr. D. Viljoen

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OPSOMMING

Die huidige navorsing was uitgevoer om te bepaal of die pryse in die Suid-Afrikaanse kapitaalmark 'n risiko-vrye koers impliseer wat nie eenders is van die teoretiese risiko-vrye koers nie. Die navorsing was uitgevoer deur middel van ŉ literatuurstudie en rekenaar gebaseerde navorsing analise, van die mark prys wat gebaseer is op die mark-opbrengskromme. Die literatuurstudie was gedoen om die belangrikheid van die risiko-vrye koers in finansiële stelsels dinamika vas te stel. Die literatuurstudie beklemtoon dat alle portefeulje teorieë en prestasie-maatstaf aanwysers, ŉ risiko-vrye koers in die kern van hulle metodiek het. Dit impliseer dat die risiko-vrye koers die belangrikste konsep is om die mark aanvraag vir verskillende instrumente vas te stel. Die navorser het ŉ vergelyking getref tussen die BESA gepubliseerde verband opbrengskromme en die mark-prys-gebaseerde opbrengskromme; wat die navorser self ontwerp het. Die bevinding is dat die mark prys risiko-vrye koers hoër is as die van die teoretiese risiko-vrye koers. Verder is daar ook bevind dat die vorm van die opbrengskromme verskil van die BESA geprojekteerde opbrengskromme, en dat dit ŉ aanduiding is van toekomstige probleme vir die Suid-Afrikaanse kapitaalmark. Die implikasie van beleggers se persepsie van ‘n hoër risiko-vrye koers word bespreek en daar deur word dit geopenbaar dat buitelandse beleggers die land risiko en standaard risiko hoër ag as wat BESA dit deurgee om te wees.

SUMMARY

The current research was undertaken to assess if the prices in the South African capital market imply a risk free rate that is not equal to the theoretical risk free rate. The research was conducted by means of a literature review and desktop-research-based analysis of the market price based yield curve. The literature review was conducted to establish the importance of the risk free rate in the financial systems dynamics. The literature review highlighted that all the portfolio theories and performance-measure indicators have the risk free rate at the core of their methodology. This implies that the risk free rate is the most important concept that determines the market demand of different instruments. Next, a comparison has been drawn between the BESA published bond yield curve and a market-price-based yield curve developed by the researcher. The findings establish that the market price derived risk free rate is higher than the theoretical risk free rate. It was also found that the shape of the yield curve is different from the BESA projected yield curve, and that it is indicative of future problems in the South African capital market. The implications of investors' perceptions of the higher risk free rate are discussed and it is revealed that the foreign investors consider the country risk and the default risk associated with the South African government as higher than the BESA may perceive it to be.

LIST OF ABBREVIATIONS

ALBI	:	All bond index
APT	:	Arbitrage pricing theory
BESA	:	Bond exchange of South Africa
CAPM	:	Capital asset pricing model
DCF	:	Discounted cash flows
EMRP	:	Equity market risk premium
GOVI	:	Government index
IRR	:	Internal rate of return
JSE	:	Johannesburg Stock Exchange
LSE	:	London Stock Exchange
MAR	:	Minimum acceptable return
MPT	:	Modern portfolio theory
MSCI	:	Morgan Stanley Capital International
NPV	:	Net present value
NYSE	:	New York Stock Exchange
SDF	:	Stochastic discount factor
US	:	United States
USD	:	United States Dollar
UK	:	United Kingdom
WACC	:	Weighted average cost of capital
YTM	:	Yield to maturity

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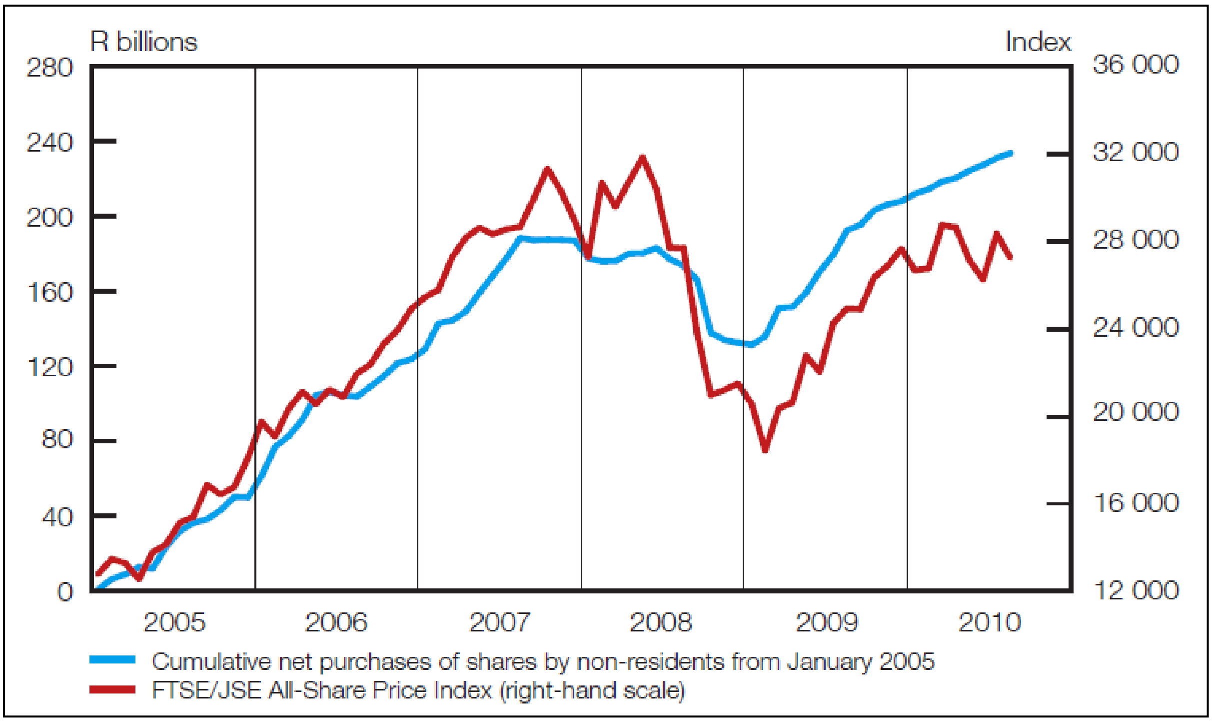
INTRODUCTION AND BACKGROUND

South Africa is an emerging country that has developed a deep capital market in the short span of time since its independence (Wajid *et al*, 2008:21). Capital markets play a crucial role in the overall development of the economy, as these provide the basic resources for large infrastructure and nation-building projects, and therefore, are essential for any country's long-term growth and progress.

Between 2002 and 2012, South Africa has made several structural and institutional changes to consolidate its capital market. These changes involved the consolidation of government bonds into benchmark bonds, the development of the secondary bond market and the establishment of the Bond Exchange of South Africa (BESA) (Wijck, 2006:61). These changes established a capital market environment conducive to long-term growth in South Africa. The conduciveness of the market environment led to increased participation from the private sector an influx of corporate bonds as a result. The introduction of private bonds in the capital market was paralled by increased involvement by the private sector in the utilities and infrastructure sector of South Africa. With the liberalisation of the capital market, there was an increase in foreign investments in South Africa.

As a result of the increasing reforms and the growth of the South African economy, more and more foreign investors began to trade on the Johannesburg Stock Exchange (JSE). Figure 1.1 illustrates the activity of foreign investors on the JSE's equity market. In 2010 foreign investors were responsible for net purchases in equities and bonds on the JSE valued at R36.4bn and R58.6bn respectively (Carte, 2011). Now, foreign investors are responsible for the majority of the activity represented in the South African markets with the London Stock Exchange (LSE) contributing to more than 50% of the activity on the JSE (Kotzé, 2011:93). Figure 1.2 illustrates the activity of foreign investors on BESA.

Figure 1.1: Non-resident activity and share prices on the JSE Limited



Source: South African Reserve Bank (SARB, 2010:62)

Figure 1.2: Non-resident activity on BESA

Bond Code	Total client trades on BESA (ZAR mn) Local Clients	Foreign Clients
R206	0.26	2
R201	0.36	294
R157	69.69	241
R203	-353.22	207
R204	209.68	88
R207	-728.85	328
R208	-109.48	701
R186	319.50	147
R213	-89.52	305
R209	-56.84	222
R214	0.00	-87

Bond Exchange Total Volumes Nominal amount traded	(ZAR bn)	latest &	Previous
Daily	18-Jul	74	227
Week to date	20-Jul	371	476
Month to date	Jul-12	1,293	2,112
Year to date	2012	12,861	20,893

Net Foreign purchases / sales of SA bonds	(ZAR mn)	latest &	Previous
Daily	18-Jul	1,799	874
Week to date	20-Jul	3,504	4,627
Month to date	Jul-12	12,663	19,280
Year to date	2012	61,133	47,359

Source: BESA (2012)

The risk free rate will play a significant role in foreign investors' investment choices as it determines the rate of return available in the market on an investment free of default risk. According to Busetti (2009:103), the risk free rate is an instrument that the investor can expect the lowest return from because the instrument is, theoretically, perfectly safe.

While foreign investors' participation in the market is good for the economic development and prosperity of South Africa, there are reservations with regard to how the risk free rate should be assessed. This is because- in a closed environment, as in the case when there are no foreign investors, the risk free rate is determined by the government securities that are held as completely free of risk, including default risk. In such an environment, the chances of a stable government defaulting are almost zero and therefore government bonds form the standard of the risk free rate (Kazemi, Schneeweis & Gupta, 2003:53).

However, in the case of foreign investors, the perceived risk related to government bonds may be higher than the theoretical risk free rate (theoretical rate of return attributed to an investment with zero risk) of government bonds. The reason for this discrepancy in the perception of risk is due to the difference in confidence that foreign investors and internal investors may have regarding the government's ability and stability (Chen & Knez, 1996:38). Foreign investors may be prone to use a different model of risk assessment for South African government bonds, the criteria for which is largely inspired by the United Kingdom (UK) perspective on governments' ability and stability.

Furthermore, the fact that foreign investors (those in London) work according to different transition matrices (transition matrices involves probabilities and percentages and calculates the probability of change in price from one state to another) from those of South African investors gives rise to growing uncertainty on whether the prices in the South African capital market imply a risk free rate that is not equal to the theoretical risk free rate (Wajid *et al*, 2008:46). This means that there is a chance that the credit assessment for government bonds by foreign investors is based on criteria that may make the South African governments' bonds riskier than the theoretical risk free rate.

As the majority of investors on the JSE appear are foreign, it is probable that the risk free rate will be established by their perception of government's bonds and their factors of evaluation. If the risk free rate is regarded as higher by the foreign investors, then there is a substantial chance that the actual risk free rate may be higher than the theoretical risk free rate (Mukherji, 2011:75). It is, therefore, essential to understand how the risk free rate in the South African market is determined in practical terms.

The assessment of the practical risk free rate and its comparison to the government risk free rate is complex in nature. The BESA methodology that is used to arrive at the zero coupon yield curves is highly complex and robust (Wajid *et al*, 2008), and beyond the scope of this project. Using zero coupon bond prices and associated yields to maturity, the BESA yield curve is compared with the market implied yield curve. The research is essential in highlighting the differences in the yield curve shapes and yields, and will, therefore, present a picture of the practical realities in the South African bond market.

PROBLEM STATEMENT

The government provided risk free rate may not be the true indicator of the capital market (Mukherji, 2011:75). Understanding the way in which foreign investors make their investment decisions will enable the government to adjust its BESA methodology with a better understanding of the investor themselves.

This research is aimed at exploring the existence of any differences between the theoretical and the actual yield curves and also at understanding the plausible reasons for these differences.

OBJECTIVES

Primary Objective

The primary objective of this research was to determine if the prices in the South African capital market imply a risk free rate that is not equal to the theoretical risk free rate.

Secondary Objectives

The second objective was to present a holistic view of the financial market, including the portfolio theories and the bond valuations methods and indices. This holistic view provided the researcher with the context needed for understanding the South African market and for conducting the analysis needed for assessing the practical risk free rate. The third objective was to present the implications of the practical risk free rate for South African economic and financial well-being (if it is found to be different from the government theoretical risk free rate). The fourth objective was show that a yield curve giving the approximate risk free interest rate for any bond duration can be obtained using the GOVI benchmark and the short term RODI rate.

RESEARCH METHODOLOGY

The research employed a literature review and desktop research to achieve the objectives outlined above. Secondary research was undertaken by perusing and critically analysing the various concepts and contents of the bond market including the risk free rate as used by the capital asset pricing model, arbitrage pricing theory and modern portfolio theory. The literature review also discussed bond valuations, sovereign ratings and bond performance measures like the Sortino and Omega ratio, and the internal rate of return. Conducting the literature review enabled the researcher to develop an understanding of the concept of the risk free rate and the various factors that are involved in its calculation.

A further aim of the literature review was to explain the various indices and factors that investors may use to value bonds or to assess the long-term performance of bonds. This insight is essential in the case of the current research, as foreign investors own perceptions or valuation of bonds are presumed to be the reason for any differences in the theoretical and the practical risk free rate.

The next part of the research aims to assess the existence of the difference in the theoretical and the actual risk free rate in the South African capital market. The research is based on data collected from the JSE on the market prices on a specific date, and then developing a yield curve. The yields to maturities are calculated by using the Equation 1 (Brigo & Mercurio, 2001:23):

$$\boldsymbol{Present\ Price} = \sum(\mathbf{1} + \boldsymbol{YTM})^{\boldsymbol{n}} \qquad \textit{(Equation 1)}$$

In Equation 1, n is the maturity period and YTM is the yield to maturity.

An easier and more appropriate method of calculating the yield to maturity is to use Equation 2 (Brigo & Mercurio, 2001:25):

$$\boldsymbol{Y(t)} = \left(\frac{\mathbf{1}}{\boldsymbol{P(t)}}\right)^{\frac{\mathbf{1}}{\boldsymbol{t}}} \qquad \textbf{\textit{(Equation 2)}}$$

Where:

Y (t) = Yield to Maturity

P (t) = cash flow at time (t) in future.

The market data were captured on 1st September 2012 (this was the date the researcher received the data from the JSE Interest Rate Department) and the current prices of the government zero coupon bonds were noted. The yields were developed using Equation 2 and a yield curve was plotted. This yield curve is presumed to reflect the practical yield curve as perceived by foreign and private investors.

The BESA Actuaries Yield Curve was obtained from the BESA publications on the 1st September 2012. The yield to maturities as provided by the government was compared with the calculated yields of current maturities to assess the difference in the perceptions with regard to future interest rate growths. The BESA yield curve and the market based yield curve were compared using the shape of the curve as the criteria. The findings are discussed in terms of yield curve theories, such as the pure expectations theory, the liquidity premium theory, market segmentation theory and the preferred habitat theory. The differences in the shape of the yield curve and the yield to maturities as calculated by the researcher and by BESA were used to assess the existence of the difference in the theoretical and the actual risk free rate in South Africa.

The research used the South African context of country and foreign exchange risk in order to explain why the investors may have a different perception and understanding of the risk free rate. The research also provides a discussion on the implications of the practical risk free rate being different from the theoretical ones, in terms of the economic and financial wellbeing of the country.

Since government bonds are considered virtually default-free, the GOVI (Government index) benchmark bonds form a term structure of nearly risk free interest rates (BESA, 2003:3). Using this term structure, a smooth continuous yield curve is fitted relating an approximated risk free interest rate to any bond duration term.

With the yield curve as a reference, credit/liquidity spreads are calculated for a selection of non-government rated bonds.

CHAPTER OUTLINE

This dissertation is presented in five chapters. An outline of each chapter is set out below.

Chapter 1 (*Introduction*) contains an overview of the South African bond market and the presence of foreign investors. It develops the rationale for doing the research and outlines the aims and objectives that were to be achieved by the completion of the research. The chapter details the research methods that were employed for the achievement of the research objectives.

Chapter 2 (*Theoretical analysis of portfolio theory*) contains findings from the literature review. Chapter 2 aims and discusses various theories, such as the capital asset pricing model (CAPM), the arbitrage pricing theory (APT), and the modern portfolio Theory (MPT). These theories were selected for discussion as the theoretical risk free rate plays an integral part in them. The above-mentioned theories are discussed in the context of the role that the theoretical risk free rate plays. In this way this chapter is useful in establishing the basis of the theoretical risk free rate by a country like South Africa.

Chapter 3 (*Bonds*) is based on developing an understanding of how private investors or third-party investors make an assessment of the risk free rate. Chapter 3 covers an overview of bond valuations, sovereign ratings, transition matrix's and tracking. This is followed by discussions on portfolio performance measures such as Sortino (which is a modification of the Sharp-Ratio), Omega, IRR (internal rate of return) and WACC (weighted average cost of capital). These indices and ratings are used to establish the role that the actual risk free rate plays in helping investors make decisions about investments.

Chapter 4 (*An analysis of the theoretical risk free rate and the perceived risk free rate*) contains the findings and analysis as conducted first hand by the researcher. It presents the calculations of the yield to maturity of the South African government bonds, using the current market prices and the development of the bond yield curve for the same data. The yield curve is compared and contrasted with the government-published yield curve (which is based on the same date of the yield curve developed for this research). The differences in yields and the difference in the shape of the BESA and market-based yield curve are used to determine if the theoretical risk free rate is different from the practical risk free rate, as implied by the prices in the market. The chapter also contains a discussion of the reasons for the differences and also elaborates on the future implications of the changes in the risk free rate for the South African economy. The final part of the chapter uses the term structure of nearly risk free interest rates (derived from the GOVI benchmark bonds). Using this term structure, a smooth continuous yield curve is fitted relating an approximated risk free interest rate to any bond duration term. With the yield curve as a reference, credit/liquidity spreads are calculated for a selection of non-government rated bonds.

Chapter 5 (*Summary, Conclusion & Recommendations*) contains a summary of the research, and gives a brief overview of all the chapters that were presented and links them to the achievement of the research purpose. The chapter also contains the main conclusions derived from the analysis set out in Chapter 4. It also highlights some of the limitations that were faced by the researcher and concludes with recommendations for future research on the topic of analysing the risk free rate in the South African capital market.

CHAPTER 2: THEORETICAL ANALYSIS OF PORTFOLIO THEORY

2.1 INTRODUCTION

The theoretical risk free rate is presumed to play a crucial role in the development of investors' expected rate of returns from various instruments available in the market (Ryan, Scapens & Theobald, 2002:10). The importance of the risk free rate in determining investors' behaviour and perception of the market is established through the modern portfolio theory (MPT), the capital asset pricing model (CAPM) and the arbitrage pricing theory (APT). In this chapter, a detailed analysis of the MPT, CAPM and APT is undertaken to establish the important role that the risk free rate plays in determining the financial market dynamics.

2.2 MODERN PORTFOLIO THEORY

Modern portfolio theory (MPT) was developed by Harry Markowitz in 1952. He assumed that most investors want to be cautious when investing and that they want to take the smallest possible risk in order to obtain the highest possible return, optimizing return to the risk ratio. MPT states that it is not enough just to look at the expected risk and return of one particular asset. By investing in more than one asset, an investor can obtain the benefits of diversification, a reduction in the volatility of the whole portfolio (Markowitz, 1959:18).

The essence of MPT is to seek optimisation of the relationship between risk and return by composing portfolios of assets determined by their returns, risks, and covariance or correlations with other assets. MPT develops a framework where, any expected return is composed of various future outcomes and are, therefore, risky. The relationship between risk and return can be optimised through diversification.

2.2.1 Markowitz's portfolio selection

One of the most important contributions made by Markowitz is the way in which he differentiated the variability of returns of the individual security and how this variability influences the risk factor in a portfolio. According to Markowitz (1959:23), if the attempt is trying to make the variance small, investing in different types of securities is not adequate. It is necessary to avoid investing in the securities that have higher levels of covariance among themselves.

MPT is a considered to be one of the first sophisticated investment approaches that Markowitz (1959:24) developed. This theory has a landmark place in the history of financial management because it was the first theory of its kind that allowed the investors by means of statistics to estimate not only the returns but also the risk involved in investment for their respective investment portfolios (Elton & Gruber, 1997:1743). Markowitz’s (1959:31) portfolio selection described how it is possible for:

- Assets to be combined into different diversified investment portfolios;
- Investors to fail to correctly account for the high correlation among the security returns; and
- The risk associated with any particular portfolio to be reduced considerably, and the excepted rate of returns increased, if different assets that had dissimilar prices were combined.

If the securities or investments are made in assets that are very similar, the risk related to investment cannot be reduced (Markowitz, 1959:34). According to modern portfolio theory, portfolio diversification is the way to reduce the risks. Diversification must be undertaken in a careful manner as risk is reduced only when various assets are combined within a portfolio and when the prices of these assets move either inversely or at different points in time relative to each security. Markowitz was one of the first to quantify risk and demonstrate how the concept of portfolio diversification can reduce the risk and improve the returns for investors. In simple terms, his approach was one of “not putting your eggs all in one basket” (Elton & Gruber, 1997:1745).

The portfolio theory works under the assumption that investors are rational, risk averse and have many varied options with regard to the choice of investments. It is necessary to understand that any investment opportunity comes with both risks and rewards and a good portfolio can be constructed where different permutations and combinations of investments work together to balance the level of risk and rewards.

