Development of a system for tracking objects in a confined space

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Declaration

I, Sarel Joubert de Wet hereby declare that the dissertation entitled “Development of a system for tracking objects in a confined space” is my own original work and has not already been submitted to any other university or institution for examination.

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Signed on the 20th day of November 2009 at Potchefstroom.
Acknowledgements

First of all, I would like to thank my Lord and Saviour, for without His mercy, I would not be where I am today.

I CAN DO ALL THINGS THROUGH CHRIST WHO STRENGTHENS ME
Philippians 4:13

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Abstract

The Pebble Bed Modular Reactor (PBMR) is one of the alternative generation options that Eskom is currently investigating to replace their old coal-fired plants. The PBMR plants are smaller than conventional plants and therefore require a smaller lead time to construct. If successfully demonstrated, the plants could comprise 20% of the country’s nuclear build programme.

The flow of fuel spheres through the Reactor Pressure Vessel (RPV), a vertical cylinder with height 27 m and diameter 6.2 m, determines the energy level in the RPV. If the flow paths of these fuel spheres are known, reactor geometries can be optimised. Currently the flowpaths of the spheres are either estimated based on previous results from different reactor geometries, or simulated using PFC-3D simulation software, based on spherical models that doesn’t correspond to the actual spheres. A system able to accurately track the spheres through the RPV will enable further research to be done into the flow of the spheres.

In this research we aim to develop a concept system that could be used as the base for a system that can track objects accurately in a confined space. Such a system could be used to track the flow of the spheres through a model of the RPV.

During our literature study on tracking systems we found that it is hard to compare different tracking systems with each other, due to the diversity in the applications of these systems. We found a few surveys and taxonomies, but these were confined to specific application domains, and thus couldn’t be used to characterise and classify systems outside of the domain. Based on our need to compare different systems from different application domains with one another we developed a characterisation and classification method based on the basic aspects of tracking systems identified during our literature study. The method enables the characterisation and classification of a tracking system to a general form. Using this method enables us to compare systems from various different application domains.
The concept system is developed based on radio interferometric positioning. We derive a mathematical model for the concept system and use this model to implement an ideal deterministic simulation of the system. We use the simulation and follow an empirical investigation to determine the effect of certain parameters on the accuracy of the system. The results obtained with these simulations are then used to make recommendations concerning the setup of the concept system. Using these recommendations as inputs, a simulation is done for a set of 20 random positions. The maximum localization error made during the simulations was 6.5 mm, much smaller than the 3 cm resolution required by the system. This implies that the concept system is a viable option for tracking the spheres through the RPV model.

**Keywords:** Tracking, Localization, Position Estimation, Radio Interferometric Positioning System (RIPS), Phase Difference
Opsomming

Die Korrelbed Modulêre Reaktor (KBMR) is ’n alternatiewe energie oplossing wat tans deur Eskom ondersoek word om hul ou steenkool generasie aanlegte aan te vul. Die KBMR is kleiner as konvensionele aanlegte en kan dus vinniger opgerig word. As die aanleg suksesvol demonstreer word, kan dit tot 20% van die land se kern program opmaak.

Die energie vlakke in die reaktor druk houer, ’n silinder met hoogte 27 m en diameter 6.7 m, word bepaal deur die vloei van die brandstofkorrels deur die druk houer. As die vloei paaie van die korrels bekend is, kan die geometrie van die reaktor optimiseer word. Tans word die vloei paaie geskat, gebaseer op resultate verkry met ander reaktor geometrie of gesimuleer deur gebruik te maak van PFC-3D simulasië sagteware. Hierdie sagteware maak gebruik van modelle wat nie ooreenstem met die werklike korrels nie. ’n Stelsel wat in staat is om die posisies van die korrels akkuraat te bepaal soos wat hul deur die reaktor beweeg sal die geleentheid bied vir verdere navorsing om gedoen te word op die vloei van die korrels.

Hierdie navorsing poog om ’n konsep daar te stel wat gebruik kan word as ’n basis vir ’n stelsel wat die posisie van objekte akkuraat kan bepaal in ’n beperkte area. So ’n stelsel sal dan ook gebruik kan word om die vloei pad van sfere deur ’n model van die reaktor druk houer te bepaal.

Gedurende ons literatuurstudie het ons gevind dat dit moeilik is om verschillende sporingstelsels met mekaar te vergelyk a.g.v. die wye verskeidenheid toepassings vir hierdie tipe stelsels. Ons het verder gevind dat die enkele opnames en taksonomieë wat in die veld gedoen is beperk is tot spesifieke toepassings en dus nie gebruik kan word om stelsels uit verschillende toepassingsareas met mekaar te vergelyk nie. Om ons behoefte om stelsels uit verschillende toepassings areas met mekaar te vergelyk aan te spreek, het ons ’n karakterisering- en klassifiseringsmetode ontwikkel gebaseer op die basiese aspekte van sporingstelsels, soos geïdentificeer gedurende die literatuur studie. Die metode stel ons in staat om stelsels uit verschillende toepassingsareas met mekaar te vergelyk.
Ons ontwikkel die konsep gebaseer op radio interferometriese posisionering. Ons lei ’n wiskundige model vir die konsep af uit die ontwerp en gebruik die model om ’n ideale, deterministiese simulasie implementering van die stelsel te doen. Deur gebruik te maak van die simulasie, volg ons dan ’n empiriese benadering om die invloed van sekere parameters op die stelsel te bepaal. Die resultate word dan gebruik om voorstelle te maak aangaande die opstelling van die stelsel. Deur die voorstelle as inset te gebruik simuleer ons die stelsel vir 20 willekeurige posisies. Die maksimum lokaliserings fout gedurende die simulasies is 6.5 mm, baie kleiner as die vereiste 3 cm resolusie van die stelsel. Die resultaat impliseer dat die konsep ’n lewensvatbare opsie is om die sporing van die korrels in ’n model van die KBMR te doen.

**Sleuteltermen:** Sporing, Lokalisering, Posisie Estimasie, Radio Interferometriese Posisionering Stelsel, Fase Verskil
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List of Acronyms

2D  Two Dimensional
3AMSA  3-Axis Magnetic Sensor Array
3D  Three Dimensional
ADC  Analog to Digital Convertor
AoA  Angle of Arrival
BS  Base Station
CLSS  Cricket Location-Support System
DAQ  Data Aquisition unit
DFT  Discrete Fourier Transform
FFT  Fast Fourier Transform
GA  Genetic Algorithm
GPS  Global Positioning System
GUID  Globally Unique Identifier
IO  Input Output
KBMR  Korrelbed Modulère Reaktor
LNA  Low Noise Amplifier
LOS  Line of Sight

LPF  Low-Pass Filter

PCB  Printed Circuit Board

PCU  Power Conversion Unit

PBMR  Pebble Bed Modular Reactor

RFID  Radio Frequency Identification

RIPS  Radio Interferometric Positioning System

RSS  Received Signal Strength

RSSI  Received Signal Strength Indicator

RPV  Reactor Pressure Vessel

RPVM  Reactor Pressure Vessel (RPV) Model

SNR  Signal-to-Noise Ratio

ToF  Time of Flight

WSN  Wireless Sensor Network
List of Symbols & Subscripts

List of Symbols

\( \gamma \)  
Phase offset relative to wavelength

\( \lambda \)  
Wavelength

\( \vartheta \)  
Measured phase offset

\( \varphi \)  
Actual phase offset

\( a \)  
Amplitude

\( B \)  
Number of quantization bits of ADC

\( c \)  
Speed of light

\( D \)  
Longest dimension of an antenna

\( d_{XY} \)  
Distance between X and Y

\( d_{XUY} \)  
T-range for quad \((X, Y, U, V)\), X unknown

\( d_{XYUV} \)  
Q-range for quad \((X, Y, U, V)\)

\( f \)  
Frequency

\( F_U(\omega) \)  
Frequency domain representation of filtered rectified signal

\( h_{XY} \)  
Hyperbola with foci X and Y

\( I_U(t) \)  
Interference signal at node U

\( N_q \)  
Number of quantization levels

\( q \)  
Quantization step size

\( R_{XY} \)  
Distance between transverse vertices of \( h_{XY} \)

\( R_{U}(t) \)  
Rectified signal at node U

\( r(t) \)  
RSSI Signal

\( s(t) \)  
Radio Signal
SR  Sampling Ratio
$t_X$  Elapsed time since node $X$ started generating a waveform
$V_{fs}$  Full scale voltage range of ADC

**List of Subscripts**

$c$  Carrier
$e$  Envelope
$cut$  Filter cut-off frequency
$IF$  Intermediate frequency
$XYUV$  Measurements made for quad $(X, Y, U, V)$
$X$  Denotes node or position $X$
Chapter 1

Introduction

This chapter starts with a brief background on the problem addressed by this work, followed by the problem statement and issues to be addressed. The methodology used in the work is then discussed and in the last section an overview of the rest of the dissertation is presented.

1.1 Background

Most of South-Africa’s electricity is generated by large scale coal-fired plants. These plants are all situated inland, close to the major coal producing areas in the country. Due to their location, long powerlines are required to supply electricity to the rest of the country. According to [1] most of these older coal-fired plants reach the end of their designed life by 2025. This, as well as the growing demand for power has prompted the country’s power utility, Eskom, to investigate alternative generation options, especially small plants, that require smaller lead times to construct than conventional plants currently being used, and can be placed near areas with high demand. One of these alternative options is the Pebble Bed Modular Reactor (PBMR). The PBMR is a nuclear reactor currently being developed and researched by PBMR (Pty) Ltd in South-
Chapter 1

Background

Africa. According to [1] the PBMR, if successfully demonstrated, will comprise about 20% of Eskom’s nuclear build programme, thus playing an important role in meeting the country’s future energy demands.

According to [2] the PBMR consists of two main units, the RPV and the Power Conversion Unit (PCU). The RPV is a vertical cylinder of 6.2 m diameter and 27 m height, lined with a 1 m thick graphite brick wall. It houses 450 000 graphite fuel spheres, each containing low enriched uranium triple coated isotropic particles. The spheres are used to heat helium gas flowing through the RPV which turns a gas turbine generator.

