Chapter 2

Pulsars and their Rotation Powered Nebulae

The two main aspects that this study focuses on are the morphological evolution of a composite supernova remnant in a homogeneous interstellar medium, and the evolution of the particle energy spectrum in a pulsar wind nebula. This chapter introduces the background required for the subsequent research chapters, and is divided into three sections that respectively focus on supernova remnants, pulsars, and pulsar wind nebulae. One important aspect that is discussed in the section on pulsars is the structure of a magnetic field frozen into a wind that originates from a rotating, magnetic dipole. Special emphasis is placed on this last topic as this has implications for the rest of the study, specifically Chapter 6.

As this chapter only touches on aspects of supernova remnants, pulsars, and pulsar wind nebulae, the following publications are recommended if a more detailed introduction to these topics is required: the review articles of, e.g., Woltjer (1972), Chevalier (1977) and McKee and Truelove (1995) provide a good overview of supernova remnants, while the textbooks of, e.g., Lorimer and Kramer (2005) and Lyne and Graham-Smith (2006) discuss many essential pulsar topics. Lastly, the widely-cited review article of Gaensler and Slane (2006) can be consulted for an introduction to pulsar wind nebulae. Additionally, the review articles of Kargaltsev and Pavlov (2008), Kargaltsev et al. (2012), Kargaltsev et al. (2013), and De Jager and Djannati-Ataï (2009) provide an overview of pulsar wind nebulae based on X-ray and very high energy gamma-ray observations.

2.1 Supernova remnants

Supernova explosions are some of the most violent events occurring in the Universe, releasing large amounts of energy into the surrounding interstellar medium. Two types of supernovae can be identified, depending on the mechanism responsible for the explosion. Thermonuclear supernovae (Type Ia) are either the result of an accretion of matter by a white dwarf from a companion star or the result of two merging white dwarfs (Schaefer and Pagnotta, 2012), while
core-collapse supernovae (Type Ib, Ic, II) are due to the gravitational collapse of a massive star. As neutron stars are not the by-product of thermonuclear supernovae, pulsar wind nebulae are not associated with this type (see, e.g., Vink, 2012). The discussion in this section will therefore focus exclusively on core-collapse supernovae.

2.1. Core-collapse supernovae

A detailed introduction on core-collapse supernova remnants can be found in the review paper of Woosley and Janka (2005). According to their description, a star with a mass \( M \gtrsim 8 M_\odot \) will go through successive stages of hydrogen, helium, carbon, neon, oxygen and silicon fusion in its centre, supplying a sufficient amount of thermal pressure to counteract the inward gravitational pressure. After the star has passed through the previous stages, a core rich in iron will be formed. At this stage iron fusion no longer supplies a net amount of energy, and the star will begin to collapse due to the reduction in thermal pressure.

As the star collapses, the electrons responsible for most of the thermal pressure are squeezed into the iron nuclei, while the radiation photo-disintegrates a fraction of the iron nuclei into helium. Both processes extract thermal energy from the star, thereby accelerating the gravitational collapse of the star. The core of the star collapses to a hot, dense, neutron rich proto-neutron star with a radius of \( \sim 30 \) km, with the short-range nuclear forces eventually halting the collapse. In the course of a few seconds, the proto-neutron star will radiate \( \sim 10^{53} \) erg of energy in the form of neutrinos, forming a neutron star with a \( \sim 10 \) km radius.

Woosley and Janka (2005) also note that the actual mechanism responsible for the observed optically luminous explosions is not well understood. It is believed that energy deposited by neutrinos into the stellar matter surrounding the proto-neutron star drives the inflation of a bubble consisting of radiation and electron-positron pairs. The outer boundary of the bubble becomes a shock wave that ejects the stellar material in the observed explosion, depositing \( \sim 10^{51} \) erg of kinetic energy into the outward moving matter. The expansion of the stellar ejecta into the surrounding interstellar medium (ISM) is supersonic, driving a forward shock that accelerates, compresses and heats the ambient matter (see, e.g., Truelove and McKee, 1999). The accelerated matter collects in a thin shell behind the shock, forming the well-known shell-type supernova remnant (SNR).