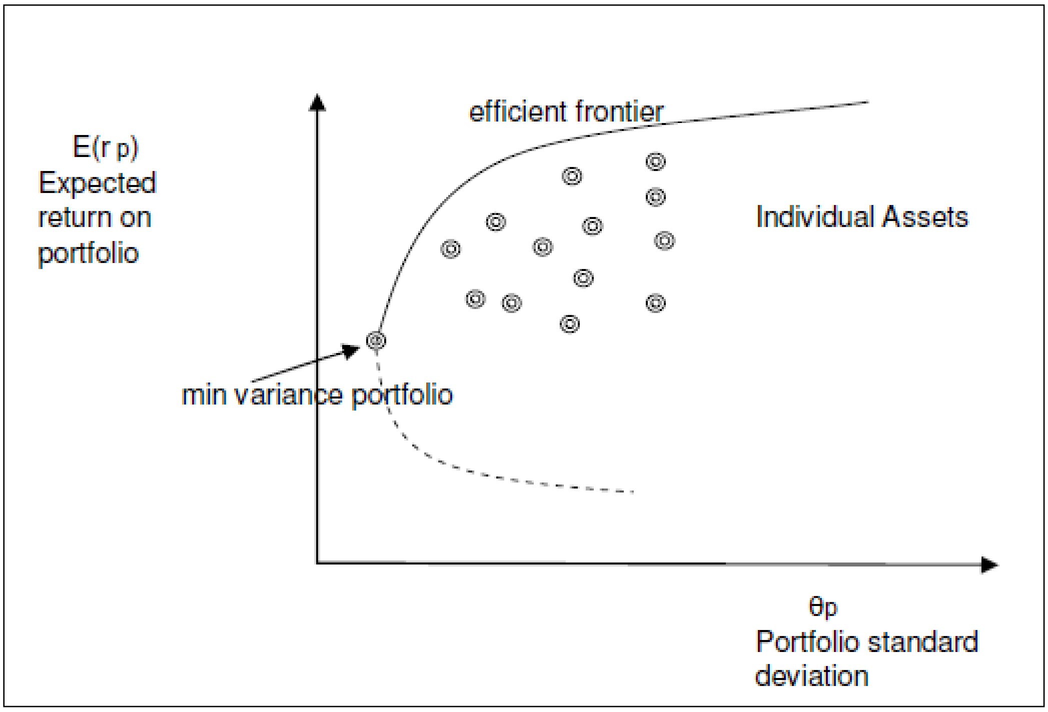
According to MPT, the assets in any portfolio can be combined to produce an "efficient portfolio" that has the capacity to give you the highest possible level of return on investment for any level of portfolio risk, which is measured by either the standard deviation or variances (Lubatkin & Chetterjee 1994:110). Those portfolios that have a combination before the efficient frontier have less chances of producing an efficient trade off. For this reason, it is necessary to have an efficient frontier for maximising the chances of a reward-risk balance (Lubatkin & Chetterjee 1994:113).

The efficient frontier is a graph representing a set of portfolios that maximize expected return at each level of portfolio risk (Bode, 2003:18). To plot an efficient frontier, it is necessary to calculate the future expected returns and standard deviation, along with the correlation coefficients between each pair of assets.

The efficient frontier describes the collection of portfolios (i.e. asset mixes) that produces the highest expected return at various levels of risk (as measured by the standard deviation of portfolio returns) (Hudson-Wilson, 1995:35). Such portfolios can be seen as efficiently diversified.

The expected return standard deviation (risk-return) combinations for any individual asset end up inside the efficient frontier, because single asset portfolios are not efficiently diversified. Figure 2.1 indicates that portfolios below the minimum variance level may be discarded, which is dominated by portfolios on the upper half of the frontier as they yield a higher expected return with an equal amount of risk. Therefore, investors should only consider portfolios on the efficient frontier above the minimum variance portfolio (Bode 2003:22). A minimum variance portfolio can be described as a portfolio of individually risky assets that, when taken together, result in the lowest possible risk level for the rate of expected return (Markowitz, 1991:41).

Figure 2.1: The efficient frontier of different assets

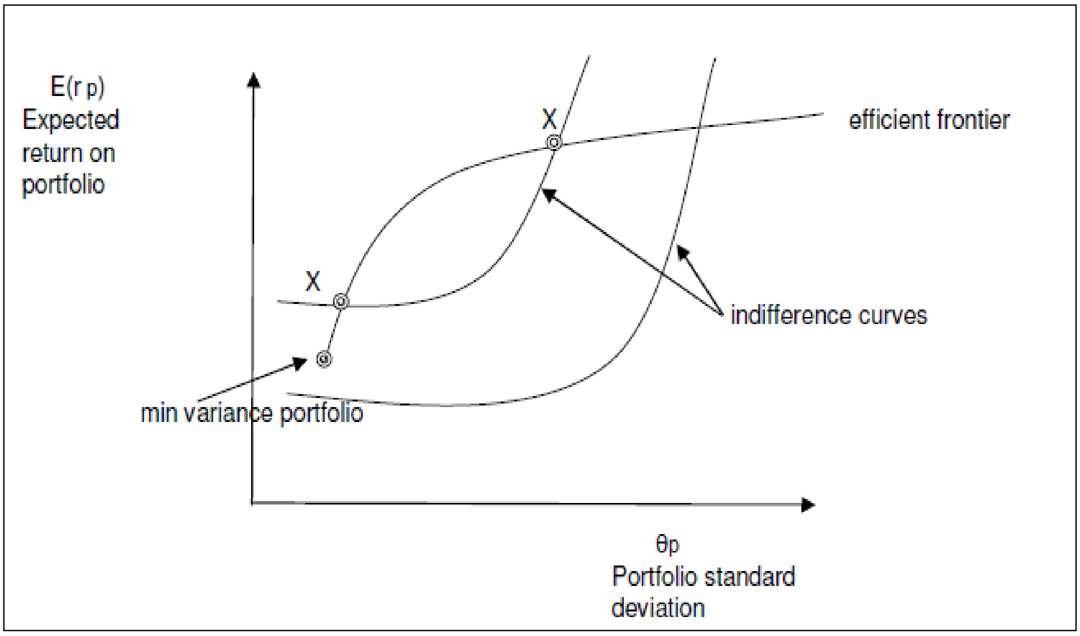


Source: Bode (2003:19)

Investor preferences and the efficient frontier can be used in order to assist an investor on choosing targeted range along the efficient frontier the optimal portfolio (Hudson-Wilson, 1995:43). Figure 2.2 shows the indifference curves superimposed on the efficient portfolio diagram. Point X in Figure 2.2 is the point where the indifference curve touches the efficient frontier. Here the investor’s preferences match up with the optimal choice (Geltner, 2007:4).

An investor should consider portfolios on the efficient frontier which lie above the investors’ indifference curve. An indifference curve is a diagram depicting equal levels of utility (satisfaction) for a consumer faced with various combinations of goods (Hudson-Wilson, 1995:43).

Figure 2.2: Indifference curves and the efficient frontier



Source: Geltner (2007:5)

One common misconception that many investors make about MPT is that diversification means a combination of individual stocks, bonds, international stocks and mutual funds. These are different types of investments, however, they belong to the same asset class and, therefore, work in concert with each other.

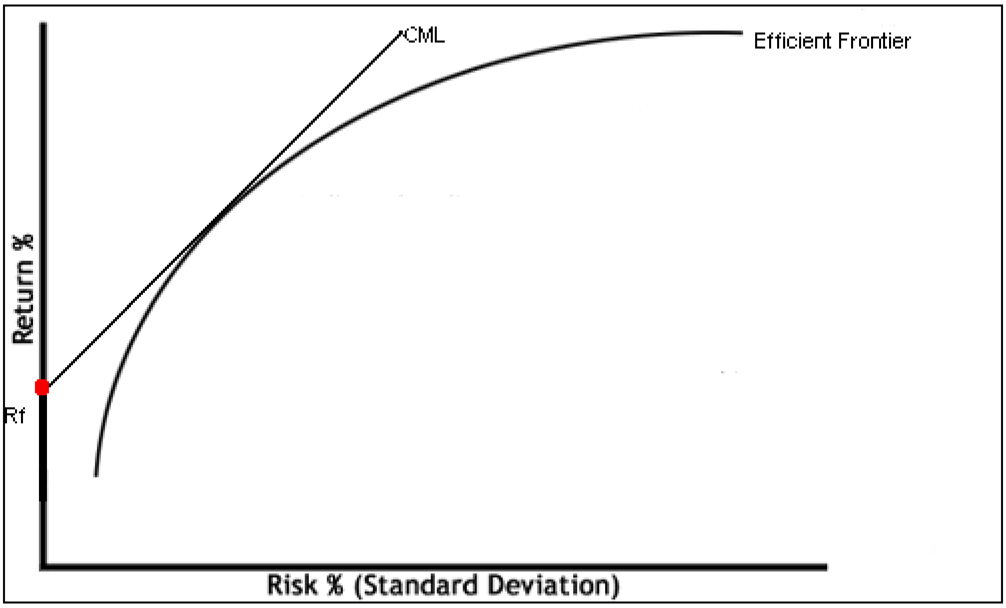
Prior to the MPT, most investors used their intuition to reach the decision that they should not put too much of their wealth in one particular type of asset or a single form of investment. However, Markowitz's theory proved mathematically how to increase profitability by deciding how much wealth needs to be put into which particular type of asset or investment. The mathematical theory related to calculations is one way in which portfolio managers are able to structure and discipline their thinking in order to reduce the risks and provide better returns (Hudson-Wilson, 1990:46).

2.2.2 Tobin's contribution to modern portfolio theory

Tobin expanded on Markowitz's work by adding the notion of leverage to portfolio theory by incorporating into the analysis an asset which pays a risk free rate (Elton &

Gruber, 1997:1749). By combining a risk-free asset with a portfolio on the efficient frontier (also known as leverage), it is possible to construct portfolios whose risk-return profiles are superior to those of portfolios on the efficient frontier. Leverage lead to the notions of a super-efficient portfolio and the capital market line (Busetti, 2009:99).

Figure 2.3: Capital market line



Source: Busetti (2009:99)

The capital market line in Figure 2.3 is the tangent line to the efficient frontier that passes through the risk free rate on the expected return axis. Through leverage, portfolios on the capital market line are able to outperform portfolio on the efficient frontier (Busetti, 2009:99).

Elton and Gruber (1997:1751) state that, by using the risk-free asset, investors who hold the super-efficient portfolio may:

- Leverage their position by short selling (simply borrowing the securities from one party and sell them to another party) the risk-free asset and investing the proceeds in additional holdings in the super-efficient portfolio, or
- De-leverage their position by selling some of their holdings in the super-efficient portfolio and investing the proceeds in the risk-free asset.

The resulting portfolios have risk-reward profiles which all fall on the capital market line. Accordingly, portfolios which combine the risk free asset with the super-efficient portfolio are superior from a risk-reward standpoint to the portfolios on the efficient frontier (Lubatkin & Chatterjee 1994: 118).

Tobin concluded that portfolio construction should be a two-step process. First, investors should determine the super-efficient portfolio. This should comprise the risky portion of their portfolio. Next, they should leverage or de-leverage the super-efficient portfolio to achieve whatever level of risk they desire. Significantly, the composition of the super-efficient portfolio is independent of the investor's appetite for risk. According to Elton and Gruber (1997:1752) the following two decisions are entirely independent of one another:

- The composition of the risky portion of the investor's portfolio, and
- The amount of leverage to use.

One decision has no effect on the other. This is called Tobin's separation theorem (Elton & Gruber, 1997:1751).

Furthermore, Tobin looks at the MPT from an advanced perspective in order to identify the aspect of efficient portfolios that should be used by the individual investors. Investors should work on dividing their funds between risk-oriented assets (equity portfolio or bond) and safe liquid assets such as cash (Goodall, 2002:46).

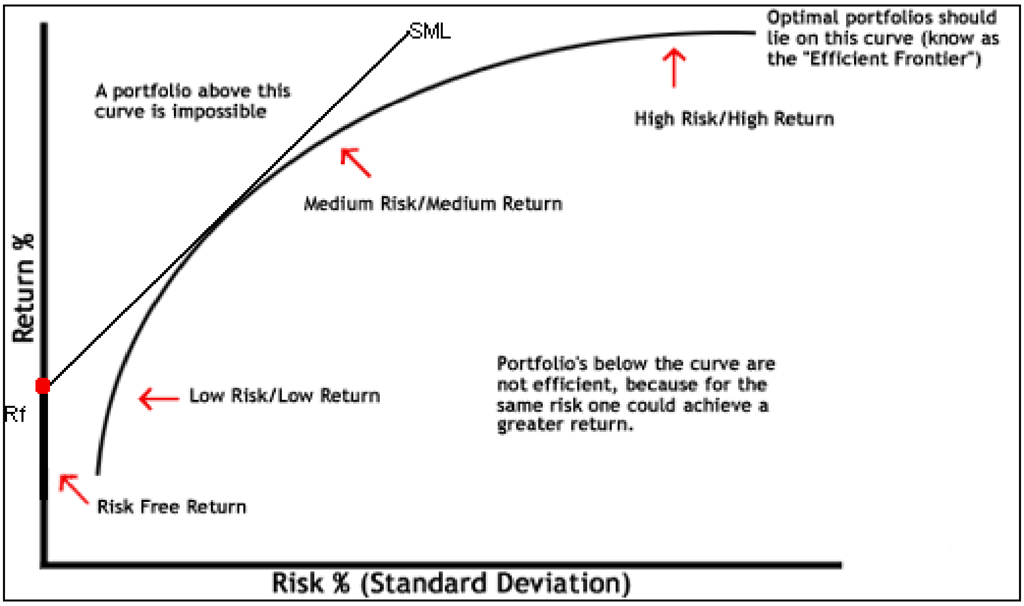
According to Goodall (2002:48), the combination of the non-cash assets is not related to their aggregate share of the investment possible. For this reason, it is possible to describe the decisions as if there were one single non-cash asset, which is combination of different types of actual non-cash assets in different fixed proportions.

2.2.3 Modern portfolio theory and the risk free rate

The risk free rate is of great importance to the MPT as the combination of assets and securities that are to be included in the portfolio need to be gauged for their interest and relative risks that will decide the overall return and risks for investors (Gibson, 2000:87).

The risk free rate forms the basic rate beyond which investors are expected to desire more returns. For example, in Figure 2.4, it can be seen that the risk free rate is the rate at the base of the curve. The risk free rate can be used by the investor as an instrument from which one can expect the lowest return from because the instrument is theoretically perfectly safe. Any perception of risk beyond this is considered to warrant a higher return on investment and, therefore, the curve slopes upwards (Busetti 2009: 103).

Figure 2.4: Risk and Return



Source: Busetti (2009:104)

The risk free rate is therefore the basic concept or attribute on which much trading activity is presumed to depend. Investors tend to assess the risk free rate from the government securities' market performance and they make their buying or selling decisions after assessing each instrument in terms of its returns and risks as compared to the risk-free government bond. The MPT, while simple in theory, is complex in practice, as the perception and calculation of the risk free rate is involved and often varies according to the inhibitions or aspirations of the investors (Gibson, 2000:89).

2.3 CAPITAL ASSET PRICING MODEL

The capital asset pricing model (CAPM) was heralded as the beginning of asset pricing theory in finance (Sharpe, 2000:75). Decades later, the CAPM is still used widely in the financial sector for estimating the cost of capital for different investment firms, as well as for evaluating the performance of different managed portfolios. One of the key attractions of CAPM is that it offers a very powerful prediction model that can be used to understand the mutual relationship between risks and returns.

According to this model, a linear relationship exists between the return on investment and the risks associated with the investment. The model views the security market line and its relationship to the expected returns as well as the systematic risks (beta coefficient) as key determinants of how the markets should price the individual securities along with the risks (Saunders & Cornet, 2006:54). The security market line enables the calculation of what is known as the return-to-risk ratio for any security related to the overall market. Hence, whenever the expected rate of return gets deflated by the beta coefficient, the return-to-risk ratio for the individual security in the market becomes equal to the market return-to-risk ratio. According to Sharpe (2000:81), this is shown in Equation 3 below:

$$\boldsymbol{R_i = R_f + \beta_i[\, R_m - R_f]} \qquad \textbf{\textit{(Equation 3)}}$$

Where:

R_i = expected return required on financial asset i

R_f = risk free rate of return, like the interest arriving from government issued bonds

β_i = beta value (i.e. beta measures how much a company's share price reacts against the market as a whole) for financial asset i

R_m = average expected return on the capital market

The CAPM model is considered to be an extension of the MPT. In the MPT, the variance analysis revolves around how the investor should focus on allocating the wealth among the various assets that are available in the market. Expanding on this premise, the Sharp-Linther CAPM model uses the characteristics of consumer wealth to arrive at the equilibrium relationship between the associated risks and the estimated returns (Saunders & Cornet, 2006:59).

2.3.1 Assumptions associated with the capital asset pricing model

According to Megginson (1996:13), the following assumptions form the basis of CAPM:

- The investors are risk-averse individuals who have the aim of maximising their expected utility;
- The investors are also 'price takers' who have homogeneous expectations with regard to their asset returns;
- A risk-free asset exists so that the investor may either borrow or lend unlimited amounts at risk free rates;
- The assets are perfectly divisible and easily marketable;
- The markets do not have any friction and market information is freely available to all investors at the same time; and
- Taxes and restrictions do not exist in the market because they are considered imperfections that affect the regular functioning.

2.3.2 Criticism associated with the capital asset pricing model

These assumptions have been the major source of criticism for CAPM because it is considered that these assumptions do not fit in the real world. Instead, they are more suitable for an idealised set up (Megginson, 1996:15).

Furthermore, the theory has attracted criticism because of its limitations. There are many simplifying assumptions involved that reflect theoretical failure. For example, according to CAPM, the risks related to shares should always be measured in accordance with a portfolio that is comprehensive and consists of variety of asset classes (Watson & Head, 2007:56). However, if the theory is limited to financial assets, it may not be legitimate to confine the theory to only one type of asset, such as shares (Kan & Zhou, 2001:32)

The empirical results have not provided good support for the model, even though many of the results support the existence of a linear and positive relationship between the returns and the systematic risks (Watson & Head, 2007:57).

2.3.3 Advantages of the capital asset pricing model

The acceptance of the concept of systematic risks is one of the advantages that this model has provided. In addition, CAPM also attempts to provide a theoretical explanation of the relationship between systematic risks and the returns. CAPM has established that the risk free rate plays a crucial role in assessing the returns on the investments and, hence, the risk free rate is a prominent factor in dictating the demand of different types of securities in the market (Saunders & Cornet, 2006:64).

2.3.4 Capital asset pricing model's risk free rate

According to Mukherji (2011:75), the risk free rate plays a pivotal part in the overall formulation of the CAPM formula. Short-term treasury bills or long-term treasury bonds are usually utilised as a risk-free security by practitioners and academics even though there is no empirical justification for doing so.

The study done by Mukherji (2011:76) analysed treasury securities with different maturity horizons and the various risks involved with these securities (i.e. market risk and inflation risk). The results of the study showed that the risks involved with treasury securities, as well as average real returns and volatility, increases as the maturity period of the treasury security increases. Further results indicated that treasury bills are free of market risk over 1 year and 5 year periods. Treasury bills also showed the lowest market risk over a period of 10 years and the lowest inflation risk over all three periods (1 year; 5 year and 10 year periods).

Furthermore, the inflation beta and explanatory power of inflation for real treasury bill returns declines with the investment horizon. Over 10 years, inflation and market risks explain only 13% of variations in real treasury bill returns, compared to 20% of intermediate government bond returns, and 23% of long government bond returns. These findings indicate that treasury bills are better proxies for the risk free rate than longer-term treasury securities regardless of the investment horizon (Mukherji 2011:81).

2.4 ARBITRAGE PRICING THEORY

The arbitrage pricing theory (APT) is one of the alternative methods or paradigms that can be used to understand and determine the equilibrium of expected returns for financial investments. The theory has the premise that any healthy, well-functioning and robust financial market should be arbitrage-free (Goldenberg & Robin, 1991:181). Arbitrage is considered to be the phenomenon by which advantages can be taken from the imbalance that occurs between different markets and therefore results in a profit that has no risks associated with it. Based on a factor model of returns on the assets, the APT helps in the understanding of an equilibrium pricing relationship (Mitchell & Pulvino, 2001:2136)

The APT is considered to be a one-price model, where every investor in the market believes that the stochastic properties of the returns on the assets are in line with a specific factor structure (Ross, 1976:64). If the equilibrium prices do not offer any chances of arbitrage over the efficient portfolio, the estimated returns on the assets can be considered to be related to the betas, or factor loadings, in a linear manner. The linear relationship between the returns and the betas or the factor loadings came to be called the "stochastic discount factor" (SDF) (Stamburg, 1983:358). Ross proved empirically that this linear relationship is a prerequisite for establishing equilibrium in a market where the investors can work towards maximising specific utilities (Goldenberg & Robin, 1991:185).

Risk-related asset returns have a factor structure and, therefore, Equation 4 holds true (Ross, 1976:66):

$$\boldsymbol{r_j = E(r_j) + b_{j1}F_1 + b_{j2}F_2 + \cdots + b_{jn}F_n + \varepsilon_j} \qquad \textbf{\textit{(Equation 4)}}$$

Where:

$E(r_j)$ is the j th expected return of the asset;

F_k is a systematic factor (assumed to have mean zero); and

b_{jk} is the sensitivity of the j^{th} asset to factor k, also called factor loading or the betas.

In addition, ε_j is the risky asset's idiosyncratic random shock with mean zero that has an assumption that they are not correlated across the different assets and factors.