The fuel spheres flow from the top to the bottom of the RPV where they are removed via exit funnels [3]. The flow path of the fuel spheres determines the energy level in the RPV. If the flow path of the fuel spheres is known, the energy levels in the RPV can be calculated and thus optimised by using more effective reactor geometries [3]. Currently the flow of the spheres is either estimated, based on experimental results, or calculated using software that simulates the flow of particles in three dimensions, PFC-3D [4]. According to [3] the experimental results upon which the estimated flow paths are based were obtained using different reactor geometries and thus might not be very accurate. Furthermore, the PFC-3D simulation code is based on spherical models that do not correspond to the spheres used in the RPV in terms of elasticity and energy conservation of collisions, thus the simulation results may not be very accurate. Another disadvantage of the PFC-3D simulation software is that it needs to apply the two particle collision model it uses recursively, resulting in very long simulation runs.

The above mentioned methods are currently the only available options to determine the flow of the spheres through the RPV [3]. A system able to accurately track the spheres through the RPV will enable further research to be done into the flow of the spheres. This research can be used to validate the existing flow models as well as the PFC-3D simulation code. Such a system will also provide a tool for future research and the testing of new reactor geometries.

The circumstances (for example the high temperatures) in the RPV make it impractical
to track the spheres in the PBMR. In [3] it was suggested that the proposed system could be implemented by tracking spheres through a smaller scale, room temperature, model of the RPV. This model would have to be made of materials that match the material of the actual RPV and fuel spheres in terms of elasticity and energy conservation of collisions, at room temperature. More specifically, according to [3], the system should be able to track the fuel spheres if it complies with the following objectives:

- The position of a specified sphere in the model will have to be determined with an acceptable accuracy.
- The system will have to monitor multiple spheres during the same time frame†.
- The positions of the tracked spheres will have to be logged in order to construct flowpaths for the spheres.
- The system will have to be adaptable to different reactor geometries in order to test and compare different geometries.
- The system will have to achieve dynamic similarity between the reactor and the experimental setup, thus implying that the system must not be dependant on the mechanical configuration.
- The system must operate at temperatures below 40°C.

A survey of tracking systems currently being used and researched revealed that there exist a plethora of different tracking systems. This is mainly due to the fact that applications for tracking and localization systems range over a very wide variety of different disciplines. For example, objects being tracked can vary from small wireless sensors [5] to an object moving through the human GI tract [6]. These diverse applications result in miscellaneous methods to track or localise these objects. In this regard a lot of research has been done on the different techniques, principles, physical phenomena and so on used to implement tracking systems. Examples include [7] in which a survey and

†Please note that although the objective is to track multiple spheres during the same time frame, this does not imply that the spheres need to be tracked simultaneously, due to the nature of the system setup, see section 5.2.1
taxonomy on tracking systems for ubiquitous computing is presented, [8] in which a survey of the tracking technology used in virtual environments are given, [9] presents a survey on visual object tracking, [10] presents a survey on localization techniques used in Wireless Sensor Networks (WSNs) and the very cited survey on wireless position estimation presented in [11]. This research unfortunately mostly focus on only one application domain, for example [8], focus only on the virtual environment. The result is that no method exist for comparing different tracking systems from different application domains.

From the above it is concluded that in order to develop a system that can be used to track the movement of spheres through a model of the RPV† of the PBMR a thorough survey of

- the aspects affecting how tracking and localization systems are implemented and,
- the tracking and localization systems currently being used and researched

needs to be done.

The survey on the aspects affecting the implementation provides an insight into the functioning of tracking and localization systems. It is also used to create a framework with which to compare different tracking and localization systems from different application domains. The survey is based mainly on taxonomies and surveys from different application domains.

The survey on the tracking and localization systems currently being used and researched is done to gain information and ideas on how to implement a tracking or localization system that address the needs of the system for tracking the spheres through the RPV Model (RPVM).

Using the the information gained during the literature survey, we design a concept system. A mathematical model is derived from this concept and used to implement

†During the rest of the dissertation we will refer to the model of the RPV as the Reactor Pressure Vessel Model or RPVM
a simulation model of the system. The mathematical as well as simulation model are then verified and validated using the guidelines provided by [12]. The verified simulation model can be used to empirically investigate the effect of various parameters on the accuracy of the system. The results from these simulations can be used to make recommendations regarding the set up of the system. Using the set up the system can be simulated to determine whether the concept system is a viable solution.

If the system proves to be a viable solution, it can be used as a base for the development of systems that aim to track objects accurately in a confined space. Such a system can then be used to:

- Research the flow of the fuel spheres in the RPV.
- Validate and update the estimated flow models currently being used.
- Validate and update the PFC-3D simulation software.

From this background the problem statement is given in the following section.

1.2 Problem Statement

The goal of this research is to develop a concept system† for accurately tracking objects in a confined space and testing this concept system for viability.

1.3 Issues to be Addressed

In this work the following issues are addressed to achieve the goal of the research:

- Design of a concept system.

†Please note that we define system as an “organized scheme or method” as found in the Oxford dictionary and thesaurus.
Chapter 1

Issues to be Addressed

- Derivation of a deterministic mathematical model of the concept system as well as the implementation of the mathematical model as a simulation model.

- Validation of the concept system using the simulation model.

The first issue that needs to be addressed is the design of a concept system that can be used to track the flow of spheres through the RPVM. We start the design by considering the objectives as proposed by [3] and given in section 1.1. We base the design on knowledge gained during the literature study on systems currently being used and researched.

From the designed concept system we derive an ideal, deterministic, mathematical model. We then implement the mathematical model as a simulation model, which we use to experiment with the effects of different parameters on the system. We verify and validate both these models using the guidelines provided in [12].

We use the results gained during the empirical investigation of the effects of different parameters on the system, to make recommendations regarding the setup of the system. Using these recommendations we evaluate the accuracy and precision obtained by simulating the position calculation of several randomly selected sphere positions in the RPVM. The main objective of the system is to track the position of the spheres as they move through the RPVM and then use these sphere positions to reconstruct their respective flowpaths. We can thus use the aforementioned simulation results to validate the system by confirming the following:

- The worst accuracy achieved is smaller than 3 cm†.

- The precision with which the above mentioned accuracy can be achieved is 100%†.

- The flowpath of multiple spheres through the RPVM can be reconstructed using the position data.

†The definitions of accuracy and precision as used here are given in chapter 2 in section 2.2
Chapter 1

1.4 Research Methodology

The core process followed throughout this work corresponds to Aristotle’s model of scientific logic, the inductive-deductive method [13] and is illustrated in figure 1.1.

The research methodology used in this work is based mainly on the engineering method as presented in [14] and shown in figure 1.2. It was however slightly adjusted for the purpose of this work as shown in figure 1.3.

The clear definition of a concrete problem is done in section 1.2 in the form of the problem statement. The important factors are given in section 1.3 as the issues to be addressed. The rest of the methodology will now be discussed, starting with the next step as shown in figures 1.2 and 1.3, the formulation of a working model in the form of the Ideal Mathematical Model.

1.4.1 Formulate Working Model: Concept System

The working model in this work is presented in the form of the concept system. The working model is created by addressing the first of the important factors as identified in section 1.3, the development of a concept system.

This issue is addressed by achieving four sub-objectives:

- A literature study on the methods, techniques and principles used in tracking and localization systems.
- A literature study of tracking and localization systems.
- Identifying a possible solution.
- Doing a conceptual design.

These sub-objectives are now discussed in more detail.
Figure 1.1: Aristotle’s model of scientific logic as applied in this work

Literature Study: Methods, Techniques & Principles

The literature study on the methods, techniques and principles used in tracking systems is presented in Chapter 2 as a taxonomy that is used to gain an insight into how tracking and localization systems work, as well as provide a framework for the classification and characterization of tracking and localization systems into a general form. Once in a general form, tracking and/or localization systems from different application domains can be compared more effectively.

Literature Study: Tracking and Localization Systems

The literature study on existing tracking and localization systems is presented in Chapter 3. The study is performed for selected systems, identified during a survey, as systems that could be used as a possible base for the proposed system.
Chapter 1 Research Methodology

Clearly Define Concrete Problem

Postulate Important Factors

Formulate Working Model

Conduct Experiment (Collect data concerning problem)

Estimate Working Model

Determine Important Factors

Revise Working Model

Conduct Confirmatory Experiment

Viable Solution

Not a Viable Solution

Figure 1.2: Research Methodology
Figure 1.3: Methodology as implemented in this work
Identify possible solution

The information and ideas gained during the literature study sub-objectives (as dis-
cussed above) are used to identify possible solutions for the concept system by char-
acterising and classifying the proposed system and then comparing it to the systems
identified during the literature survey in 3. The system identified as the base from
which the concept system is designed, is investigated in more detail in Chapter 4

Conceptual Design

The conceptual design is done based on the above three sub-objectives and is discussed
in detail in Chapter 5.

1.4.2 Conduct Experiment: Simulate Concept System

In this part of the methodology the second issue from section 1.3 is addressed. The
issue is again divided into sub-objectives. These are:

- Derive an ideal deterministic model of the concept system.
- Use the model to implement a simulation of the concept system.
- Validate and verify the mathematical model as well as the simulation.
- Simulate concept system.

These sub-objectives are achieved as follows.

Derivation of a Mathematical Model

The mathematical model is derived for the concept system presented in Chapter 5. The
model is presented in Chapter 6.
Use Model to Implement a Simulation of the Concept System

The derived mathematical model is implemented in a mathematical simulation program. The implementation is discussed in Chapter 6.

Validate and Verify the mathematical model and simulation

The mathematical model and simulation are verified in chapter 6 using the guidelines presented in [12].

Simulate Concept System

In chapter 7 the validated models are used to perform a set of empirical investigations on the effect of various parameters on the system. The simulations are presented in sections 7.2 to 7.9.

1.4.3 Recommendations, Revised Setup and Optimised Simulation

These three steps in the methodology address the third issue from section 1.3. The recommended parameters as used for the revised simulation can be found in table 7.47. A more detailed discussion on the parameters are given in chapter 8. The revised simulation setup as well as the results obtained for the optimised simulation is presented in section 7.10. A conclusion regarding the viability of the system is presented in chapter 8.
Chapter 2

Location System Taxonomy

In this chapter a method for characterising and classifying tracking and localization systems in a general form is presented. The method is based on a literature study of the basic aspects of tracking systems as well as surveys and taxonomies previously done for specific application domains.