2.1.2 The reverse shock

The matter swept up by the forward shock expands supersonically into the ejecta, leading to the formation of a reverse shock that compresses, heats, and decelerates the ejecta, with the colder, higher density shocked ejecta separated from the hotter, lower density shocked ISM by a contact discontinuity (see, e.g., Truelove and McKee, 1999). As a result of the adiabatic losses suffered by the ejecta during the expansion of the remnant, the pressure inside the shell
decreases with time (see, e.g., Chevalier, 1977). When the pressure in the remnant becomes smaller than the pressure behind the forward shock, the reverse shock is driven back into the ejecta (McKee, 1974).

The time needed for the reverse shock to return to the centre of the shell has been derived by Ferreira and de Jager (2008) as

\[
t_{\text{rev}} = 4 \times 10^3 \left( \frac{\rho_{\text{ism}}}{10^{-24} \text{ g cm}^{-3}} \right)^{-1/3} \left( \frac{E_{\text{snr}}}{10^{54} \text{ erg}} \right)^{-45/100} \left( \frac{M_{\text{ej}}}{3 \text{ } M_{\odot}} \right)^{3/4} \left( \frac{\gamma_{\text{ej}}}{5/3} \right)^{-3/2} \text{ yr}, \tag{2.1}
\]

where \( \rho_{\text{ism}} \) is the density of the ISM and \( E_{\text{snr}} \) the kinetic energy released during the SNR explosion, while \( M_{\text{ej}} \) and \( \gamma_{\text{ej}} \) are respectively the mass and adiabatic index of the ejecta. A very similar expression has also been derived by Reynolds and Chevalier (1984). Using the fiducial values \( E_{\text{ej}} = 10^{51} \text{ erg} \) and \( \gamma = 1.67 \), together with \( M_{\text{ej}} = 5 \text{ } M_{\odot} \) and \( \rho_{\text{ism}} = 10^{-24} \text{ g cm}^{-3} \), leads to the estimate \( t_{\text{rs}} \approx 6000 \text{ yr} \). Using a smaller value for the ISM density, \( \rho_{\text{ism}} = 10^{-25} \text{ g cm}^{-3} \), increases the reverse shock time scale to \( t_{\text{rs}} \approx 11\ 000 \text{ yr} \).

2.1.3 Evolutionary phases of a core-collapse supernova remnant

According to the review papers of, e.g., Woltjer (1972) and McKee and Truelove (1995), the evolution of a shell-type SNR can be broadly divided into four phases:

- **Free expansion** phase: \( t \sim 10^2 - 10^3 \text{ yr} \) - \( M_{\text{ej}} \) is larger than the swept-up mass \( M_{\text{sw}} \) and the dynamics of the system are determined by the explosive process. The velocity of the forward-shock is determined by the requirement that the total kinetic energy of the ejecta \( E_{\text{snr}} \) is equal to the mechanical energy of the SNR.

- **Sedov-Taylor** phase: \( t \sim 10^3 - 10^4 \text{ yr} \) - the remnant is dominated by \( M_{\text{sw}} \) while radiative losses remain unimportant. If \( E_{\text{snr}} \) is injected instantaneously into the surrounding ISM, then the radius and velocity of the forward shock evolve in a self-similar fashion (see, e.g., Taylor, 1950).

- **Pressure-driven** phase: \( t \sim 10^4 - 10^5 \text{ yr} \) - radiative cooling becomes important and the swept-up shell collapses to a thin, dense layer, while the material in the interior of the SNR continues to expand adiabatically. The expansion of the SNR is no longer governed by the conservation of energy, but rather by the conservation of momentum. Blondin et al. (1998) estimated that the pressure-driven stage should start at a time

\[
t_{\text{rad}} = 2.9 \times 10^4 \left( \frac{E_{\text{ej}}}{10^{51} \text{ erg}} \right)^{4/17} \left( \frac{\rho_{\text{ism}}}{10^{-24} \text{ g cm}^{-3}} \right)^{-9/17} \text{ yr}, \tag{2.2}
\]

- **Merging** phase: \( t \gtrsim 10^5 \text{ yr} \) - the expansion velocity of the remnant and temperature behind the shock become comparable to the random motions and temperature of the ISM, with the SNR eventually merging with the ISM.
2.2 Pulsars