According to the APT, if an asset's returns follow the factor structure, then the relationship that occurs between the expected rewards or returns and the factor sensitivities can be described as shown in Equation 5 (Pastor & Stambaugh, 2000: 336).

$$E\left(r_j\right) = rf + b_{j1}RP_1 + b_{j2}RP_2 + \cdots + b_{jn}RP_n \qquad \textit{(Equation 5)}$$

Where:

RP_k is the risk premium of the factor

r_f is the risk free rate

Equation 5 proves that any expected returns or rewards of a particular asset can be considered as the linear function of the asset's sensitivities with respect to the *n*-factors involved (Pastor & Stambaugh, 2000: 338).

2.5 COMPARING THE CAPITAL ASSET PRICING MODEL AND ARBITRAGE PRICING THEORY

The APT is often regarded as a substitute for CAPM. These theories both work on a linear relationship between the expected returns or rewards of the assets and their covariance along with other variables (Kan & Zou, 2001:20). It can be said that the goal of APT is to analyse the equilibrium that exists between the asset risks, as well as the rewards. This is similar to CAPM.

2.5.1 Similarities between the capital asset pricing model and the arbitrage pricing theory

In both these models, two basic assumptions remain constant: (1) a perfect and efficient market and (2) homogeneous expectations. In addition to these, APT includes an assumption that the portfolio is diversified and, therefore, the contribution to the total risk of assets to the unique unsystematic risk is almost close to nil (Kan & Zou, 2001:41).

2.5.2 Differences between the capital asset pricing model and the arbitrage pricing theory

In addition to these similarities, some differences exist. The first main difference is that CAPM had its source in a single-factor model (Chen, 1983:5). This means that the derivation of CAPM was through a process of generating asset returns that was a function of returns specific to the asset and returns two different factors – the market portfolio and the zero-beta portfolio. However, APT is multi-index, which means that here the returns-generating process is a combination of many different factors that generally exclude the market portfolio. These factors are not predetermined and the choice is made based on the issue at hand (Chamberlain, 1983:28).

The CAPM and APT models also have another differentiating factor related to the notion of equilibrium. While CAPM works on the assumption of an efficient market portfolio, the APT largely relies on the absence of free arbitrage in the market Goldenberg & Robin, 1991: 194).

Overall, it can be noted that the MPT, APT and CAPM models originated from the same premise related to the emphasis on risk-free investment opportunities.

2.6 SUMMARY AND CONCLUSION

Chapter 2 is based on a discussion of the portfolio theory and the role that the risk free rate plays in this theory. The modern portfolio theory (MPT) is discussed in detail in terms of the assumptions and the utility of the theory for market investors. The MPT proposes that a diversified portfolio – consisting of different maturities and different types of assets – is more profitable, as it helps to mitigate risks associated with investing only in one instrument or in instruments that have a close covariance. While the MPT provides a rationale for why rational investors choose a diverse portfolio, the basic calculations of the risk premiums are based on an assessment of the risk free rate.

It can be seen, therefore, that the MPT is based on the basic premise of a risk free rate of return and uses the same for assessing the risk premium for other riskier instruments. The risk free rate, therefore, lies at the core of the MPT and it plays a crucial role in the investment activity of investors.

The chapter also discusses the capital asset pricing model (CAPM), which is an extension of the MPT. The CAPM model is based on the premise that investors need to be compensated both for the time value of their money as well as for the risks associated with the loss of investment value in any way. Equation 3 is used to calculate the expected rate or return on any given security:

$$\boldsymbol{R_i} = \boldsymbol{R_f} + \boldsymbol{\beta_i}[\,\boldsymbol{R_m} - \boldsymbol{R_f}] \qquad \textbf{\textit{(Equation 3)}}$$

The CAPM postulates that the risk free rate forms the basis of the time value (without the involvement of risk consideration). To this a risk factor is added in order that the expected rate of return can be arrived at. The risk factor too is dependent on the risk free rate (R_f) as it is calculated by subtracting the expected return on the bond market (R_m) and the risk free rate. This risk premium (R_m - R_f) is multiplied by a beta coefficient that is calculated for an individual instrument. The expected return on that particular security is therefore a sum of the risk-free return and the risk premium (the calculation of which again involves the risk free rate of return).

The final theory discussed in Chapter 2 is the arbitrage pricing theory (APT). The basic premise of this theory is the same as that of the CAPM model. The APT uses the risk free rate as the core factor for calculating the expected return for any of the securities under consideration. The APT differs from the CAPM, however, in terms of its assumptions, with the APT stating that the market is arbitrage free. Further, the APT also shows that the CAPM is simplistic, as it uses only a single beta factor associated with the security to arrive at the expected rate of return. According to the APT, the market complexities cannot be captured in any one factor but a variety of factors. For each factor that is selected by the investor to be important, a risk premium is associated and this risk premium needs to be multiplied by the beta coefficient of the security for that risk factor. The expected return is therefore calculated by adding the risk free rate to the sum of the products of the beta factors and risk premiums for a variety of factors, as depicted in Equation 6 below:

$$\boldsymbol{R_i = R_f + \sum[B_i(n) \times R_m(n)]} \qquad \textbf{\textit{(Equation 6)}}$$

Chapter 2, therefore, establishes the importance of the risk free rate in the calculations of the expected rate of return on market investments. The financial markets operate using one or other of the premises established by the abovementioned theories. For this reason, the role of the risk free rate becomes crucial in determining the market dynamics. It is noted that any variation in the actual risk free rate as perceived by investors has the ability to distort the expected rate of returns by them and affect the demand for securities that are available in the market.

CHAPTER 3: BONDS

3.1 INTRODUCTION

Chapter 3 focuses on the valuation of the bonds and the various indices and indicators that investors employ to develop their expectations of a desirable interest rate for any given bond. The underlying purpose of bond valuation is to assess if the present value of the bond as per the expected rate of return of investors is attractive enough to warrant investment. The risk free rate forms the basis of the calculation of the expected rate of return and the evaluation of the present value. It will be shown that the risk free rate is a determining factor in market demand and dynamics.

Sovereign ratings will be discussed to show that the risk free rate available in the country is a strong deciding factor for foreign investors. Another tool that is employed in assessing the value of the bonds is the transition matrix, or a matrix of probable expected cash flows from any given assessment. The assessment of this risk settles the risk free rate for the investors and based on their perception of the risk free rate, they tend to develop the cash flow matrix with their expected rate of returns.

The Sharpe ratio, the Sortino ratio and the Omega ratio are discussed and the importance of the risk free rate in the composition of the ratios is highlighted.

In addition, the chapter also highlights the internal rate of return (IRR) and the weighted average cost of capital (WACC), which are corporate indices used by alternate investment companies, venture capitalists and companies moving ahead with new projects and investments.

The ratios and equations covered in Chapter 3 will give an indication of the role the risk free rate plays in each of these measures.

3.2 BOND VALUATIONS

3.2.1 Bonds defined

A financial bond can be considered as a financial instrument that is either issued by the government or organisations when they need to borrow money from the general public for a long term in order to financially support some of their projects (Duffee, 1996:96). In return for the borrowed money, these bodies issue interest payments on a regular basis. The entire loan amount, called the face value, is returned at the end of the stipulated term. The bond-holder has the option of purchasing the bond in order to collect the interest at regular periods or holding until maturity to get the initial investment (Castillo, 2004:346).

At any time before the bond matures, the bondholder also has the option to sell the bonds in the market. Bonds are of different types and the functioning, rate of interest, and the terms and conditions differ on the basis of these different types. For example, a level coupon bond pays the interest during period intervals and then pays the face value on the maturity of the bond, whereas a convertible bond grants permission to the bondholder to exchange it for a specific number of shares (Castillo, 2004:350).

3.2.2 The bond valuation process

Bond valuation can be defined as the process by which the fair price of a bond is determined. As with a security investment, the fair price or fair value of the bond is considered to be the current value of the flow of money that the bond is estimated to bring in. Therefore, the fair price of a bond is determined by applying a discounter to the estimated cash flow by using a predetermined discount rate (Ho, Stapleton & Subrahmanyam, 1997:1488). By referring to other similar instruments that exist in the market, the discount rate can be derived.

Many bonds have embedded options. These embedded options are additional features that give the bondholder, as well as the issuer, the option to take action at any given point. This makes it difficult to determine the value of the bond as both the discounter and option pricing need to be taken into consideration (Duffee, 1998:2227).

For a bond that does not come with any particular embedded options, the fair price is determined by using the discounter. The formula used in this regard works in the following manner (Castillo, 2004:253):

$$\boldsymbol{P = C\left(\frac{1-(1+i)^{-N}}{i}\right) + M(1+i)^{-N}} \qquad \textit{\textbf{(Equation 7)}}$$

Where:

P = the price of the bond in the market

C = the regular interest that is paid to the bondholders, or the cash flow

N = the total number of payments

i = the market interest rate

M = face value

If the resultant P is less than the face value, the bond sells at a discount. If the resultant P is higher than the face value, the bond sells at a premium (Ho *et al*, 1997:1490). Keeping Equation 7 as a premise, there are two different approaches taken for bond evaluation, namely, the relative pricing approach and arbitrage-free pricing.

The relative pricing approach prices the bond with regard to a benchmark. This benchmark is usually determined either by a government security or by relative valuation. According to Duffee (1998: 2230), the credit rating that is determined by the benchmark determines the yield to maturity (YTM). If the bond does well the gap between the required return and the YTM would be smaller.

The arbitrage-free pricing approach is based on the premise that the bond price is arbitrage free. The interest value (face value) is discounted separately on the basis of the rate that a corresponding zero interest bond has with almost similar credit worthiness. Therefore, the interest dates and the interest amount of the bond are fixed and known well in advance. Owing to this, the price of the bond on a particular day should correspond to the total of its cash flows that have been discounted with the help of the discounter that is implied by the value of zero interest bonds (Duffee, 1996:2232). If the total does not add up, the arbitrageur can finance the purchase by means of short selling. Short selling can be defined as the technique where securities are borrowed from one party and sold to another party (this creates a net negative position).

In cases where the future rates are not certain and where the discount rate cannot be denoted with the help of a specific entity, stochastic calculus is used. The bond valuation, therefore, depends upon the various types of bonds as well as the approaches taken to evaluate them (Ho *et al*, 1997:1490). It can be seen from the discussion on bond valuation that the risk free rate plays a central role in developing investor expectations of rate of return for individual securities.

3.3 SOVEREIGN RATINGS

3.3.1 Sovereign ratings defined

A sovereign credit rating can be described as the credit rating of a sovereign body, such as that of a government. Sovereign credit risks consider the political risk and are, therefore, a good indicator of the level of risk involved in investing in another country. During the last decade (Killian & Manganelli, 2008:1104), sovereign ratings have gained much popularity, as they have the potential to reduce the uncertainty related to risks. This has ensured that many governments with records of debt defaults gain access to the previously inaccessible international bond scenario.

Sovereign ratings are considered to be an assessment of the relative chance that borrowers may default on their obligations. Many governments focus on credit ratings so that they are able to easily access the international share and bond markets. Access to these markets implies that these governments can participate in the share domains where many investors prefer the option of rated securities over the unrated securities (Suter, 1992:21). Earlier, governments used to look for ratings on the foreign currency obligations because these bonds had a higher likelihood of being placed with big international investors than domestic currency obligations did.

However, international investors have also shown interest in bonds that are issued in domestic currencies or in currencies that do not belong to the category of traditionally established global currencies. Therefore, many sovereigns have also been trying to obtain other domestic currency bond ratings (Killian & Manganelli, 2008:1106).

3.3.2 Features of sovereign ratings

One of the key features of sovereign ratings is that the credit ratings do not exceed the ratings of the domestic currency obligations and are in fact, much lower. Governments have the power to print domestic currency, as well as tax any domestic income. For this reason, they are always in a better position to fulfil any domestic currency obligations. Despite this domestic currency bonds can also result in default. This occurs if the government decides to avoid the political results of increased tax rates or debasement of the domestic currency (Arthur & Sheffrin, 2003:36).

Sovereign ratings attract much attention, not just because some of the biggest issuers in the international bond market are governments, but also because of the fact that these assessments also influence the ratings of a high number of borrowers from the same country (Bulow & Rogoff, 1989:16). Ratings are not provided by the agencies to private sector issuers that are higher than the sovereign rating of the home country. Hence, sovereign ratings impact those given to local bodies such as local government, as well as private organisations, which have their headquarters in the same country (Arthur & Sheffrin, 2003:39).

3.3.3 Rating agencies

The rating agencies have a very critical role to play with regard to sovereign ratings. The main rating agencies in the market are Moody’s, Standard and Poor, Duff and Phelps and Thomson Bank Watch. Even though the provision of sovereign risk ratings is relatively new, ratings agencies have been providing ratings since the early 1900s. Moody’s, one of the large credit agencies, has been providing ratings to bonds issued by foreign governments since 1919 (Mintz, 1951:42). In the first half of the 20th century, the international bond market was very popular. For this reason, by 1929 Moody’s had rated bonds for more than 50 government bodies. However, as a result of the Great Depression of the 1920s, as well World War I, the global financial industry experienced a severe setback that affected the international bond market. It took a very long time for the international bond market to revive which only became active again during the 1970s. Even after these crises, the demand for sovereign rating was not very high and, only a small number of foreign governments expressed their desire to borrow in the US share and capital markets and, therefore, required ratings from credit agencies (Mintz, 1951:46).

Most of the governments had clear credit records; therefore, their sovereign ratings were always straightforward. Some financially strong governments also entered the international capital market through Euromarkets where ratings were not required. Those governments, which were usually less creditworthy, obtained individual credits from privately issued bonds that did not need a credit rating. During the early 1990s the sovereign credit market increased in popularity because market conditions became favourable to issue debt. Many of the governments had an affinity towards the US bond market, where credit rating was mandatory. Initially, most of the ratings were either AAA/Aaa. However, later ratings such as BBB/Baa and so on came into existence (Killian & Manganelli, 2008:1110).

With sovereign credit ratings, the risk of is determined largely by prevailing economic conditions. Historically, it has been noted that sovereign borrowing and lending have both caused defaulting – during the Great Depression of the 1920s, for example (Bulow & Rogoff, 1989). Sovereign ratings, while taking note of the government-provided risk free rate, take into consideration various risk factors such as political instability, economic stability or economic changes, the introduction and impact of new legislation, environmental hazards, corruption, and income disparities in determining the expected risk free rate of return. Sovereign ratings are one of the most robust and credible criteria that foreign investors use in making their decisions and, therefore, the rating agencies' perception of the risk free rate becomes a crucial factor in their own decisions (Arthur & Sheffrin, 2003:46).

3.4 TRANSITION MATRICES

A tool that that can be employed in assessing the value of the bonds is the transition matrix, or a matrix of probable expected cash flows from any given assessment. The assessment of this risk settles the risk free rate for the investors and based on their perception of the risk free rate, they tend to develop the cash flow matrix with their expected rate of returns (Saunders & Cornett, 2006:102).

It is necessary to rely on a logical, mathematical forecasting system that can assist in predicting trends in financial markets in order to minimise loss and maximise gains. In order to arrive at a rational forecasting model, it is necessary to rely on probability theories based on historical performance to influence future predictions. Most rational prediction patterns are based on observation of previous outcomes to arrive at a specific mathematical pattern.

3.4.1 Markov's chain

Andrey Markov developed one of the earliest and the most popular forecasting models. Markov's chain found widespread usage in the field of mathematics, statistics, physics, economics and finance. The key reason for using this chain is to build a model that can effectively predict the state of any particular object of a particular period of time in the future, based on the probability vector of both the initial stage as well as the state transition probability matrix (Feng, 1994:45).

Markov's chain plays a critical role in modern-day finance due to its inhirent properties which include: no after-effect properties, weak demands on previous data, as well as a unique forecasting methodology (Latouche & Ramaswami, 1999:32). The main difference between Markov's chain and other prediction models, such as regression analysis and time series, is that it does not rely on mutual laws among the various factors. Instead it considers the main characteristics of historical events in order to make predictions of the internal state by understanding the concept of 'state transition probability', which is the probability of change from one state to another (Feng, 1994:46).

3.4.2 Transition matrix

On the basis of Markov's chain, the transition matrix can be described with the help of a mathematical formula. This matrix is a stochastic process that does not have any after-effect properties (Hao, 2006:78). In any balanced system, the transition probability vector forms a transition matrix that can be described by:

$$\boldsymbol{P} = \boldsymbol{P}\left[\boldsymbol{P}_{\boldsymbol{IJ}}\right]_{\boldsymbol{m}\times\boldsymbol{n}} \qquad \textbf{\textit{(Equation 8)}}$$

Where:

P_{IJ} = the probability of the system from a stage i to another stage j, and

n = a non-negative integer characterised by n_1, n_2, n_3, n_m.

As the transfer matrix is characterised by a probability matrix whose operation is similar to that of a conventional matrix, its properties can be described as below (Hao, 2006:79):

$$\boldsymbol{P}^{(\boldsymbol{K})} = \boldsymbol{P}^{(\boldsymbol{K}-\boldsymbol{1})} \times \boldsymbol{P} = \boldsymbol{P}^{\boldsymbol{K}} \qquad \textbf{\textit{(Equation 9)}}$$

Where:

K = an arbitrary natural number (the next larger natural number is $^{k+1}$ and the one following that is $^{k+2}$).

On the basis of the composition of Markov's chain, certain assumptions need to be made so that the transition matrix can be effectively applied to stock market analysis, namely (Hao, 2006:80):

- The functioning of the stock market is affected by numerous random factors such as economy, global or local politics, and the society. However, the macro policies related to the securities exchange remains constant and is not impacted by these factors;
- The fluctuating trends in the share market on any particular day are highly dependent on the trends just before closing and it does not have much effect because of the past; and

- The chances or probability of determining the state of the stock market ‘i’ moving to the state of ‘j’ by a specific time interval is not associated with the movement of state ‘i’.

When constructing or designing a transition matrix based on the Markov Chain, there are four different steps involved (Latouche & Ramaswami, 1999:37):

- The first is to construct or determine the state and find out the relative state probability;
- The second step is to state the probability of the transition matrix through the state transfer;
- The third step is to derive the different state vector based on the matrix and; and
- The final step is to analyse, forecast, and arrive at a conclusion.

It is also necessary to understand that the transition matrix based on the Markov’s chain is simply a method of probability forecasting that has been found to be very effective in stock markets in different economic environments. However, the predictions or forecasts are only an expressed probability based on the certain state of stock prices and it can be by no means considered as an absolute state (Latouche and Ramaswami, 1999:41). The stock market is dynamic and is therefore susceptible to the influence of numerous factors such as trade, policy changes, and the psychology of investors.

The transition matrix for the projected cash flows for any given bond or security is dependent on the perceptions of the financial and economic stability of the environment in which the bonds are traded. This implies that within the context of a country, the transition matrix can be simple and reflect a lower perception of risk associated with the government securities and, for this reason, a lower set of cash flows. On the other hand, external credit rating agencies or foreign investors may have an altered transition matrix with higher expected cash flows from government bonds and, for this reason, an indicator of a higher perception of the risk free rate (Saunders & Cornett, 2006:113).

3.5 PORTFOLIO PERFORMANCE MEASURES

A portfolio's performance is measured to understand and evaluate how that particular asset class performing when compared to its peers and the related benchmark. It is this comparison that helps investors and companies identify the performance of fund managers and the funds they manage. The comparison comes under the conventional method of evaluation of bonds (Hendriksson, 2005:2).

The same performance measure can be conducted through risk-adjusted methods, such as the Sharpe ratio, the Jensen ratio, the Treynor ratio, the Sortino ratio, and the Omega ratio. These risk-adjusted methods are preferred to the conventional methods such as making use of bond valuations and transition matrices (Chen & Knez 1996:24).

Portfolio performance measurement is vital in the financial domain for the following reasons (Chen & Knez 1996:26):

- From an investor's point of view, the fund's performance as compared to its peers or benchmark needs to be known. This information helps in making decisions on making an exit on the investments or rebalancing the portfolio to generate higher returns; and,

- The fund manager has been performing based on the returns generated by the fund and its performance compared to others in the peer space. This information helps the management to decide on the fund manager's remuneration, as it is linked to portfolio performance.

3.5.1 Common approaches to portfolio measurement

Some of the most common approaches used for portfolio measurement are the benchmark comparison and the thematic comparison (Kilian & Manganelli, 2008:39). In benchmark-comparison, the most commonly used indices are S&P 500, the MSCI Index or the Russell's Index. However, a major drawback with the benchmark-comparison approach is that it does not consider the entire risk associated with the portfolio as compared to the benchmark. The level of risk in the portfolio and the level of risk in the benchmark differ because the portfolio might generate higher returns than the benchmark (Kilian & Manganelli, 2008:42). For this reason, a simple comparison does not provide valid results for investors and fund managers.

The thematic comparison approach occurs when comparison is done on the basis of investment thematic-growth or value-investing. Portfolios that are based on the value-investing approach pick up undervalued shares based on valuation for specific parameters. Portfolios that aim to capture growth invest in growth shares by evaluating the revenues and earnings generated and growth prospects involved in it. Growth-oriented portfolios are compared to growth benchmarks. There is a similar drawback here, as the benchmark may not be comparable or two similar thematic funds might differ in various terms. This is when most companies select the risk-adjusted approach for evaluating performance (Stutzer, 2000:53).

The following risk-adjusted models will be discussed in detail, namely the Sortino ratio, the Sharpe ratio and the Omega ratio. The following corporate finance indicators will also be discussed, namely the internal rate of return (IRR) and the weighted average cost of capital (WACC).

