Chapter 3

Hydrodynamic Simulations of Spherically-Symmetric Composite Remnants

One of the aims of this study is to simulate the evolution of a composite supernova remnant in a homogeneous interstellar medium, with an emphasis on the evolution of the pulsar wind nebula. For a study of this kind, one would ideally use magnetohydrodynamic simulations. However, these simulations tend to be computationally intensive, and a hydrodynamic model is used for the present investigation.

The functionality of this hydrodynamic model is extended by including the magnetic field in a kinematic fashion. This implies that the effect of the fluid on the evolution of the magnetic field is taken into account, while neglecting the counter-reaction of the magnetic field on the fluid. Vorster et al. (2013a) showed that the kinematic approach is a good approximation when the ratio of electromagnetic to particle energy in the nebula is $\sigma < 0.01$. Although it is generally believed that $\sigma \lesssim 1$ for pulsar wind nebulae, the only stringent calculation of this ratio for any nebula was performed by Kennel and Coroniti (1984a,b), where the authors derived a value of $\sigma \sim 0.003$ for the Crab Nebula. From the spectral modelling of Tanaka and Takahara (2011), one may additionally infer the values $\sigma \sim 0.001 - 0.01$ for four additional nebulae. Therefore, the $\sigma < 0.01$ limitation of the hydrodynamic simulations does not necessarily represent a severe shortcoming of the model.

In this chapter, the effects of the pulsar’s initial luminosity and spin-down rate, the supernova ejecta mass, and the density of the interstellar medium on the evolution of a spherically-symmetric composite remnant are investigated. The first part of the results presented will focus on the evolution of the termination shock, outer boundary, and average magnetic field in the PWN, while the second part will focus on the radial profiles calculated for the various fluid quantities in a composite remnant. The dependence of these radial profiles on $\sigma$ is also briefly discussed at the end of this chapter. As far as is known, this is the first time that such a comprehensive study has been undertaken, and a large part of the work presented in this chapter has been published in Vorster et al. (2013a).
Before proceeding, it is important to point out a limitation inherent to all (magneto)hydrodynamic PWN models. These models, based on the assumption that the energy spectrum of the particles is described by a Maxwellian, are often used to study the formation of the termination shock. However, the energy spectrum of the particles escaping through the pulsar’s light cylinder is believed to be non-thermal, with the particles only attaining a Maxwellian spectrum after their acceleration at the termination shock (see Section 2.3.3). With this self-contradiction taken into account, it seems very difficult to justify the use of (magneto)hydrodynamic models. Nevertheless, these models, including those presented by Blondin et al. (2001), Van der Swaluw et al. (2001b), Bucciantini (2002), and Volpi et al. (2008), have successfully explained a number of PWN evolutionary features, and are therefore still widely used.

### 3.1 The hydrodynamic model

SNR evolution in a uniform and non-uniform interstellar medium has been modelled extensively by, e.g., Tenorio-Tagle et al. (1991), Borkowski et al. (1997), and Jun and Jones (1999), while simulations focusing on selected aspects regarding the evolution of PWNe and composite remnants have been presented by, e.g., Blondin et al. (2001), Van der Swaluw et al. (2001b), Bucciantini et al. (2003), and Chevalier (2005). Similar to many of these studies, the present hydrodynamic model is based on the well-known Euler equations for inviscid flow describing respectively the balance of mass, momentum and energy

$$\frac{\partial}{\partial t}\rho + \nabla \cdot (\rho \mathbf{V}) = 0,$$

$$\frac{\partial}{\partial t}(\rho \mathbf{V}) + \nabla \cdot (\rho \mathbf{VV} + P \mathbf{I}) = 0,$$

$$\frac{\partial}{\partial t}\left(\frac{\rho}{2} \mathbf{V}^2 + \frac{P}{\gamma - 1}\right) + \nabla \cdot \left(\frac{\rho}{2} \mathbf{V}^2 \mathbf{V} + \frac{\gamma}{\gamma - 1} \mathbf{VP}\right) = 0.$$  \hspace{1cm} (3.1)

Here $\rho$ is the density, $\mathbf{V}$ the velocity, $P$ the pressure, and $\gamma$ the adiabatic index of the fluid, while $\mathbf{I}$ represents the unit matrix. The evolution of the magnetic field $\mathbf{B}$ is calculated by solving the ideal MHD equation

$$\frac{\partial \mathbf{B}}{\partial t} - \nabla \times (\mathbf{V} \times \mathbf{B}) = 0,$$  \hspace{1cm} (3.2)

in the kinematic limit, as discussed by Ferreira and de Jager (2008).