2.1 Introduction

In Section 1.1 of the previous chapter it was stated that during our literature survey on tracking and localization systems, it was found that a plethora of tracking systems from different application domains are currently in use and being researched. Due to the fact that these systems are intended for diverse applications, they are based on different principles and physical phenomena. In order to better understand these systems and for them to be compared it is necessary to understand the aspects underlying these systems. In this chapter a literature study on these aspects is presented in the form of
The work is based mainly on previous work done in [7] and [8] as it was found that these articles, although focusing on specific application domains, are general enough to address different systems from different application domains. The aspects presented in the articles respectively are combined and expanded on. Other aspects identified as important from the survey are also discussed. These aspects and their relationships are then used to propose a method of characterising and classifying tracking/localization systems. Once classified in this general form it is easier to compare different systems with one another as well as make informed choices with regards to implementing new systems.

This chapter addresses the first sub-objective as stated in section 1.4.1 and is organised as follows: In Section 2.2 the definitions of ambiguous terms from the field of tracking/localization will be defined as they are used in this work. In Section 2.3 a discussion on what to keep in mind when analysing and comparing tracking/localization systems will be given. In Section 2.4 important aspects affecting tracking systems, as found during our literature survey, and their relationships are described, followed by the proposed method for the classification of tracking/localization systems.

### 2.2 Definitions of Ambiguous Terms

During our literature study we found that some terms one would expect to be standard are either used interchangeably or are used to describe different things/processes. This can be attributed to the fact that there are no standards in the field, due to the variety of different applications tracking and localization systems are used for. To avoid any ambiguity we define the following terms as we use them in this dissertation.

**Definition 1** Localisation refers to the process of determining the position of an object while the object is stationary.
Definition 2  Tracking refers to the process of determining the position of an object while it is moving. Tracking is normally not as accurate as localization, as it requires a finite amount of time to determine the position of an object and the position of the object will change in this time. Note that in this chapter the term Tracking will be used to refer to both Tracking and Localization.

Definition 3  Accuracy defines the error of the measured position relative to the actual position of the object and is given as a distance in meter.

Definition 4  Precision is a measure of how often the stated accuracy can be obtained and is usually expressed as a percentage.

2.3  Initial Considerations when Analysing a Tracking System

It was found that when analysing a tracking system one needs to ask three important questions. These are What, Why, and Where.

What is going to be tracked? This could be people, nodes in a network, cars, and other objects. It is important to know what is being tracked as this imposes limitations on the system and determines certain properties of the system. For example, if a car is being tracked, as the case may well be in a Global Positioning System (GPS), power for the device can be obtained from the vehicle. If the object being tracked is a tag in a crate, issues may arise concerning the power source of the tag. What is being tracked also influences a variety of other factors for example size and computational power. All this needs to be kept in mind when analysing and comparing tracking systems.

Why is the object being tracked? The answer to this question affects the properties, principles and physical phenomena used in the system. For example, if a system
tracks the position of devices in a network to determine what computer is nearest what printer or projector or other peripheral device, many properties, such as the reference grid and resolution of the system, are affected.

**Where** is the object that is being tracked? This refers to the environment in which the object is being tracked. This is important as it has a big effect on the physical phenomena used in the tracking system. For example, a magnetic tracking system can not be used in an environment with lots of magnetic interference or with lots of reflective materials as the results obtained would be adversely affected.

Keeping these three questions in mind will simplify the classification process as it will help with the identification and understanding of the systems requirements.

## 2.4 Classification of Tracking Systems

From the literature survey done it was found that there are four important aspects that need to be understood and considered when evaluating and comparing tracking systems. These are the **Properties**, **Principles**, **Location Computing Techniques** and the **Physical Phenomena** used. The properties as well as the location computing techniques described here are gathered mostly from [7]. The principles are gathered from [8]. Note however that although the aspects as described here draw strongly on the above mentioned articles, it differs in certain areas.

The aspects will now be discussed in the order they are mentioned above.

### 2.4.1 Properties

According to [7], the properties of a tracking system deal with a set of issues that arise when characterising tracking systems and are normally not related to technology and
techniques used in the system. We however found that in some instances the properties may be affected by aspects related to the technology and techniques used in by the system. For example the “limitations” property may be influenced by the physical phenomena used. The properties that are recommended for the characterisation of tracking systems are Resolution, Accuracy and Precision (Accuracy and Precision), Reference Grid (Absolute and Symbolic Location), Physical Position and Symbolic Location, Computational Power (Localised Location Computation), Scale, Recognition, Cost, and Limitations (Note that the properties in brackets correspond to those used in [7]).

Resolution, Accuracy and Precision

Resolution, accuracy and precision are very important characteristics of a tracking system. In order to function correctly a system needs to provide accurate results consistently as stated in [7]. Accuracy is a measure of the difference in the measured position of the object being tracked relative to the actual position of the object. Precision is a measure of how often this accuracy can be achieved and is usually expressed as a percentage. For example a system may yield an accuracy of 10 cm for 95 % of the measurements made. Resolution is defined as the minimum accuracy of the system, which can be obtained with an acceptable precision, in order for the tracking system to achieve its goal.

Reference Grid

In order for the position information yielded by a tracking system to be of any use, it has to be given in terms of a reference grid. Example are the Cartesian grid as used in math and physics, or the grid used for GPS coordinates given in terms longitude, latitude and altitude. Reference grids are mostly used in one of two ways. Position information can be given in terms of a fixed shared reference grid, like the one used by GPS systems. These systems are said to give an Absolute location. Alternatively position information can be given relative to the object tracking other objects or relative
to the object being tracked. In this case the zero position of the grid is not fixed at a specific point. These systems are said to give Relative location. An example of such a system is a system where office equipment knows their location and that of equipment around them. For example, if a computer terminal knows the location of other equipment relative to its own position, it can print to the nearest printer. Note that position information can be converted from absolute to relative for a known grid as well as vice versa if enough relative locations are known.

**Physical Position and Symbolic Location**

A system provides a physical position if it gives position in terms of a physical reference grid, for example, GPS gives a physical position in terms of latitude, longitude and altitude.

A symbolic location system provides an abstract location for the object being tracked. For example, the Active Badge System [15] tracks the location of employees to an abstract location within a building, like a certain room or corridor and can thus be classified as a system providing symbolic location.

Usually a system that provides a physical position has a higher resolution than a system providing symbolic location. If a physical position system has a high enough resolution it may be used to convert the physical position to a symbolic location. For example, if a warehouse is divided into certain 20 m x 20 m zones a physical position system with a resolution of 10 m can be used to determine the symbolic location of an item.

**Computational Power**

The computational power property refers to the computational resources available for implementing the system. For example the RIPS tracking system [16] is used for determining the position of nodes in a wireless sensor network without adding any ad
ditional hardware to the nodes. This imposes certain limitations on the complexity of the computations that can be done by the system.

The two main factors to keep in mind when identifying this property are the computational power of the object being tracked, as well as the computational power of the support hardware or external tracking hardware. These two factors influence aspects like where the location computation of the object can be done e.g. on the object self, *self-positioning* \[11\], or on the external hardware, *remote-positioning* \[11\]. It also influences other properties such as the accuracy and precision that can be obtained.

**Scale**

The scale of a system is affected mainly by two factors, the area covered by the system and the number of objects that can be tracked within infrastructure and time constraints. For example, GPS can be used by an unlimited number of receivers and covers the whole earth, while a system that tracks objects using Radio Frequency Identification (RFID) tags may only be able to deal with one object at a time and in a small space like a room \[7\]. The scalability of a system is thus influenced by how easy it is to increase the infrastructure as well as the cost of the increased infrastructure.

**Recognition**

In some tracking systems the object being tracked needs to be identified. Recognition is usually achieved by designating each object being tracked or classes of objects of the same sort with a Globally Unique Identifier (GUID) \[7\]. An example of a system that uses recognition is the Active Bat system \[17\].
Cost

According to [7] the cost of a tracking system can be assessed according to three different factors, time, space, and capital.

*Time cost* refers to the amount of time the installation and setup of the system requires as well as the amount of time it will take to administrate the system.

*Space cost* refers to the physical dimensions of the tracking system (form factor and size) as well as the amount of infrastructure that needs to be installed.

*Capital cost* refers to all financial costs of the system like the manufacturing and installation price of the system.

Limitations

The limitations of a tracking system refer to what the system cannot do. These limitations are mainly influenced by the environment and can differ from size constraints, to the effect of the environment on the physical phenomena that may be used.

Note that the properties discussed above may influence one another. For example, if a tracking system is to be implemented in a limited space the limitation property of the system is influenced by the cost property (space cost).

2.4.2 Principles

Principles describe the basic method a system uses to track objects [8] and are thus one of the most important aspects to consider when analysing and comparing tracking systems. It determines the core working of the tracking system and is therefore closely linked to the physical phenomena used to implement the system, which in turn
is largely influenced by the environment the system is used in.

The principles as discussed here are based on [8], with the exception of Received Signal Strength (RSS), a principle found to be used often. Note that although [8] focus on principles used in tracking systems for augmented reality applications, it was found that the principles are applicable to most tracking systems.

The principles that will be discussed are **Time of Flight (ToF)**, **Spatial Scan**, **Inertial Sensing**, **Mechanical Linkage**, **Phase Difference Sensing**, **Direct Field Sensing** and **RSS**.

It is also important to note that many systems are based on a combination of these principles and are then referred to as **hybrid** systems.

**Time of Flight (ToF)**

Systems based on the ToF principle determine the distance between two points by measuring the travelling time of a wave from one point to the other. Thus the ToF principle is always implemented using the physical phenomenon of propagation. In order for the principle to be used accurately the propagation speed of the wave used should be constant (or as close to constant as possible) in the medium the tracking is implemented in.

Most ToF systems are implemented using sound waves; usually in the ultrasound range (greater than 20 kHz, normally 40 kHz) as these waves cannot be heard by humans. According to [8] other waves used to implement ToF systems are light waves, using for example pulsed infrared diodes as well as electromagnetic waves.