Pulsars are rapidly rotating neutron stars with strong dipole magnetic fields. The magnetic field consists of open and closed field lines, with the outermost closed field line defining the light cylinder of the pulsar. Any (hypothetical) particle co-rotating with the last closed field line will have a velocity equal to the speed of light. Electromagnetic radiation from pulsars is observed in the form of two conical beams, with distant observers who happen to lie on the path of these ‘lighthouse’ beams detecting a pulse every time a beam sweeps past them (see, e.g., Lorimer and Kramer, 2005). Although it is generally believed that the radio and X-ray beams are directed along the magnetic axis, recent observations indicate that this is not necessarily the case for the γ-ray emission (see, e.g., Venter et al., 2012). A simplified pulsar model is illustrated in Figure 2.1.

\[ L \equiv -\frac{dE_{\text{rot}}}{dt} = -\frac{d(I\Omega^2/2)}{dt} = -I\Omega\dot{\Omega} = \frac{4\pi^2 I \dot{P} P^{-3}}{3}, \]  

(2.3)

Figure 2.1: A simplified model (not drawn to scale) of a pulsar showing the open and closed magnetic field lines, light cylinder, and various axes (Lorimer and Kramer, 2005).

2.2.1 Energy loss rate

Shortly after the discovery of the first pulsar, it was observed that the rotational periods of pulsars \( P \) increases with time (see, e.g., Richards and Comella, 1969). The rate of increase, \( \dot{P} = dP/dt \), can be related to the loss rate of rotational kinetic energy (see, e.g., Lorimer and Kramer, 2005)
where $\Omega = 2\pi/P$ is the angular frequency and $I$ the moment of inertia. The quantity $L$ (often denoted by $E$) is referred to as the spin-down luminosity of the pulsar. The largest part of the spin-down power is carried away from the pulsar in the form of a pulsar wind, with a small fraction $\eta_{\text{rad}}$ of the spin-down power converted into pulsed emission. The exact value of $\eta_{\text{rad}}$ is difficult to determine, but the results from the first Fermi-LAT pulsar catalogue (Abdo et al., 2010) suggest that $\eta_{\text{rad}} \sim 1\% - 10\%$, with $\eta_{\text{rad}} \sim 1\%$ estimated for the well-known Crab pulsar.

The spin-down luminosity evolves as (see, e.g., Pacini and Salvati, 1973)

$$L(t) = L_0 \left(1 + \frac{t}{\tau}\right)^{-(n+1)/(n-1)},$$

where $L_0$ is the initial luminosity at the pulsar’s birth and

$$n = \frac{\Omega \ddot{\Omega}}{\dot{\Omega}^2}$$

is the pulsar’s braking index (see, e.g., Lorimer and Kramer, 2005). Measured values range from $n = 1.4 \pm 0.2$ for the Vela pulsar (Lyne et al., 1996) to $n = 2.91 \pm 0.05$ for PSR 1119-6127 (Camilo et al., 2000). For modelling purposes it is often assumed that the pulsar is a pure dipole radiator with $n = 3$ (see, e.g., Blondin et al., 2001). The variable

$$\tau = \frac{4\pi^2 I}{(n-1)P_0^2 L_0},$$

is known as the spin-down time scale of the pulsar, with $P_0$ the spin period at the birth of the pulsar. Migliazzo et al. (2002) estimated the birth period for a number of pulsars, finding that $P_0 = 14 - 90$ ms, while Faucher-Giguère and Kaspi (2006a) estimated that the birth periods for a number of Galactic pulsars range between $P_0 = 11 - 139$ ms. The latter authors also noted that spin periods of $P_0 \sim 50 - 150$ ms may be the general rule.

Using the rough approximation that $L(t)$ is constant when $t < \tau$, the spin-down energy loss of a pulsar with an initial luminosity of $L_0 = 10^{38}$ erg/s and a spin-down time scale of $\tau = 3000$ yr over a $t = 3000$ yr interval will be $\sim 10^{49}$ erg, or about 1% of the kinetic energy released by the supernova explosion. Using $n = 3$, the evolution of the luminosity can be approximated as $L(t) \propto t^{-2}$ for subsequent times $t > \tau$. During this phase the amount of energy lost by the pulsar rapidly declines with time. The amount of energy lost by the pulsar over its lifetime is therefore insufficient to influence the evolution of the SNR.

### 2.2.2 The pulsar wind and frozen-in magnetic field

Current theory states that the rotation of a magnetised neutron star induces an extremely strong electric field, which is capable of extracting charged particles from the pulsar’s surface. The electric field accelerates the extracted particles to relativistic energies, with the accelerated particles in turn initiating an electron-positron cascade, filling the magnetosphere with plasma...
Although the exact mechanism driving the process is not fully understood, the plasma escapes along the open magnetic field lines of the pulsar in the form of a relativistic supersonic wind, with the flow velocity $V \approx c$ (see, e.g., Arons, 2004).

