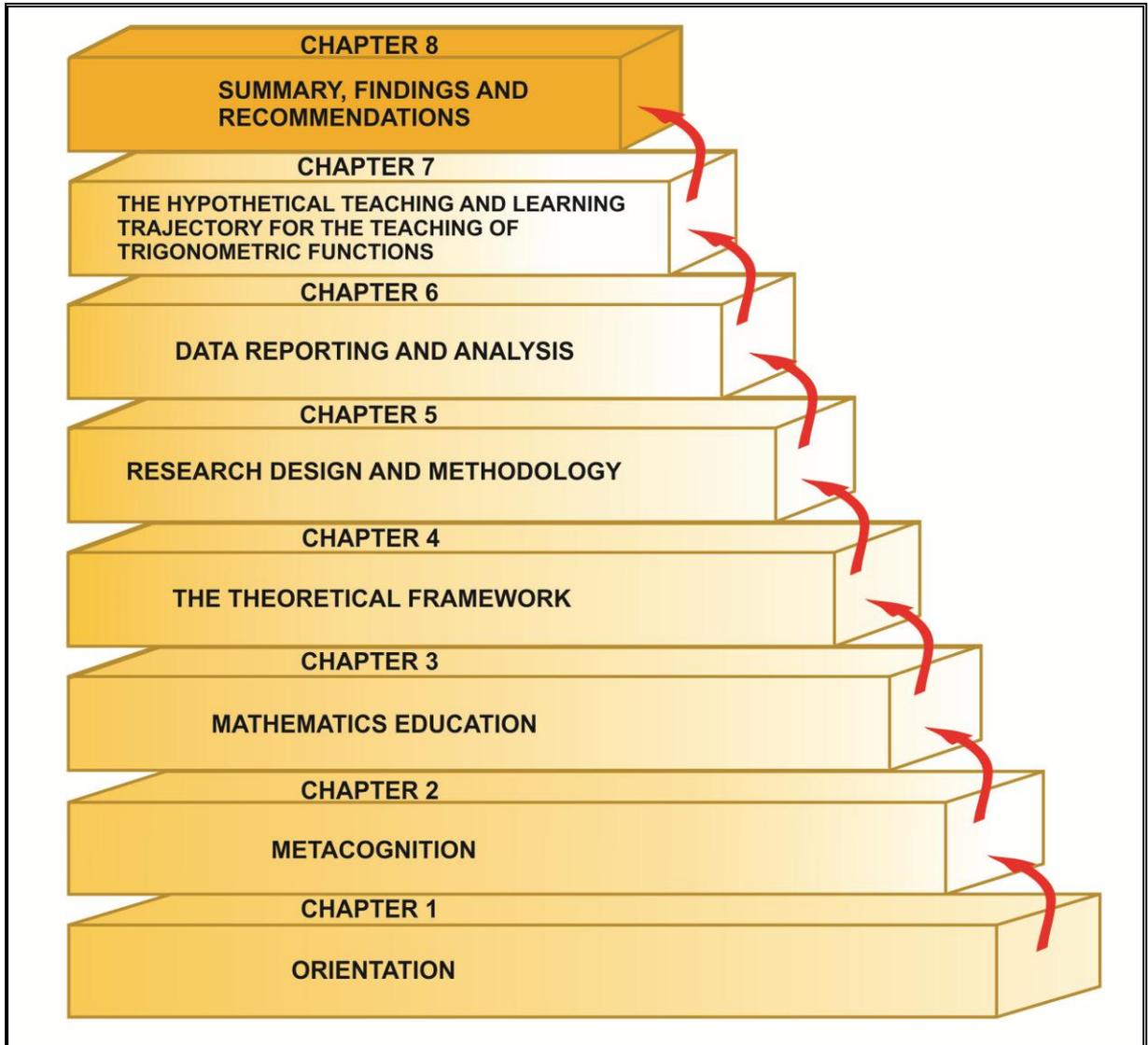
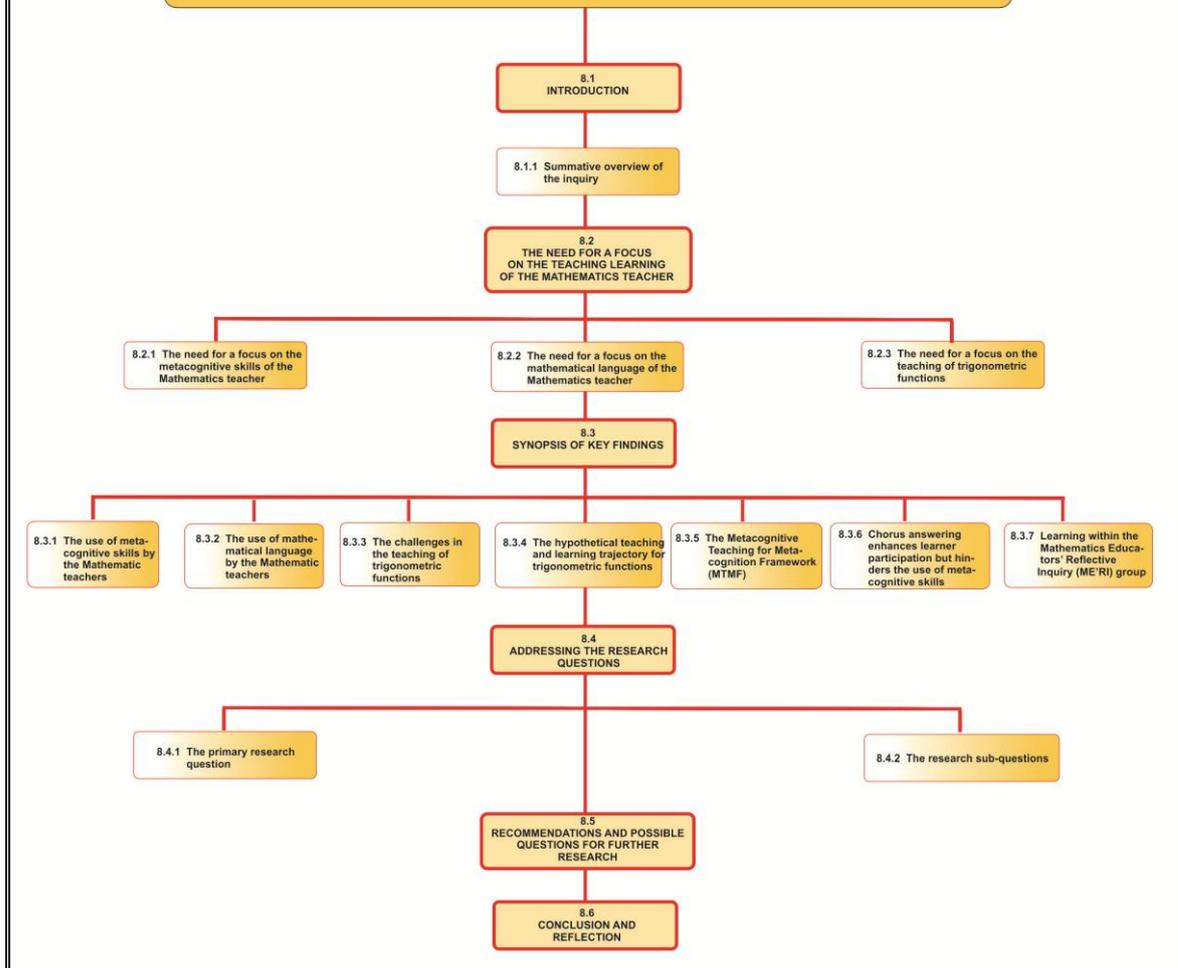