As ToF systems yield distances and from the physical setup angles can be obtained, it is normally used with triangulation techniques to determine position. A special case of ToF, Angle of Arrival (AoA), estimates the angle between two nodes. It is usually implemented using antenna arrays, by measuring the different arrival times of the signal at different antenna elements for which the geometry of the array is known [11].
Spatial Scan

Spatial scan systems use optical tracking methods, usually for the recognition of known features and their positions, from which angles and distances can be computed. Another form of optical tracking is implemented by measuring the time between a light beam passing one sensor and then another. In [8], spatial scan systems are divided into two categories, outside-in and inside-out.

Outside-in systems comprise external hardware looking for features on the object being tracked.

Inside-out systems use hardware on the object being tracked to identify features or reference points on/in the surroundings.

It should be noted that a main drawback of spatial scan systems is that a direct line of sight is necessary for the system to operate, implying that environment plays a big role in the viability of using a spatial scan system.

Inertial Sensing

Inertial sensing uses the physical phenomena of inertia to determine the orientation and acceleration of the object being tracked. It is usually implemented using accelerometers. From the acceleration data the distance the object has moved can be obtained by double integration over time.

A main drawback of these systems is the fact that the inertial sensors used in the implementation suffer from drift [18] and thus need to be calibrated very often.

Mechanical Linkage

Mechanical linkage systems are systems where the objects being tracked are physically linked to each other in some mechanical way. These mechanical links usually comprise
some sort of arms that can rotate and extend. From the angles and distances the location of the tracked object as well as its orientation can then be determined [8]. We found during our literature survey that mechanical linkage systems are not that common in modern tracking systems.

**Phase Difference Sensing**

Systems based on the phase difference sensing principle use the phase shift of a signal travelling through space to determine distance, as the phase shift is a function of the distance the signal has travelled. According to [8], these systems usually measure the phase of an incoming signal and compare it to a signal of the same frequency on a fixed reference. The implementation of RIPS [16] is a very good example of an innovative use of phase difference sensing, implementing it with an interference signal. Phase difference systems can achieve high resolutions, in the order of centimetres [18]. According to [8], systems based on phase difference sensing can obtain a higher accuracy than ToF based systems due to their ability to generate high data rates.

**Direct Field Sensing**

Systems based on direct field sensing use measurements taken directly from some field e.g. a magnetic field or gravitational field to determine an object’s distance from another and can in some cases also detect its orientation.

These systems are usually implemented using the physical phenomena of magnetic coupling by creating an orthogonal field [8]. Magnetic trackers are inexpensive, lightweight and compact; as a result they are widely used in the augmented and virtual reality realms for tracking body and head movement. Other magnetic phenomena may also be used as well as gravitation. An example of a position and orientation tracking system based on magnetic field sensing is discussed in [19].
RSS

As a wave propagates through space from the point where it is emitted, its power/energy decreases. Systems based on the RSS principle use the signal strength of the received signal to estimate the distance from the receiver to the transmitter. According to [11] the received signal strength can be used with path-loss and shadowing models to determine distance estimates.

2.4.3 Location Computing Techniques

The implementation of a tracking system using one or a combination of the principles mentioned above usually provides some sort of measurement e.g. the distance from one object to another, but normally the position of the object is still unknown.

Location computing techniques are used to determine the position of the object(s) being tracked according to the reference grid used by the system.

The three principle techniques identified in [7] are Triangulation, Scene Analysis, and Proximity.

In [11] location techniques are categorized into three types, Mapping (Fingerprinting), Geometric, and Statistical.

**Mapping (Fingerprinting)** refers to techniques where previous position data, for known locations, are used to train a system to determine or estimate the location of a target node.

**Geometric** techniques estimate the position of the target node from positional data obtained from the principle used to implement the system, e.g. distance and angle information if a ToF system is considered, using geometric relationships.

**Statistical** techniques estimate the position of the target node from positional data using statistical approaches.
The techniques from [7] are mainly geometric, with scene analysis, a mapping technique, the exception. Statistical techniques are not discussed here, but is a very interesting alternative to geometric techniques.

The three principle techniques will now be discussed in more detail.

**Triangulation**

The triangulation technique is based on the geometric properties of triangles and uses distance and angle measurements to compute the position of an object [7]. This technique can be divided into two categories, *Lateration* and *Angulation*.

*Lateration* uses only distance measurements from known reference points to determine an object’s location [7]. In two dimensions three non-collinear measurements are needed to locate an object’s position. The position is determined by finding the intersection of the three circles with the reference point as centre and the distance from the reference point to the object as radius (see figure 2.1). This technique can be extended to obtain the Three Dimensional (3D) position of an object. Lateration for the 3D case require three non-coplanar distances from the object to known locations. The position of the object is at the intersection of the four spheres with reference point as centre and radius, the distance from the reference point to the object. The amount of known ranges required may be reduced by domain specific knowledge; for example in the Active Bat System [17] measurements are made from an array of receivers in the ceiling of a building, three dimensional position can thus be determined from only three distance measurements as one of the two points of intersection (the one above the array of receivers) can be ignored [7].

*Angulation* uses a combination of angles and distances between reference points and the object to determine its position. Two dimensional angulation uses two angles and the distance between the reference points to determine the position of the
object (see figure 2.2). Three dimensional angulation requires one length measurement, one azimuth measurement, and two angles to determine the location of an object [7].

The triangulation techniques as described above can be used with all systems that yield distance and angle information. Systems that give these results are usually based on ToF, Mechanical Linkage, and Phase Difference Sensing. Some beam scanning systems based on the Spatial Scan principle could also yield the results required to use triangulation techniques.
Scene Analysis

According to [7] scene analysis uses features in a scene to estimate the position of the observer. The scenes are usually simplified to make feature recognition and comparison easier. There are two main forms of Scene Analysis, static and differential.

**Static** scene analysis compares the features from the observed scene to a dataset. The dataset contains the location from where specific features were recognised. A features match in the dataset thus yields the approximate location of the observer.

**Differential** scene analysis uses the change in the scene to compute movement, as changes in the scene correspond to movement of the observer. The position of known features enables the observer to determine its position relative to the feature [7].

This technique is used with optical spatial scan systems, as they yield scene results that can be used as a way to implement scene analysis with feature recognition and comparison.

Proximity

According to [7], systems using the proximity technique determine an object’s location near a known location using a physical phenomenon with limited range. The methods usually used for the implementation of this technique are Physical Contact, Monitoring wireless access points, and Observing automatic ID systems [7]. It was also found that another common method for implementing proximity systems is by using beacons, placed in predetermined zones (like rooms in a building). Examples of systems using this technique are the Active Badge system [15] as well as the Cricket location support system [20].

Using physical contact to determine an object’s position can be done by using pressure
sensors or touch sensors [7]. As soon as physical contact is made it can be assumed that the object is near the location of the object that contact was made with.

By monitoring wireless cellular access points it can be determined when the object being tracked is in range of one or more of the access points. The position of the object is then known with an accuracy that corresponds to the size of the area serviced by the access point [7].

Automatic ID systems include systems like credit card point of sales terminals and land line telephone records. By determining the location of the credit card point the location of the person using it can be tracked.

2.4.4 Physical Phenomena

It was found that the physical phenomena used to implement a tracking system, although closely related to the principle used, is an important aspect of the system. It is influenced by the environment and, vice versa, is the factor determining in which environments the system will be able to function. Physical phenomena are divided into four main categories: Propagation, Optic, Inertia and Magnetic.

Propagation

Propagation refers to all the physical phenomena using the propagation of waves through space to determine position. The distance a wave travelled through space is a function of the travelling time as well as the propagation speed of the wave and can thus be used to determine distance. The phase offset as a result of the travelling time of the wave can also be used to determine distance travelled.
Optic

Optic phenomena are used to implement spatial scan systems. Systems using these phenomena require a clear line of sight to or from the object.

Inertia

Inertia refers to the physical phenomenon as described by Newton’s first law of motion. It is usually implemented with gyroscopes or accelerometers. The main drawback of systems using these technologies is that they suffer from drift [18] and thus need to be calibrated often to ensure accurate results. This physical phenomenon is used to implement the inertial sensing principle.

Magnetic

Magnetic phenomena are mainly used in the form of magnetic field sensing (magnetic coupling) where magnetic fields radiated by a source (usually comprised of three coils placed perpendicular to each other to create an orthogonal field) induce a flux in a receiver. The flux is a function of the distance and orientation between the source and the receiver [8]. Systems can also use magnetic phenomena to measure its orientation with respect to a known magnetic field, for example the earth’s magnetic field.

2.4.5 Relationship between the Main Aspects

All the main aspects of tracking systems as discussed are related to each other. The principle that is used affects the location technique that can be used. The physical phenomenon used is in turn directly linked to the principle used. The properties of the system affect the choice of principle, location technique and physical phenomenon used by a system. Table 2.1 gives an indication of the aspects affected by the different properties of a system.
Table 2.1: Relationship of properties with other aspects

<table>
<thead>
<tr>
<th>Property</th>
<th>Principles</th>
<th>Location Techniques</th>
<th>Physical</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resolution, Accuracy &amp; Precision</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>Reference Grid</td>
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<tr>
<td>Physical Position &amp; Symbolic Location</td>
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<td>Cost</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>Limitations</td>
<td></td>
<td></td>
<td>x</td>
</tr>
</tbody>
</table>

2.4.6 Proposed Classification Method

We propose the following method for classifying a tracking system:

1. Determine the system requirements
   The system requirements can be determined by answering the three questions in section 2.3.

2. Characterise the system according to the system properties
   Using the requirements identified in the previous step, the system can be characterised according to the properties of tracking systems as discussed in section 2.4.1.

3. Identify the most important properties
   Identify the properties most important to the success of the system. For example, if it is important that the system has a high accuracy and is intended to work in a highly reflective environment, the most important properties are the Resolution, Accuracy & Precision and Limitations properties.

4. Determine the Principle, Location Technique and Physical Phenomenon
   Once the most important properties affecting the system are known, it can be used to determine the best principle, location technique and physical phenomenon to use. For example, if the most important property of the system is recognition,
it would be best to first decide on a physical phenomenon that can be used to implement GUID. The next step would then be to determine which principle and location technique can be used with the physical phenomenon.

### 2.5 Summary

In this chapter the important aspects of tracking systems as identified during our literature survey, were presented. The aspects are:

- **Properties** - Used to characterise the system.
- **Principles** - The basic method a system use to track an object(s).
- **Physical Phenomena** - Used to implement the tracking system.
- **Location Computing Techniques** - Computes the object(s)’ positions with the data obtained from the combination of principle and physical phenomenon used.