It is generally believed that the “force-free” condition $\mathbf{E} + \mathbf{V} \times \mathbf{B} = 0$ holds in the pulsar’s magnetosphere (see, e.g., Bogovalov, 1999; Spitkovsky, 2006), leading to the conclusion that the pulsar’s magnetic field is frozen into the out-flowing plasma wind. As a result of the rotation of the pulsar, this frozen-in magnetic field is wound into an Archimedean spiral (see, e.g., Bogovalov, 1999), as illustrated schematically in Figure 2.2. As the pulsar has an extremely large angular velocity, the spirals are wound up so tightly that the magnetic field beyond the light cylinder can effectively be approximated as purely azimuthal (see, e.g., Arons, 2004; Kirk et al., 2009).

Figure 2.2: A magnetic field frozen into a plasma wind that originates from a rotating, magnetized object leads to an Archimedean spiral structure, or Parker spiral in the parlance of heliospheric physics. The illustration is taken from Hattingh (1998).

The structure of a magnetic field frozen into a plasma wind originating from a rotating, magnetized body has been derived by a number of authors, including Parker (1958) and Michel (1973, 1974). As the structure of the magnetic field plays a role in various parts of this study, especially in Chapter 6, a derivation is provided below, with the presentation following that of Steenkamp (1995).

In an axisymmetric system ($\partial / \partial \phi = 0$) with the rotation $\Omega$ and magnetic $\mu$ axes aligned, the $r$ and $\theta$-components of the ideal MHD limit in the steady-state

$$\nabla \times \mathbf{V} \times \mathbf{B} = 0$$

(2.7)
are
\[ V_r B_\theta - V_\theta B_r = \frac{f_1(r)}{\sin \theta} \] (2.8a)
and
\[ V_r B_\theta - V_\theta B_r = \frac{f_2(\theta)}{r}, \] (2.8b)
where \( f_1 \) and \( f_2 \) are arbitrary functions, \( V \) the wind velocity, and \( B \) the magnetic field. As the two equations in (2.8) are simultaneously valid, it follows that
\[ V_r B_\theta - V_\theta B_r = \frac{C}{r \sin \theta}, \] (2.9)
for some constant \( C \). Assuming that \( V \) has no \( \theta \)-component, while imposing the restriction \( \nabla \cdot B = 0 \), simplifies (2.9) to
\[ \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 B_r) = -\frac{1}{\sin \theta} \frac{\partial}{\partial \theta} (r \sin \theta B_\theta) \]
\[ = -\frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \left( \frac{C}{V_r} \right). \] (2.10)
Under the additional assumption that \( V_r \) is independent of \( \theta \), the radial component of the magnetic field follows from the solution of (2.10)
\[ B_r = B_{rs} \left( \frac{r_s}{r} \right)^2, \] (2.11)
where the symbol * denotes the rotating, magnetised body. An expression for \( B_\phi \) can be found by expanding the corresponding component of the MHD limit. With the boundary condition \( V_{rs} = 0 \) imposed, the \( \phi \)-component of (2.7) is
\[ r (V_\phi B_r - V_r B_\phi) = r_s V_{\phi s} B_{rs}. \] (2.12)
From the conservation of angular momentum density, \( L = r \times \rho V \), it follows that
\[ r V_\phi = r_s V_{\phi s}, \] (2.13)
where \( V_{\phi s} = \Omega r_s \sin \theta \) is the co-rotation velocity on the source surface. The azimuthal component of the magnetic field in (2.12) can therefore be written as
\[ B_\phi = -B_{rs} \left( \frac{r_s}{r} \right)^2 \tan \psi \left[ 1 - \left( \frac{r_s}{r} \right)^2 \right], \] (2.14)
with
\[ \tan \psi = \frac{\Omega r \sin \theta}{V_r}, \] (2.15)
where \( \psi \) is the angle between the magnetic field and the radial direction, also known as the spiral or garden-hose angle. The last term in (2.14) becomes insignificant beyond a few source radii, and the magnetic field can be written as
\[ B = B_s \left( \frac{r_s}{r} \right)^2 (e_r - \tan \psi e_\phi), \] (2.16)
with its magnitude given by
\[ B = B_s \left( \frac{r_s}{r} \right)^2 \frac{1}{\cos \psi}. \] (2.17)
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If \( V_r = V_0 \) is a constant, as is the case for a pulsar wind, then \( B_\phi \propto 1/r \). On the other hand, the velocity \( V_r \propto 1/r^2 \) leads to the scaling \( B_\phi \propto r \). As \( B_r \propto 1/r^2 \) for both these scenarios, the radial component is only important at the poles where \( B_\phi \) disappears. Furthermore, when \( \Omega \) becomes very large, \( \psi \to \pi/2 \), leading to a magnetic field that can be regarded as purely azimuthal.