The Euler equations (3.1) are solved using a hyperbolic scheme that is based on a wave propagation approach, as discussed in detail in LeVeque (2002), while the scheme presented by Trac and Pen (2003) is used to solve the induction equation, (3.2). The numerical code used for the modelling was initially developed by Kausch (1998), where a complete derivation and discussion of the numerical scheme can be found. This numerical code has also been used extensively for the hydrodynamic modelling of the heliosphere (see, e.g., Fahr et al., 2000; Ferreira and Scherer, 2006) and other astrospheres (see, e.g., Ferreira and de Jager, 2008). A short overview of the history of the model, as well as a very basic introduction to the wave propagation approach, can be found in Appendix A.
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One feature of the model is that it is solves (3.1) in the spherical coordinates $r$ and $\theta$. For the present simulations, the spherical grid is chosen to extend between $0.01 \, \text{pc} \leq r \leq 25 \, \text{pc}$ and $0^\circ \leq \theta \leq 180^\circ$, with a grid resolution of $2400 \times 25$. The low $\theta$ resolution is possible as the complexity of the problem reduces to a one-dimensional, spherically-symmetric scenario when the remnant evolves into a homogeneous ISM.

In solving the equations, any relativistic effects are neglected, whereas a more realistic model should take into account that the velocity of the particles between the pulsar and the PWN termination shock is comparable to the speed of light $c$. As the post-shock flow is subsonic, $V$ must be less than the sound speed in a relativistic plasma, $V_s \sim c/\sqrt{3}$ (see, e.g., Reynolds and Chevalier, 1984). Furthermore, $V$ decreases from the termination shock towards the outer boundary of the PWN, and the fluid velocity in the PWN will change from mildly relativistic to non-relativistic.

A second possible limitation arises from considering a single-fluid scenario in which only a single value for the adiabatic index $\gamma$ is specified. A more realistic model would treat the stellar ejecta and PWN as two separate fluids, with $\gamma_{\text{ej}} = 5/3$ for the ejecta and $\gamma_{\text{pwn}} = 4/3$ for the PWN. Van der Swaluw et al. (2001b) showed that this limitation can be compensated for in a single-fluid model by setting $\gamma = 5/3$, while choosing the value for the mass deposition rate at the inner boundary of the computational domain $\dot{M}_{\text{pwn}}$ as

$$\sqrt{L(t)/\dot{M}_{\text{pwn}}} = c,$$

where $L(t)$ is the spin-down luminosity of the pulsar, defined by (2.4). Using a two-fluid, one-dimensional, relativistic magnetohydrodynamic (MHD) model, Bucciantini et al. (2003) found that the size of the PWN is only $\sim 10\%$ smaller compared to single-fluid simulations using $\gamma = 5/3$.

A further simplification in the present model is related to radiation losses being neglected in the evolution of the SNR. The main emphasis of this study is on the interaction of the PWN with the reverse shock of the shell and the subsequent evolution of the PWN. The only limitation is that the time needed for the reverse shock to propagate back towards the PWN must be smaller than the time that it takes for radiation losses to become important, i.e. $t_{\text{rev}} < t_{\text{rad}}$, with the expressions for these time scales given by (2.1) and (2.2) respectively. As discussed above, the present model uses $\gamma = 5/3$, along with the fiducial value $E_{\text{ej}} = 10^{51} \, \text{erg}$. The time scales (2.1) and (2.2) are thus reduced to functions of the ejecta mass and ISM density. For any permutation of the values $M_{\text{ej}}$ and $\rho_{\text{ism}}$ chosen from the ranges $M_{\text{ej}} = 4 \, M_{\odot} - 16 \, M_{\odot}$ and $\rho_{\text{ism}} = 10^{-25} - 10^{-23} \, \text{g cm}^{-3}$, one has $t_{\text{rev}} < t_{\text{rad}}$.