3.5.2 Sharpe ratio

The Sharpe ratio calculates risk premium of an investment class or a portfolio as per a unit of total risk of portfolio (Sharpe, 2000:16). The risk premium is the return of the portfolio less the risk free rate of return on treasury rates. The total risk of the portfolio is the standard deviation of the entire portfolio returns. This can be calculated by using Equation 9:

$$S = \frac{R_p - R_f}{\sigma_p} \qquad \textit{(Equation 9)}$$

Where:

S = Sharpe ratio

R = expected portfolio return

R_f = risk free rate

Sigma in the denominator = standard deviation

Here, the numerator captures the returns earned for investing in a risky portfolio (risk premium = expected rate of return from the portfolio – the risk free rate) and the denominator determines the variability of portfolio rate of returns. The Sharpe ratio is, therefore, the ratio of the risk premium and the standard deviation of the portfolio. It indicates whether the risk premium is appropriate compensation for the risk, or if the risk premium is a result of unnecessary high risk associated with the securities in the portfolio. A high Sharpe ratio indicates that the investment is sound, as the premium is the result of the decision to mix the securities judiciously (Sharpe, 2000:21).

The Sharpe ratio is also called the reward to variability ratio (Sharpe, 2000:26). For example, if a managed portfolio earns a return of 25% over a specified time with a standard deviation of 30%, one can assume the risk free rate as 5%. On applying the formula, (25-5/30), the Sharpe ratio is 0.66. During the same time period, the share market generated returns of 20%, with a standard deviation of 25%. On applying the same formula here (20-5/25), the Sharpe ratio is 0.60, which means that the managed portfolio has outperformed the benchmark (Sharpe, 2000:28).

The major setback for the Sharpe ratio is that the asset or portfolio returns have to be normally distributed as standard deviation does not have the same effectiveness for abnormal distribution like kurtosis or skewness. Generally, if the standard deviation is high it does not affect the leverage on the portfolio. Whereas, if the standard deviation is too high, then leverage on a share of 5:1 could easily go for margin call, once the share falls down 5% to 10% (Bernardo &Ledoit, 2000:145).

The Sharpe ratio can only be used as a comparative tool and it does not signify anything on an individual basis. The Sharpe ratio would be appropriate for an investor who invests in a single portfolio using standard deviation, which measures the total risk associated with the portfolio. It is, however, advisable to use the Sharpe ratio along with other inherent measures like fund strategies and fund theme (Bernardo & Ledoit, 2000:147).

The biggest advantage of using the Sharpe ratio is that it does not pertain to a particular time period of investment, and there is no correlation between the time frame and the Sharpe ratio. The reason is that both required returns and risk factors tend to change as time passes by (Sharpe, 2000:35).

However, it can again be seen that the Sharpe ratio is dependent on the risk free rate. It can be observed from the formula that the risk free rate of return forms an essential part of the risk premium and, hence, the ratio. Here, the average expected return on an investment is deducted from the risk free rate, and then divided with standard deviation to arrive at the ratio. The same application was further developed as the CAPM, where again the risk free rate plays a significant role, as it provides the net additional return earned by an investor for the extra risk being taken (risk premium) (Bernardo & Ledoit, 2000:161).

3.5.3 Sortino ratio

The Sortino ratio is a modification of the Sharpe ratio which measures the risk-adjusted return of an investment asset (Sortino & Price, 1994:60). The major differentiator from the Sharpe ratio is that the Sortino ratio only uses downside risk whereas the former takes into consideration both upside and downside volatility on the portfolio. The advantage of using downside risk is that it can measure negative volatility as well. The ratio can be calculated as (Sortino & Price, 1994:62):

$$\boldsymbol{S = \frac{R-T}{DR}} \qquad \textbf{\textit{(Equation 10)}}$$

Where:

R = asset / portfolio realised return

T = required rate of return

DR = downside risk

On comparing the above-mentioned ratios, it is understood that standard deviation has been replaced with downside risk of portfolio. However, dependency on the Sortino ratio over Sharpe’s depends on the required rate of return assigned to the portfolio. Here the positive and negative volatility is differentiated and a minimum acceptable return (MAR) is set for downside volatility that is generally zero. Moreover, this ratio is not considered effective for funds with low or no volatility. The downside risk of a portfolio can be calculated using the following formula (Hudson-Wilson, 1990:57):

$$\boldsymbol{DR} = \left(\int_{\infty}^{T}(\boldsymbol{T}-\boldsymbol{x})^2 \ \boldsymbol{f}(\boldsymbol{x})\boldsymbol{dx}\right)^{1/2}$$ ***(Equation 11)***

Where:

T = risk-free interest rate

f(x) = the probability density function of returns which denotes the relative likelihood of the variable to appear at any given point.

This can be called the root mean squared of underperformance, where underperformance relates to the proportion by which a return is below the target. Hence, this ratio denotes the actual rate of return against the investors' target rate of return (Dowd, 2000:210).

The Sortino ratio provides an alternative measure of investment performance that is more suitable when applied to asymmetric return distributions. But at the same time, it also has some shortcomings, as the data can be manipulated to achieve artificially high values. The reason is, for portfolios generating frequent small positive returns and more substantial losses; there arises a requirement of a long historical track record to capture true risk (Henriksson, 2005:4).

The Sharpe ratio would be considered as the most widespread and frequently used approach for portfolio performance, except for certain funds that have been using the Sortino ratio due to low tolerance. Moreover, large Sortino ratios indicate low risk or a risk-adverse investment method, which is preferred by investors with a low appetite for risk (Chen & Knez, 1996:6).

The Sortino ratio emphasises the importance of the risk free rate, as it is calculated by deducting the risk free rate from the return of the portfolio and then dividing by the downside deviation. The total returns generated by the portfolio when deducted from the risk free rate yield the risk premium earned by the investor for the amount of additional risk taken over and above the risk free rate (Dowd, 2000:216).

3.5.4 Omega ratio

The Omega ratio is the measure of risk of an investment asset or a portfolio. It is considered to be an extension of the Sharpe or Sortino ratios and apportions gains or losses relative to the required rate of return. The ratio can be calculated as:

$$\Omega(\boldsymbol{r}) = \frac{\int_{r}^{\infty}(\boldsymbol{1}-\boldsymbol{F}(\boldsymbol{x}))\,\boldsymbol{dx}}{\int_{\infty}^{r}\boldsymbol{F}(\boldsymbol{x})\boldsymbol{dx}} \qquad \textit{(Equation 12)}$$

Where

F = cumulative distribution function, and

(r) = required rate of return (threshold).

Equation 12 is a ratio of the probability of having a gain opposed to the probability of having a loss. A higher Omega ratio is always preferred by fund managers and investors, as it signifies a higher weighted-gain-ratio or a lower weighted-loss-ratio (Keating & Shadwick, 2002:3).

The two great advantages of the Omega ratio is that, first, it sums up all information on the risk and return of a portfolio within the distribution and, second, the precise value is directly derived from the risk appetite of the investor. The Omega ratio has certain features that have to be noted by investment analysts or fund managers on deriving results. At any given level of return a portfolio with a higher Omega value has to be preferred to a lower one, as the former has a higher probability of generating better returns. The Omega ratio is considered to be more reliable compared to a traditional mean-variance approach as investors consider return distribution of funds, which is a more information rich, easy assessment of return versus risk and as it does not require estimates of higher movements. This provides a tailored performance measure, as the return threshold is set on the basis of the investor’s risk appetite (Kazemi, Schneeweis & Gupta, 2002:23).

The Omega ratio usually shows a different ranking of funds, portfolios or assets from those derived from the Sharpe ratio. In those cases where higher movements are concerned and when their effects have significant impact on financials, this ratio provides corrections to these simpler theories. It also depicts that at different levels or returns and market conditions the allocation among assets also tends to change (Keating & Shadwick, 2002:4).

At times, the Omega ratio is also used to calculate the persistence, skills and performance of an investment manager, which are closely linked to the returns generated by the portfolio. The model is also used for portfolio construction where different weights are derived under standard mean variance analysis through other models like the Markowitz model. The scalability and flexibility obtained from a general Omega ratio is considered more efficient than those portfolios that are generating higher returns and that are shown to be limited special case approximations. Another feature of the Omega ratio is to provide error tracking and analysis for benchmark-relative portfolios (Kazemi *et al*, 2002:29).

3.5.5 Sharpe-Omega ratio

A new concept called the Sharpe-Omega ratio is also employed by investors. This is a variation of the Omega ratio but preserves all the features of the same and provides a measure of risk that is similar to that found with the Sharpe ratio. The formula can be written as:

$$\textbf{Sharpe - Omega} = \frac{\bar{x} - L}{P(L)} \qquad \textit{(Equation 13)}$$

Where:

$\bar{x}$ = expected rate of return on investment

L = threshold selected by investor, and

P(L) = put option price

The only difference this model provides when compared to the Omega ratio is that it represents a measure of return risk that is more intuitive than that of the Omega model. Here too, the threshold return is taken into account and if $\bar{x}$ < L then the Sharpe-Omega ratio will be negative. In this case, the higher the put price (cost of protecting an investment's return), the better the investment. Higher volatility tends to increase the put price, thereby increasing the value of the Sharpe-Omega ratio. On the other hand, if $\bar{x}$ > L, then the Sharpe-Omega ratio will be positive. Unlike the previous case, the higher put price reduces the Sharpe-Omega ratio which is similar to the Omega ratio as it is greater than one (Kazemi *et al*, 2002:31).

From the example above, it can be inferred that the Omega ratio could be more widespread for the calculation of a portfolio's risk-return ratio compared to other ratios, as the numerator-denominator factor provides good insight into the cost of acquiring a return above threshold and the cost of protecting the return below the threshold. Also, investments with the same expected returns will have the same Omega based on the price of puts and the denominator can be analysed from the market price of put option for those investments. The Omega ratio is therefore considered as a primary tool within portfolio management companies and asset management firms (Keating & Shadwick, 2002:5).

The Omega ratio and the risk free rate relationship can be describes as the threshold return or required rate of return based, on which the probability of gains over loss is calculated. This required return is considered as a minimum / risk-free return, which acts as a benchmark level based on which gains and losses in the portfolio are evaluated.

In all the above risk-adjusted ratios (namely the Sharpe ratio, the Sortino ratio and the Omega ratio) it can clearly be seen that the risk free rate of return is vital for an analysis of portfolio performance as it always provides a benchmark for investors based on which they evaluate the risk-return ratio (additional return gained on extra risk taken).

In addition to the abovementioned risk-adjusted ratios there are several corporate finance indicators that play an integral part in a firm's investment decision or expansion activities. These indicators are of substantial importance for the South African bond market as most of the bonds that the government has issued are in sectors that are related to development and construction, with the objective of attracting large international investment organisations.

3.5.6 Internal rate of return

The internal rate of return (IRR) is a form of capital budgeting and is used to determine the interest rate at which the net present value of costs of investment equals the net present value of benefits of investment (rate at which cost nullifies with benefit). The IRR is generally used by companies when they take decisions in terms of new investments or projects, as in the case of foreign investors in South Africa. A higher IRR definitely means that the company should move forward with its investment plans, as it can cover the cost of capital (Kelleher & MacCormack, 2004:21). IRR can be calculated by using Equation 14:

$$NPV = \sum_{n=0}^{N} \frac{C_n}{(1+r)^n} = 0$$

(Equation 14)

Where:

N = total number of periods

n = time period (positive integer)

C_n = cash flows

NPV = net present value; and

r = internal rate of return

Apart from companies using the IRR for evaluating project viability, this tool is also widely used among private equity and venture capital funds to analyse their IRR and determine payback period on investments made at different stages and in different time frames. The IRR is mostly used only for individual projects, as it does not consider the cost of capital.

Consider the following examples (Kelleher & MacCormack, 2004:24):

An initial investment of USD 400 is required for a project and the cash flow for three years is USD 50, 100, 120,150 respectively for years 1, 2, 3 and 4. Here, the NPV will be calculated as:

NPV: $-400 + 50/(1+r)^1 + 100 / (1+r)^2 + 120/ (1+r)^3 + 150/(1+r)^4 = 0$

Here IRR (r) can be calculated between 6% and 7% approximately to obtain the value as zero. The rule of thumb in IRR is that if the IRR is greater than 1, the project or investment should be accepted. If the IRR is less than 1, then reject the project.

In certain cases, if one project has an initial investment higher than a second, mutually exclusive investment, the first project may have a lower IRR but a higher NPV. In such cases, the first project has to be accepted as a higher NPV helps in attracting more investors to the project due to the increased returns generated on it. In other cases, if the IRR overstates equal rates of return on investment made by reinvesting cash flows than the calculated IRR, then it leads to overestimation as the formula assumes that the company has additional projects available that can attract higher returns.

One of the major disadvantages of the IRR is that it assumes that all cash flows are reinvested at the same discount rate. It could be much more useful for comparing projects with equal risk rather than fixed returns. Therefore, the IRR is mostly considered appropriate by private equity and venture capital funds, as there are several investment outlays that gain on maturity through initial public offering (IPO) activity or mergers and acquisitions activity (Kelleher & MacCormack, 2004:26).

Investment companies prefer to use the IRR over the NPV, as it is easier to compare investments of different sizes in terms of percentage basis rather than use the NPV and compare with dollar values. Even though the NPV gives better accuracy in terms of a business proposition, the IRR can give better insight into various situations where companies are affected with capital constraints. However, for mutually exclusive projects, the NPV is still used as a better measure (Grayson, 2011:3).

The calculation process of the IRR includes the discounted cash flow method (DFC), which is used to calculate the NPV using IRR. The DCF method analyses projects, the value of company, or a particular asset by using the time value of money approach. The future cash flows generated from a business asset is discounted to NPV, which is the output as a price. (Kelleher & MacCormack, 2004:32).

The concept of IRR is used in different ways. For example, in a lottery business when there is a pay out of USD 50 million is simply a series of small payments from various sources eventually leading to a pay out of USD 50 million. The payments are made either on a lump sum basis or in periodic payments. The IRR can also be used for computation of portfolio or individual share returns (Grayson, 2011:5).

At times a simple IRR model may create discrepancies due to variations in cash flows (positive and negative cash flows in consecutive years). Therefore, the modified internal rate of return (MIRR) is used, which is calculated as (Ryan, Scapens & Theobald, 2002:15):

$$\boldsymbol{MIRR = \sqrt[n]{\frac{FV(positive\ cash\ flows, reinvestment\ rate)}{PV(negative\ cash\ flows, finance\ \ rate)}} - 1} \qquad \textit{\textbf{(Equation 15)}}$$

Where:

n = time period

FV = future value; and

PV = present value

Reinvestment rate = discount rate

Finance rate = finance cost

The MIRR a geometric average of the compounded future value of positive cash flows over the discounted present value of negative cash flows. Each positive cash flow is compounded at the reinvestment rate (WACC or discount rate) to find future value, and each negative cash flow is discounted at the finance rate (financing cost) to find the present value. For example, using the sample from the above case (NPV calculation) and calculating the IRR for the above given formula of MIRR the following results are derived.

$-400 + -50 / (1+ r)^1 = - 447$

$100 (1+r)^1 + 120 (1+r)^2 + 150 = 390$

MIRR = $\sqrt[4]{390 / 447} - 1$ = (-3.4%), which clearly means that cash outflows (negative values) are higher than cash inflows (positive) and therefore the project is not viable. The same project derives a positive rate of 6% - 7% IRR, as seen earlier (NPV calculation).

The example set out above (NPV calculation) demonstrates the variation in values with the two methodologies. However MIRR should always be negative and could yield higher returns compared to the same generated by IRR, which gives a double assurance for companies to go ahead with their projects.

As is the case with ratios, similarly the risk free rate is equally important for capital budgeting used by companies for their projects, investment companies for deciding on investments, and the broking community to provide guidance to investors on shares. In terms of IRR, cash flows are discounted to arrive at NPV of companies for which the discount rate is generally considered to be based on the risk free rate of return. It is this discount rate that is used in DCF calculations to arrive at a fair value. In this way, it is the risk free rate of return that acts as a base for many valuation parameters to initiate and show true results (Ryan *et al*, 2002:21).

Generally, for comparison of projects a more comprehensive tool is preferred to IRR. This tool is called Weighted Average Cost of Capital (WACC).

3.5.7 Weighted average cost of capital

The weighted average cost of capital (WACC) can be defined as the minimum rate of return that a company earns on its existing asset base to satisfy the stakeholders. Capital can be raised by companies by two means – equity and debt. WACC takes into account the relative weights of both these sources of capital pooled into the company. WACC is generally calculated as (Watson & Head, 2007:131):

$$\boldsymbol{WACC = \frac{MV_e}{MV_d+MV_e}.R_e + \frac{MV_d}{MV_d+MV_e}.R_d.(1-t)} \quad \textbf{\textit{(Equation 16)}}$$

Where:

MV_e = market value of equity

MV_d = market value of debt

R_e = return on equity

R_d = return on debt, and

t = corporate tax rate.

Assume MV_e as USD 100, MV_d as USD 150, the cost of equity as 15% and the cost of debt as 7%, with corporate tax rate at 30%. On applying values to the formula:

WACC = 100 / 150 +100 * 0.15 + 150/ 150 + 100 * .07 * (1- .30)

= 0.06 + 0.042 * 0.70

= 0.089, i.e., 8.9%

WACC is a very useful corporate budgeting tool used to analyse the cost of capital from various sources of finance, i.e. debt and equity. Capital raised by the company when combined with debt, calculates to the total market value of the firm. WACC can be calculated providing equal weights to both these sources of finance along with their respective cost of raising funds (interest for debt and expected returns by shareholders for equity). This is the opportunity cost for investors for risking their money in a particular project as opposed to any other (Ross, Westerfield & Jaffe, 1996:43).

WACC is used for decision-making purposes regarding investment. A negative sign for investment would be if a company generates returns less than WACC. Suppose the annual return generated by a company is 13% whereas WACC is 15%, then for every rand invested the company has created negative value of 2 cents to its investors.

WACC is based on certain assumptions, such as (Watson & Head, 2007:136):

- The project meant for investment is small compared to the company's overall size;
- The project identified is similar to the business activity currently carried out by the company (non-diversification factor);
- The capital structure (debt / equity mix) for the new project is similar to the existing capital structure of the ongoing business; and
- The required rate of return, which currently exists, does not change for the new project as this is being decided by external funders.

Some of the major disadvantages involved in WACC is the use of market prices for shares that are not static. The market price is highly volatile and tends to change based upon the demand-supply scenario. This leads to change in expectations from investors and, therefore, may not depict a true value. The WACC can only be considered as a measure of capital distribution rather than operating cost, as reduction of one particular aspect of cost of capital in the company can lead to increase in the other cost of capital. Similarly, changes in interest rate of loans (if the company has opted for a floating rate of interest) can be misleading, as this automatically reflects in the WACC (Ross *et al*, 1996:43).

WACC in general, can be only used as a tool to analyse the cost of capital rather than to evaluate the feasibility of an investment, as it involves factors like management capability, responsiveness of the business to customers and investors, marketability and demand, quality time required for pay back and turnaround strategies developed by the company to gear at high growth path. Hence, an investor, upon analysing all these qualitative factors, can only consider investing in the project apart from the calculations that has been done above, which acts as a benchmark or preliminary evaluation tool. WACC should be calculated periodically using real values of debt and equity and it would not be advisable for it to be calculated on the basis of book value (Brealey & Myers, 1996:31).

With WACC the risk free rate is used both for calculation of cost of debt and cost of equity. The cost of equity is basically what it costs the company to maintain a share price theoretically satisfactory to investors. On this basis, the most commonly accepted method for calculating cost of equity comes from the CAPM (Watson & Head, 2007:136):

$$R_e = R_f + \beta\,(R_m - R_f) \qquad \textbf{\textit{(Equation 3)}}$$

Where:

R_f = the risk free rate;

ß = beta; and

R_m – R_f = the equity market risk premium

In Equation 3, the risk free rate is the amount obtained from investing in securities considered free from credit risk, such as government bonds. Beta measures how much a company's share price reacts against the market as a whole. A beta of one, for instance, indicates that the company moves in line with the market. If the beta is in excess of one, the share is exaggerating the market's movements; less than one means the share is more stable. Occasionally, a company may have a negative beta (e.g. a gold-mining company), which means the share price moves in the opposite direction to the broader market. The equity market risk premium (EMRP) represents the returns investors expect to compensate them for taking extra risk by investing in the stock market over and above the risk free rate. In other words, it is the difference between the risk free rate and the market rate (Brealey & Myers, 1996:42).

3.6 SUMMARY AND CONCLUSIONS

Chapter 3 focuses on the valuation of the bonds and the various indices and indicators that investors employ to develop their expectations of a desirable interest rate for any given bond. The bond valuation is undertaken by calculating the current price of the bond using the par value and the cash flows stream to maturity for that bond, as shown in the Equation 7:

$$P = C\left(\frac{1-(1+i)^{-N}}{i}\right) + M(1+i)^{-N} \qquad \textbf{\textit{(Equation 7)}}$$

The present value P is determined by adding the discounted cash flows (C discounted by the market interest rate i) and the Face Value or the Par Value (M) discounted by market interest rate i.

The underlying purpose of bond valuation is to assess if the present value of the bond as per the expected rate of return of investors is attractive enough to warrant an investment in it. As can be seen from Equation 7, the expected rate of return is the key factor in how the bond is valued by investors. The expected rate of return here again is determined by the relative risk that investors perceive they are incurring in investing in a non-risk-free instrument. The risk free rate therefore forms the basis of the calculation of the expected rate of return and the evaluation of the present value.