Although (2.16) is qualitatively similar to the results of Michel (1973), the above derivation, as well as the one presented by Michel (1973), is limited by the assumption that \( \Omega \) and \( \mu \) are aligned. However, a more recent derivation has been presented by Bogovalov (1999) for an oblique rotator, where the same equations were derived as those found by Michel (1973).

2.2.3 The neutral sheet

Since pulsars have a dipole magnetic field, the magnetic polarity must reverse between the northern and southern hemispheres. As discussed in, e.g., the review article of Kirk et al. (2009), this implies that the frozen-in magnetic field must also have two regions of opposing polarity, with these regions separated by a magnetically-neutral sheet. For an aligned rotator, this neutral sheet is a flat plane in the equator. On the other hand, if the magnetic axis \( \mu \) is inclined by an angle \( \alpha \) with respect to the rotational axis \( \Omega \), the sheet will obtain a wave-like structure. In the parlance of pulsar physics, this wavy neutral sheet is generally referred to as the striped wind, a term possibly first introduced by Coroniti (1990). This structure is in essence the same as the neutral sheet that is observed in the heliosphere.

With the directional cosine defined as

\[
A \equiv \cos \alpha = \frac{\Omega \cdot \mu}{|\Omega \mu|},
\]

Jokipii and Levy (1977) and Jokipii and Thomas (1981) have shown that the amplitude of the neutral sheet oscillations can be described by

\[
\theta_{ns} = \frac{\pi}{2} + \sin^{-1} \left[ \sin \alpha \sin \left( \phi - \phi_* + \frac{\Omega r}{V_r} \right) \right],
\]

where \( \theta_{ns} \) is the angular position of the wavy neutral sheet for a given radius, and \( \phi_* = r_* \Omega / V \) is an arbitrary phase constant. Figure 2.3 shows how \( \theta_{ns} \) is influenced by different values of \( \alpha \), as well as different wind profiles. As \( \alpha \) increases, the amplitude of \( \theta_{ns} \) also increases. When \( V_r = V_0 \), the spatial separation between the oscillation remains constant, while decreasing as a function of distance when \( V_r \propto 1/r^2 \).

Jokipii and Levy (1977) have further shown that with the neutral sheet taken into account, the magnetic field (2.16) is modified to

\[
B = B_* \left( \frac{r_*}{r} \right)^2 (e_r - \tan \psi e_\phi) \left[ 1 - 2H(\theta - \theta_{ns}) \right],
\]

with \( H \) representing the Heaviside step function.
2.3 Pulsar wind nebulae

For a completely isolated pulsar, the relativistic wind would extend to arbitrarily large distances. However, pulsars are surrounded by supernova ejecta, and a characteristic radius $r_{ts}$ should exist where the ram pressure of the wind is equal to the pressure of the confining medium. This leads to the formation of a termination shock at $r_{ts}$ (Rees and Gunn, 1974) which can accelerate the leptons in the wind (see, e.g., Kennel and Coroniti, 1984a). Results from particle-in-cell simulations suggest that the accelerated energy spectrum has the form of a relativistic Maxwellian with a non-thermal power law tail (Spitkovsky, 2008; Sironi and Spitkovsky, 2011).

Downstream of the termination shock the leptons of the pulsar wind interact with the frozen-in magnetic field, leading to the emission of synchrotron radiation ranging from radio to X-rays. Additionally, the leptons can also inverse Compton scatter the cosmic microwave background radiation (CMBR), infra-red photons from dust, starlight, and even synchrotron photons, to GeV-TeV energies (see, e.g., Gaensler and Slane, 2006; Kargaltsev and Pavlov, 2008; De Jager and Djannati-Ataï, 2009). This non-thermal emission creates a luminous nebula around the pulsar, referred to as a pulsar wind nebula (PWN).