It was mentioned in Section 2.3.2 that pulsars are often born with large kick-velocities, leading to the formation of an asymmetric PWN. Additionally, pulsars with a sufficiently large proper motion can also break through the SNR shell and escape into the ISM, with the escape time for a 1000 km s$^{-1}$ pulsar estimated to be around $\sim 40,000 \, \text{yr}$ (Van der Swaluw et al., 2003a). Although the simulations in this study exclude the high-velocity pulsar scenario, this need not limit the
One limitation of the kinematic approach is that it does not reproduce the required PWN morphology in the initial evolutionary phase. Axisymmetric, two-dimensional MHD simulations, extending up to $t = 1000$ yr, have shown that the magnetic field in the PWN leads to an elongation of the nebula from a spherical to an elliptic-like shape, with an eccentricity of $1.2 - 1.5$ \citep{VanderSwaluw2003b}. This non-spherical structure will lead to time delays between the interaction of the reverse shock with the polar/equatorial regions of the PWN. A similar feature is predicted by the axisymmetric, two-dimensional, relativistic MHD simulations of \cite{DelZanna2004}. However, these authors found that the elongation increases to a maximum value at around $t = 400$ yr, before decreasing again. When the ratio of magnetic to particle energy is $\sigma = 0.01$, the elongation ratio decreases to a value that is less than 1.2 after $t = 1000$ yr. As these simulations only extend up to $t = 1000$ yr, it may be possible that the elongation will reduce further. Additionally, \cite{DelZanna2004} also found that the elongation ratio becomes unity after $t = 1000$ yr for the case $\sigma = 0.003$.

It is unclear what role this asymmetric structure plays in the long-term evolution of the PWN, as such simulations have never been performed. It is heuristically argued that the effect would not be as pronounced as one would expect. When the reverse shock reaches the polar boundary, the nebula will be compressed in these regions and the thermal pressure increased. Due to the large sound speed in a PWN, the nebula will reach pressure equilibrium in a very short time. The larger pressure in the PWN will lead to an accelerated expansion in the regions where the reverse shock has not yet interacted with the PWN, specifically in the equatorial regions. This in turn reduces the time needed for the reverse shock to reach the equator of the PWN, leading to a more “symmetric” interaction between the PWN and reverse shock. Naturally this is a very simplified argument, as many other factors, such as the pulsar’s motion and asymmetries in the interstellar medium, will ultimately determine how the reverse shock interacts with the PWN.

Simulations performed by, e.g., \cite{Blondin2001}, \cite{VanderSwaluw2004}, and \cite{Bucciantini2004a} have shown that the interaction of the reverse shock with the PWN leads to the formation of Raleigh-Taylor instabilities. Additionally, the same instabilities are expected to occur at the contact discontinuity between the forward and reverse shocks of young SNRs. An initial test of the present numerical scheme found that these instabilities can be produced if a sufficiently fine computational grid is used. However, to avoid numerical dissipation, the grid has to be refined to such a degree that the code becomes extremely computationally intensive. As the aim of this chapter (and the next) is to simulate the large-scale evolution and structure of the PWN, these instabilities are neglected in subsequent discussions.
3.2 Initial and boundary conditions

The initial configuration of the system is similar to the one used in the models of, e.g., Van der Swaluw et al. (2001b), Bucciantini et al. (2003), and Del Zanna et al. (2004). The SNR is spherical with a radius \( R_{\text{snr}} = 0.1 \) pc and a radially increasing velocity profile

\[
V_{\text{ej}} = \frac{r}{t} = \frac{V_{\text{snr}} r}{R_{\text{snr}}(t)},
\]

while the density profile of the ejecta is constant,

\[
\rho_{\text{ej}}(t) = \frac{3M_{\text{ej}}}{4\pi R_{\text{snr}}^3(t)}.
\]

In these expressions \( R_{\text{snr}} \) and \( V_{\text{snr}} \) respectively represent the position and velocity of the forward shock associated with the SNR. A more correct model would have an inner density plateau for the ejecta, followed by a steeply declining density profile (see, e.g., Chevalier, 1982; Blondin and Ellison, 2001). However, as the PWN is not expected to expand beyond the region of constant density, (3.5) is sufficient for modelling purposes. If the total kinetic energy of the ejecta is equal to the mechanical energy of the SNR, it follows that

\[
E_{\text{snr}} = \int_0^{R_{\text{snr}}} \frac{1}{2} V_{\text{ej}}^2(r) 4\pi r^2 \rho_{\text{ej}} dr.
\]

Evaluating the integral with (3.4) and (3.5) taken into account leads to

\[
V_{\text{snr}} = \sqrt{\frac{10 E_{\text{snr}}}{3 M_{\text{ej}}}}.
\]

Note that in the initial configuration the PWN is absent, with the particles injected by the pulsar only included after the first time step.