It was found that the factors are all related to one another, with the properties affecting the principle, location computing technique and physical phenomena that can be used to implement the system.

These aspects and their relationships were used to form the base for a method that can be used to characterise and classify tracking systems.

This method provides a tool used in the next chapter to characterise and classify tracking systems in order to better understand the capabilities and limitations of the systems as well as compare the systems with each other, as well as enabling us to determine an appropriate set of principles, techniques, and physical phenomena to be used for the proposed tracking system.
Chapter 3

Characterisation and Classification of Selected Tracking and Localisation Systems

In this chapter a literature study of Tracking and Localisation Systems that could be used as a base for the proposed tracking system are discussed within the framework of the characterisation and classification method presented in the previous chapter.

3.1 Introduction

In the previous chapter a method for characterisation and classification of tracking systems is presented. In this chapter the method is used to characterise the proposed tracking system enabling us to determine an appropriate set of principles, techniques, and physical phenomena to be used. The characteristics of the proposed system are then used to identify existing tracking/localisation systems that can be used as a base for the proposed system (note that only the systems that was identified as the most
viable will be discussed here). The method is then used to characterise and classify these systems to

1. better understand the systems and how they work and

2. compare the systems with each other.

After the systems are discussed, they are compared with one another as well as with the proposed system. Finally a conclusion as to which system(s) are to be used as a base for the concept solution is made.

This chapter address the second sub-objective as stated in 1.4.1. The characterisation of the proposed tracking system is now done.

### 3.2 Characterisation and Classification of Proposed System

The first step in the process of characterisation and classification as presented in the previous chapter is to determine the system requirements by answering the three questions in section 2.3.

1. **What** - The system needs to track spheres with 6 cm diameter.

2. **Why** - The spheres need to be tracked to determine their flowpaths through a model of the RPV of the PBMR, the RPV Model (RPVM).

3. **Where** - The spheres are to be tracked in the RPVM, a cylinder with height 2 m and radius 0.5 m.
Characterisation and Classification of Proposed System

3.2.1 Characterisation of Proposed System

The answers form section 3.2 are now used to characterise the proposed system as shown in table 3.1.

<table>
<thead>
<tr>
<th>Property</th>
<th>System Characteristic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resolution, Accuracy &amp; Precision</td>
<td>Resolution smaller than 3 cm</td>
</tr>
<tr>
<td>Reference Grid</td>
<td>Absolute 3D Cartesian Grid</td>
</tr>
<tr>
<td>Physical Position &amp; Symbolic Location</td>
<td>Physical Position</td>
</tr>
<tr>
<td>Computational Power</td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>Internal</strong></td>
</tr>
<tr>
<td></td>
<td>Limited space available</td>
</tr>
<tr>
<td></td>
<td><strong>External</strong></td>
</tr>
<tr>
<td></td>
<td>Good</td>
</tr>
<tr>
<td></td>
<td><strong>Position Computation</strong></td>
</tr>
<tr>
<td></td>
<td>Remote Positioning</td>
</tr>
<tr>
<td>Scale</td>
<td><strong>Size of Cylinder, One object</strong></td>
</tr>
<tr>
<td>Recognition</td>
<td>No</td>
</tr>
<tr>
<td>Cost</td>
<td><strong>Time</strong></td>
</tr>
<tr>
<td></td>
<td>Unknown</td>
</tr>
<tr>
<td></td>
<td><strong>Space</strong></td>
</tr>
<tr>
<td></td>
<td>Limited space on sphere</td>
</tr>
<tr>
<td></td>
<td><strong>Capital</strong></td>
</tr>
<tr>
<td></td>
<td>Unknown</td>
</tr>
<tr>
<td>Limitations</td>
<td><strong>No visual Line of Sight (LOS)</strong></td>
</tr>
</tbody>
</table>

For the proposed system the resolution, accuracy and precision is a very important property. From the requirements it is known that the spheres need to be tracked with a resolution of at least 3 cm to achieve its goal of describing the flowpaths of the spheres. This implies that an accuracy of 3 cm or better needs to be obtained with a high precision (90 % to 100 %).

The proposed system will need to yield positional data in terms of a fixed reference grid that can be used to relate the positions of the objects to one another as well as their position in the cylinder. We propose that the system yields an absolute location given in terms of a 3D grid. Due to the requirement that the system be adaptable to different reactor geometries we decide to use a cartesian rather than cylindrical grid due to the fact that it will be easier to relate positions from the cylinder to a cartesian grid, than to relate positions in a non-cylindrical setup to a cylindrical grid. The use of an absolute location given in terms of a 3D cartesian grid will enable the logging of the
movement of the spheres as well as determining their flowpaths through the cylinder.

The proposed system will need to yield Physical Position, as a symbolic location would be too vague to determine the flowpaths of the spheres.

The proposed system will need a fair amount of computational power to provide the high resolution required for the system to achieve its goal. It is highly likely that the location computation of the system will need to be done on the external hardware (remote-positioning) as there is a limited amount of space on the object (6 cm diameter sphere).

The scale of the proposed system is limited to the size of the RPVM, a cylinder with height 2 m and radius 0.5 m. It is only necessary to be able to track one object in this area at a time, due to the fact that the configuration of the spheres in the cylinder only change once a sphere is removed from the cylinder via the exit funnel and thus the position of all the spheres can be determined before a sphere is removed and then the process can be repeated. Although this might seem to conflict with the second objective as given in section 1.1, the same time frame, can be interpreted as the time elapsing between one sphere being removed (and thus the configuration changing) and the next (the configuration changing again). In other words, although the spheres need to be tracked during the same time frame, this does not imply they need to be tracked simultaneously.

As discussed above, the system will only need to track one object at a time and thus the proposed system will not need recognition capability.

As no decision has yet been made about how the system will be implemented it is not possible to determine the cost of the system. It is however necessary to keep in mind that there is limited space available on the spheres that are being tracked, thus the space cost of the hardware to be installed on the spheres are to be kept as small as possible. The capital cost for the implementation of the system may turn out to be quite high as specialised hardware might be needed to obtain the high resolution and small size of the proposed system.
Chapter 3
Characterisation and Classification of Proposed System

The most important limitation on the system is imposed due to the environment. A wireless tracking solution that doesn’t require line of sight will need to be implemented due to the environment (a closed cylinder filled with spheres).

3.2.2 Important Properties of Proposed System

The most important property of the proposed system is the “Resolution, Accuracy & Precision” property, as the system will not be able to accurately enough track the spheres to determine their flowpaths if a Resolution of 3 cm or smaller cannot be obtained. Another important property of the system is the “limitations property“ as it limits the possible Principles and Physical Phenomena that can be used to implement the proposed system.

3.2.3 Principle, Location Technique & Physical Phenomena

Principle

The principle that can be used to implement the tracking system is limited by the environment as was discussed in the previous section. The only principles that can be used are those based on wireless, non LOS physical phenomena. Thus the only principles that can be used for the proposed system are:

- ToF
- Inertial Sensing
- Phase Difference Sensing
- Direct Field Sensing
- RSS
Although the system can be based on the principle of inertial sensing, it will not be considered due to the fact that inertial sensors suffer from drift, as stated in the previous chapter, and frequent calibration will be difficult for the proposed system [18]. Other solutions may also be possible, but will entail adding extra functionality to the device, which is unwanted, due to the size constraint imposed by the sphere.

During the literature survey it was also found that the resolution of RSS systems is not small enough for this application (resolution typically around 2 to 3 m). Different systems implemented on various platforms report that one of the main factors influencing accuracy by causing variations in the RSS is the orientation of the transmitters. For example, the RADAR system [21], implemented using nodes equipped with Digital RoamAbout Network Cards, reported RSS fluctuations as big as 5 dBm due to orientation, while a location tracker by Yun and Kim [22], implemented using active RFID tags, noted the same effect. This error due to orientation is a major issue for the proposed system, as no control over the orientation of the object is possible while it is in the RPVM.

The remaining three principles are used to identified systems that can be used as a possible base for the concept solution.

Physical Phenomena

As with the Principle that can be used, the Physical Phenomenon is also limited by the environment and is restricted to:

- Propagation
- Inertia
- Magnetic

No direct LOS to the sphere is possible, thus eliminating optical phenomena.
Location Technique

The technique used to determine the location of the spheres in the proposed tracking system is limited by the data provided by the combination of Principle and Physical Phenomena that can be used to implement the proposed system. The combinations of Principle and Physical Phenomena identified in the previous two sections provide mainly distance and angle data, although in some cases proximity data may also be yielded. Based on this we assume that the location technique to be used in the proposed system will most likely be either Triangulation or Proximity. For example if a ToF system is used, the range data will be in the form of distances or distances and angles, which implies a triangulation techniques will have to be used.

In the following sections we will discuss three different tracking/localization systems, each based on one of the three principles identified above. The systems were identified as the most viable solutions to be used as a base for the proposed for each of the different identified principles.

3.3 The Cricket Location-Support System

3.3.1 Overview

The Cricket Location-Support System (CLSS) [20] was designed as a support system for in-building location dependant applications with the following main design goals:

- User privacy
- Decentralised administration
- Network Heterogeneity
- Cost
- Room-sized granularity
The system enables *Listeners* that can be attached to mobile or static nodes to determine their own location based on information transmitted by *beacons* placed throughout the building. The beacons transmit concurrent radio and ultrasonic waves. The beacon transmits its location information using the RF signal, while simultaneously also transmitting an ultrasonic pulse. The listener receives both the RF and ultrasonic signals and use them to estimate the distance to the different beacons it can “hear” by using the difference in propagation time. Using this information it can infer the space it is in. Using this location information, a node can make its location known to an advertised service.

### 3.3.2 Characterisation of CLSS

From the overview it is easy to answer the three questions from section 2.3 as follows:

1. **What** - The system tracks mobile and static nodes, irrespective of what they are, by attaching a device called a listener to the node. The nodes may be anything from a person to a printer.

2. **Why** - The nodes use their position information to make their own location known to advertised location based services.

3. **Where** - The system tracks the position of the nodes in a building/indoors.

The system can be characterised as shown in table 3.2.

The resolution of the system is described in [20] as “room sized granularity”, which is defined as being able to determine an object’s position to portions of a room. Nothing is stated about the precision with which this accuracy can be obtained.