The present value, in comparison with the par value, forms the basis of the buying or selling decisions. (if the present value is more than the par value the bond is sold at a premium; if the present value is less than the par value the investor may want to hold on to the investment). The risk free rate is a determining factor in the market demand and dynamics.

This is further proved by the fact that international investors rely on the credit rating agencies that provide sovereign ratings to countries based on the political, legal, economic, social or environmental risks associated with the country. The credit rating agencies take into consideration these risks in providing their assessment of the risk free rate that should be considered as appropriate for the level of country risk. This implies that the risk free rate available in the country is a strong deciding factor for foreign investors.

Another tool that is employed in assessing the value of the bonds is the transition matrix, or a matrix of probable expected cash flows from any given assessment. As it is not possible to generate a future cash flow stream for an investment, investors develop probability scenarios for different expected cash flow returns from a security. These probabilities are in turn dependent on a variety of considerations involving the risks associated with the political, social, economic, and legal environment of a region. The assessment of this risk settles the risk free rate for the investors and based on their perception of the risk free rate, they tend to develop the cash flow matrix with their expected rate of returns. Here again it can be noted that the perception of the risks is subjective and could be substantially different for internal investors or the government of a country or external credit rating agencies and foreign investors.

Chapter 3 explains the different indices that investors employ in assessing their portfolios and in making a judicious investment. The Sharpe ratio, the Sortino ratio and the Omega ratio are discussed and the importance of the risk free rate in the composition of the ratios is highlighted. The portfolio performance measures explained in Chapter 3 help to gauge an understanding of these models and the importance of the risk-free role that is inherent in them. These ratios are used as effective tools by financial investors, especially those delving in fund management and portfolio management. Hence, once again the importance of the risk free rate for the dynamics of the financial market is emphasised.

In addition, the chapter also highlights the internal rate of return (IRR) and the weighted average cost of capital (WACC) that are corporate indices used by alternate investment companies, venture capitalists and companies moving ahead with new projects and investments. WACC and IRR are two basic and most important tools used to calculate the viability of projects and the time taken for the investments to break even and start generating returns for the company. Both these factors have the risk free rate of return at their core and, in this way, establish the crucial role that the risk free rate plays in the development of a country’s financial market.

CHAPTER 4: ANALYSIS OF THE THEORETICAL RISK FREE RATE AND THE PERCEIVED RISK FREE RATE

4.1 INTRODUCTION

This chapter contains an analysis of and discussion on government bonds valuation and assessment of any differences in the theoretical and the actual yield curves of zero coupon bonds in South Africa. An explanation of the methodology used is provided. A discussion of how the yield curves are calculated and the factors that impact the calculation of yields follows. Next, the chapter will present data from the Bond Exchange of South Africa (BESA) on the yields and the yield curve that shows the theoretical risk free rate as presumed by the government.

Data from the trading of bonds will be presented and a yield curve based on the market data will be developed**.** The comparison of the two yield curves will highlight the differences in the actual risk free rate – the one that is presumed by government data. The chapter contains an in-depth analysis of the yield curves. This chapter aims to answer the research question, "is there a difference between the theoretical risk-free interest rate and the actual risk free rate as presumed by the foreign investors in the South African bond market?"

In the final part of this chapter (Section 4.6) a yield curve is generated that approximates the risk free yield curve. Since government bonds are considered virtually default-free, the GOVI (Government index) benchmark bonds (a selection of the most liquid fixed rate government bonds) form a term structure of nearly risk free interest rates. Using this term structure, a smooth continuous yield curve is fitted relating an approximated risk-free interest rate to any bond duration term.

With the yield curve as a reference, credit/liquidity spreads are calculated for a selection of non-government rated bonds. The width of these spreads compares well to each bond's respective credit rating.

4.2 YIELD CURVE

A yield curve is a graphical depiction of the relationship between the yields of a particular class of securities and their time of maturity (Symes, 2011:14).

Yield curves are calculated in diverse ways and are dependent on the instruments that are employed in the Equation 2 (Brigo & Mercurio 2001:25):

$$\boldsymbol{Y(t) = \left(\frac{1}{P(t)}\right)^{\frac{1}{t}}} \qquad \textbf{\textit{(Equation 2)}}$$

Where:

Y (t) = Yield to Maturity

P (t) = cash flow at time (t) in future.

The yield curves developed by different organisations (governmental and non-governmental) for the same set of instruments may also differ, depending on the different methodologies or different assumptions that each use (Duffee, 1998:2241).

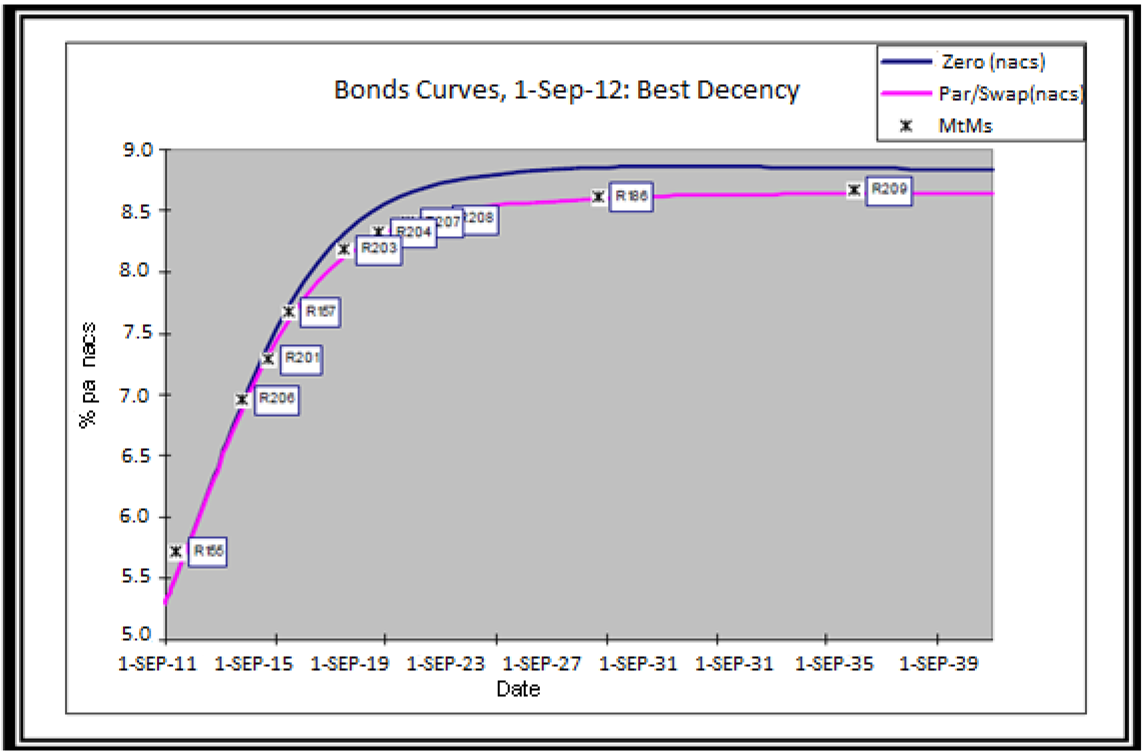
Section 4.3 presents a comparison between the yield curves developed by BESA and those developed for this study. This was to show how foreign investors develop yield curves. The comparison is expected to provide the answer to the current research question: is there a difference between the theoretical risk-free interest rate and the actual risk free rate as presumed by the foreign investors in the South African bond market?

4.3 THEORETICAL RISK FREE RATE ACCORDING TO THE BOND EXCHANGE OF SOUTH AFRICA

The South African capital market is relatively small and illiquid. BESA needed to develop a strategy that best suited the local market in order to capture trends completely (Caims, 2004:19). Owing to the perceived complexities that result from the illiquid nature of the market, BESA developed a robust methodology that can be used to develop yield curves for bonds, swaps, futures or any other instruments for which the cash flows and the present values are available. The yield curves developed by BESA can be interpolated to show the trends for hypothetical instruments that may enter the market in the future.

The yield curve presented in Figure 4.1 shows the BESA-methodology-based yield curve for the coupon market. This curve is calculated from government data and the perception of interest rates. Table 4.1 is prepared on the basis of the liquid zero coupon government bonds that are being issued by the government. This means that the data do not include the actual market performance data, which should ideally be based on all the instruments that are being traded in the market, and not just the zero coupon liquid bonds.

Figure 4.1: BESA zero coupon bonds yield curve



(Source: BESA, 2012)

Table 4.1: Zero coupon government bonds

Bond	MTM	Previous
Primary Dealer		
R 157	7.67	7.7
R 186	8.61	8.7
R 196	9.435	9.435
R 201	7.29	7.315
R 203	8.18	8.215
R 204	8.32	8.36
R 206	6.965	6.99
R 207	8.41	8.445
R 209	8.665	8.78

Source: BESA (2012)

Bonds are issued for different maturities and, as a general rule, the larger the number of years to maturity the greater the interest rate or the coupon on it. This is expected, because money invested for an extended period of time may be tied with more uncertainty than money that is invested for the short term only. The further away the future maturity date, the greater the chance of circumstances arising for which the investor cannot be ready at the time of investing. Government securities, in spite of the risks associated with the future time uncertainties, are considered to be relatively safe by investors. This perception of safety is seen as a focal point when the South African Reserve Bank projects the yield curve for the government's zero coupon bonds (Caims 2004:31).

The data referred to in Table 4.1 and Figure 4.1 is the data that is available from the South African government. However, this study aims to assess if the actual zero coupon yield curve (that is calculated using the performance data on government bonds in the market where foreign investors are operant) is different from the government-published yield data.

In the actual market, there are a large number of bonds that are in reality non-zero coupon, but the yield on these is determined by a blend of yields of the zero coupon bonds. If such a difference is observed, it will have a significant impact on the market, as the present values of the instruments available in the capital market will be affected by any shift in the zero coupon rates (Cwik, 2005:36). This shift will indicate that there is a need for the government to revise its data and to use additional risk factors, as perceived by the foreign investors who form a large majority of investors on the JSE.

The following section analyses whether a difference exists in the government yield curve and the one that is generated using first-hand market data.

4.4 MARKET-BASED YIELD CURVE

In contrast to the complex nature of factors and methodology adopted by BESA, the international calculation of the yield or the percentage return is based on calculating the current values of the bond derived from a summation of all the future cash flows (Cwik, 2005:37). The simple formula for calculating yield to maturity is presented in Equation 17 below (Brigo & Mercurio 2001:25):

$$\boldsymbol{Y(t) = \left(\frac{1}{P(t)}\right)^{\frac{1}{t}} - 1} \qquad \textbf{\textit{(Equation 17)}}$$

Here, P is the present value of the cash flow that may be received after a time (t). The present value is calculated for all futures at different time intervals and is summated to arrive at the yield or the annualised interest rate. This is the interest rate at which money can be borrowed or the maturity time of that particular bond (Brigo & Mercurio 2001:26).

The calculation of the yield curve is, therefore, dependent on the future cash flows of financial instruments over specific interval times. A matrix is developed in the following manner:

Table 4.2: Sample table of hypothetical cash flow matrix

Instruments	Time Interval 1	Time Interval 2	Time Interval 3	Time Interval 4
Instrument 1	I1 Cash flow 1	I1 Cash Flow 2	I2 Cash Flow 3	I1 Cash Flow 4
Instrument 2	I2 Cash Flow 1	I2 Cash Flow 2	I2 Cash Flow 3	I2 Cash Flow 4
Instrument 3	I3 Cash Flow 1	I3 Cash Flow 2	I3 Cash Flow 3	I3 Cash Flow 4

Source: Burger (2012)

The data in Table 4.2 are derived from the price available in the bond or money market. The bond market yield curves restrict the instruments to government bonds such as zero coupon bonds, while those derived for the money market include a variety of instruments. A comprehensive money-market yield curve contains the current prices of cash, current prices of futures that are mid-term maturity instruments, and swaps that are long-term security instruments. This ensures that the curve is representative of a wide range of maturity instruments and can be used for assessing the yields from different types of instruments (Caims, 2004:45).

However, a problem that is encountered in developing a meaningful yield curve is to calculate the future cash flows of all securities operating in the market. The future cash flows cannot accurately be known in advance and, for this reason, there is no direct way of assessing what the overall yield to maturity is by using Equation 2 in Section 4.2. This is an impossible task and therefore Equation 2 cannot be used directly. Instead, the calculation of the yield to maturity for different instruments is carried out in a complex manner using the iteration method. This is undertaken in Section 4.5.1, where the JSE data on present prices and coupon rates of the same set of bonds as used in the BESA yield curves are developed.

4.4.1 Calculating the market-based yield curve

In Table 4.3 the current value of the bonds on 1st September 2012 is provided along with their coupon rates. In order to develop a yield curve, the yield to maturity of each of the individual bonds is needed. An easy method of calculating this would be to use the cash flow streams and to arrive at the yield to maturity (YTM) using the formula:

$$Y(t) = \left(\frac{1}{P(t)}\right)^{\frac{1}{t}} - 1 \qquad \textit{(Equation 17)}$$

Table 4.3: Market data using present values on September 1, 2012

Bonds	Present Value Price	Maturity	Interest rate
R 157	121.22	24 years	13.50%
R 186	115.42	28 years	10.50%
R 196	N/A	N/A	N/A
R 201	104.41	11 years	8.75%
R 203	99.98	13 years	8.25%
R 204	97.89	14 years	8%
R 206	101.14	9 years	7.50%
R 207	92.55	15 years	7.25%
R 209	74.95	30 years	6.25%

Source: BESA (2012)

However, as the future cash flows are unavailable, the YTM of the entire bond in Table 4.3 is calculated using an iteration method employing the formula of annualised compound interest over the period of maturity (Equation 1).

$$\textit{Present Price} = \sum(1 + YTM)^{n} \qquad \textit{(Equation 1)}$$

For example, in the case of R206, with 101.14 as the present value, a 7.5% interest rate and a maturity period of nine years; the YTM can be calculated by using 101.14 as the price and using tentative YTM values.

Through multiple iterations for each of the bonds, the YTMs for the coupon bonds listed in Table 4.3 are calculated. These are provided in Table 4.4.

Table 4.4: Yield to maturities and expected rates of returns

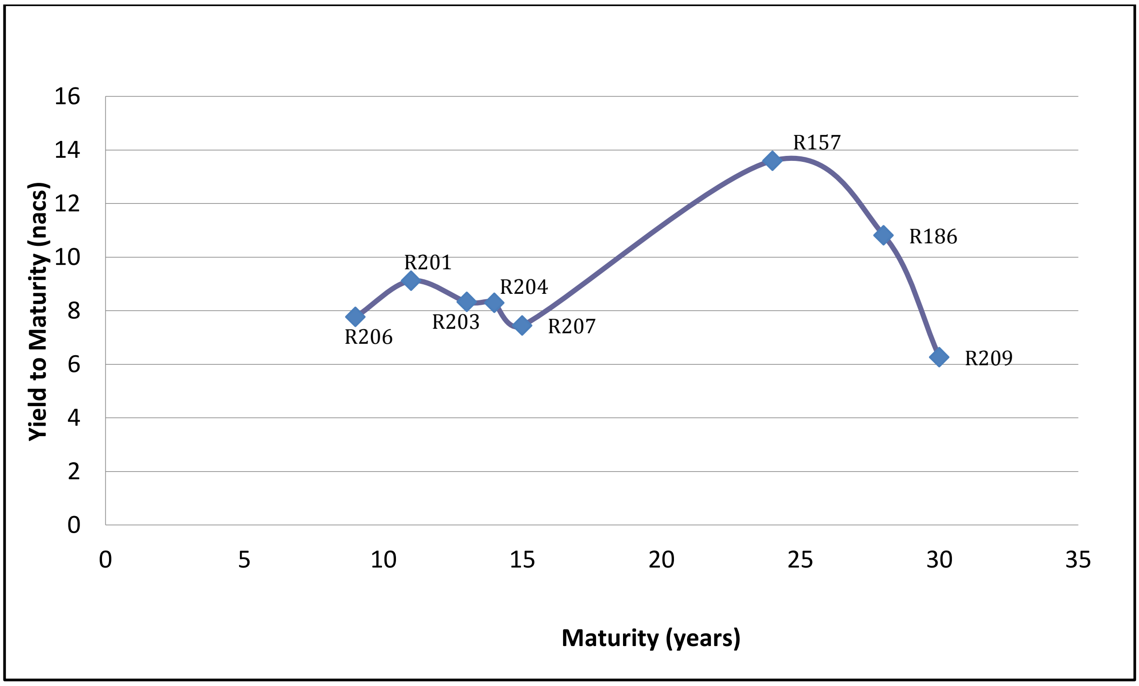
Bonds	YTM	Coupon rate
R 157	13.61%	13.50%
R 186	10.81%	10.50%
R 196	N/A	N/A
R 201	9.12%	8.75%
R 203	8.33%	8.25%
R 204	8.30%	8%
R 206	7.77%	7.50%
R 207	7.45%	7.25%
R 209	6.27%	6.25%

Source: BESA (2012)

According to basic assumptions taken for bond yields and coupon rates, the YTMs are always higher than the coupon rates. The difference depends on the magnitude of the spread between the two. In Table 4.4, the spread between YTM and coupon rate of the bond R201 is the highest at 0.37%. This indicates that among all bonds listed in Table 4.4, R201 is riskier to invest in. The larger spread shows the magnitude of the risk undertaken by an investor in order to achieve higher returns on their investment (Hagen & West, 2006: 27).

The YTMs in Table 4.4 are used along with the maturity dates to arrive at the yield curve, which is presented in Figure 4.2:

Figure 4.2: Yield curve using market data

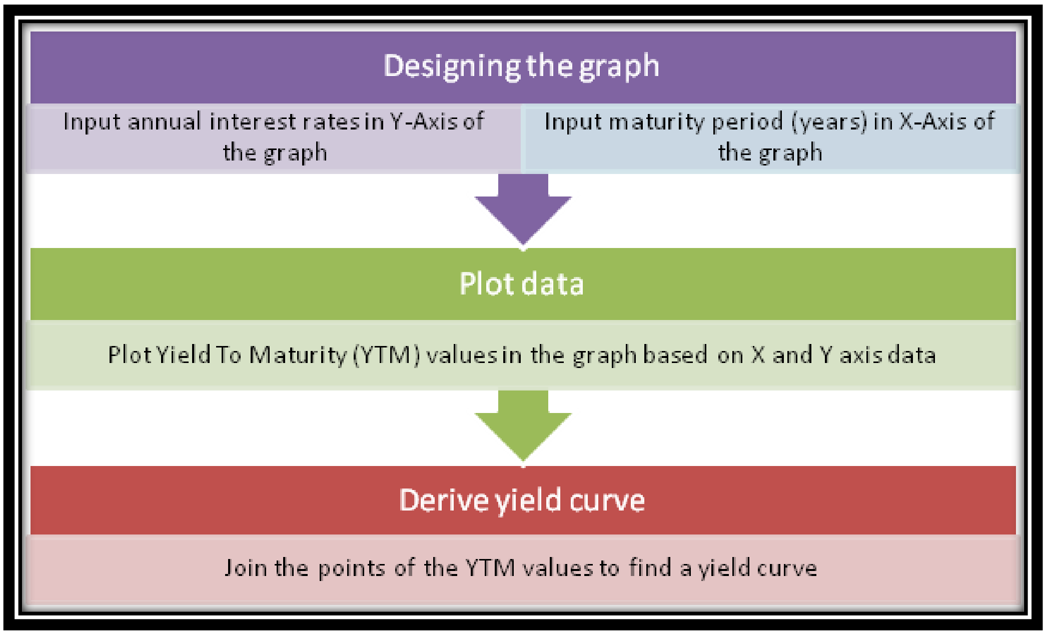


Source: Burger (2012)

The yield curve illustrated in Figure 4.2 is prepared in using the steps outlined in Figure 4.3. The inverted yield curve is largely attributed to the anticipation of a fall in the risk rates in the long-term future. This could well be the case in South Africa, as foreign investors are optimistic of the region's stability and, therefore, a reduction in country risk (Hagen & West, 2006:91). In addition, most of the investment that South Africa attracts is through construction and infrastructure development bonds that are expected to be highly stable long-term investments.

However, according to Harvey (1986:26) the literature and research on yield curve shapes indicates that an inverted curve is a preceding factor or an indicator of a coming recession in the longer term. Investors are still willing to invest in such a market because they presume that, in the longer term, the falling interest rates will be compensated by the falling inflation prices and, therefore, they can still expect to make profits. This outcome is because the low bond yields can still be offset by a low inflation rate and, in this way, lead to substantial profits for investors over the longer term.

Figure 4.3: Steps in designing a yield curve



Source: Burger (2012)

A comparison of the yield curves in Figure 4.1 and Figure 4.2 showcases the differences in the shape of the curve. It can be seen that the government-issued yield curve is a fairly smooth line that has accelerated growth towards the centre and then stabilises across the maturity period. Dissimilarly, Figure 4.2 shows that the curve is artificially discontinuous and that the actual rates are much higher than the original BESA curve (Hagen & West, 2006:96).

The BESA yield curve is 'normal' as it indicates an upward slope in the yields to maturity, indicating that the long-term bonds are considered to be riskier than the short-term bonds. A normal curve is considered to be an indicator of investors' expectations that the coming future will see an increase in economic activity and growth, with higher interest rates as a result. The BESA-based curved shows a sharp slope and then a flat line, which indicates that over the much longer term, there is uncertainty about the way in which the interest rates may change (James & Webber, 2001:56).