CHAPTER 8: SUMMURY, FINDINGS AND REDOMMENDATIONS



CHAPTER 8: SUMMARY, FINDINGS AND RECOMMENDATIONS



8.1 INTRODUCTION

Teaching involves, amongst other activities, the careful planning of lessons before the teaching, selecting appropriate strategies and examples for imparting the knowledge to the learners, using resources to teach the lesson, managing/controlling the lesson, monitoring the learners, assessing learners' work and giving feedback to learners. All these activities require teachers to think and reflect carefully about the lesson and the learners. Reflecting on this thinking may happen prior to the lesson, during the lesson or even after the lesson and can be referred to as metacognition (Hartman, 2002b). This kind of reasoning supports the notion that teaching is in essence a metacognitive act. If teaching is in essence a metacognitive act then it can be assumed that the end product should be effective teaching. Surprisingly this is not the case; therefore this inquiry endeavoured to find ways of how the metacognitive skills and the mathematical language of the Mathematics teacher can be applied to facilitate the effective teaching and learning of trigonometric functions in the FET phase of township schools in the Dr Kenneth Kaunda District of the North West Province. The question which directed the study therefore was:

What are the characteristics of an in-service arrangement that facilitates the implementation of lesson activities focusing on the metacognitive skills and mathematical language of Mathematics teachers for the teaching of trigonometric functions in township schools in the Dr Kenneth Kaunda district?

Before any results are discussed, a short summary is provided of each chapter in order to illustrate the line of thinking within this inquiry.

8.1.1 Summative overview of the inquiry

Chapter One positioned the research problem against the background of the South African education system (§1.2) and provided the reasons why the study was needed (§1.3). The research question was provided (§1.4), followed by the purpose and objectives (§1.5) with this inquiry. Next the study was contextualized (§1.6) and showed the developmental relevance for South Africa. Chapter One also contained a short review of the relevant literature (§1.7). The conceptual think-piece at the onset of this inquiry was also provided, followed by my paradigmatic assumptions and perspectives as researcher (§1.8). Some limitations and ethical considerations were also discussed before an outline of the chapters within this inquiry was provided.

Chapter Two discussed metacognition as concept (§2.2) and provided insight into the link between metacognition and cognition, and secondly a distinction between cognitive and metacognitive skills. This chapter also highlighted the works of theorists such as Vygotsky (§2.3.1) and Piaget (§2.3.2) as forerunners to metacognition. Chapter Two also shed more light on the development of a description of metacognition over four generations (§2.3.3) and discussed four models underpinning metacognition, *viz.* Flavell's model (1979), Paris and Winograd's model (1990), Hartman and Sternsberg's model (1993) and lastly Tobias and Everson's model (2001). Components of metacognition were critical for this inquiry and were discussed in Chapter Two (§2.5). The next section concentrated on the development of metacognitive skills (§2.6), since this study was focusing on the metacognitive skills of the teacher. Teaching *for* and teaching *with* metacognition (§2.2.5) was also discussed as metacognitive instruction and brought to the fore a comparison, firstly between metacognitive and non-metacognitive learners (Table 2.3) and secondly between metacognitive and non-metacognitive teachers (Table 2.4). Two frameworks *viz.* the Framework for Analysing Mathematics Teaching for the Advancement of Metacognition (FAMTAM) from Ader (2013) and the Teacher Metacognitive Framework (TMF) from Artzt & Armour-Thomas, (2002) were discussed as possible conceptual frameworks to examine metacognitive instruction within this inquiry.

The literature review in **Chapter Three** facilitated the creation of draft design guidelines that informed the design and development of the hypothetical teaching and learning trajectory that was implemented in this inquiry by sketching firstly the South African education landscape (§3.2) followed by learning theories and conceptions of teaching (§3.3). Simon's Mathematics Teaching Cycle model (1997) (§3.4.1) and Steinbring's two-ring model of teaching and learning Mathematics (1998) (§3.4.2) were discussed because they could shed more light on trajectories in Mathematics teaching and learning. Chapter Three also featured a discussion on the knowledge of Mathematics teachers (§3.5) which included mathematical language. The learning of the teacher versus teacher change (§3.6), as well as their instructional practices (§3.7) was then addressed because this study is mainly concerned with Mathematics teachers. As this study used adapted lesson study to enable understanding of how Mathematics teachers use their metacognitive skills and mathematical language in lessons, a discussion on adapted lesson study (§3.8) followed. Finally a discussion of the teaching and learning of Trigonometry (§3.9) resulted in some guiding principles for the teaching of trigonometric functions.