The system uses an absolute reference grid, based on the floorplan of the building it is used in.
Table 3.2: Properties of CLSS

<table>
<thead>
<tr>
<th>Property</th>
<th>System Characteristic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resolution, Accuracy &amp; Precision</td>
<td>Room sized granularity</td>
</tr>
<tr>
<td>Reference Grid</td>
<td>Absolute, Office floorplan</td>
</tr>
<tr>
<td>Physical Position &amp; Symbolic Location</td>
<td>Symbolic location</td>
</tr>
<tr>
<td>Computational Power</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Internal Hardware capable of performing location computation</td>
</tr>
<tr>
<td></td>
<td>External Hardware providing location information Self positioning</td>
</tr>
<tr>
<td>Position Computation</td>
<td>Size of Office building with unlimited amount of nodes</td>
</tr>
<tr>
<td>Scale</td>
<td></td>
</tr>
<tr>
<td>Recognition</td>
<td>Yes</td>
</tr>
<tr>
<td>Cost</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Time Low installation and administration time cost</td>
</tr>
<tr>
<td></td>
<td>Space Presumed small, can be attached to object as well as carried on person</td>
</tr>
<tr>
<td></td>
<td>Capital US$10.00 per device (listener or beacon)</td>
</tr>
<tr>
<td>Limitations</td>
<td>Beacon range limited to 9.14 m</td>
</tr>
</tbody>
</table>

The system yields position in terms of parts of a room, thus a Symbolic Location is given.

Although the system uses external hardware to assist, the object being tracked computes its own position using the information provided by the external hardware, thus the system is self-positioning.

The system can cover the area of a building, and is easily expandable by adding more beacons. Due to the fact that the listeners are self-positioning, an unlimited number of objects can be tracked, as an unlimited number of listeners have access to the information provided by the beacons, without affecting each other.

The system incorporates recognition for the objects being tracked (the listener can choose to make its position and thus identity known) as well as the beacons. The beacons transmit its location, thus enabling the listeners to “identify” it.

**Time Cost** The time cost of the system is low due to the fact that the owner of each
space is responsible for the setup of the beacons in that space (thus, placing the beacon and setting it up to transmit its location) and it is easy to set up a node for tracking, by simply attaching a listener to the node. These factors implies setup time is low. The decentralised administration cause administration time to be a minimum.

**Space Cost** Although no direct mention of the size of the beacons or listeners are given, it is assumed that the listeners are relatively small due to the fact that they have to be attached to the nodes.

**Capital Cost** The capital cost of the system is given as less than US$ 10.00 per device (whether listener or beacon).

The most important limitation imposed on the system is that the area that can be covered due to the RF radio used is given in [20] as 30 feet (approximately 9.14 m) in a building.

### 3.3.3 Important Properties

The most important property of the system is its computational power as it enables the system to reach its main goal, user privacy.

### 3.3.4 Classification of System

**Principle**

The system uses the ToF principle as a base for the system.
Physical Phenomena

The ToF principle is implemented in the system using the physical phenomena of propagation, using both radio and ultrasonic waves.

Location Computing Technique

The location computing technique used in the system is proximity.

3.3.5 Conclusion on CLSS

CLSS satisfy the requirements of the proposed system in terms of:

Scale  The area covered by CLSS is an office building, this is a large area relative to the area that needs to be covered by the proposed system. CLSS is also able to track multiple objects, which is a positive in terms of the proposed system.

Limitations  The main limitation of CLSS is the area covered per beacon, given as 9.14 m. This limitation will not have a negative effect on the system as it is large compared to the distances involved in the proposed system, the longest dimension being 2 m.

The downside to CLSS in terms of the proposed system is:

Resolution  The system yields room sized resolutions, which is large relative to the required 3 cm. This is a major drawback to using this system as a base for the proposed system, as the resolution of the system was identified as the most important property.

Reference Grid  Although the system yields absolute locations, the location information is given relative to a floor plan, the proposed system requires 3D cartesian coordinates.
Physical Position & Symbolic Location  The system yields symbolic location, in contrast to the physical position required by the proposed system.

The precise dimensions of the listener to be attached to the object being tracked is not provided, thus it could be possible that the listener will be too large to fit inside the sphere. This problem may be addressed by redesigning the listener to fit. It should also be possible to obtain the necessary accuracy and precision, which will enable the system to match the requirements of the proposed system in terms of Resolution, Reference grid and Physical Position, by redesigning the system, but using the basic concepts (e.g. combination of RF and Ultrasound ToF) of the system.

3.4 Radio Interferometric Positioning System

3.4.1 Overview

Radio Interferometric Positioning System (RIPS) [16] is a system used for determining the position of nodes in WSNs. The system uses only the hardware on the node/mote to determine the position of the nodes. The nodes are divided into groups of four nodes, called quads, with two nodes transmitting pure sine waves on close frequencies. These waves interfere causing an interference signal with a high frequency carrier and low frequency “beat” or envelope. The remaining two nodes in the quad measure the phase offset of this envelope using the Received Signal Strength Indicator (RSSI) signal. The relative phase offset between the two receiving nodes is a function of the distances between the four nodes and the wavelength of the carrier signal and can be used to compute the distances between the nodes. By repeating this process for multiple linearly independent quads, the 2 or 3 dimensional position of the nodes relative to one another or relative to an absolute grid (if the position of some nodes, called anchor nodes, are known) can be determined. The authors claim that the system can achieve an average localization error of 3 cm and a range of 160 m.
3.4.2 Characterisation of RIPS

The first step in the characterisation of RIPS is answering the three questions from section 2.3:

1. **What** - The system determines the location of nodes in a WSN.

2. **Why** - It is often helpful or even necessary to know the location of nodes in WSN for example in systems as presented in [23] and [24]. This system gives a generic method for determining the location of nodes in a WSN for any application.

3. **Where** - The experimental setup of the system as presented in [16] was done in a flat, grassy area.

The system can be characterised as shown in table 3.3.

The authors give the average accuracy of the system as 3 cm and the largest error as 6 cm.

The system can yield either relative locations or absolute locations, depending on whether the positions of a set of anchor nodes are known. The system yields these locations using a cartesian grid, but it should be able to convert the locations to other grids as well.

The system yields physical positions. The accuracy of these positions are high enough to enable conversion to symbolic locations, as the dimensions of most symbolic locations (e.g. rooms) are large, compared to the accuracy that can be obtained with the system.

The system uses the hardware on the mote to estimate the frequency and phase of the RSSI signal, thus limited computational power is available for this part of the localization computation. The estimated frequency, phase and a quality indicator is sent to the base station for further computations. The base station performs the range calculations.
The area covered by the system is limited by what is defined as the *interferometric radio range*, which was found as twice the digital communications range of the motes used [16]. This interferometric radio range determines the maximum distance between the transmitters and a receiver. The amount of motes that can be localised is unlimited (thus all the motes in the WSN can be localised), although the complexity as well as the time needed to perform the localization might become a constraint for larger networks.

Due to the fact that the system is implemented in a WSN, recognition is automatic, as each of the motes have an identifier as part of their function in the WSN.

**Time Cost** The system has no time cost in terms of physical installation, but will re-
quire some time to configure in terms of setting up the software to transmit pure sine waves as well as the algorithms needed for synchronisation and quad selection.

**Space Cost**  The system has no space cost if it is implemented in a WSN, as it uses only the resources available on the motes self, as well as the base station.

**Capital Cost**  The system uses the resources of the WSN to perform localization, thus no extra capital cost is incurred.

The complexity of the phase and frequency estimation techniques that can be used is limited by the resources of the motes used in the WSN. This can affect the accuracy that can be achieved by the system due to the fact that errors in especially phase measurement will affect the accuracy of the calculations to determine the distances between the four nodes in a quad.

### 3.4.3 Important Properties

The property that places the largest constraint on this system is the cost property, as all localization functions are performed with the resources available in the WSN, thus no extra costs are incurred.

### 3.4.4 Classification of System

**Principle**

The system is based on the phase difference sensing principle. The principle is implemented in a novel way, eliminating the need for synchronisation and a reference signal.
Physical Phenomena

The physical phenomena used to implement the system is radio wave propagation.

Location Computing Technique

The system uses a form of triangulation to determine the position of the nodes.

3.4.5 Conclusion on RIPS

RIPS satisfies the requirements of the proposed system in terms of:

**Reference Grid** RIPS yield absolute locations relative to a 2D or 3D cartesian grid, as required by the proposed system.

**Physical Position & Symbolic Location** The system yields physical positions.

**Scale** RIPS cover an area defined in [16] as the interferometric radio range, which is twice the communication range that can be obtained by the motes used. For the motes used to implement the experimental system in [16] this range is 160 m, which is large relative to the dimensions of the proposed system. RIPS is also able to localise multiple objects.

Although the accuracy of RIPS is in the same order as the resolution required by the proposed system, the precision needs to be much higher. This is connected to the main limitation of the system on the complexity of calculations that can be done using the mote’s resources. These issues can be addressed by considering more powerfull motes, or by designing custom hardware for the proposed system, which will also take into consideration the size constraints of the spheres.
Chapter 3 3-Axis Magnetic Sensor Array

3.5 3-Axis Magnetic Sensor Array

3.5.1 Overview

The 3-Axis Magnetic Sensor Array (3AMSA) tracking system [6] was developed to track an object through the human GI tract. It uses a small permanent magnet as the excitation source, as this is smaller than magnetic coils and no additional power source is needed. The position of the object is determined using a grid of commercially available 3-axis magnetic sensors. The data sampled from these sensors used to calculate the position as well as the orientation of the magnet.

3.5.2 Characterisation of 3AMSA

1. What - Small object containing a magnet.
2. Why - To determine the movement of the object through the human GI tract.
3. Where - In the human GI tract.

The system can be characterised as shown in table 3.4.

The authors [6] claim an average accuracy of 3.3 cm, but accuracy is influenced by a variety of factors, including the number of sensors used in the grid, with more sensors yielding better results.

The system yields absolute locations in terms of a 3D or 2D cartesian reference grid.

The system yields Physical Positions.