For the boundary conditions, the speed of the particles injected at the inner boundary of the computational domain is equal to \( c \), with the density of particles calculated from the mass deposition rate (3.3). Lastly, following the derivation presented in Section 2.2.2, the magnetic field frozen into the wind is assumed to be purely azimuthal. A realistic model should also take into account the temporal evolution of the magnetic field at the inner boundary, which is derived as follows: the spin-down luminosity \( L(t) \) at any given position \( r \) upstream of the pulsar wind termination shock can be written as (Kennel and Coroniti, 1984a)

\[
L(t) = cr^2 B(t)^2 \frac{1 + \sigma}{\sigma},
\]

where \( \sigma \) is the ratio of electromagnetic to particle energy and \( L(t) \) is defined by (2.4). This is effectively a statement of the conservation of energy, and is valid for any value of \( \sigma \). As (3.8) should hold for all times \( t \) one may also state that

\[
L_0 = cr^2 B_0^2 \frac{1 + \sigma}{\sigma}.
\]
Inserting (3.9) into (2.4) leads to the expression

\[ L(t) = \frac{cr^2 B_0}{\sigma} 2 \left( 1 + \frac{t}{\tau} \right)^{-(n+1)/(n-1)}. \]  

(3.10)

To calculate the time-dependence of \( B \), the right-hand sides of Equations (3.8) and (3.10) are set to be equal, while it is assumed that \( \sigma \) has no time dependence at the inner boundary, leading to the result

\[ B(t) = B_0 \left( 1 + \frac{t}{\tau} \right)^{-(n+1)/(n-1)}. \]  

(3.11)

For the simulations, the initial luminosity ranges between \( L_0 = 10^{37} - 10^{40} \) erg s\(^{-1}\), and the spin-down time scale between \( \tau = 500 - 5000 \) yr, in agreement with the \( L_0 \) and \( \tau \) values that have been derived by Zhang et al. (2008), Fang and Zhang (2010), and Tanaka and Takahara (2011). As discussed in Section 2.1, core-collapse supernovae (where pulsars are produced as a by-product) are expected to take place for stars with masses \( \gtrsim 8 \) \( M_\odot \). For example, a value of \( M_{ej} \approx 8M_\odot - 16M_\odot \) has been derived for SN 1987A (Shigeyama and Nomoto, 1990).\(^1\) In the simulations, the ejecta mass ranges between \( M_{ej} = 8M_\odot - 16M_\odot \). Furthermore, core-collapse supernovae occur in star-forming regions in the ISM where the density ranges between \( \rho_{\text{ism}} = 10^{-26} - 10^{-23} \) g cm\(^{-3}\) (see, e.g., Thornton et al., 1998), and a similar range is chosen for the model. Lastly, the fiducial values \( E_{ej} = 10^{51} \) erg and \( n = 3 \) are used for the simulations.

Using hydrodynamic simulations, Van der Swaluw and Wu (2001c) investigated the role of the pulsar’s initial spin-period \( P_0 \) in the evolution of a composite remnant. These authors found that if \( P_0 \leq 4 \) ms, the energy in the pulsar wind exceeds the total mechanical energy of the SNR, resulting in the SNR being blown away. This result is indirectly supported by the findings of Faucher-Giguère and Kaspi (2006b). Based on observations, these authors calculated \( P_0 \) for a number of galactic pulsars, with \( P_0 = 11 \) ms being the lowest value found. The values of \( L_0 \) and \( \tau \), related through (2.6), must therefore be chosen in such a way so as not to result in an unrealistic \( P_0 \), with the definition of “realistic” being (loosely) taken to be \( P_0 \gtrsim 4 \) ms. Using the fiducial values \( I = 10^{45} \) g cm\(^2\) and \( n = 3 \) for the moment of inertia and braking index of the pulsar, the smallest \( P_0 \) value for the present simulations is found for the \( L_0 = 10^{40} \) erg, \( \tau = 5000 \) yr scenario, where \( P_0 = 3.5 \) ms. Although this scenario is technically below the accepted limit, it is nevertheless included for the purposes of illustration.

3.3 Evolution of a PWN inside a spherically-symmetric SNR

In this section the effects of the parameters \( L_0, \tau, M_{ej}, \) and \( \rho_{\text{ism}} \) on the evolution of a spherically-symmetric PWN in a homogeneous ISM are investigated. In order to better compare the various scenarios, it is useful to define a reference set of parameters, chosen as \( L_0 = 10^{39} \) erg s\(^{-1}\).\\footnote{It should be noted that SN 1987A does not have a pulsar associated with it.}
for the initial luminosity, $\tau = 3000$ yr for the spin-down time scale, $M_{\text{ej}} = 12 M_\odot$ for the mass of the stellar ejecta\(^2\), and $\rho_{\text{ism}} = 10^{-25}$ g cm\(^{-3}\) as the density of the ISM.