Figure 2.4 illustrates the different regions of a PWN system, as well as the non-thermal emission typically associated with every region. The first region contains the central pulsar, the second region, observed as under-luminous, extends from the light cylinder to the termination shock, while the last region forms the PWN.

2.3.1 Characteristics of a pulsar wind nebula

In one of the initial reviews, Weiler and Panagia (1978) defined a PWN as having the following, defining characteristics:
- The nebula exhibits a filled morphology, in contrast to the shell structures of the supernova remnants (see Section 2.1). This quality is observed at all wavelengths, and is the result of the continual injection of energy into the PWN by the central pulsar.

- Polarisation measurements show that the nebula has a structured magnetic field.

- The observed radio synchrotron spectrum is unusually hard. If $N_e$ is the particle number density, then the particle spectrum producing the radio emission is described by $N_e \propto E^{-\alpha}$, with $\alpha = 1 - 1.6$.

Based on more recent X-ray observations, De Jager and Djannati-Ataï (2009) expanded the list of characteristics to include the following:

- A number of PWNe have a torus and jet near the pulsar. The direction of the jet is aligned with the pulsar’s rotation axis, while the torus displays an under-luminous region inside
a radius \( r_{ts} \approx 0.03 - 0.3 \) pc, believed to signify the position of the termination shock (see, e.g., Ng and Romani, 2004).

- X-ray observations show that a part of the particle spectrum accelerated at the termination shock is described by a power law \( N_e \propto E^{-\alpha} \), with \( \alpha \sim 2 - 3 \) (see, e.g., Bocchino et al., 2005; Mangano et al., 2005; Schöck et al., 2010).

- Observations show evidence for synchrotron cooling at distances \( r > r_{ts} \), leading to softer particle spectra with \( \alpha = 3 - 4 \) (see, e.g., Bocchino et al., 2005; Mangano et al., 2005; Schöck et al., 2010). The synchrotron cooling also leads to the size of the PWN decreasing with increasing X-ray energy.

Taking VHE gamma-ray observations into account, the list of characteristics can be expanded even further (see, e.g., De Jager and Venter, 2005):

- The pulsar wind magnetisation parameter \( \sigma \) is less than unity for a PWN. For the Crab Nebula, Kennel and Coroniti (1984b) calculated a value of \( \sigma \approx 0.003 \), while Bogovalov et al. (2005) estimated that \( 0.01 \leq \sigma \leq 0.1 \) for the Vela PWN.

- The rapid expansion of a PWN in the first few thousand years can lead to a relatively weak magnetic field. This weak field will allow for the survival of VHE gamma-ray producing electrons from the earliest pulsar epochs. If the magnetic field drops below a few \( \mu \)G, it can in principle lead to a source that is undetectable at synchrotron energies, while remaining bright at TeV energies\(^1\). This scenario can possibly explain a number of unidentified TeV sources observed by H.E.S.S. (Aharonian et al., 2008; De Jager, 2008).

- If the radiation lifetime of the particles is comparable to or longer than the age of the system, adiabatic losses will be the main energy loss mechanism. The size of the PWN, when observed through VHE gamma-rays, will then be independent of energy.

### 2.3.2 Pulsar wind nebula evolution

In Section 2.2.1 it was shown that the evolution of the pulsar’s spin-down luminosity, and by implication the evolution of the PWN, does not significantly influence the dynamics of the SNR shell. However, the dynamics of the PWN are significantly influenced by the reverse shock of the SNR:

- In the initial evolutionary phase the pulsar injects energy at a constant rate into the nebula, resulting in the PWN expanding supersonically into the slow-moving stellar ejecta (see, e.g., Van der Swaluw et al., 2001b; Gaensler and Slane, 2006). Theoretical models predict that the expansion of the PWN can be described by

\[
R_{\text{pwn}} \propto t^\beta,
\]

\(^1\)This scenario requires that the interaction between the reverse shock and the PWN does not lead to a significant increase in the nebular magnetic field. This topic is investigated and discussed in more detail in the next chapter, specifically Section 3.3.3.
where $R_{pwn}$ is the outer boundary of the PWN, and $\beta \sim 1.1 - 1.2$ (see, e.g., Reynolds and Chevalier, 1984).