However, the market-database yield curve is not even and cannot be strictly classified as being 'normal' or inverted. The curve shows that the short-term bonds are expected to provide lower risk and lower returns, mid-term bonds a relatively higher risk and return, and much longer-term bonds lower returns. This is indicative of the inhibitions of investors with regard to the growth prospects of South Africa over the next few decades. It indicates that any governmental initiative and investment in construction and development activities are seen as attractive by foreign investors. However, investors are not yet sure of the sustainability of the growth momentum over a longer period of time (Wets & Bianchi, 2006:32).

The main objective of this study is to assess if the theoretical risk free rate is different from the actual risk free rate. When comparing the yield curves, it is seen that there is not only a difference in the shape but also the placement of the curves. The market-based curve is positioned a little higher, indicating higher interest rates or YTMs than those depicted by the BESA curve.

The reason for the difference in shape is due to the differences between the YTMs calculated by the South African government and by independent investors.

Table 4.5: Yield to maturities using the BESA method and JSE market prices

Bond	Market	BESA	Differences
R 157	13.60	7.67	5.93
R 186	10.81	8.61	2.20
R 196	N/A	9.43	N/A
R 201	9.12	7.20	1.92
R 203	8.33	8.18	0.15
R 204	8.30	8.32	-0.02
R 206	7.77	6.96	0.81
R 207	7.45	8.41	-0.96
R 209	6.27	8.66	-2.40

Source: Burger (2012)

As can be seen in Table 4.5, the theoretical risk free rate or the BESA rate is lower than the actual rates. This is the cause of the difference in the diagrammatic representation of the yield curves shown in Figures 4.1 and 4.2. It is also seen in

Figure 4.2 that, in the case of the yield curve prepared using the market data, the risk-free interest rates fall between 6% and 14%, while for the BESA curve the interest rates are much lower with all falling below 10%. These discrepancies are the direct result of the way the government perceives the risks associated with the coupon bonds and how the risks are perceived by foreign or independent investors. In Section 4.5, the reasons for these differences are discussed in detail so that the reader can understand the risks or advantages that the South African capital market and economy face.

4.5 REASONS FOR DIFFERENCES IN THE THEORETICAL RISK FREE RATE AND THE MARKET RISK FREE RATE

Differences between the two may occur for various reasons such as macro-economic factors like changes in the political scenario, changes in interest rates, changes in government policy and regulations, inflation and other major financial indicators, along with growth drivers of the economy. Other factors that must be taken into account are the expectations of investors to earn a higher return on each asset class and a comparison of the same with inflation rates and the interest rates of banks, based on which the investors earn interest income (Wajid *et al,* 2008:8).

4.5.1 Expectations of investors

The most popular and prominent explanation for the differences in the yield curve is given by the market expectations (pure expectations) hypothesis theory. According to this theory, all available maturities are a perfect substitute for each other. Investors can invest in any of the maturities with complete ease whenever they wish to invest. Another assumption on which this theory is based is that the opportunities to arbitrage are non-existent. This assumption works well in the case of a market such as South Africa’s where there is a unified and centralised securities exchange (Watson & Head, 2007:67).

The theory proposes that the yield curve is dependent on what the investors expect the future interest rates to be. According to the expectations theory, the expectations of investors are one of the major influencing factors for future interest rate movements. When short-term interest rates are expected to fall there will be an increased demand for long-term maturity demands. Similarly, when long-term interest rates fall, then an inverse demand for short-term maturities arises, where there will be less demand for long-term maturity bonds (Wijck, 2006:42).

The mathematical representation of the YTM of any given instrument is as follows (James & Webber, 2001:21):

$$(1 + i_{lt})^n = (1 + i_{st}^{year\ 1})(1 + i_{st}^{year\ 2}) \ldots (1 + i_{st}^{year\ n}) \qquad \textit{(Equation 18)}$$

The expected rates are considered to be the rationale for all the investment activity and are also considered to be appropriate for the development of the yield curve. The yield curve is developed on the basis of the investors' perception of the interest rate over a period of time. The mathematical calculations to find the expected yield to maturity are undertaken by using the compounded interest for short-term securities. In the case of long-term securities, the calculation of the yield to maturity is undertaken by finding the geometric mean of the yields on the short-term instruments. The theory therefore is consistent with the assumption that the yields tend to move together and in this way lead to the creation of a normal yield curve (James & Webber, 2001:22).

The inherent flaw in this theory is that all the instruments or maturities that are available in the market are, in reality, not perfect substitutes for each other and instead, investors have varying level of risk perceptions for each. Investors also perceive bonds as riskier, as the calculation of the yield to maturity is based on the forward rates and not the exact future rates. In addition, there is the risk associated with the changes in the long-term interest rates being neglected. There is a practical possibility that the future interest rates may fluctuate or rather increase and thus lead to a lowering of the present market value of the instrument. Another risk that is ignored is the reinvestment risk or the risk that the instrument may be cancelled or paid out earlier, leaving the investor with no appropriate option to reinvest the money. In this case, the investor may not find a suitable or similarly attractive instrument to invest in at the time his current instrument is liquidated (Rebonato, 1998:15).

While this theory provides a direct indication about the fact that the investors' perceptions are responsible for shaping the yield curve, it is simplistic and neglects the risks that investors may be taking into account while developing the yields to maturities (Caims, 2004:102). This is one of the basic reasons for a difference in the BESA yield curve and that developed within this study on the basis of market data. The BESA yield curve is based on the presumption that investors expect long-term stability in interest rates and that they have perfect faith in the bonds remaining viable until their maturities are realised. This, in fact, is the perception of the government and not of foreign investors, who may be more vigilant about and sensitive to the expected changes in the interest rates in the future (Watson & Head, 2007:72).

4.5.2 Liquidity premium theory

The liquidity premium theory is an extension of the pure expectations theory. According to this theory, investors always prefer liquid investment instruments to illiquid ones. Here, debt instruments (including bank financial derivatives) are considered as highest priority, followed by gold investments and then equity markets. Investors also follow a similar exit-strategy for churning the portfolio. They will prefer to sell the illiquid assets first and hold on to the more liquid ones (Rebonato, 1998:38).

The investors' expectations of the interest rates are still based on the understanding that long-term maturities should bring higher interest rates than short-term holdings, owing to the commitment or opportunity cost that they incur by keeping their money tied in for a longer duration. However, in addition to this rationale, investors are also concerned about the reinvestment risk and the interest rate risks (which are ignored by the pure expectations theory). Investors largely associate the long-term maturities with greater risk and, therefore, add a risk premium to their expected yield to maturity. The risk premium is based on the assumption that it will protect against price uncertainties and will safeguard against changes in interest rates (Caims, 2004:116).

These assumptions again determine a yield curve that is normal or sloping upward (positive slope). This is a reflection of investors' assumption that the long-term maturities' yield to maturity will be higher than the short-term yields. The yields are calculated using the same formula employed by the expectations theory, but with the addition of risk premium factors (Brigo & Mercurio, 2001: 97).

$$(\mathbf{1}+\ \boldsymbol{i_{lt}})^{\boldsymbol{n}}\ \mathbf{=}\ \boldsymbol{rp_n} + \langle\langle \mathbf{1} + \boldsymbol{i_{st}^{year1}}\rangle\left(\mathbf{1} + \boldsymbol{i_{st}^{year2}}\right)\ldots\left(\mathbf{1} + \boldsymbol{i_{st}^{yearn}}\right)\rangle \qquad \textbf{\textit{(Equation 19)}}$$

Where:

rp_n =risk premium.

This theory is, however, not significantly different from the expectations theory, as it too places investors' expectations as the core of the yield curve (Brigo & Mercurio, 2001:103). It nevertheless establishes that investors' own perceptions regarding the future interest are the most important criteria and, hence, highlights the importance of the formation of these expectations and the factors that lead to the development of expectations.

It also indicates that the government-prepared yield curve or risk free rate may have a conservative normal shape, as the official data tend to reflect the stability of long-term economic growth and interest rates. This may, however, not be perceived similarly by foreign investors, who may have their own criteria and factors that they employ in developing their expectations of the future interest rates of a country. The difference in perception of the future trends in interest rates is therefore the main cause of the difference in the shapes of the two yield curves as discussed in the previous sections (Rebonato, 1998:42).

4.5.3 Market segmentation theory

The market segmentation theory is more realistic and it places the supply-and-demand dynamics of the market at the core of the interest rate changes. The market segmentation theory takes a realistic stance and postulates that, unlike the pure expectations theory's assumption, the instruments available in the market are not a perfect substitute for each other. This is especially true in the case of short-term- and long-term securities. According to this theory, short-term- and long-term securities markets are distinct and the supply and demand for either are exclusive of each other. This means that the availability and demand for the instruments are not dependent on the existence or supply of other instruments but on a variety of market-dictated factors (Brigo & Mercurio, 2001:113).

The liquidity factor on each bond creates a demand-supply-scenario of its own, thereby allowing investors to decide whether to stay invested or exit the investment. In the case of short-term instruments the perceived liquidity is large and, hence, these instruments are expected to be more in demand by investors who prefer liquidity. More demand tends to increase the price while lowering the yields and in this scenario where the preferred security is short term, the yield curve will again be a normal curve (James & Webber, 2001:35).

The longer-term securities that are of a relatively less liquid nature may have a reduced demand and, hence, lower prices and larger yields. This theory therefore provides an explanation of the existence of the normal yield curve, based on the assumption that the demand for short-term securities is higher at any given time. However, there is a substantial probability that this assumption may not hold true for all markets or for all investors (Deventer, Imai & Mesler, 2004:27).

Instead of preferring liquidity, investors may prefer a longer-term investment, and hence create a large demand for long-term bonds. A large demand for the long-term bonds may ultimately lead an increase of the long-term maturities prices and a lowered yield to maturity as a consequence. This leads to a yield curve where the short-term securities may have a higher yield than the long-term ones or to a yield curve that is inverse, instead of being normal (Brigo & Mercurio, 2001:117).

The South African BESA yield curve is based on the assumption that investors have a preference for liquid or short-term securities and that the demand is an upward-sloping curve. However, as the South African market is generally considered small and less liquid compared to other mature markets, investors consider the short-term investments as riskier and demand a higher premium for investment, even in short-term bonds. This is the reason why the market based yield curve developed in this study shows the yields between 6% and 14% while that developed by BESA has a smaller range of yields of between 5% and 10% (Hogan & West, 2007:104).

The market segmentation theory also does not indicate what happens in the case of changing interest rates or the changed perceptions of investors about the future interest rates. The interest rate fluctuations create a new demand-supply-scenario for the respective bonds. In these instances, the yield curve shapes and yield expectations are changed (Deventer, Imai & Mesler, 2004:34).

According to the market segmentation theory, the short-term securities are expected to have lower risks and lower yields than the long-term bonds but, in the case of the South African market, the foreign investors may perceive more risks associated with mid- and short-term securities than with the long-term securities. This is definitely indicated by the yield curve presented in Figure 4.2. Here again, it can be noted that it is the perception and the expectations of the investors that are responsible for the shape of the yield curve (Hogan & West, 2007:106).

4.5.4 Preferred habitat theory

4.5.4.1 Differences in expectations of future interest rates

Another theory that can be employed to explain the difference in the BESA yield curve and that provided by this study is the preferred habitat theory. According to this theory, investors not only have their expectations regarding liquidity and interest rates, but they also assign a risk premium according to the region they are investing in. This theory takes the calculation of the yield curve into the realm of subjectivity, as it emphasises that the investors have their minds set regarding what they considered a safe maturity time and what they believe is a safe investment destination. These risks are added to the interest rate expectations and increase the expected YTM as a result (Brigo & Mercurio, 2001:121).

Investors expect a premium if they buy securities from outside of their preferred comfort zone. This presumption indicates that the investors may perceive a short-term security to be riskier than a long-term one for a particular region, based on their own preference and criteria of selection. This is the probable reason why, in the case of the South African market, the yield curve developed in this study shows an inverse curve towards the longer-term maturities (Deventer, Imai & Mesler, 2004:34).

The analysis of the BESA and the market based yield curve using the all the above mentioned theories has revealed that the basic differences in the yields and the shape of the yield curve are due to the difference in perceptions and expectations of the future interest rates of the foreign investor and that of the BESA analyst. A prominent point that is highlighted by this is that by identifying the reasons for the difference in the expectations of the foreign investors the South African government may gain better insights into the practical assessment of the market by its foreign investors (Watson & Head, 2007:72).

The reason that the foreign investors may perceive higher risks with their investments and more fluctuating interest rates over the long term is their perception and assessment of the country's financial and economic health (Arthur & Sheffrin, 2003:105).

The changes in interest rates generally take place because of government-initiated policy changes or macro-economic factors affecting the banking sector. These macro-economic factors enable the sector to make adjustments in the risk free rates so as to keep inflation and the value of currency intact (Arthur & Sheffrin, 2003:106).

4.5.4.2 Country risks

Country risk is a term generally used to refer to the expected or probable changes that may happen in the target country, which may lead to changed economic and financial scenarios. These changes range from changes in the political environment and political stability, changes in economic policies, inflation, and changes in the financial setup of the country (South Africa.info, 2008).

Changes in the political-economic scenario are expected to affect, both positively and negatively, the growth rate and the returns on investment in various asset classes. In addition to the presumptions of any changes that may occur in that country, the investors are also aware of other factors, such as the level of corruption, the presence of a regulatory environment, and legal and technical restrictions (Busetti, 2009:170).

Country risk is, therefore, closely associated with the expectations that investors have of the returns or interest rates and the risk free rates provided by that particular economy. The risk free rate is ascertained by the government treasury independently of the perceptions or expectations of foreign investors. For this reason, there may be a difference in the theoretical and the actual risk free rate (Hagan & West, 2006:120).

4.5.4.3 Foreign exchange risk

Foreign exchange risk is associated with the risk of a currency appreciating or depreciating against another currency. Such movement in the currency causes fluctuations and affects general business activities (especially exports and imports). In the case of South African market, on appreciation of the South African Rand, importers tend to benefit. However exports tend to weaken, which brings about a reduction in income for the country's economy. This can create a trade imbalance, which leads to a deficit increase that can put pressure on input costs in the economy, leading to consumers paying more for the same commodity. Currency movements are linked directly with the inflation factor (Arthur & Sheffrin, 2003:114).

In South Africa, inflation plays a major role in determining interest rates of bonds in the economy. The annual inflation for 2011 was 6% (reported in March 2012) which lead to an increase in food and fuel prices and a slowing down of the economy. The risks perceived by the foreign investors are reflected in the yield curve formulated in this study. Inflation in the long term is expected to stabilise and the much longer-term securities like the R209 are expected to give lower yields than the mid-term ones, where the inflation is expected to be high (South Africa.info, 2012).

4.5.4.4 Default risk

Default risk is essentially dependent on the chance that a given country's government that issues risk-free bonds may default on payment. Credit rating agencies across the world study the historical and geographical context of the country or the region as well as the government that has issued the bonds in order to provide investors with a factor for the default risk for that particular country.

South Africa has been observed as a region of much turmoil and change in the past few decades, due to the economic sanctions levied against the country due to Apartheid and later land redistribution and rebellion. However, the South African government has been stable and has led the country through difficult times with determination. It is a reflection on the stability of the government and the economic growth that the recent financial crisis of 2008 did not affect the country (Wajid *et al*, 2008:62).

However, it is also observed that there is a growing trend towards credit crunch which may signify that the government may not be able to avert the crisis due to its own widening current account deficits. The credit market collapse and the inability of the government to find a way out, translates into a higher perception of risk associated with the government’s ability to pay its own debts (Wajid *et al*, 2008:64).

The current attitude of the South African government appears aggressive, as it is involved in the rapid development of infrastructure and utilities projects, which has necessitated making changes to its financial legislation. The uncertainty associated with fluctuating financial regulations, excessive foreign capital and a large current account are expected to make the default risk associated with the government’s bonds higher for foreign investors, many of whom are still reeling from the recent recession (Wajid *et al*, 2008:69).

4.5.4.5 Implications for economic development

The findings based on the yield curve differentials between actual yield curve and government yield curve clearly indicate that the risk free rate is more than the theoretical rate. The factors discussed in Section 4.5.5 play a major role in a market such as South Africa’s, which is small in comparison and has liquidity constraints. Investors would naturally tend to demand more returns from this market to compensate for the higher risk (Busetti, 2009:206).

This study has established that the theoretical risk free rate and the practical risk free rate used by foreign investors in the South African bond market are different. The overall YTMs are found to be higher than the BESA actuaries curve, and also the shape of the market-based curve is different from that of the BESA yield curve. These findings highlight the fact that foreign investors perceive the country risk inherent in South Africa and the foreign exchange risk as high (Stutzer, 2000:61).

The yield curve developed using market data shows a humped slope with an increase in yields for 10-year maturity; lowered yields between 10 and 15 years; increased yields between 15 and 25 years; and then an inverse slope beyond 25 years. The shape of the yield curve is indicative of the fact that the risk free rate is high, and also that long-term bonds are perceived to be riskier than short-term bonds (Burger, 2012:59).

An increase in risk free rates is not considered a good sign for the economy in the long term as this can touch down at a micro level to affect each and every company operating in the economy. As risk free rates rise, the assets that generate positive cash flows tend to decrease in value, especially for a growing firm compared to a mature firm. This affects business performance and may lead investors to withdraw from the particular industry or even that country's market. The shape of the yield curve is indicative of this fact and highlights that in the longer term the South African economy may suffer from large-scale withdrawals of foreign investments (Wijck, 2006: 24).

In addition, there is a historical precedent that an inverse yield curve is a credible indicator of a recession in the future. According to Cwik (2005:21), all the depressions in the US in the past 56 years have been predicted by an inverse yield curve. This finding indicates that the South African market may be heading for a recession in the future, even though in a shorter-term context it may see inflation on the rise accompanied by rapid growth.

The inverse yield curve indicates that the short-term securities are higher-yielding ones and banks will have to pay more on the maturity of these securities than they would have been prepared for. As the long-term securities are less yielding, the banks do not make a profit on these; while at the same time have to pay out cash for the short-term securities. This offsets banks' balance sheets and also makes them reluctant to lend any more. Banks' refusal to provide credit and funds leads to a credit crunch and brings down the economy to a recession (Cwik, 2005:27).

In Section 4.6 it will be shown that a yield curve giving the approximate risk free interest rate for any bond duration can be obtained using the GOVI benchmark and the short term RODI rate.

4.6 THE RISK FREE YIELD CURVE

Since government bonds are considered virtually default-free, the GOVI (Government index) benchmark bonds (a selection of the most liquid fixed rate government bonds) form a term structure of nearly risk free interest rates (BESA, 2003:3). Using this term structure, a smooth continuous yield curve is fitted relating an approximated risk free interest rate to any bond duration term.

With the yield curve as a reference, credit or liquidity spreads are calculated for a selection of non-government rated bonds. The width of these spreads compares well to each bond's respective credit rating.

4.6.1 The government yield curve

Historically, most market participants construct yield curves from their observations of market yields on government bonds. The reasoning is that these bonds are from the same issuer and considered free of default risk. Due to their high tradability, these bonds are also considered very liquid and therefore low on liquidity risk. The GOVI benchmark, which represents the very liquid all-government sector of the All Bond Index (ALBI), is an example of such government bonds (BESA, 2003:3). The constituents of the GOVI and their corresponding coupon rates and maturity dates on 01/09/2012 are listed in Table 4.6

Table 4.6: The GOVI benchmark as on 1/9/2012

Bond	Coupon Rate	Maturity
R 157	13.50	15/09/2015
R 186	10.50	21/12/2026
R 201	8.75	21/12/2014
R 203	8.25	15/09/2017
R 204	8.00	21/12/2018
R 207	7.25	15/01/2020

Source BESA (2012)

All bonds in the GOVI benchmark have different coupon rates leading to coupon bias, as was the case with the BESA Actuaries yield curve. The key effect of coupon bias is that due to the varying coupon rates of different bonds in the yield curve, some bonds with similar maturity terms may not trade at the same yield. The government yield curve might therefore not be an exact representation of the risk-free rate for a given maturity term.

This effect of the coupon bias can be partly offset by expressing the government yield curve in terms of duration and not maturity. Duration is a measure of the cash-weighted maturity terms of a bond. There are two types of duration - Macaulay duration and Modified duration. The former is useful in immunisation, where a portfolio of bonds is constructed to fund a known liability. The latter is an extension of the Macaulay duration and is a useful measure of the sensitivity of a bond's price (the present value of its cash flows) to interest rate movements (BESA, 2003:5).