### 3.3.1 The evolution of the outer boundary of the PWN

Figure 3.1 shows the effect of the parameters on the evolution of the outer boundary of the nebula $R_{\text{pwn}}$. The solutions were obtained using the reference values, only varying the appropriate variable as stated in the legends of the figures. The reference solution in the various panels of Figure 3.1 is represented with a short-dashed line. Figures 3.1(a)–(b) also show the evolution of the forward shock of the SNR. As the forward shock does not depend on the values of $L_0$ and $\tau$, the evolution is the same for all scenarios shown in these two panels. For Figures 3.1(c)–(d), the evolution of the forward shock is the same as that described by Ferreira and de Jager (2008).

Before the interaction with the reverse shock (indicated by the $\Diamond$ symbols), the rate of expansion of the PWN can be described by a power-law function, $R_{\text{pwn}} \propto t^{1.1 - 1.3}$, similar to the prediction $R_{\text{pwn}} \propto t^{1.2}$ made by Reynolds and Chevalier (1984). For the different $L_0$ scenarios in Figure 3.1(a), a larger PWN is predicted when $L_0$ is increased. This difference in PWN size can be attributed to the larger $L_0$ scenarios having a faster expansion rate at the earliest times, $t < 100$ yr, with the expansion rate eventually evolving to $R_{\text{pwn}} \propto t^{1.2}$. Larger $L_0$ values not only lead to a larger PWN, but also imply that more particles are injected into the PWN per time interval, thereby increasing the pressure in the PWN. For the $L_0 = 10^{40}$ erg s\(^{-1}\) scenario, the interaction with the reverse shock does not lead to a compression of the PWN, but only a slower (subsonic) expansion rate. For the $L_0 = 10^{37}$ erg s\(^{-1}\) scenario (where the PWN pressure is smaller), the reverse shock noticeably compresses the PWN, with the post-interaction expansion rate having an oscillatory nature.

At times $t \lesssim \tau$, the value of $L_0$ is almost constant, and any variation in $\tau$ should not affect the evolution of $R_{\text{pwn}}$. This can be seen from the results in Figure 3.1(b) where the evolution of $R_{\text{pwn}}$ at times $t \lesssim 5000$ yr is very similar for the various scenarios. The exception is the $\tau = 500$ yr scenario where a smaller PWN is predicted. The largest influence of $\tau$ is seen after the interaction with the reverse shock. As a smaller $\tau$ leads to a faster decrease in $L(t)$, the PWN pressure should correspondingly decrease, leading to a larger compression of the PWN. However, as shown in Figure 3.1(b), this effect is not as prominent for a PWN where the reverse shock time scale is of the order of $\tau$.

Figure 3.1(c) shows that a smaller $M_{\text{ej}}$ initially leads to a larger PWN when $t < 3000$ yr. A larger mass for the stellar ejecta $M_{\text{ej}}$ causes an increase in the density of the matter that the PWN expands into, and consequently a slower expansion rate for $R_{\text{pwn}}$. However, it follows

\(^2\)After the numerical simulations were performed it was pointed out that this reference mass may not have been the most optimal choice. Most stars that will end their lives as a supernova explosion are believed to be near the low-mass threshold of $8 M_\odot$. Furthermore, $M_{\text{ej}} = 12 M_\odot$ is close the maximum possible ejecta mass (see, e.g., Woosley et al., 2002). Nevertheless, a number of ejecta masses are investigated, and should be sufficient in illustrating the effect of this parameter on the evolution of the composite remnant. This is also the main aim of this chapter.
from (2.1) that the reverse shock time scale increases as a function of $M_{ej}$, thereby allowing for the development of a larger PWN in the initial expansion phase.

From the reverse shock time scale (2.1), it can be seen that a larger value of $\rho_{ism}$ markedly reduces the time needed for the reverse shock to reach the PWN. This decreases the duration of the supersonic expansion phase of $R_{pwn}$, resulting in a PWN that is noticeably smaller, as shown in Figure 3.1(d). From the $\rho_{ism} = 10^{-24}$ g cm$^{-3}$ and $\rho_{ism} = 10^{-23}$ g cm$^{-3}$ scenarios it can also be seen that the PWN eventually reaches a stage where $R_{pwn}$ approaches a constant value.

### 3.3.2 The evolution of the termination shock radius

The next quantity investigated is the termination shock radius $R_{ts}$. Figure 3.2 shows the corresponding evolution of $R_{ts}$ for the scenarios shown in Figure 3.1. Note that the striations shown in Figure 3.2 are due to numerical effects, resulting from the chosen grid resolution. For the various scenarios, $R_{ts}$ initially expands, reaching a maximum value shortly after the initial interaction between the PWN and the reverse shock (cf Figure 3.1). The subsequent slower (subsonic) expansion and compression of $R_{pwn}$ leads to an increase in the pressure in the PWN, resulting in $R_{ts}$ being pushed back towards the pulsar. The effect of the various parameters on the evolution of $R_{ts}$ mirrors the results of Figure 3.1 as a larger PWN also has a larger $R_{ts}$ before the reverse shock interaction. After the reverse shock interaction, a faster decrease in $R_{ts}$ is predicted for a larger compression of the PWN.