- The next phase begins when the reverse shock, propagating towards the centre of the SNR shell, reaches $R_{pwn}$. The reverse shock initially compresses the PWN, followed by an unsteady contraction and expansion of $R_{pwn}$ (see, e.g., Van der Swaluw et al., 2001b; Bucciantini et al., 2003).

- After the initial compression and oscillation of $R_{pwn}$, the PWN enters a second phase of steady expansion. The ejecta surrounding the PWN have been heated by the reverse shock, leading to a subsonic expansion of $R_{pwn}$ (see, e.g., Van der Swaluw et al., 2001b; Bucciantini et al., 2003). This expansion can again be described by a power law, $R_{pwn} \propto t^\beta$, with $\beta \sim 0.3 - 0.7$ (see, e.g., Reynolds and Chevalier, 1984).

The above summary tacitly assumes that both the reverse shock and PWN are spherically-symmetric, and while it provides a useful starting point, the evolution of a PWN can be considerably more complex:

- Simulations by Blondin et al. (2001) show that an SNR expanding into an inhomogeneous ISM, e.g., due to the presence of a molecular cloud, will lead to an asymmetric reverse shock. The interaction of the PWN with this reverse shock leads to a displacement of the nebula away from the pulsar. The resulting PWN morphology is often described as “bullet” or “cigar”-shaped, with the pulsar located at the tip of the “cigar”. The archetypical example of such a system is the Vela PWN, with the cigar morphology clearly visible in X-rays (Markwardt and Ögelman, 1995) and VHE gamma-rays (Aharonian et al., 2006a).

- Pulsars are often born with large kick-velocities, possibly as a result of an asymmetric SNR explosion. The PWN is initially created at the centre of the SNR, but will be dragged along by the pulsar as it moves through the SNR. This reduces the distance between the shell and the PWN in the direction in which the pulsar is moving, thereby decreasing the time needed for the reverse shock to reach the forward boundary of the PWN. For the opposite boundary, the reverse shock interaction time is correspondingly increased. The different reverse shock interaction times again leads to the formation of a cigar-shaped nebula with the pulsar located at the tip (Van der Swaluw et al., 2004). The PWN continues to expand, while a bow-shock eventually forms at the nebula’s edge closest to the pulsar when approximately two-thirds of the SNR radius has been traversed (Van der Swaluw et al., 2003a).

- Additionally, pulsars with a sufficiently large proper motion can break through the SNR shell and escape into the ISM. The motion of the pulsar in the ISM is expected to be highly supersonic, leading to the formation of a PWN bow-shock system (Van der Swaluw et al., 2003a; Gaensler and Slane, 2006).
Lastly, magnetohydrodynamic (MHD) simulations have shown that the magnetic field of the PWN can lead to an elongation of the PWN from a spherical to an elliptic-like shape, with an eccentricity of $1.2 \sim 2.6$ (Van der Swaluw, 2003b; Del Zanna et al., 2004).

### 2.3.3 The two-component lepton spectrum

The PWN characteristics presented in Section 2.3.1 leads one to the conclusion that two leptonic populations are required to explain the non-thermal emission typically observed from PWN: (1) a low-energy component to account for the radio synchrotron and GeV inverse Compton emission, and (2), a high-energy component to account for the X-ray synchotron and TeV inverse Compton emission. As discussed in the same section, it follows from observations that these two components can both be described by a power law, $N_e \propto E^{-\alpha}$, with $\alpha \sim 1 - 1.3$ for the low-energy component, and $\alpha \sim 2$ for the high-energy component. The value of $\alpha$ quoted for the latter component is only possible in the vicinity of the shock (Bocchino et al., 2005; Mangano et al., 2005; Schöck et al., 2010), as synchrotron losses and diffusion will lead to an evolution of the high-energy component. As will be shown in Chapter 5, a similar evolution is not expected for the low-energy component as diffusion and synchrotron losses are markedly more effective at higher energies. Although it has never been directly observed, the particle evolution models presented by Zhang et al. (2008), Fang and Zhang (2010), and Tanaka and Takahara (2011) predict that the transition between the two components should occur at an energy of $E \leq 0.3$ TeV.

Motivated by the above-mentioned considerations, particle evolution models often use a broken power law to describe the spectrum of electrons injected into the PWN at the termination shock (see, e.g., Venter and de Jager, 2007; Zhang et al., 2008; Tanaka and Takahara, 2011). While these models typically assume that the two components connect smoothly, i.e. have the same intensity at the transition, De Jager et al. (2008a) found that this is not the case for the Vela PWN, but that the low-energy component should cut off steeply in order to connect to the high-energy component. It will be seen in Chapter 4 that a similar conclusion is also drawn for the young PWN G21.5-0.9.