Chapter Four explained the theoretical framework, complexity theory, employed in this inquiry to provide “a unique way of perceiving reality” and “a fresh and different perspective” (Anfara & Mertz, 2006, p. xiv) on the endeavours of the Mathematics teachers in this inquiry. This explanation started with a definition of “theoretical framework” (§4.2), followed by an overview of the study (§4.3) as basis for the discussion of Complexity theory. Next, an explanation of how Complexity theory was found (§4.4) and why I have selected Complexity theory rather than Hierarchic Interactionism (HI), or Constructivism or the Realistic Mathematics Education (RME) (§4.7). Subsequently Complexity theory, its effects on this inquiry, as well as the discipline in which it originated (§4.5), is discussed in detail. The chapter concluded with a critique of Complexity theory as theoretical framework in this inquiry. This chapter was a very relevant one as design-based research has always been criticized as stripped from any theory because of its interventionist and pragmatic nature (Bakker & Van Eerde, 2012).

Chapter Five not only outlined the instrumentation and methods that were employed in the study to gather and analyse the data, but also provided the motivation for the choices that were made in this regard. I have explained my role as researcher (§5.2) within the study. Design-based research was compared to action research and experimental research in order to illustrate its relevance to this study as most effective design for the inquiry (§5.3). Before the population and sampling were addressed, an explanation (§5.3.5) and an illustration (Figure 5.5) were provided of how the three phases of this design-based research study were merged with the five cycles of adapted lesson study within this inquiry (Table 5.2). The biographical data of the teachers and lecturers were given in table form (Table 5.6) which formed part of the population and sampling (§5.4). The data collection instruments used in the study (§5.5) were described before the data-analysis process in each phase was provided (§5.6). Lastly some ethical issues (§5.7) and the trustworthiness of the study (§5.8) were discussed.

In **Chapter Six** constituted the first part of the data analysis and described how the data gathered in the first phase were reported and analysed using ATLAS.Ti 7. The data collection instruments that were used to gather the data in the first phase included the individual interviews with Teacher A and Teacher B (§6.2.1), the lesson observations from lesson one and two presented by Teacher A and Teacher B respectively (§6.2.2), the assessment tasks completed by Teacher A and Teacher B (§6.2.3) and lastly, focus group discussions one and two (§6.2.4). The data analysed in this phase informed the metacognitive performance profile for Teacher A (§6.3.1) and Teacher B (§6.3.2), the two grade 10 teachers, (one from each school in this inquiry). Data analysed in this first phase

also informed the design of the hypothetical teaching and learning trajectory for trigonometric functions (§6.3.3) after it had been implemented in the first two cycles (§6.2.5.1 and §6.2.5.2). The systematic reporting and analysis of the data gathered in this first phase gave effect to the first two sub-questions in this inquiry viz.:

1. *How do Mathematics teachers apply metacognitive skills and mathematical language in the teaching of trigonometric functions?*
2. *Which challenges do Mathematics teachers face in the teaching of trigonometric functions?*

Various network views, which can be seen as the visual diagram of relationships between codes, quotations, and memos, were generated by ATLAS.Ti from the data gathered which underpin the discussion of the data analysis. The first three themes emerged out of the analysis of data in the first phase.

Chapter Seven contained the second part of the data reporting and analysis and gave an analytical account of the data gathered in the second (§7.2) and third phase (§7.3) of this design-based research study. In the second phase of the study, cycle three and four of the adapted lesson study were performed, while the last cycle happened in phase three. The data reported and analysed in this phase gave effect to the third sub-question viz.

3. *How can the teaching of trigonometric functions be improved focusing on the metacognitive skills and mathematical language used by Mathematics teachers?*

Data reporting and analysis within the second phase included the reporting of design experiment three (§7.2.1) and design experiment four (§7.2.2). The analysis of the data in this second phase (§7.2.3) gave birth to theme four, the Metacognitive Teaching for Metacognitive Framework (MTMF) and theme five, chorus answering and metacognition. Similarly the data gathered in phase three were reported and analysed (§7.3) from which theme six, Learning that happened within The Mathematics Educators' Reflective Inquiry (ME'RI) group, emerged. Chapter Seven also included Table 7.7 which portrayed how each lesson played out in the adapted lesson study before the chapter concluded with an analysis of all the data using the lens of complexity theory (§7.4).

In **this chapter** I now proceed, against the background of the above summative overview, to explain why it was necessary to focus on the metacognitive skills and mathematical language of Mathematics teachers teaching trigonometric functions in township schools (§8.2). I then provide a synopsis of the key findings (§8.3), portray the findings against each of the three

sub-questions (§8.4) and then reflect on recommendations and possible questions for further research (§8.5) before I finally conclude this thesis.

8.2 THE NEED FOR A FOCUS ON THE TEACHING-LEARNING OF THE MATHEMATICS TEACHER

8.2.1 The need for a focus on the metacognitive skills of the Mathematics teacher

Research in teacher metacognition is critical in that it provides valuable insight into how teachers are thinking about their teaching practices. Thinking takes time and teachers need to give think-time to their learners to think, which all boils down to the effective use of their metacognitive skills in order to teach *for* metacognition (and to teach *with* metacognition.) Hartman (2002) postulates that the use of metacognitive skills by teachers enable them to know what instructional strategies they have in their toolbox, when to use them, why to use them and how to use them. Teachers should attempt to not only teach the content but balance the teaching of the key content with teaching students how to think. If teachers model the use of metacognitive skills, then learners will follow as the teacher usually sets the example for the learners in the classroom. Teachers who model good metacognitive thinking in action, such as prompting learners to do a metacognitive task, for example to evaluate all the possible outcomes of an experiment, will also facilitate learners to become more metacognitive (Schoenfeld, 1987).

8.2.2 The need for a focus on the mathematical language of the Mathematics teacher

Although Mathematics is usually thought of in terms of numbers, it still needs language to communicate its content which in itself contains a lot of words. Teachers need to make their thinking visible by using correct mathematical language in the Mathematics classroom. Teachers have several misconceptions which are mostly due to the incorrect use of the mathematical language in the teaching of Mathematics. This challenge is exacerbated in South Africa by the fact that teachers and their learners have to teach and learn Mathematics in most cases in their second language, sometimes even in their third language.. Van der Walt *et al.* (2008) and Kassiem (2004) advise that factors related to language in particular be taken into account for effective Mathematics teaching and learning. It is therefore imperative that the Mathematics teacher should model the correct use of terminology as these terms are the same terms that would be used by their learners.