### 3.3.3 The evolution of the average magnetic field

Figure 3.3 shows the temporal evolution of the average magnetic field $\bar{B}/B_0$ in the PWN, defined as the region $R_{ts} \leq r \leq R_{pwn}$. The solutions correspond to those shown in Figures 3.1 and 3.2. If the magnetic energy in the PWN is not transferred to the particles, then the conservation of magnetic flux implies that the evolution of $\bar{B}$ should be related to the evolution of $R_{pwn}$. A comparison of Figures 3.1 and 3.3 shows that an increase in $R_{pwn}$ leads to a decrease in $\bar{B}$, and vice versa. Furthermore, the rate of change in $R_{pwn}$ is also reflected in the rate of change in $\bar{B}$.

Before the interaction with the reverse shock, it is possible to derive a simple time-dependence for $\bar{B}$, as the expansion of $R_{pwn}$ is described by a power law. From the conservation of magnetic flux it follows that

$$\int_{0}^{t} \eta_B L dt' = \mathcal{V}_{pwn} \frac{\bar{B}^2}{8\pi},$$

where $\eta_B$ is the (time-independent) fraction of the spin-down luminosity converted to magnetic energy and $\mathcal{V}_{pwn} \propto R_{pwn}^3$ is the volume of the PWN. Note that this integral represents the total amount of magnetic energy injected into the PWN over the time interval $0 \leq t' \leq t$. According to (2.4), the pulsar’s spin-down luminosity can be approximated as constant ($L = L_0$)
Figure 3.1: Model solutions for the evolution of the outer boundary of a spherically-symmetric PWN in a homogeneous ISM. A reference parameter set is chosen as $L_0 = 10^{39}$ erg s$^{-1}$, $\tau = 3000$ yr, $M_{ej} = 12 M_\odot$, and $\rho_{ism} = 10^{-25}$ g cm$^{-3}$, with the solution represented by a short-dashed line in Figures (a)–(d). The other solutions were obtained using the reference parameters while varying the parameters listed in the legends of the figures. The $\bigodot$ symbol represents the time when the reverse shock reaches $R_{pwn}$. Also shown in Figures (a) and (b) is the evolution of the forward shock of the SNR.
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Figure 3.2: The same as Figure 3.1, but for the termination shock radius \( R_{ts} \).
Figure 3.3: The same as Figure 3.1, but for the average magnetic field in the PWN.
for times $t \lesssim \tau$. Lastly, using $R_{pwn} \propto t^\beta$, the integral can be simplified to

$$\eta_B L_0 \int_0^t dt' \propto t^{3\beta} \frac{\bar{B}^2}{8\pi},$$

which evaluates, after rearranging the terms, to

$$\bar{B} \propto t^{-(3\beta-1)/2}.$$  

At times prior to the interaction of the PWN with the reverse shock, the values $\beta = 1.1 - 1.3$ are derived from Figure 3.1, and the evolution of the average magnetic field in Figure 3.3 can thus be described by $\bar{B} \propto t^{-1.1} - t^{-1.5}$. The same arguments have also been used by Reynolds and Chevalier (1984) to derive the evolution of $\bar{B}$, finding that $\bar{B} \propto t^{-1.3}$. For reference, the line $\bar{B} \propto t^{-1.3}$ is also shown in Figure 3.3. For later times, it is difficult to characterise the evolution of $\bar{B}$ with a simple expression, as the evolution of $R_{pwn}$ becomes more complex.

The fact the HD results in Figure 3.3 and (3.12) both predict a similar evolution of $\bar{B}$ at times $t < t_{rev}$ shows that the integral (3.12) can be used for times $t > t_{rev}$, provided that the evolution of $R_{pwn}$ can be described by an analytical expression. This approach is employed in the next chapter where a spatially independent particle transport model is presented.

Although similarly-themed evolution studies have previously been performed by, e.g., Van der Swaluw et al. (2001b), Bucciantini et al. (2003), and Bucciantini et al. (2004b), the present simulations represent the first comprehensive study of the effects that various parameters have on the evolution of the PWN. Additionally, the present simulations also represent the first (known) study of the long-term evolution of the average magnetic field in the PWN. The evolution of $R_{pwn}$ and the average magnetic field are of particular importance to the spatially independent particle evolution models that have been developed by, e.g., Zhang et al. (2008), Fang and Zhang (2010), and Tanaka and Takahara (2011), as well as the model that will be presented in the next chapter.