4.6.2 The zero coupon yield curve

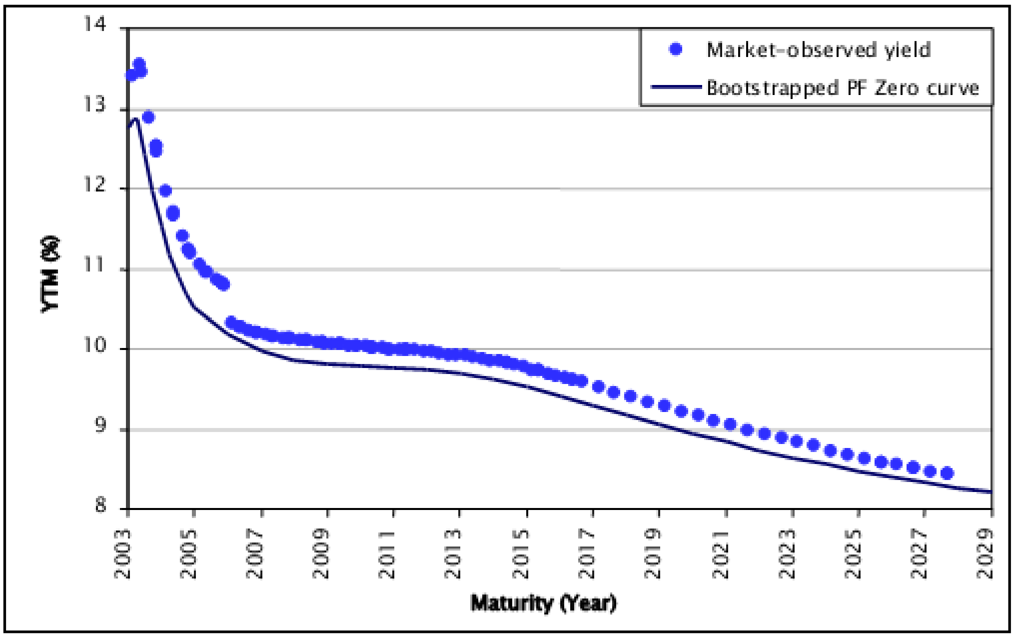
Late in 2001 BESA introduced a standardised zero coupon yield curve for the SA debt market that is derived from the most liquid coupon-bearing government bonds. The zero coupon yield curve is constructed using the yields from the coupon bearing bonds by an iterative process known as bootstrapping. This determines an appropriate discount rate associated with a unique maturity, which solves the unknown zero rate (Fabozzi & Fabozzi, 1995:122).

In addition to this bootstrapped zero curve, the National Treasury implemented a formal bond stripping program early in 2002 in which the most liquid government bonds' coupon and principal payments are stripped and traded as separate zero coupon bonds on BESA (Redelinghuys, 2001:5). Since only principal payments occur on these bonds, it is free from coupon bias and due to the credit quality of the issuer, considered to be riskless. In theory, the zero coupon yield curve can then be observed directly from the market-based YTM values. In practice, however, these bonds are not frequently traded, have fairly wide bid-ask spreads and are not suitable as an indicator for zero rates.

A comparison of the market-observed and bootstrapped zero coupon yield curve is shown in Figure 4.4. Although not the smoothest zero curve, the "perfect fit" curve that was bootstrapped from a selection of government bonds represents the zero curve that prices its generating instruments exactly. The wide bid-ask spreads between these two curves are evident, indicative of the liquidity issues surrounding the trading of these stripped government bonds.

One area where a yield curve is frequently used is in the calculation of credit spreads for non-government traded bonds. When buyers invest in a non-government bond, the buyer expects to be compensated for the additional default and liquidity risks taken on by demanding a higher yield. The yield difference between this risky bond and an identical but riskless government bond is referred to as credit spread.

Figure 4.4: A comparison between the market-observed zero coupon yield curve and the bootstrapped perfect fit zero curve



Source: Burger (2012)

Credit spread consists of two separate components, one attributable to default risk and the other to liquidity risk. The ratio of these two components in the credit spread can vary for a number of market and issuer-related reasons. For example, the credit spread for a very default-risky bond, but which has good liquidity, will have large default and small liquidity components. Conversely, the credit spread for a bond that has a small default risk, but which is extremely illiquid will have small default and large liquidity components. Although these examples adequately illustrate the concept of the credit and liquidity components in the credit spread, the exercise of quantifying these components is more difficult. The proper separation of these components will be the topic for future research, but for the time being, the credit spread will be assumed to be wholly attributable to credit risk.

Due to the long investment horizon for bonds as well as possible variations in coupon rates, a non-government bond will rarely, if ever, have an exact matching government bond in terms of its respective cash flow structures. It therefore warrants the creation of a synthetic government bond, which matches the cash flow structure of the non-government bond exactly. The generation of this synthetic government bond can be done in one of two ways.

The more correct method prices the non-government bond using the zero curve, which is bootstrapped from a collection of liquid government bonds. This price then represents the risk free price for the bond, free of default or liquidity risks. Once the price is determined, the corresponding YTM for the bond is calculated. Again, this YTM then represents the risk-free YTM for the bond. Although this method is the most acceptable method to calculate risk free rates, it is very calculation intensive. Calculations rely upon the availability of correctly interpolated zero rates corresponding to the bond's cash flow dates. Depending on the compounding method used in the generation of the zero curve, a discount vector can then be generated to discount all cash flows to their present values, considering all the peculiarities regarding the SA bond pricing formula (BESA, 1997:2). This price is then used to derive an YTM value using a proper numerical iteration method.

The other, less accurate, method is by approximating the risk-free rate by interpolating the interest rate term structure from a collection of government bonds for the non-government bond's duration. The reason a duration-axis instead of a maturity-axis is used for the yield curve is to counteract the effects that different coupon rates of the government bonds have on the yield curve in general. For bonds with different coupon rates, the YTM and price eventually settled on will impound this coupon rate. In the calculation of the Macaulay duration, each maturity term is discounted by the weight of the cash flow divided by the price of the bond. Although this interpolating method is not as accurate as the re-pricing method, it can still be used as a guide to estimate the credit spread.

4.6.3 The government term structure of interest rates

The interest rate term structure consists of a collection of identical debt instruments that only differ by maturity term or in this case, duration. For the government term structure, a collection of government bonds called the GOVI benchmark, which is a subset of the ALBI benchmark and consists of the most liquid government bonds, was chosen. As an approximation, the coupon bias between these bonds will be ignored in order to construct a term structure of risk free interest rates.

4.6.4 Extending government term structure of interest rates

Referring to the maturity terms of the different government bonds listed in Table 4.6, the R201 bond matures on 21/12/2014. Although this bond has the shortest maturity term in the GOVI benchmark, it is still longer than many of the maturity terms for non-government bonds listed on BESA. Therefore in order to obtain a comprehensive yield curve, especially for maturity terms shorter than that of the R150 maturity term, two additional short-term financial instruments are used for the term structure at shorter durations. These instruments are a risk-adjusted version of the Rand Overnight Deposit rate Index (RODI) and the 91-Treasury bill discount rate. Since these instruments have maturity terms of one business day and 91 days respectively, two extra node points to the left of the R201 node point are introduced, totalling eight instruments for the government term structure.

4.6.5 The 91-day treasury bill

A Treasury bill is a short-term debt obligation of the government and should therefore carry no risk premium or credit spread on the quoted discount rates relative to sovereign debt. The discount rate can be readily converted into a yield rate by using the following expression:

$$y = \frac{d}{1 - \left(\frac{91}{365}\right) d}$$

(Equation 20)

Where:

y = yield

d = discount rate

In Equation 20 the relationship between the yield and discount rate can be easily verified using the fundamental definitions of present value (PV) and future value (FV) for investments:

$$PV = \frac{FV}{1 + yt}$$

(Equation 21)

and

$$PV = FV(1 - dt)$$ ***(Equation 22)***

With t representing a fraction indicating whether the yield or discount rate is a simple, monthly, quarterly, semi-annual or annual rate. Combining Equations 21 and 22 and solving for the yield for the 91-day Treasury bill leads to Equation 20.

Since the YTM for bonds listed on BESA is quoted on a NACS (nominal amount compounded semi-annually) basis, the 91-day Treasury bill can be converted into a synthetic 3-month maturity term bond by changing the yield y from NACQ (nominal amount compounded quarterly) to NACS:

$$\frac{Y}{2} = \left(1 + \frac{y}{4}\right)^2 - 1$$

(Equation 23)

With Y the YTM. In Equation 23 the quarterly yield is compounded for two three-month periods assuming that the Treasury bill can be re-invested at the same rate after the first three-month period.

4.6.6 The rand overnight deposit rate index

The RODI is the weighted average of the overnight call deposit rates paid by A1-rated local and F1-rated foreign financial institutions where the South African Futures Exchange (SAFEX) places its daily margin deposits received by members (SAFEX, 2003:5). The RODI is quoted on a nominal amount compounded monthly (NACM) basis, which can be converted to NACS using the following expression:

$$\frac{Y}{2} = \left(1 + \frac{y}{12}\right)^6 - 1$$

(Equation 24)

Where:

y = YTM

Since this rate is between non-government entities, it also carries a narrow implicit credit spread and because it is derived from a very liquid instrument, the credit spread is assumed to be mostly due to default risk.

4.6.7 Adjusting the RODI for the implicit credit spread

To account for the implicit credit spread in the RODI, the NACS yield spread between two short-term money market instruments, the 91-day Treasury bill and the 3-month JIBAR rate can be used as proxies for the credit spread between short-term government and non-government debt. By subtracting this yield spread from the RODI, a ‘risk free’ RODI is obtained that can be used as the first node point in the term structure and yield-curve calculations.

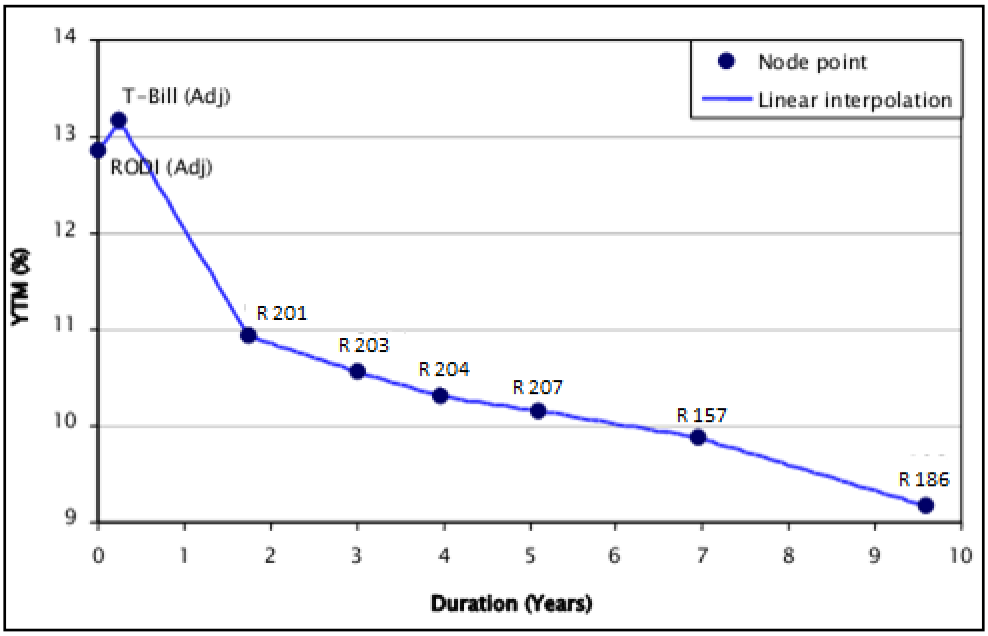
4.6.8 Generating a continuous government term structure

The discrete interest rates of the various instruments in government term structure need to be interpolated to obtain a continuous government term structure. From here, the "risk free" rate for any duration can be determined.

4.6.8.1 Term structure interpolation methods

The simplest method to interpolate these discrete interest rates is to put the YTM for the duration term in question equal to the weighted average of the YTM's for its two nearest neighbours. Mathematically, this comes down to linear interpolation, the result of which is shown in Figure 4.5. The drawback of this method is that the yield curve is not smoothly interpolated and may lead to major interpolation errors, especially at the short-end of the yield curve where large differences between neighbouring instruments occur.

Figure 4.5: The government yield curve constructed from the term structure using linear interpolation



Source: Burger (2012)

4.6.8.2 The cubic spline interpolation method

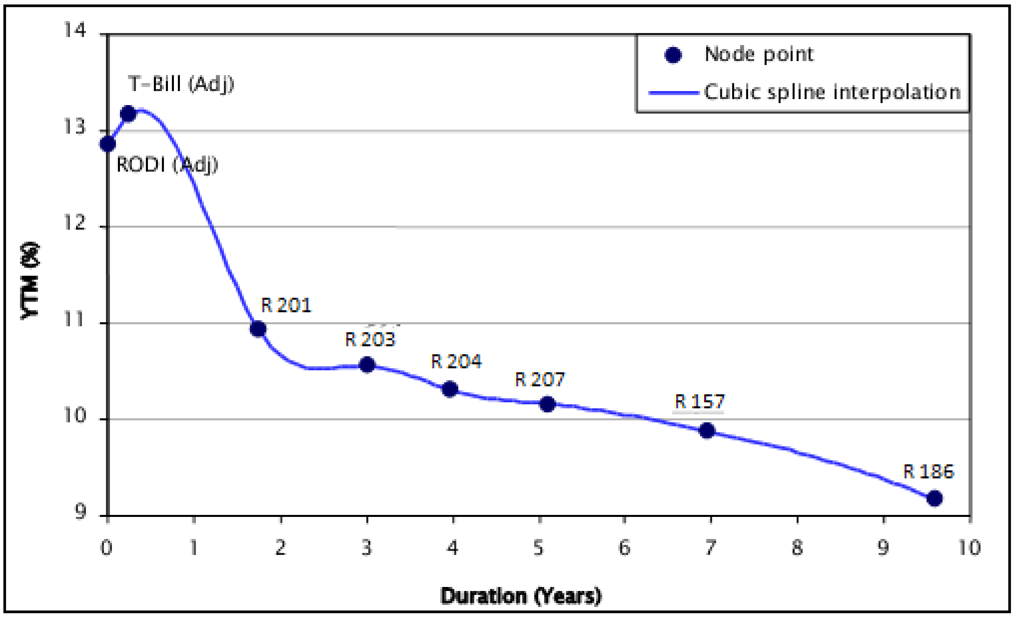
To fit a smooth curve through the node points in the government term structure, a technique called cubic spline interpolation is used (Burden & Faires, 1993:132). Supposing that an appropriate number of evenly spaced node points is available, a cubic polynomial can then be fitted through each pair of node points, starting at the first and ending at the last node. When these polynomials are joined together at the node points, they make up a curve called a cubic spline, which represents a smooth curve that intersects the entire given collection of node point (Weisstein, 2003:4). The cubic spline interpolation method is basically the same method applied by BESA (BESA, 2003:9) in calculating the yield curves published daily. However, since the BESA yield-curve is based upon the ALBI, the benchmark's bonds are clustered along the maturity axis and require a technique called cluster analysis to spread the node points more evenly along it.

By assigning certain constraints to the end-points of the cubic spline, it is possible to alter the curve's shape. The most general cubic spline is the natural cubic spline, which is constructed in such a way that the second derivative at both end points is zero. This results in the ends of the spline being unconstrained so that the free ends are linear. For the yield-curve, it is possible to constrain the fitted cubic spline so that the gradient at the right-hand end is zero. The effect of this constraint on the yield-curve is to flatten the curve at longer maturities. A financial cubic spline denotes a cubic spline that is constrained so that its first derivative at its right-hand end is zero as well as its second derivative at the left-hand end (Adams, 2001:7). These additional constraints will change the shape of the curve slightly compared to the natural cubic spline passing through the same set of nodes, so it will not be not be the smoothest curve to interpolate those points.

In Figure 4.6, the government yield curve is interpolated using a financial cubic spline. Comparing this version of the yield curve with the previous one shown in Figure 4.5, it is clear that using the cubic spline interpolation method results in a curve with a high degree of smoothness.

However, the cubic spline interpolation also introduces two undesirable artefacts between the nodes for the Treasury bill, R201 and R204 bonds. Although the cubic polynomials between the Treasury bill and the R204 nodes represent the smoothest curves, the YTM predicted by these curves most probably under- and overestimate the market-representative yield.

Figure 4.6: The government yield curve constructed from the term structure using cubic spline interpolation

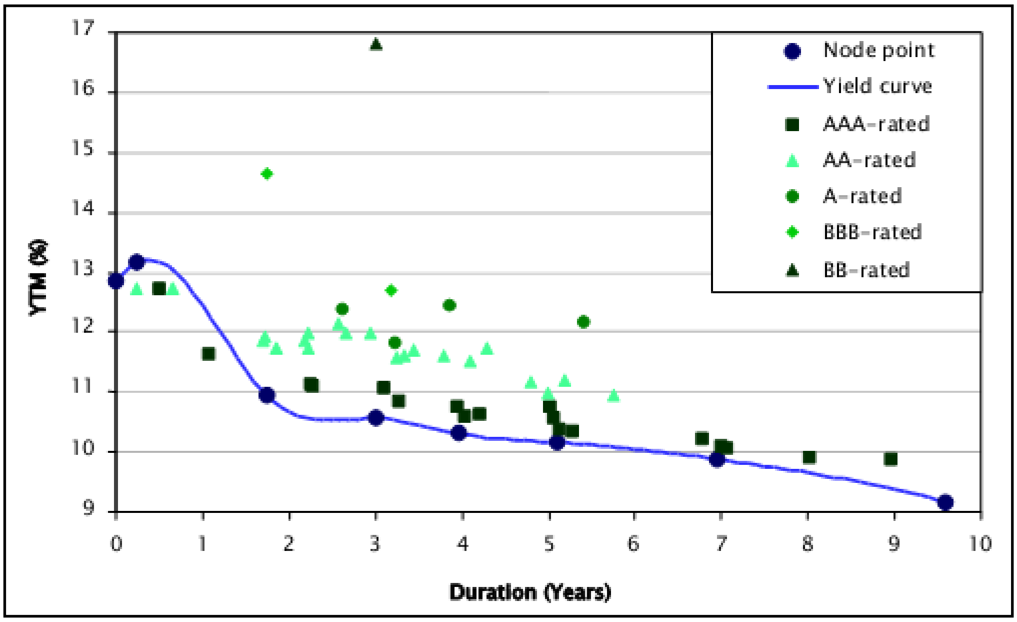


Source: Burger (2012)

4.6.8.3 Comparing the GOVI yield curve

As the government yield curve is supposed to approximate the lowest market rate for any maturity term, an obvious test of the applicability of this curve would be to compare it to the YTM's for non-government rated bonds. All non-government issuers rated by Fitch, Moody or Standard & Poors were included in the selection. The result of this comparison is shown in Figure 4.7, with the bonds colour indexed according to their respective credit ratings. For the mid and long duration end of the curve, the YTM for all the non-government rated bonds are above the yield curve. However at the short end, some bonds actually have YTM's below the yield curve. From an investment perspective this clearly does not make sense as few investors would be willing to accept a yield lower than the risk-free yield, while simultaneously taking on default and/or liquidity risk.

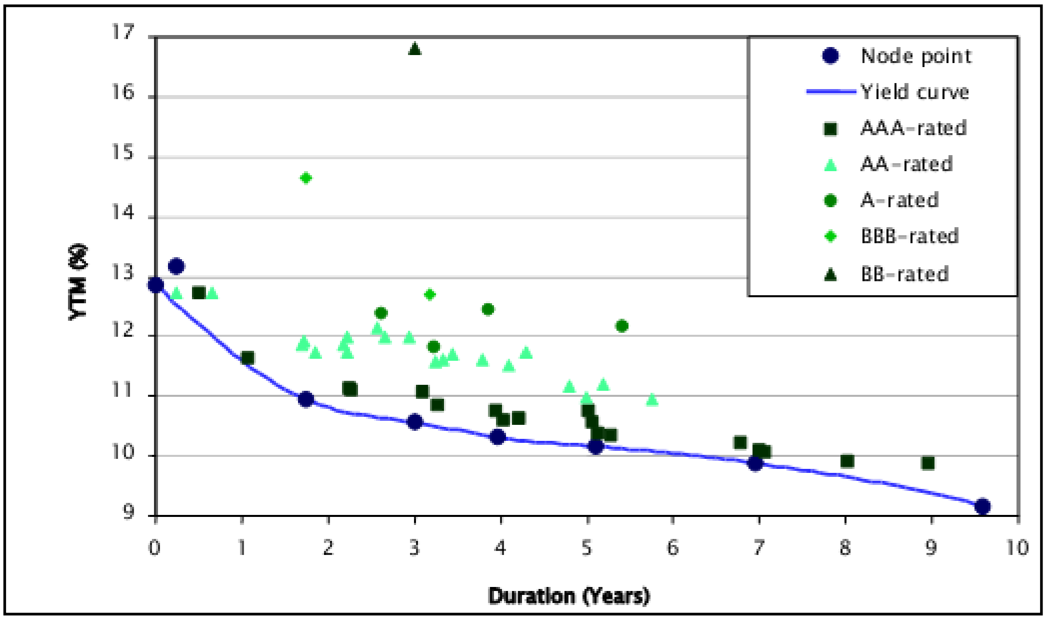
Figure 4.7: The government yield curve compared to the YTM's for a selection of non-government bonds.



Source: Burger (2012)

Ignoring the fact that this effect might be attributable to market-related issues, including liquidity, a better description of the yield curve with regards to the YTM's of the non-government rated bonds may be obtained by simply excluding the 91-day Treasury bill yield from the government term structure. Interpolating this modified term structure using the financial cubic spline interpolation method results in the yield curve shown in Figure 4.8. Having excluded the second node point from the yield curve, the YTM's of all the non-government rated bonds are now above the yield curve as intended. Furthermore, this also eliminates the artefacts pointed out in Figure 4.6 regarding the predicted YTM.

Figure 4.8: The modified yield curve compared to the YTM's for a selection of non-government bonds.