8.2.3 The need for a focus on the teaching of trigonometric functions

In the current Mathematics CAPS (Curriculum and Assessment Policy Statements) document (Department of Basic Education, 2011) Trigonometry is only introduced in grade 10. It is only then when grade 10 learners are confronted with a myriad of new terms and concepts that they did never previously encountered in the curriculum. Furthermore the literature on Trigonometry revealed that learners do not have a coherent understanding of Trigonometry (Demir, 2012). Some teachers also find it difficult to teach trigonometric functions.

In the next section the findings are presented.

8.3 SYNOPSIS OF KEY FINDINGS

8.3.1 The use of metacognitive skills by the Mathematics teachers

Based on the metacognitive performance profile of Teacher A (§6.3.1) and that of Teacher B (§6.3.2), it appears as if the teachers in the inquiry are using metacognitive skills to a certain extent, but this can be greatly enhanced if teachers are more aware of reflection in the Mathematics classroom. This finding concurs with findings by Van der Walt (2008) that teachers are using metacognitive skills, but are not aware that they are using them. Having said that, it must be acknowledged that to assess the use of metacognitive skills was not an easy task. It was also evident that teachers want to use their metacognitive skills more as they verbalized their need to reflect more on their teaching practices. Teachers also acknowledged the value of metacognitive skills in the teaching of not only Trigonometry, but also other areas in Mathematics. Although teachers have a reasonable good understanding of metacognition, the conceptualization of metacognition as concept is important (Hartman, 2002b). If teachers do not know what metacognition is, they will not be able to exercise metacognitive skills in their classrooms. Teachers need to get workshops, examples and practice in metacognitive strategies that they can use to not only teaches with metacognition but also to teach for metacognition. Teachers need to work out mathematical problems in real life context more frequently, both for their own practice of their subject knowledge but also for the practice of their metacognitive skills.

8.3.2 The use of mathematical language by the Mathematics teachers

The teachers in this inquiry used mathematical language fairly well. However, there is a need for teachers to focus more explicitly on difficult or new concepts, especially since trigonometric functions is part of a new area in the Mathematics curriculum for the grade 10 learner and considering that in South Africa Mathematics is taught and learnt mostly not in the mother tongue of the teacher or the learner. At the onset of this inquiry, the teachers most of the time used content language as their teaching was primarily of an informative nature. Later, as the adapted lesson study progressed, academic language as well as symbolic language was used. Together with the context of any new concept, the fundamental knowledge should also be addressed when introducing the new concept. The focus on required fundamental or prior knowledge when introducing Trigonometry in grade 10 for the first time (§3.9.5) seems to have been also of great importance in this inquiry and concurs with findings by Blackett and Tall (1991) and Weber (2005). If teachers use using mathematical language effectively, which is also referred to as “the register of Mathematics” by Halliday (1978), it can prevent learners from becoming more confused with unnecessary jargon.

8.3.3 The challenges in the teaching of trigonometric functions

From the literature review on the challenges of the Trigonometry teaching and learning, the following problem areas were identified: Order of operations, value and place of the sine and cosine functions, improper use of equation, distorted definitions, misused data, misinterpreted language, logically invalid inferences and technical mechanical errors (Gür, 2009). The following challenges were gathered from the perspectives by teachers as well as from observations during lessons, and from discussions in the focus groups that were conducted in this inquiry:

Learner related challenges: The negative attitude of learners towards Mathematics, learners who are failing to see the big picture, the poor learner performance in Mathematics as well as the fact that learners do not practise Mathematics at home seem to have a hindering effect on the Mathematics teacher when teaching trigonometric functions.

Teacher related challenges: Not only does the attitude of learners affect the teaching of trigonometric functions, but also the negative attitude of some Mathematics educators towards Mathematics is seen as a stumbling block for the teaching of trigonometric functions. Other factors which can be laid in front of the door

of the Mathematics teacher include lack of content knowledge of some Mathematics educators as well as the pride of some Mathematics teachers not to ask help in cases where they do not understand the content.

Subject related challenges: Lastly the fact that Trigonometry contains a lot of definitions, and the language in the form of the many words as opposed to numbers involved in the real life situations, also seem to be challenging for the teaching of trigonometric functions. This challenge concurs with literature (Department of Basic Education, 2011a).

8.3.4 The hypothetical teaching and learning trajectory for trigonometric functions

The hypothetical teaching and learning trajectory for trigonometric functions came about as a result of implementing the guiding principles that were identified in Chapter Three from Quinlan (2004: 20). In order to understand the efficacy of these principles, it was carried out through instruction in the lessons and adjusted. The trajectory was also influenced by some valuable instructional points from the literature about metacognition and mathematical language following advice from Van den Akker *et al.* (2010) that design researchers “search for already available interventions that can be considered useful examples or sources of inspiration for the problem at stake”. The trajectory was further adjusted after every design experiment depending on its success, whether it did indeed elicit the anticipated reactions from not only the learners, but also the teacher in terms of the use of metacognitive skills and mathematical language. The reactions from the learners and the teacher, combined with what literature was saying, informed the adjustment of the trajectory. The focus group discussions in which lessons were reflected upon and new lessons were planned resulted in new ideas as the one idea ignited another in line with complexity thinking where knowledge is socially created in the community of Mathematics educators. The hypothetical teaching and learning trajectory for trigonometric functions can be seen in Figure 7.4. Although it can be regarded as the end product in this study, it is acknowledged that it can be adjusted even more in future research endeavours.