The evolution of the average magnetic field is also important when one attempts to understand so-called “dark sources”, i.e., TeV $\gamma$-ray sources without a synchrotron counterpart (Aharonian et al., 2008). From Figure 3.3 it can be seen that a rapid expansion of the PWN leads to a significant decrease in $\bar{B}$ as a function of time, which in turn will lead to a rapid decline in the synchrotron luminosity. Figure 3.3 further shows that the compression of the PWN by the reverse shock does not always lead to an order-of-magnitude increase in $\bar{B}$. Therefore, if the PWN had a low magnetic field before the interaction, $\bar{B}$ will remain low even after the interaction with the reverse shock, leading to a faint synchrotron source. Although these PWNe will not be detected at radio and X-ray energies, it is nevertheless possible for these sources to remain bright at TeV energies (De Jager, 2008).

3.3.4 Radial profiles of the fluid quantities

It is also useful to investigate the evolution of the different fluid quantities, especially for times before and after the interaction of $R_{pwn}$ with the reverse shock. The radial profiles of the fluid
quantities at three different times are shown in Figure 3.4, corresponding to the scenarios (a) $L_0 = 10^{38} \text{ erg s}^{-1}$, $\tau = 3000 \text{ yr}$, $\rho_{\text{ism}} = 10^{-24} \text{ g cm}^{-3}$, $M_{\text{ej}} = 8 M_\odot$, and (b) $L_0 = 10^{39} \text{ erg s}^{-1}$, $\tau = 3000 \text{ yr}$, $\rho_{\text{ism}} = 10^{-25} \text{ g cm}^{-3}$, $M_{\text{ej}} = 8 M_\odot$.

It follows from the Rankine-Hugoniot jump conditions that a velocity decrease across the shock will not only compress the magnetic field, but will also lead to an increase in density. If the magnetic field is perpendicular to the flow velocity at the shock (as is the case for the present model), then the maximum compression ratio for a monatomic gas is 4 (see, e.g., Choudhuri, 1998). The magnetic profile (top panel) is therefore useful for identifying the various components of the PWN. In Figure 3.4 a sharp increase of about a factor 4 is visible in $B$, indicative of the position of $R_{\text{ts}}$. At the same radial distance, an increase in $\rho$ can also be seen in Panel 2, while Panel 4 shows a decrease in $V$. On the other hand, the radial distance where $B$ falls off to zero indicates the position of $R_{\text{pwn}}$.

One important result that follows from Panel 1 is that the hydrodynamic effects lead to only small variations in the radial profile of the magnetic field when the reverse shock has not yet interacted with the PWN. Upstream of the shock, where $V$ is independent of radial position, $B \propto 1/r$, while scaling essentially as $B \propto r$ downstream of the shock, where $V \propto 1/r^2$. The numerical results thus confirm the derivation of the magnetic field structure presented in Section 2.2.2.

The density profile in Panel 2 allows one to distinguish between the different components of the composite remnant. The low-density region represents the PWN, and the region of greater density the ejecta. The $t = 1000 \text{ yr}$ profile shows two peaks at $r \sim 5 \text{ pc}$ and $r \sim 7 \text{ pc}$, with these peaks respectively representing the reverse and forward shock of the SNR. For larger $r$ values, the constant-density region corresponds to the ISM. Comparison of the density profiles at the various times with the position of $R_{\text{pwn}}$, as defined by Panel 1, shows that there is a region just behind $R_{\text{pwn}}$ where the density increases significantly. This can be heuristically viewed as being a result of $R_{\text{pwn}}$ interacting with the denser ejecta, leading to a pile-up of PWN material. Such an explicit distinction is, however, difficult to make using a single-fluid model.

Panel 3 shows that the pressure behind the reverse shock is lower than both the pressure in the PWN and the SNR forward shock. It is the pressure difference between the ejecta and shock that eventually leads to the reverse shock propagating back towards the pulsar. After the interaction with the reverse shock, the difference in pressure between the PWN and the ejecta decreases, reducing the PWN expansion rate, as shown in Figure 3.1.

Apart from compressing the PWN (and consequently enhancing $B$), the interaction between the reverse shock and $R_{\text{pwn}}$ drives a reflection wave through the PWN, as illustrated in Panel 5. Lastly, Panel 6 shows that the outer edge of the PWN initially expands supersonically into the ejecta, becoming sub-sonic once the reverse shock has passed.