The question naturally arises as to how these two electron populations are formed. As demonstrated by Axford et al. (1977), Krymskii (1977), Bell (1978), and Blandford and Ostriker (1978), diffusive shock acceleration leads to a power law energy spectrum, $N_e \propto E^{-\alpha}$, with $\alpha = 2$ representing the maximum possible value. It is therefore possible to associate the origin of the high-energy component of the spectrum with this process. To explain the low-energy component is more difficult, as $\alpha = 1$, and one would not naturally associate the origin of this component with diffusive shock acceleration. As was recently shown by Summerlin and Baring (2012), it is nevertheless possible for relativistic magnetohydrodynamic shocks to produce this hard spectrum if particles are subjected to shock drift acceleration.

An alternative explanation for the origin of the low-energy component has also been pro-
posed by Spitkovsky (2008). Results from particle-in-cell simulations show that the acceleration of particles at the termination shock leads to a relativistic Maxwellian spectrum with a non-thermal power law tail. This result is also indirectly supported by the modelling of Fang and Zhang (2010) and Grondin et al. (2011), where the authors were able to reproduce the non-thermal emission from four PWNe using the spectrum predicted by Spitkovsky (2008). The advantage of this spectrum is that it provides a natural explanation for the discrepancy in intensity between the two components.

Extending the simulations of Spitkovsky (2008), Sironi and Spitkovsky (2011) found that magnetic reconnection occurring in the striped pulsar wind (see Section 2.2.3) can accelerate particles at the termination shock, leading to a deviation from a Maxwellian spectrum. From the simulations it follows that this modified, low-energy component can be described by a power law, with \( \alpha_R \sim 1.5 \). These results may be supported by the observations of Dodson et al. (2003b). Focusing on the very inner regions of the Vela PWN, these authors found radio lobes in the equatorial plane of the nebula. As magnetic reconnection will also occur around the equatorial plane, it is possible that these lobes are formed by the accelerated particles that have been injected into the nebula.

Apart from the uncertainties regarding the nature of the two electron populations, observations of G21.5-0.9 show a radio and X-ray PWN that are spatially coincident (Slane et al., 2000; Bock et al., 2001; Bocchino et al., 2005), whereas observations of the Vela PWN show that the radio/GeV emission is only partially coincident with the X-ray/TeV emission (Grondin et al., 2013). It is therefore not an a priori requirement that the two electron components should be spatially coincident. It should, however, be noted that the former source is a young (\( \sim 1000 \) yr) PWN (Bietenholz and Bartel, 2008), whereas the latter source is an evolved PWN that is believed to have interacted with the reverse shock of the SNR (see, e.g., Blondin et al., 2001; LaMassa et al., 2008).

### 2.4 Summary

Apart from an introduction to supernova remnants, pulsars, and pulsar wind nebulae, this chapter also introduced concepts such as composite remnants and the supernova reverse shock. Both these concepts play an important role in the morphological evolution of a pulsar wind nebula, as will be demonstrated by the hydrodynamic simulations of the next chapter. Furthermore, emphasis was placed on Section 2.2.2, where it was shown that the magnetic field frozen into a plasma wind originating from a rotating magnetic dipole has an Archimedean spiral structure. In the hydrodynamic simulations of Chapter 3, this derived structure is used to motivate the choice of magnetic field used at the inner boundary of the computational domain, while Chapter 5, where the evolution of the particle energy spectrum in a spherically-symmetric system is calculated, uses this spiral field geometry to motivate the choice of radial profiles used for the velocity and magnetic field. Lastly, Chapter 6 almost exclusively focuses
on the effect that the Archimedean spiral magnetic field has on the evolution of the particle energy spectrum.

Another important concept that was also discussed in this chapter is that the particle spectrum in a pulsar wind nebula consists of both a low and high-energy component. Although it can not be stated with certainty, the low-energy component is believed to be a Maxwellian distribution. It is precisely this assumption that allows one to calculate the morphological evolution of a pulsar wind nebula with (magneto)hydrodynamic simulations, as models of this type inherently assume that the distribution of particles responsible for the evolution have a Maxwellian character.