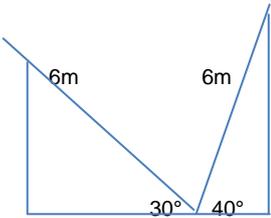
THE HYPOTHETICAL TEACHING AND LEARNING TRAJECTORY FOR TRIGONOMETRIC FUNCTIONS

Learning goal: To enhance the use of metacognitive skills and mathematical language for trigonometric teaching

Guiding principles (Quinlan, 2004, p. 20) adjusted

1. *Go from the concrete to the abstract, while explicitly mentioning the outcome/s of the lesson. Avoid starting with definitions*
2. *Go from particular to general.*
3. *immerse students in the context of any new concept before explicating its technicalities and intricacies and mathematical jargon.*
4. *The actions of the teacher should elicit favourable reactions from the learners.*

Outcomes for the learners: (i) To know the basic concepts of triangle Trigonometry
 (ii) To use trigonometric ratios to solve mathematical real world problems

Content	Teacher Activities	Learner activities
<p>Introduction</p> <p>Real world application</p> <p>A 6m long ladder with its foot in the street makes an angle of 30° with the surface of the street when its top rests on a building on one side of the street. The same ladder makes an angle of 40° with the street when its top rests on a building on the other side of the street while the foot of the ladder remains in the same position. How wide is the street (to the nearest meter)?</p>	<p>Teacher gives the problem to learners after they are handed charts and markers to make a drawing to show their understanding of the problem.</p> <p>Instructions:</p> <ul style="list-style-type: none"> • Work in pairs • Make a drawing • Show your drawing • Solve the problem <p>Teacher has to move between the pairs and assist where needed.</p> <p>Teacher guides learners toward correct drawing and correct calculations</p> <p>When pairs are finished, teacher asks one pair to come to the board and explain their solution to the</p>	<p>Anticipated actions:</p> <p>Learners might make the correct drawing</p>  <p>and use the correct ratio: Cosine theta:</p> $\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$ <p>Learners understood that in order to answer the question which was that the width of the street needed to be calculated, two distances had to be found: First distance:</p>

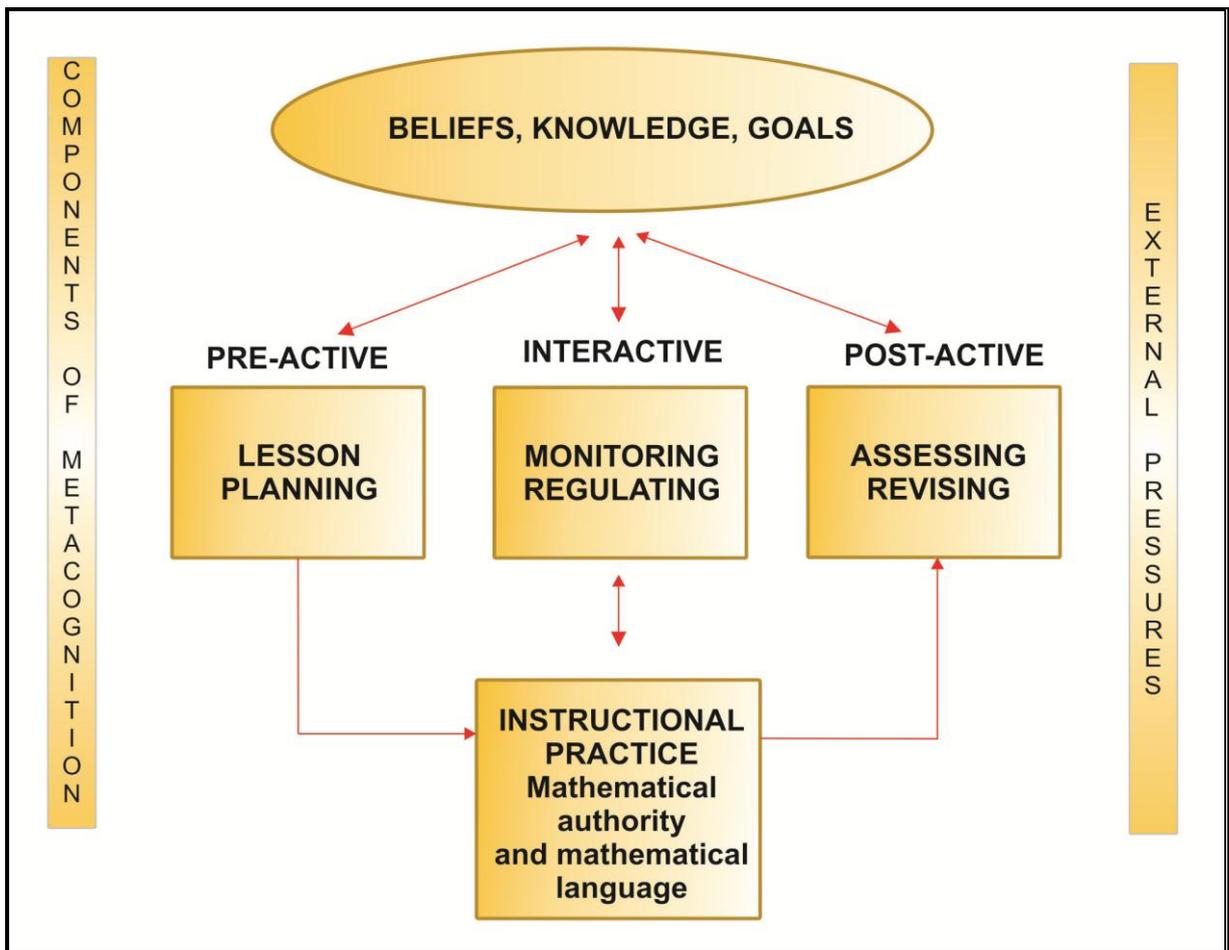
Content	Teacher Activities	Learner activities
	<p>class.</p> <p>Teacher must see that learners draw the correct drawing.</p>	$\frac{\text{distance}}{\cos 30^\circ} = 6m$ <p>Distance = $\cos 30^\circ \times 6m$ = 5.96152423</p> <p>Second distance:</p> $\frac{\text{distance}}{\cos 40^\circ} = 6m$ <p>Distance = $\cos 40^\circ \times 6m$ = 4.5962666659</p> <p>Learners understood the two distances should be added: 5.96152423 + 4.5962666659 = 10.55779089</p> <p>Therefore the street is about 11m wide.</p> <p>Learners might have the drawing incorrect and use correct ratio.</p> <p>Learners might have drawing incorrect and use incorrect ratios.</p>
<p>List of basic concepts: Hypotenuse side, opposite side, adjacent side, right-angled triangle</p> <p>Sine theta: $\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$</p> <p>Cosine theta: $\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$</p> <p>Tangent theta: $\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$;</p> <p>Pythagoras theorem</p>	<p>Teacher revised the basic terminology by question and answer method.</p> <p>Conceptualizing these ratios is important for both the teacher and the learners.</p> <p>Teacher need to let learners explain what the ratio mean.</p>	<p>Learners raised their hands and respond individually.</p> <p>Learners might answer wrongly.</p> <p>Learners might answer correctly.</p>