All of the system’s computational power is on the external hardware, as the object being tracked is a permanent magnet. The computational power therefore is only limited by the hardware used externally. The system can thus be characterised as a remote-positioning system.
Table 3.4: Properties of 3AMSA

<table>
<thead>
<tr>
<th>Property</th>
<th>System Characteristic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resolution, Accuracy &amp; Precision</td>
<td>Average 3.3 mm</td>
</tr>
<tr>
<td>Reference Grid</td>
<td>Absolute 2D or 3D Cartesian Grid</td>
</tr>
<tr>
<td>Physical Position &amp; Symbolic Location</td>
<td>Physical Position</td>
</tr>
<tr>
<td>Computational Power</td>
<td></td>
</tr>
<tr>
<td>Internal</td>
<td>None with hardware on mote</td>
</tr>
<tr>
<td>External</td>
<td>Hardware captures sensor readings and calculate position</td>
</tr>
<tr>
<td>Position Computation</td>
<td>Remote positioning</td>
</tr>
<tr>
<td>Scale</td>
<td>Relatively small areas (cm range), single object</td>
</tr>
<tr>
<td>Recognition</td>
<td>No</td>
</tr>
<tr>
<td>Cost</td>
<td></td>
</tr>
<tr>
<td>Time</td>
<td>Low installation time on object, external hardware dependant on magnetic sensor array size</td>
</tr>
<tr>
<td>Space</td>
<td>Low space cost on object, external hardware space cost dependant on sensor array size</td>
</tr>
<tr>
<td>Capital</td>
<td>Low cost for the object, external hardware capital cost dependant on amount and type of sensors used</td>
</tr>
<tr>
<td>Limitations</td>
<td>Susceptible to magnetic interference, could be expensive to cover large areas</td>
</tr>
</tbody>
</table>

The areas covered during the experimental setup for which results are presented are:

- A rectangular area of 240 x 240 x 100 mm.
- A circular plane with diameter 240 mm.
- A set of lines 94 mm above the sensor plane.

The areas covered are all relatively small. Due to the fact that a permanent magnet is used as excitation only one object can be tracked at a time.
As stated above, this system tracks only one object at a given time, recognition is not implemented.

**Time Cost** The system has a low time cost in terms of installation on/in the object being tracked due to the fact that it is only necessary to attach a small permanent magnet to the object. Installation time of the external support hardware in the form of a sensor array may take longer, but will depend on the size of the array.

**Space Cost** The space cost of the system on the object being tracked is low, again due to the small size of the permanent magnet used. The space cost of the external infrastructure depends on the size of the sensor array.

**Capital Cost** The capital cost in terms of the object being tracked is small (the price of a permanent magnet) and once again the cost of the external hardware will depend on the size of the sensor array used as well as the price of the individual sensors (which increase with the accuracy of the sensors).

The main limitation of the system is its susceptibility to magnetic interference/noise. Another limitation of the system is that it could be expensive to cover large areas, as a lot of magnetic sensors will be needed to maintain a high accuracy.

This system has two important properties, the first is the accuracy and the second is the computational power. The accuracy of the system needs to be small enough to provide useful data on the movement of an object through a human GI tract. The computational power is important due to the fact that a small excitation source is necessary.

### 3.5.3 Classification of System

**Principle**

The system is based on the principle of direct field sensing.
Physical Phenomena

The system is implemented using the magnetic physical phenomena, using a permanent magnet as excitation source.

Location Computing Technique

The system yields distance vectors and thus a form of triangulation is used to determine the object’s position.

3.5.4 Conclusion on 3AMSA

3AMSA satisfies the requirements of the proposed system in terms of:

Reference Grid The system yields absolute locations relative to a 2D or 3D cartesian grid, satisfying the requirements of the proposed system.

Physical Position & Symbolic Location The system yields Physical Position.

Space Cost The system uses a small permanent magnet to track the object, thus it should be able to easily fit inside the sphere.

The average accuracy of the system is 3.3 mm, implying that a resolution of 3 cm should be easily attainable. There remains doubt whether the system will be able to achieve this for large areas, as the areas used to obtain the above mentioned results in [6] is in the range of centimetres, small relative to the area covered by the proposed system. The main downfall of 3AMSA is the fact that only one object can be tracked, as it is not possible to switch a permanent magnet on or off. This can be overcome by replacing the permanent magnet with a magnetic coil, but this negates the most attractive aspect of the system, the fact that no power source is needed on the object being tracked and the fact that a small magnet should easily fit inside the sphere.
3.6 Comparison & Summary

This chapter started with the characterisation and classification of the proposed system. It was found that the system should be based on one of the following principles:

- ToF
- Phase Difference Sensing
- Direct Field Sensing

From this, a system based on each of the identified principles, that can be used as a possible base for the proposed system, was found in literature and then characterised and classified using the method presented in chapter 2. It should be noted at this point, that although many systems were investigated during the literature survey and study, only those deemed most likely to be used as a possible base (classified according to principle) was discussed here.

A brief summary and comparison of the characterisation and classification of the systems is given in figure 3.1. The comparison shows that CLSS doesn’t achieve the desired resolution. It should however be possible to achieve the desired resolution using the same principle, but this will involve redesigning the system. RIPS achieves accuracy in the order needed (around 3 cm) but not consistently enough. It should be possible to get better accuracy and precision from the system if the hardware used to track the object can perform more complex calculations, making the phase and frequency extraction more accurate. Finally 3AMSA obtains an accuracy well below the required 3 cm, but the setup of the system is done for very small areas compared to the area that will be covered in the proposed system, and from the information provided in [6] it is not possible to determine whether this accuracy will be possible for larger areas.

The reference grid of all the systems can yield absolute locations. RIPS and 3AMSA yield these absolute locations in terms of a 2D/3D cartesian grid, and thus also provide
### Summary and Comparison of Systems

<table>
<thead>
<tr>
<th>System</th>
<th>Proposed CLSS</th>
<th>RIPS</th>
<th>3AMSA</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Properties</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Resolution, Accuracy &amp; Precision</td>
<td>Smaller than 3 cm</td>
<td>Room Sized Granularity Average 3 cm, largest 6 cm</td>
<td>Average 3.3 mm</td>
</tr>
<tr>
<td>Reference Grid</td>
<td>Absolute, 3D Cartesian Grid</td>
<td>Absolute, Floorplan Grid</td>
<td>Relative/Absolute depending on information available. 2D/3D Cartesian Grid</td>
</tr>
<tr>
<td>Physical Position and Symbolic Location</td>
<td>Physical Position</td>
<td>Symbolic Location</td>
<td>Physical Position</td>
</tr>
<tr>
<td>Computational Power</td>
<td>Internal</td>
<td>Limited Space available Hardware capable of performing location computation</td>
<td>Limited, all computation done with hardware available on mote</td>
</tr>
<tr>
<td>Position Computation</td>
<td>Remote Positioning</td>
<td>Self Positioning</td>
<td>Remote Positioning</td>
</tr>
<tr>
<td>Scale</td>
<td>Cylinder with Height 2m and Radius 0.5. One object at a time (ability to turn tracking on/off)</td>
<td>Office Building with unlimited amount of objects being tracked</td>
<td>Twice the digital communications range of the motes used in the system</td>
</tr>
<tr>
<td>Recognition</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Cost</td>
<td>Time</td>
<td>Unknown</td>
<td>Low Installation time cost, low administration time cost</td>
</tr>
<tr>
<td>Space</td>
<td>Limited Space available on sphere/object Presumed to be small, due to fact that it can be attached to objects as well as carried on person</td>
<td>No Space cost if implemented in existing WSN, if implemented in other system, size of motes needed.</td>
<td>Low space cost on object (small magnet). External hardware space cost dependant on size of sensor array.</td>
</tr>
<tr>
<td>Capital</td>
<td>Unknown</td>
<td>US$10.00 per device (Listener and Beacon)</td>
<td>No additional cost if implemented in existing WSN, otherwise cost determined by the cost and quantity of motes used</td>
</tr>
</tbody>
</table>

### Limitations
- No direct line of sight to the object
- Each beacon has a range limited to 9.14 m
- Complexity of phase and frequency estimation limited by resources of mote.
- Susceptible to magnetic interference.
- Could be expensive to cover large areas, due to large amount of sensors that will be needed.

### Principle
- ToF/ Phase Difference Sensing/Direct Field Sensing

### Physical Phenomena
- Propagation/Magnetic Propagation (RF and Ultrasonic) Propagation (RF) Magnetic

### Location Computing Technique
- Triangulation/Proximity Proximity Triangulation (Lateration) Triangulation

---

**Figure 3.1: Comparison of Systems**
physical position. If a better resolution can be obtained with CLSS as discussed above, it should be able to yield the location information in terms of a 3D Cartesian grid and thus physical position, rather than symbolic location.

The internal computational power of the proposed system is affected by the space available on the object (sphere with 3 cm radius). The hardware added, if any, on the object self has to be small enough to fit in the space as well as light enough not to affect the weight of the spheres in such a manner as to change their flow. In this regard, if a small enough magnet can be used, 3AMSA will work for the proposed system. An added benefit of 3AMSA is that no power source is required. For both CLSS and RIPS the hardware will have to be modified to achieve this goal. Concerning the external computational power, the proposed system doesn’t have any major constraints. It was assumed that due to the limited space available on the sphere, remote-positioning would be the best option for the proposed system. If the size of the CLSS listener can be made small enough, this will still be a viable option, as it is not necessary to track the object in real time, the position information can be downloaded from the device once the experiment is complete and the sphere has been removed from the RPVM model.

The area covered by the proposed system is a cylinder with height 2 m and radius 0.5 m, with only one object being tracked at a time. It is however necessary to track more than one object for each configuration, thus it must be possible to switch between tracking different objects. Both CLSS and RIPS will be able to cover the area, as well as track multiple objects. The results provided in [6] for 3AMSA are all for areas much smaller than that of the proposed system, and from the information it cannot be inferred whether the system will be scalable to the larger size. Another problem with 3AMSA is the fact that it won’t be possible to switch between objects, as there is no way to “switch on/off” a permanent magnet.

Time cost is not a very important factor in the proposed system, and a compromise can be made to work with a system that requires longer installation and administration time if it results in better accuracy. The same is applicable to the cost of the system (although if two systems yield results with accuracy in the same range, the one with
lower cost will be used). The space cost of the proposed system for the object being tracked is important, due to space being limited, as discussed in the paragraph on internal computational power above.