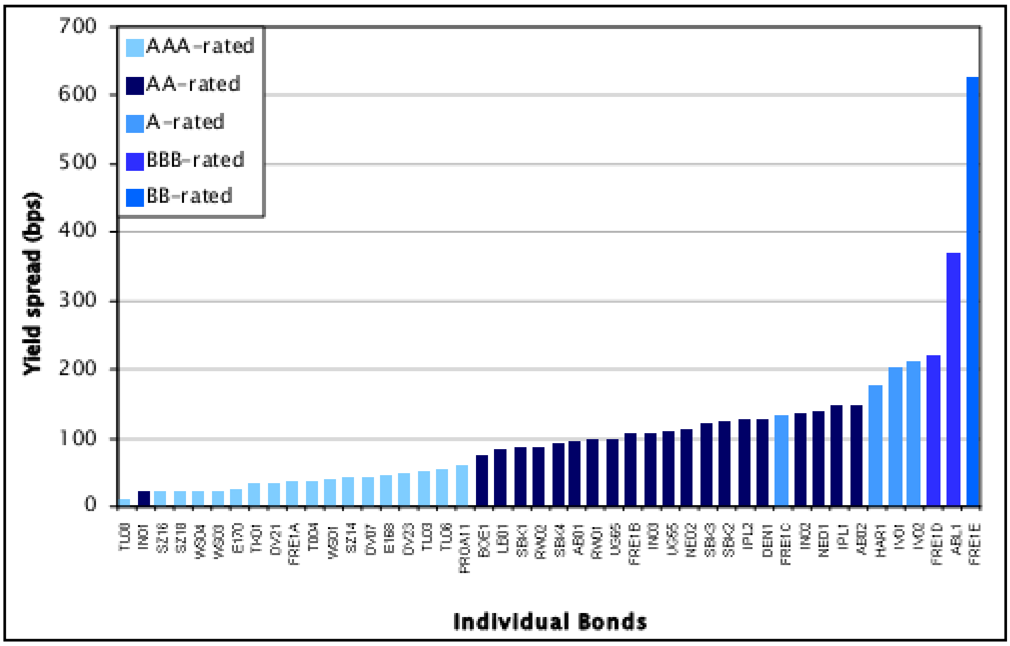
Source: Burger (2012)

4.6.9 Credit Spread Comparison

The width of the credit spread is determined by both the default and liquidity components. Since these rated bonds represent the more liquid portion of the non-government bond market, the credit spread should be largely attributable to default risk with the liquidity risk component being subsequently smaller. Clearly this however also depends on the size of the investor as some foreign pension funds and banks may be prejudice of liquidity, even our most liquid instruments. Therefore, in this case, a direct correlation between the width of the credit spread and the bond's corresponding credit rating should exist.

Having determined an approximate continuous risk-free yield curve, the credit spreads between the non-government rated bonds and the modified GOVI yield curve, shown in Figure4.8, were calculated and ranked in ascending order based on the width of these spreads. This credit spread distribution is shown in Figure 4.9 and is colour indexed according to each bond's particular credit rating. The credit spreads for only two bonds, the INCA and FRESCO securitisations IN01 and FRE1C, do not correlate well with their assigned credit rating. Clearly liquidity issues need to be contemplated as well. The remaining bonds tend to group together according to their respective rating, indicating that a high degree of correlation between the credit spread and credit rating does indeed exist as speculated.

Figure 4.9: The distribution of credit spreads as calculated using the YTM and yield curves as on 1/9/2012



Source: Burger (2012)

4.7 SUMMARY AND CONCLUSIONS

Chapter 4 represents the crux of the dissertation as it presents the analysis that was carried out by the researcher to develop the market-based yield curve. The yield curve was developed by calculating the yields to maturity, using the current market values of the bonds and using an iteration method to arrive at the rate of interests. The yield curve was plotted for the same date as the BESA curve; i.e. 8 April 2011. The difference in the shapes of the two yield curves is apparent on comparison and also the change in the yields in the market-price-based yield curve.

The shape of the yield curve that is presented by BESA is a simple normal curve that shows that short-term maturities are less risky and, therefore, have lower yields than the long-term ones. The upward sloping curve is indicative of rising inflationary pressures in the mid- to long term, which raises the expectations of investors. This curve, therefore, appears to be a strong correlate as well as a predictor of the development and growth in the economy as proposed by the government.

The market-price-based yield curve has a upward slope until the mid-term maturities and then sharply declines, giving an inverse curve. This curve can be interpreted to mean that in the longer term, there is expected to be a slowdown of the economy or even an onset of a recession. The long-term securities are expected to fetch less than the mid- and the short-term ones and this may lead to a large-scale withdrawal of foreign investments from the country, pushing it into a depression as was seen in the Asian economic crisis of the late 1990s. The current study, therefore, has provided insight into how foreign investors may perceive the economic progress of South Africa and has also provided evidence for the financial institutions and the government in South Africa to adjust their assessment of the risk free rate to reflect investors' sentiments better.

The chapter also highlighted some of the reasons for the possible difference in the theoretical risk free rate and the actual one, as perceived by foreign investors. The differences in the yield curves are explained using popular theories such as the pure expectations theory, the liquidity premium theory, the market segmentation theory and the preferred habitat theory.

The pure expectations theory is a basic theory that proposes that all the instruments available in the market are substitutes and that investors expect a rate of return consistent with the holding time of their maturities. However, investors' perception of economic growth and interest rates in the future also affects what instruments they will buy or avoid.

Thus, it is the perception of the future movement in the interest rates that dictates the expected yields to maturity and decides the shape of the yield curve. In addition to the pure expectations theory, the liquidity premium theory also indicates that investors' perception of risks associated with the future of the bonds plays a role in developing their yields' expectations. This theory, as well as the expectations theory, is therefore applied to South Africa to explain that investors in the South African market may perceive the future financial environment to be unstable or the interest rates to fluctuate and, hence, they demand a higher return from the risk free rate as well.

The market segmentation and the preferred habitat theories also dwell upon investors' or the credit ratings agencies' perception of the risks associated with the country or the region where investments are to be made.

It is shown in the final part of this chapter that a yield curve giving the approximate risk free interest rate for any bond duration can be obtained using the GOVI benchmark and the short term RODI rate. Using this yield curve the credit spreads between a selection of non-government rated bonds and the yield curve have been calculated. Assuming that these credit spreads are largely due to the bond's perceived default risk, the credit spreads compared well with the bond's credit rating. This indicates that although the GOVI yield curve is an approximation of the risk-free yield curve, it seems to approximate the expected credit spread well and is a useful tool in quickly analysing the market's perception of a bond's default risk

CHAPTER 5: SUMMARY, CONCLUSIONS & RECOMMENDATIONS

5.1 SUMMARY

A capital market primarily comprises long-term funds and these are in the form of bonds, equity, and bank loans and deposits. For an efficient balance in investment capacity and inflation, it is essential that the three components – bonds, equity and loans – work complementarily to each other and that all three are equally developed. In South Africa this is indeed the case, as the capital market is fully developed and consists of all three means of funding.

This situation places South Africa in a unique position, as – in case of large developmental projects – the bonds are issued and to maintain the liquidity position equity is added in adequate amounts. Bank loans serve as a bridge to additional or temporary needs and, thus, the South African financial market has the opportunity to continue in a smooth manner.

Chapter 1 of this dissertation highlighted the background to and the rationale for believing that there is a difference in the theoretical- and the actual risk free rate. It set out the research objectives as an assessment of the existence of such a difference in the perception of the risk free rate, and exploration of the reasons (if any) for the existence of this difference.

The chapter also set out the importance of the risk free rate and the economic and financial implications that a higher risk free rate may have for South Africa. The chapter outlined the research methodology and the strategy used to conduct the analysis, which comprised developing the yield curve using the market prices and then comparing the market-based yield curve with that of the BESA curve.

Chapter 2 is based on a discussion of the portfolio theory and the role that the risk free rate plays in this theory. The Modern Portfolio Theory (MPT) is discussed in detail in terms of the assumptions and the utility of the theory for market investors. The MPT proposes that a diversified portfolio – consisting of different maturities and different types of assets – is more profitable, as it helps to mitigate risks associated with investing only in one instrument or in instruments that have a close covariance.

While the MPT provides a rationale for why rational investors choose a diverse portfolio, the basic calculations of the risk premiums are based on an assessment of the risk free rate.

It can be seen, therefore, that the MPT is based on the basic premise of a risk free rate of return and uses the same for assessing the risk premium for other riskier instruments. The risk free rate, therefore, lies at the core of the MPT and it plays a crucial role in the investment activity of investors.

The chapter also discusses the Capital Asset Pricing Model (CAPM), which is an extension of the MPT. The CAPM model is based on the premise that investors need to be compensated both for the time value of their money as well as for the risks associated with the loss of investment value in any way. Equation 3 is used to calculated the expected rate or return on any given security.

$$\boldsymbol{R_i} = \boldsymbol{R_f} + \boldsymbol{\beta_i}[\, \boldsymbol{R_m} - \boldsymbol{R_f}] \qquad \textit{(Equation 3)}$$

The CAPM postulates that the risk free rate forms the basis of the time value (without the involvement of risk consideration). To this a risk factor is added in order that the expected rate of return can be arrived at. The risk factor too is dependent on the risk free rate (R_f) as it is calculated by subtracting the expected return on the bond market (R_m) and the risk free rate.

This risk premium (R_m - R_f) is multiplied by a beta coefficient that is calculated for an individual instrument. The expected return on that particular security is therefore a sum of the risk-free return and the risk premium (the calculation of which again involves the risk free rate of return).

The final theory discussed in Chapter 2 is the Arbitrage Pricing theory (APT). This theory's basic premise is the same as that of the CAPM model. The APT uses the risk free rate as the core factor for calculating the expected return for any of the securities under consideration. The APT differs from the CAPM, however, in terms of its assumptions, with the APT stating that the market is arbitrage free.

Further, the APT also shows that the CAPM is simplistic, as it uses only a single beta factor associated with the security to arrive at the expected rate of return. According to the APT, the market complexities cannot be captured in any one factor but a variety of factors. For each factor that is selected by the investor to be important, a risk premium is associated and this risk premium needs to be multiplied by the beta coefficient of the security for that risk factor. The expected return is therefore calculated by adding the risk free rate to the sum of the products of the beta factors and risk premiums for a variety of factors, as depicted in the Equation 6 below:

$$\boldsymbol{R_i} = \boldsymbol{R_f} + \sum[\boldsymbol{B_i}(\boldsymbol{n}) \times \boldsymbol{R_m}(\boldsymbol{n})] \qquad \textit{(Equation 6)}$$

Chapter 2, therefore, establishes the importance of the risk free rate in the calculations of the expected rate of return on market investments. The financial markets operate using one or other of the premises established by the abovementioned theories. For this reason, the role of the risk free rate becomes crucial in determining the market dynamics. It is noted that any variation in the actual risk free rate as perceived by investors has the ability to distort the expected rate of returns by them and affect the demand for securities that are available in the market.

Chapter 3 focuses on the valuation of the bonds and the various indices and indicators that investors employ to develop their expectations of a desirable interest rate for any given bond. The bond valuation is undertaken by calculating the current price of the bond using the par value and the cash flows stream to maturity for that bond, as shown in the following formula:

$$\boldsymbol{P} = \boldsymbol{C}\left(\frac{\boldsymbol{1}-(\boldsymbol{1}+\boldsymbol{i})^{-\boldsymbol{N}}}{\boldsymbol{i}}\right) + \boldsymbol{M}(\boldsymbol{1}+\boldsymbol{i})^{-\boldsymbol{N}} \qquad \textit{(Equation 7)}$$

The present value P is determined by adding the discounted cash flows (C discounted by the market interest rate i) and the Face Value or the Par Value (M) discounted by market interest rate i.

The underlying purpose of bond valuation is to assess if the present value of the bond as per the expected rate of return of investors is attractive enough to warrant an investment in it. As can be seen from the formula shown above, the expected rate of return is the key factor in how the bond is valued by investors. The expected rate of return here again is determined by the relative risk that investors perceive they are incurring in investing in a non-risk-free instrument.

The risk free rate therefore forms the basis of the calculation of the expected rate of return and the evaluation of the present value. The present value, in comparison with the par value, forms the basis of the buying or selling decisions. (If the present value is more than the par value the bond is sold at a premium; if the present value is less than the par value the investor may want to hold on to the investment). The risk free rate is a determining factor in the market demand and dynamics.

This is further proved by the fact that international investors rely on the credit rating agencies that provide sovereign ratings to countries based on the political, legal, economic, social or environmental risks associated with the country. The credit rating agencies take into consideration these risks in providing their assessment of the risk free rate that should be considered as appropriate for the level of country risk. This implies that the risk free rate available in the country is a strong deciding factor for foreign investors.

Another tool that is employed in assessing the value of the bonds is the transition matrix, or a matrix of probable expected cash flows from any given assessment. As it is not possible to generate a future cash flow stream for an investment, investors develop probability scenarios for different expected cash flow returns from a security. These probabilities are in turn dependent on a variety of considerations involving the risks associated with the political, social, economic, and legal environment of a region.

The assessment of this risk settles the risk free rate for the investors and based on their perception of the risk free rate, they tend to develop the cash flow matrix with their expected rate of returns. Here again it can be noted that the perception of the risks is subjective and could be substantially different for internal investors or the government of a country or external credit rating agencies and foreign investors.

Chapter 3 explains the different indices that investors employ in assessing their portfolios and in making a judicious investment. The Sharpe ratio, The Sortino ratio and the Omega ratio are discussed and the importance of the risk free rate in the composition of the ratios is highlighted. The portfolio performance measures explained in Chapter 3 help to gauge an understanding of these models and the importance of the risk-free role that is inherent in them. These ratios are used as effective tools by financial investors, especially those delving in fund management and portfolio management. Hence, once again the importance of the risk free rate for the dynamics of the financial market is emphasised.

In addition, the chapter also highlights the Internal Rate of Return (IRR) and the Weighted Average Cost of Capital (WACC) that are corporate indices used by alternate investment companies, venture capitalists and companies moving ahead with new projects and investments. WACC and IRR are two basic and most important tools used to calculate the viability of projects and the time taken for the investments to break even and start generating returns for the company. Both these factors have the risk free rate of return at their core and in this way establish the crucial role that the risk free rate plays in the development of a country’s financial market.

Chapter 4 represents the crux of the dissertation as it presents the analysis that was carried out by the researcher to develop the market-based yield curve. The yield curve was developed by calculating the yields to maturity, using the current market values of the bonds and using an iteration method to arrive at the rate of interests. The yield curve was plotted for the same date as the BESA curve; i.e. 1 September 2012. The difference in the shapes of the two yield curves is apparent on comparison and also the change in the yields in the market-price-based yield curve.

The shape of the yield curve that is presented by BESA is a simple normal curve that shows that short-term maturities are less risky and, therefore, have lower yields than the long-term ones. The upward sloping curve is indicative of rising inflationary pressures in the mid- to long term, which raises the expectations of investors. This curve, therefore, appears to be a strong correlate as well as a predictor of the development and growth in the economy as proposed by the government.

The market-price-based yield curve has a upward slope until the mid-term maturities and then sharply declines, giving an inverse curve. This curve can be interpreted to mean that in the longer term, there is expected to be a slowdown of the economy or even an onset of a recession. The long-term securities are expected to fetch less than the mid- and the short-term ones and this may lead to a large-scale withdrawal of foreign investments from the country, pushing it into a depression as was seen in the Asian economic crisis of the late 1990s. The current study, therefore, has provided insight into how foreign investors may perceive the economic progress of South Africa and has also provided evidence for the financial institutions and the government in South Africa to adjust their assessment of the risk free rate to reflect investors' sentiments better.

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The market segmentation and the preferred habitat theories also dwell upon investors’ or the credit ratings agencies’ perception of the risks associated with the country or the region where investments are to be made.

Chapter 4 also discusses some of the specific risks – such as the political economic stability, currency and foreign exchange rates, legislation changes, social unrest, and environmental hazards – that foreign investors may associate with South Africa and that may not figure strongly in the BESA methodology.

It is shown in the final part of Chapter 4 that a yield curve giving the approximate risk free interest rate for any bond duration can be obtained using the GOVI benchmark and the short term RODI rate. Using this yield curve the credit spreads between a selection of non-government rated bonds and the yield curve have been calculated. Assuming that these credit spreads are largely due to the bond's perceived default risk, the credit spreads compared well with the bond's credit rating. This indicates that although the GOVI yield curve is an approximation of the risk-free yield curve, it seems to approximate the expected credit spread well and is a useful tool in quickly analysing the market's perception of a bond's default risk

Chapter 5 presents a summary of the content of the chapters and also sets out the main conclusions that are drawn from the current research. These conclusions are presented in the next section.

5.2 CONCLUSIONS

The risk free rate of return simply means the rate of return on an investment with zero or no risk involved over a period of time. In other words, it is the minimum return expected by investors for their investment, as accepting additional risk is always linked to an increased rate of return over and above the risk free rate. The risk free rate acts almost as a benchmark for other investment options to perform and generate higher returns and lure investors with these greater returns. In practice, there is no such thing as risk-free returns or investment-yielding returns without risk, as there is always a small element of risk attached to each investment option. Even then, in general, the rate of treasury bills can be considered as risk free rates

(Kaplan & Knowles, 2004:14). This rate also signifies to an investor that the return is inflation adjusted with low volatility and guaranteed for a particular time period.

However, in the case of foreign investment, as in the South African capital market, the perceptions of investors about the risk associated with the country's government may be different from what internal investors or the government itself perceives. This translates into a higher expectation of the returns on the risk-free government bonds and, hence, an overall higher expectation of the interest rates for all the instruments. The investor yield curve, therefore, reflects a higher perceived risk free rate than the one developed by BESA.

The main conclusion from the current research is therefore:

5.2.1 The risk free rate as perceived by the foreign investors is higher than the theoretical risk free rate in the South African market

A further aim of the research was to ascertain the reasons behind the difference in the theoretical risk free rate and the perceived risk free rate. A literature review enabled the researcher to arrive at the plausible reason for this discrepancy. For an investor or a financial analyst, it is necessary to consider certain qualitative factors apart from the tools (like the Sortino, the Sharpe, the Omega, IRR or WACC) used for evaluating portfolios or investment opportunities. These factors include primarily the management's reputation, the growth prospects of the country, the economic model and its scalability, corporate governance, political and economic stability, default risk, currency movements, financial legislation, and social or environmental risks. The qualitative and quantitative factors taken together are used to provide a true and reliable understanding of any business and better insights into investments in that business.

Foreign investors, therefore, may perceive the abovementioned factors as higher than the South African government perceives them while presuming the risk free rate.

5.2.2 The difference in the perceptions of the risks associated with investment in South Africa

The next research aim was to highlight the relevance of the risk free rate by providing evidence of it being an integral aspect of all financial assessments. This was undertaken in a discussion of the portfolio theories (the Modern Portfolio Theory, the Capital Asset Pricing Model, the Arbitrage Pricing Theory) and performance measures like (the Sortino, the Sharpe and the Omega) and by highlighting the crucial role played by the risk free rate. It was established that: The risk free rate forms the crux of all investment decisions and, therefore, has the potential to affect the dynamics of financial markets.

5.2.3 Implications of the differences between the actual and theoretical risk free rate

The final research aim was to explore the implications of the risk free rate being higher than the theoretical risk free rate. It was seen that, in the case of South African financial market, foreign investor sentiments are crucial for the future development and growth of the nation, owing to the large emphasis that the government is placing on foreign investment. In the case where investors perceive the risk free rate to be higher, they will expect higher returns from all the instruments available in the market (as the risk free rate is the core factor on which expected rate of return is calculated). In addition, investors will expect a higher return from government security bonds. In the event that they perceive that the rate of return is not going to be on a par with their expectations, they may decide against investment or withdraw from the market, which could bring the country into a state of economic slowdown or even a recession in the coming future. This is the trend that is being highlighted by the market-based yield curve.

Investors might also perceive that in the longer run the rate of returns will fall and hence they may be prepared to sell their long-term securities on a large scale in the coming years, which situation would also lead to an economic slowdown.

The government of South Africa needs to address foreign investors' perceptions of and concerns about risks associated with investment in the country, if it desires a sustainable long-term growth for the nation.

5.3 RECOMMENDATIONS FOR FUTURE RESEARCH

The research has highlighted the crucial role that the risk free rate plays in dictating the demand for the bonds and securities in any given market and, hence, its impact on expected interest rates. The research has also provided evidence about the existence of the difference between the theoretical risk free rate and the market-based risk free rate as perceived by foreign investors. However, the research employed the conventional discounted cash flow formulas to arrive at the yield to maturities for each of the government issued bonds as used in the BESA curve.

The scope exists to use more sophisticated methods or to calculate the yields using different methods so that a more thorough conclusion about the difference in perceptions of the risk free rate can be obtained. In addition, the research also suffered from the limitation that there was no background information available on how BESA actually calculates the yield curve.

It was noted in the research that the BESA method is complicated and that it has been developed on the basis of years of research. However, in the absence of a full understanding of the factors and the probabilities that BESA uses in determining the risk free rates, it was impossible to make a substantial comparison about the reasons for the differences in the risk free rate.

The research was able to present a set of probable factors that could be the reason that foreign investors may judge the South African risk free rate as different from the theoretical one. There is scope to conduct research that explores BESA methodology and then compare it with the specific strategies that foreign investors may be employing to assess the risk free rate. In addition, there is also scope for primary research via interviews with foreign investors or international credit rating industries so that a better understanding of the perceptions of foreign investors can be obtained.

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