Figure 7.4: The Hypothetical teaching and learning trajectory for trigonometric functions

8.3.5 The Metacognitive Teaching for Metacognition Framework (MTMF)

Schofield (2012) is of the opinion that, in order to teach metacognitive skills, teachers need to have a broad understanding of the metacognitive process, but Hartman (2002b) argues that teachers need to use metacognitive skills themselves first before they can develop these skills in their learners. I argue that using only one of the frameworks will be inadequate and therefore the two frameworks were merged to form a new framework: the Metacognitive Teaching for Metacognition Framework (MTMF) to analyse the metacognitive skills used by mathematics teachers TwM as well as TfM (Figure 8.1).

The MTMF was not only a simple merging of the two frameworks but the components in each of the frameworks have been carefully put together to avoid repetition in the final framework. From the FAMTAM only mathematical authority in the classroom and external pressures perceived by teachers were selected and added to the TMF, as the knowledge component in TMF already includes the teachers' conceptualization of metacognition as well as the teachers' perceptions of their learners' needs and features. Lastly, mathematical language is added in the final framework as important component in instruction.



Sources: Artzt & Armour-Thomas (2002, p. 130) and Ader (2013, p. 24)

Figure 8.1: The Metacognitive Teaching for Metacognition Framework (MTMF)

Schofield (2012) is of the opinion that, in order to teach metacognitive skills, teachers need to have a broad understanding of the metacognitive process, while Hartman (2002b) argues that teachers need to use metacognitive skills themselves first before they can develop these skills in their learners. I have used the Teacher Metacognitive Framework (TMF) from Artzt and Armour-Thomas (2002: 130) to analyse whether the teachers were teaching *with* metacognition on the one hand, and also the Framework for Analysing Mathematics Teaching for the Advancement of Metacognition (FAMTAM) from Ader (2013: 24) to analyse whether teachers were teaching *for* metacognition. From the analysis (§7.2.3.1) I found that using only one of the frameworks proved to be inadequate. The components of both frameworks were shown to be present in the lessons and therefore the two frameworks were merged to form a new framework: the Metacognitive Teaching for Metacognition Framework (MTMF). The MTMF was not only a simple merging of the two frameworks but the components in each of the framework were carefully put together because it was evident from the lessons and focus group discussions that teaching *with* metacognition is as important as teaching *for* metacognition. Lastly, mathematical language was added in the final framework as an important component in instruction, because mathematical language was found to be a valuable component towards metacognitive instruction within the lessons.

8.3.6 Chorus answering enhances learner participation but hinders the use of metacognitive skills

One of the guiding principles that were used in the Hypothetical teaching and learning trajectory was that the actions from the teacher should elicit favorable reactions from the learners. Chorus answering emerged from the data as a prominent reaction that was elicited from the learners (§7.2.3.2 and §6.3.3). Literature on chorus answering seems to be non-existent. A possible reason for this phenomenon might be that this is the case due to the unpopularity of chorus answering as reaction from the learners: “Too much speaking and indiscipline in class is an on-going problem for any teacher’ (McBain, 2011, p. 1). The analysis in this inquiry showed that chorus answering in actual fact ensured participation and involvement from learners. Whether this involvement is in any way productive is another avenue for research. However, the only literature that could be nearly in line with chorus answering was found in an article by McBain (2011) titled: “Speaking in the classroom” in which McBain (2011, p. 2) mentions the predicament “when other students are talking at the same time”. In this inquiry it appeared as if those questions that require only a yes or a no, elicit chorus answering. Also the questions for which the answers require content language appeared to be the ones that elicit chorus answering. The teacher’s input in the focus group discussions concurred with literature (McBain, 2011) in that the teacher has to prevent

chorus answering because for these teachers chorus answering was not helping towards metacognition. In fact, the teachers decided that it was hard to understand what learners are thinking when they are chorus answering the questions posed by the teacher.

8.3.7 Conceptualization happened within the Mathematics Educators' Reflective Inquiry (ME'RI) group

Conceptualization, in the broad sense of the word, refers to the process of the development and clarification of concepts. It is important to note that this implicates a cognitive action and involve a process of clarifying one's concepts with words and examples. Conceptualization within this inquiry had the meaning of finding the bigger picture. The ultimate goal with Mathematics instruction is by no means training, but that learners get the big picture, that they can make sense of Mathematics. I argue that for this to happen the teacher must be able to conceptualize first, to make a cognitive picture and guide learners to do likewise. The teacher is therefore instrumental in this regard. Teachers need to conceptualize daily. They need to model good conceptualization skills in order for their learners to draw mindmaps for themselves. Schoenfeld (1992) shares with us how he experiences Mathematics:

The wonderful thing about mathematics is that it coheres: when you understand a mathematical idea, everything fits in place beautifully (Schoenfeld, 1992).

The ME'RI group was the platform where this conceptualization happened in this study. It meant that teachers and lecturers could share their knowledge, expertise and experiences to conceptualize. Conceptualization was made possible within this inquiry and for that matter within the ME'RI group by placing an explicit focus on metacognitive skills and mathematical language. Teacher A, for example could form for himself the bigger picture of trigonometric functions and could come to an understanding why it makes no mathematical sense to divide by Tan only. From Teacher A's misconception, he could conceptualise Tan theta and what the meaning was of the concept Tan theta.