A comparison between the profiles of Figure 3.4(a) and (b) shows that similar profiles are obtained, regardless of the scenario investigated. Apart from the obvious differences, such as
Figure 3.4: From top to bottom: radial profiles of the magnetic field strength $B$, density $\rho$, pressure $P$, speed $v$ (−$v$ indicates a velocity in the opposite direction), and Mach number at three different times. The profiles in Figure (a) are for the $L_0 = 10^{38}$ erg s$^{-1}$, $\tau = 3000$ yr, $\rho_{\text{ism}} = 10^{-24}$ g cm$^{-3}$, and $M_{\text{ej}} = 8 M_\odot$ scenario, while the profiles in Figure (b) are for the $L_0 = 10^{39}$ erg s$^{-1}$, $\tau = 3000$ yr, $\rho_{\text{ism}} = 10^{-25}$ g cm$^{-3}$, and $M_{\text{ej}} = 8 M_\odot$ scenario.

the size of the PWN, the only notable difference between the two scenarios is that the reflection wave, created as a result of the interaction of the reverse shock with $R_{\text{pwn}}$, travels deeper into the PWN in scenario (a), compared to scenario (b). This was found to be true for even longer time scales than those shown in Figure 3.4.

3.3.5 Small $\sigma$ vs. large $\sigma$ PWNe

It was mentioned in the introduction to this chapter that the kinematic approach is valid when the magnetic pressure is negligible, which translates into the requirement that $\sigma < 0.01$ for PWNe. Note that when this condition is satisfied, the fluid is essentially hydrodynamic. In this respect, the velocity and magnetic profiles in Figure 3.4 are comparable to the profiles derived by Kennel and Coroniti (1984a) from their steady-state MHD model. While the present
simulations are certainly useful, it must be kept in mind that they do have a limitation as the
values $0.01 \leq \sigma \lesssim 1$ are also possible for PWNe (see, e.g., De Jager and Djannati-Ataï, 2009).
When $\sigma > 0.01$, Kennel and Coroniti (1984a) found that both the velocity and magnetic field
decrease as a function of position, a result that has also been illustrated by Vorster et al. (2013a).
Here it is also shown that the compression ratio decreases from $\sim 4$ to $\sim 2$ when the value of $\sigma$
increases from the hydrodynamic limit to $\sigma \sim 1$.

### 3.4 Summary

This chapter commenced with a brief description of the spherically-symmetric, hydrodynamic
model that was used to simulate the evolution of a composite supernova remnant, with emphasis
placed on the evolution of the pulsar wind nebula. One notable feature of this model
is that the magnetic field is included, albeit in a kinematic fashion. This model is solved numerically
using a hyperbolic numerical scheme based on a wave propagation approach. A
brief introduction to this solution technique can be found in Appendix A. For a more comprehensive
discussion of this model, Kausch (1998), Snyman (2007), and Ferreira and de Jager (2008)
should be consulted. As the model used in this chapter has been previously developed, a short
history of the model is given in Appendix A.5.

One of the main aims of this chapter was to investigate the effects that the pulsar’s initial lu-
minosity and spin-down rate, the supernova ejecta mass, and the density of the interstellar
medium have on the evolution of a spherically-symmetric, composite remnant. These sim-
ulations focused specifically on the evolution of the outer boundary, termination shock, and
average magnetic field of the PWN. Although similar studies can be found in the literature,
the present results represent the most comprehensive investigation of its kind.

In the next chapter a spatially independent particle evolution model for PWNe is presented.
This model requires that the evolution of $R_{\text{pwn}}$ be known in order to calculate the adiabatic
energy changes that the particles in the nebula are subjected to. It will be shown that the
hydrodynamic simulations in Figure 3.1 are compatible with values derived from data, and
that these simulations can be used to interpret the unidentified TeV sources discovered by the
H.E.S.S. telescope (Acero et al., 2011).

The simulations in this chapter were also used to calculate the radial profiles of the various
fluid quantities. The spatially dependent transport models used in Chapters 5 and 6 to cal-
culate the evolution of the PWN particle spectra require the specification of radial profiles for
both the velocity and magnetic field. The profiles used in the above-mentioned chapters are
subjected to the prescription $V Br = \text{constant}$, as derived from the steady-state version of the
ideal MHD limit, (3.2). This is supported by the results of Figure 3.4, from which an identical
prescription can be derived. The same figure also confirms the magnetic field geometry de-
vised in Section 2.2.2, and used in Chapter 6.