Teachers need platforms such as the ME'RI group to give them opportunities to focus on their metacognitive skills and mathematical language to help them make meaning of concepts, to conceptualize. The ISPF for TED (Department of Basic Education, 2011b, p. 16) recommends that:

Time for teachers to participate in professional learning communities and engage in quality teacher development must be deliberately and formally scheduled.

This recommendation indicates the need for effective opportunistic collaboration amongst educators who share the same goal of quality teaching. In this inquiry the ME'RI group which was formed by the community of Mathematics teachers from two township schools and the lecturers from the nearby university proved that this collaboration could realize if the members of the group are committed and could stay faithful to the goal. Although time was a big challenge and it was not easy to get all the members of the group together each time, it was possible to meet for five times in the two years while five of the six teachers got the opportunity to present a lesson. Not only did the teachers learn from this collaboration, the lecturers also appeared to have gained from this collaboration (§7.3.2.2). It appears as if the ME'RI showed great promise for future professional development endeavours for the teachers and the lecturers in this inquiry.

8.4 ADDRESSING THE RESEARCH QUESTIONS

8.4.1 The primary research question

What are the characteristics of an in-service arrangement that facilitates the implementation of lesson activities focussing on the metacognitive skills and mathematical language of Mathematics teachers for the teaching of trigonometric functions in township schools in the Dr Kenneth Kaunda District?

Addressing this primary question entailed firstly some kind of guidelines for the instructional practise of how to teach trigonometric functions focussing on the metacognitive skills and mathematical language used by Mathematics teachers (HTLT). Secondly it hinted on some form of professional community of Mathematics educators who could bounce off ideas, share and discuss in order to improve on their instructional practise (ME'RI). The endeavours embarked upon in this enquiry in order to address the primary research question resulted in an in-service arrangement that indeed facilitated the implementation of lesson activities focussing on the metacognitive skills and mathematical language of Mathematics teachers for the teaching of trigonometric functions (Figure 8.2). Following the work of one of the more popular mathematicians in the field of mathematics sense-making, Alan Schoenfeld (2012), I refer to this in-service arrangement as a microcosm (small society, place or activity which has all the typical features of a much larger one) of mathematical practice in which the interaction between teachers and lecturers focussing on the metacognitive skills and the mathematical language used by teachers when teaching Trigonometric functions brought about conceptualization in general firstly for the mathematics teachers, secondly their learners and also even to the lecturers. Therefore the simple mathematics equation:

Metacognitive skills + mathematical language = enhanced conceptualization skills

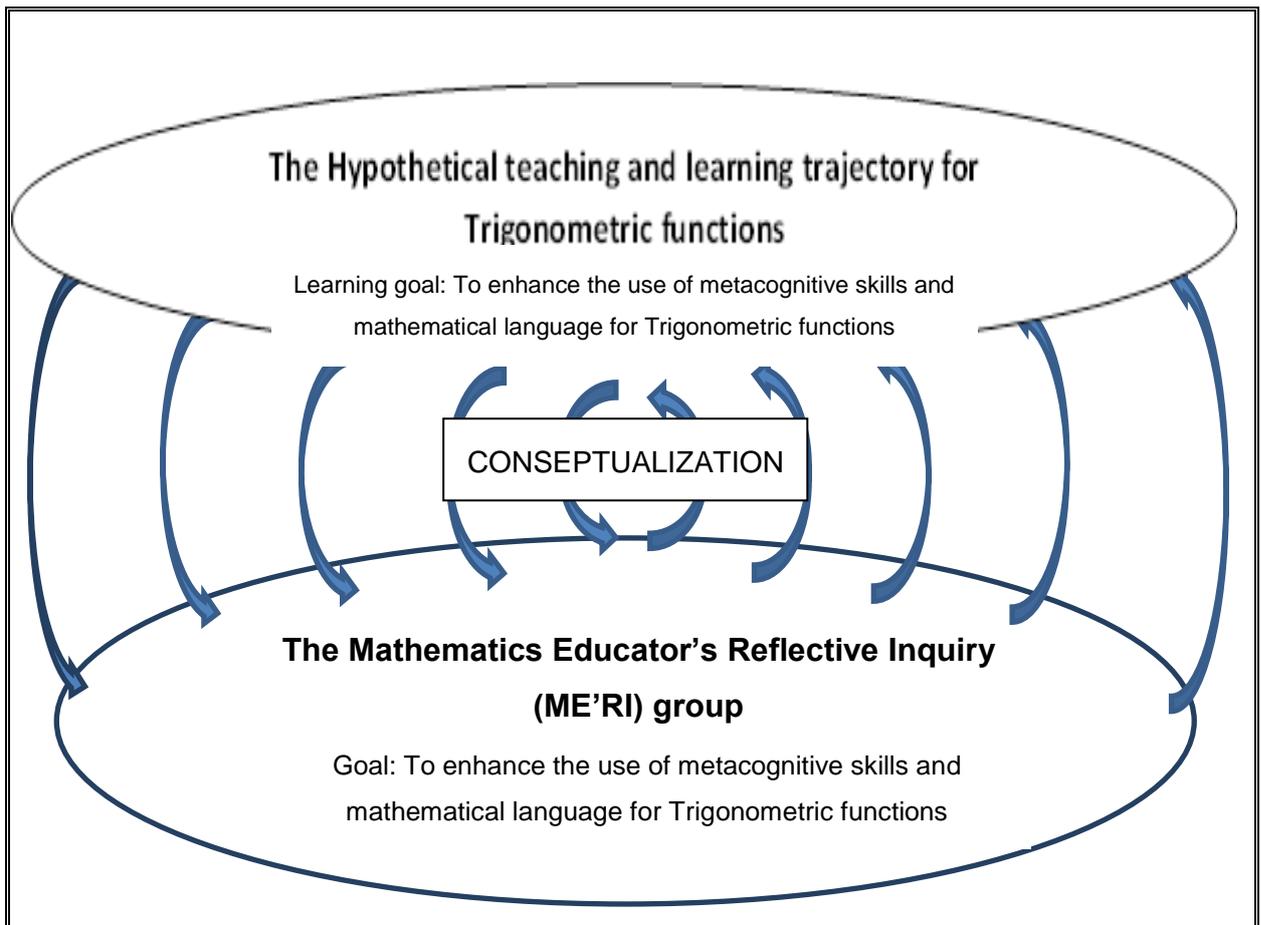


Figure 8.2: A microcosm of mathematical practice for enhancing the teaching of trigonometric functions

The interaction between the teachers presenting the lessons and the research team who in this inquiry is referred to as the ME'RI, happened iteratively in order for each iteration to inform the next one. The characteristics of such an arrangement are therefore as follows:

- It is iterative of nature.
- It focuses on the metacognitive skills and mathematical language of the Mathematics teacher.
- It entails one common goal shared by each member in the group.

8.4.2 The research sub-questions

The primary research question had been broken down into three sub-questions, *viz.:*

1. How do Mathematics teachers apply metacognitive skills and mathematical language in the teaching of trigonometric functions?
2. What are the challenges Mathematics teachers are facing in the teaching of trigonometric functions?
3. How can the teaching of trigonometric functions be improved when focussing on the metacognitive skills and mathematical language used by Mathematics teachers?

Table 8.1 shows how these questions were addressed in this inquiry:

Table 8.1: Summary and findings of all the data with reference to the research questions

Primary Research Question				
What are the characteristics of an in-service arrangement that facilitates the implementation of lesson activities focussing on the metacognitive skills and mathematical language of Mathematics teachers for the teaching of trigonometric functions in township schools in the Dr Kenneth Kaunda District?				
Secondary Research Questions				
Data collection instruments	1. How do Mathematics teachers apply metacognitive skills and mathematical language in the teaching of trigonometric functions??	2. What are the challenges Mathematics teachers are facing in the teaching of trigonometric functions?	3. How can the teaching of trigonometric functions be improved when focussing on the metacognitive skills and mathematical language used by Mathematics teachers?	
Findings	Individual interviews	The metacognitive performance profile of Teacher A (6.3.1) and that of Teacher B (6.3.2) that teachers in the inquiry are using metacognitive skills to a certain extend but this can be enhanced to a great extend more if teachers are more aware of reflection in the Mathematics classroom. The use of mathematical language of the teachers was reasonable although they were using content language most of the time.	<ol style="list-style-type: none"> 1. The negative attitude of learners towards Mathematics. 2. The negative attitude of some Mathematics educators towards Mathematics. 3. Lack of content knowledge of some Mathematics educators. 4. The pride of some Mathematics teachers not to ask help in cases where they do not understand the content. 5. More definitions in Trigonometry. 6. Learners are failing to see the big picture. 7. Poor learner performance in Mathematics. 8. Learners do not practise Mathematics at home. 	
	Lesson observations	The Hypothetical teaching and learning trajectory (HTLT) for trigonometric functions.		
	Focus group discussions	The Mathematics Educators' Reflective Enquiry (ME'RI) group		
Assessment task	Teachers did not exercise good metacognitive skills Mathematical language proved to be difficult to comprehend when a problem has to be solved in real life situation.	Language is problematic Misconceptions of teacher and learners		
Relevant Literature	Literature on metacognition of teacher quite scarce. Mathematical language is becoming increasingly important. Code switching	Order of operations, value and place of the sine and cosine functions, improper use of equation, distorted definitions, misused data, misinterpreted language, logically invalid inferences and technical mechanical errors (Gur, 2009)	Guidelines for the introduction of trigonometric functions (Quinlan (2004:24)	

8.5 RECOMMENDATIONS AND POSSIBLE QUESTIONS FOR FURTHER RESEARCH

Firstly the in-service arrangement (Figure 8.2) is recommended to be used in any situation that asks for collaborative interaction between teachers and lecturers (§8.4) in order to improve instructional practise in Mathematics education or even the education of any other subject where the following needs might arise.

1. The need of teachers to gain more subject knowledge on the teaching of trigonometric functions;
2. The need of teachers to gain more subject knowledge on a particular topic in the syllabus which was not sufficiently addressed during the training of teachers provided that the trajectory is adjusted with the new topic;
3. The need of teachers to gain more in-depth knowledge on how to teach a specific topic in the syllabus which proved to be difficult for learners to understand;
4. The need of lecturers or researchers to gain more or new understanding on specific phenomena that require research on.

Possible avenues for future research include the following:

- The Hypothetical teaching and learning trajectory (Figure 7.4) could be improved in further research studies in metacognition using a mixed method approach, as this inquiry employed a purely qualitative approach.
- The in-service arrangement (Figure 8.2) can be employed in future research endeavours in order to be adjusted to suit the particular context.
- The hypothetical teaching and learning trajectory might be used to inform code switching in Mathematics instruction.

8.6 CONCLUSION AND REFLECTION

The findings in this inquiry show noteworthy stimulation of circumstantial variables in challenging the practicality and effectiveness of the intervention for the improvement of instructional practices in Mathematics education and in particular the teaching of trigonometric functions. The focus on the metacognitive skills and mathematical language of the teacher in particular is unique as other similar studies usually concentrate on the learners and were not connecting metacognition with mathematical language. It seems that teachers need to structure their thinking more clearly around what it is they actually want their learners to learn. In this regard the mathematical language that teachers are using plays an integral role in the instructional practise of the teacher. The discussion of chorus answering as a prominent feature in South African Mathematics classrooms which we need to embrace, also stands out as new and different and can be seen as a way of democratizing culture in the South African Mathematics classroom.

I must acknowledge though, that there are various other factors that influence the effective teaching of trigonometric functions and that no clear-cut answer to the question on how the teaching of trigonometric functions could be enhanced, is possible. However, collaborating with other Mathematics educators might be the first step towards quality in Mathematics education.

Through this study I had the opportunity to do some valuable reflection on my own teaching practises as well here at the North-West University. I have gained valuable experiences and “critical friends” who left me with the desire to continue working with my teachers in the ME’RI group for the betterment of Mathematics education.

I conclude this study with a proposition made by Alan Schoenfield (2012, p. 3), renowned academic and one of the greatest mathematicians of our time (in my mind):

I propose that we all, each time we teach, stop to think about how and why the mathematics fits together the way it does and how we can help our students to see it that way. We owe our students no less.

DOCEAMUS - let us teach