The main limitation on the proposed system is that there is no line of sight to the object being tracked. The main limitation of CLSS is its range, limited to 9.14 m. This is not a problem, as the area the proposed system will need to cover has much smaller dimensions. The complexity of the calculations to estimate phase and frequency is the main limitation of RIPS. If better accuracy is to be achieved, this limitation will have to be addressed, by providing more computational resources to the system. Finally, 3AMSA is susceptible to magnetic interference, limiting its operation to environments with little or no magnetic interference. Another possible limitation of 3AMSA is its scalability, as the sensor array may become very large as the area that needs to be covered grows.

### 3.7 Conclusion

Considering the above it is apparent that not one of the systems completely satisfy the requirements of the proposed system. Using CLSS as a base would require a redesign of most of the system, making it an unattractive option as the base for the proposed system. Although 3AMSA yields the most accurate locations, it is not clear whether the system will be scalable to cover the area needed for the proposed system neither what the effect of scaling the system will be on the accuracy. RIPS also doesn’t satisfy all of the requirements of the system, but may be considered to come the closest. We propose that by using different motes or doing a custom design for the motes (focusing not on communication as the main objective, but rather on localization) it should be possible to modify RIPS to achieve the needed resolution, while also satisfying the size constraints. Although a hybrid option may also be viable, this will complicate the design, and increase the amount of hardware needed on the object. Thus it should be a better option to use a customised form of RIPS as the base for the proposed system.
Chapter 4

Radio Interferometric Positioning System

In this chapter RIPS is discussed. An overview on the basics of interferometric positioning is given, followed by an in depth discussion of the concepts and mathematics of the system. Possible error sources that could affect the accuracy of the system are then briefly listed. The implementation of the system is then discussed, followed by a section on dealing with $q$-range ambiguity as well as a section on determining the position of the nodes in the WSN.

4.1 Introduction

In the previous chapter various tracking and localization systems were investigated and compared and it was decided that the proposed concept solution be based on RIPS. RIPS is a system developed for the localization of nodes in WSNs using the hardware of the nodes (or motes) [16]. The system uses the principle of phase difference.
sensing in a novel way. Two nodes in the network are used to transmit unmodulated sine waves at very close frequencies, causing an interference signal, measured at two other nodes in the network. From multiple measurements using different transmitter/receiver pairs, the position of nodes in the network are determined. The system claims to yield average localization errors as small as 3 cm over a range of 160 m [16].

In this chapter more detail on RIPS as well as related work is presented. This chapter addresses part of the third sub-objective as stated in section 1.4.1 and is organised as follows. Section 4.2 provides a basic overview of RIPS. In section 4.3 the theorems on which RIPS is based are discussed. Section 4.4 discusses possible sources of errors. In section 4.5 the implementation of the system as presented in [16] is given. Section 4.6 presents a method to deal with q-range ambiguity. In section 4.7 two methods for localization are presented. Finally, in section 4.8 a brief summary of the chapter is presented.

4.2 Interferometric Positioning

RIPS is a positioning system specifically developed for use in WSNs. The main problem when tracking nodes in WSNs is the limited resources available, if external support hardware is not added, as well as the fact that the available resources are intended for a very specific goal, not for tracking. Often the radio capabilities of the nodes are used to perform tracking and localization by using the RSS as described in section 2.4.2, as this means no extra hardware is needed. RIPS however makes use of the radio capabilities of the nodes in a very innovative way to gain better accuracy than systems using RSS.

RIPS is implemented by using groups of four nodes, divided into transmitter and receiver pairs. The two nodes in the transmitting group transmit unmodulated sine wave signals on very close frequencies. These signals interfere, causing a signal with a high frequency carrier and a low frequency envelope (beat). The interference signal is shown in figure 4.1.
In figure 4.1 the envelope is indicated by the dotted line. The frequency of this envelope is the difference between the frequencies on which the two nodes transmit. The signal indicated by the solid line represents the interference signal and has carrier frequency equal to half the sum of the two transmitted frequencies.

The receiving nodes reconstruct the low frequency envelope using their RSSI signal. The phase offset of the envelope at each of the receiving nodes respectively is a function of a variety factors, including the time transmission was started at each of the transmitting nodes, the frequency of the carrier and the distances between the four nodes. The relative phase offset between the two receiving nodes however is only a function of the four distances between the nodes (as indicated in figure 4.2) and the carrier frequency/wavelength.

By making multiple measurements in the network, using different node groups and different transmitter receiver pairs, it is possible to determine the relative positions of the nodes with regards to each other, using the distance information gained from the measurements. If the positions of some nodes are known (called anchor nodes), it is possible to determine the absolute position of the nodes, on the grid for which the positions of the anchor nodes are known.
4.3 RIPS Theorems

In this section the mathematical theory to substantiate the above, will be discussed.

In the theorems the RSSI signal is denoted as \( r(t) \) and modelled as the power of the incoming signal, in dBm, after being mixed down to an intermediate frequency, \( f_{IF} \) and and low pass filtered with cutoff frequency, \( f_{cut} \), where \( f_{cut} \ll f_{IF} \) [16]. Capital roman letters will be used to denote nodes. The distance between nodes \( X \) and \( Y \) will be denoted with \( d_{XY} \). The speed of light is denoted by \( c \).

The proofs of theorems 1 to 3 can be found in [16]

**Theorem 1** [16] Let \( f_2 < f_1 \) be two close carrier frequencies with \( \delta = (f_1 - f_2)/2 \), \( \delta \ll f_2 \), and \( 2\delta < f_{cut} \). Furthermore, assume that a node receives the radio signal

\[
s(t) = a_1 \cos(2\pi f_1 t + \varphi_1) + a_2 \cos(2\pi f_2 t + \varphi_2) + n(t)
\]

(4.1)

where \( n(t) \) is Gaussian noise.

Then the filtered RSSI signal \( r(t) \) is periodic with fundamental frequency \( f_1 - f_2 \) and absolute phase offset \( \varphi_1 - \varphi_2 \).
Chapter 4 RIPS Theorems

**Theorem 2** [16] Assume two nodes, A and B transmit pure sine waves at two close frequencies $f_A > f_B$ such that $f_A - f_B < f_{\text{cut}}$, and two other nodes C and D measure the filtered RSSI signal. Then the relative phase offset of $r_C(t)$ and $r_D(t)$ is

$$2\pi \left( \frac{d_{AD} - d_{AC}}{c/f_A} + \frac{d_{BC} - d_{BD}}{c/f_B} \right) \pmod{2\pi}. \quad (4.2)$$

Due to the limited range and high carrier frequency (relative to their cutoff frequency) of wireless nodes the formula for the measured relative phase offset can be simplified as shown in the following theorem from [16].

**Theorem 3** [16] Assume that two nodes A and B transmit pure sine waves at two close frequencies $f_A > f_B$, and two other nodes C and D measure the filtered RSSI signal. If $f_A - f_B < 2$ kHz, and $d_{AC}$, $d_{AD}$, $d_{BC}$, and $d_{BD} \leq 1$ km, then the relative phase offset of $r_C(t)$ and $r_D(t)$ is

$$2\pi \frac{d_{AD} - d_{BD} + d_{BC} - d_{AC}}{c/f} \pmod{2\pi} \quad (4.3)$$

where $f = (f_A + f_B)/2$.

To simplify the result from Theorem 3 the following is defined:

**Definition 5** [25] The ordered quadruple of distinct nodes A, B, C, and D is defined as a **quad**.

**Definition 6** [25] For any quad (A, B, C, D) the **q-range** is defined as the linear combination of the distances between the nodes,

$$d_{ABCD} = d_{AD} - d_{BD} + d_{BC} - d_{AC}. \quad (4.4)$$

From Theorem 3 and definitions 5 and 6, it follows that the relative phase offset for the quad (A, B, C, D) is given by the equation:

$$\varphi_{ABCD} = 2\pi \frac{d_{ABCD}}{c/f} \pmod{2\pi} \quad (4.5)$$

This equation can be rewritten in the form
Chapter 4

RIPS Theorems

\[ d_{ABCD \text{mod} \lambda} = \phi_{ABCD} \frac{\lambda}{2\pi} \]  

(4.6)

where \( \lambda = c/f \) is the wavelength of the carrier frequency \( f \).

From equation 4.6 it follows that the q-range \( d_{ABCD} \) for the quad \((A, B, C, D)\) can be determined from the relative phase offset measured at nodes \( C \) and \( D \), as the wavelength of the carrier frequency, \( \lambda \), is known. Note however that a single measurement of the relative phase offset \( \phi_{ABCD} \) will not yield a unique q-range, because of \( \text{mod} \lambda \). This result can be expected from the physics of the system. As soon as the nodes are more than one wavelength apart the phase offset may be on any of a number of different wavelengths, and thus the ambiguity arises. Measurements of the relative phase offset at different frequencies can be made to narrow down the q-range solution space until it contains a single q-range satisfying the maximum radio range constraint (for a more detailed description on how to overcome this ambiguity refer to section 4.6). According to [26] good results can be obtained in a moderate multipath environment by making measurements at eleven different channels (frequencies).

Once the q-range is determined it can be used to determine the 2D as well as 3D position of the nodes. According to [25] at least 6 nodes are required to determine the 2D position of all the nodes in a network and at least 8 nodes for 3D. This brings us to the final theorem, the upper bound on the number of linearly independent q-range measurements that can be made for a \( n \) node network.

**Theorem 4** [27] \textit{The dimension of the vector space spanned by the measurements } \( d_{ABCD} \) \textit{on a set of } \( n \) \textit{nodes, } \( n \geq 3 \), \textit{is}

\[ n(n - 3)/2. \]  

(4.7)

The proof of this theorem can be found in [27].

Theorem 4 implies that in a \( n \) node network, with \( n \) at least 3, at most \( n(n - 3)/2 \) linearly independent measurements can be made.

For q-range measurements to be linearly independent, the quads need to be mutually
independent. In [27] 6 disjoint classes are presented. Determining the quads from these classes, will yield linear independent q-range measurements. The classes are:

- **Class 0**:  
  \( \{012D|2 < D\} \), containing \( n - 3 \) elements.

- **Class 1**:  
  \( \{0B1D|1 < B < D\} \), containing \( (n - 2)(n - 3)/2 \) elements.

- **Class 2**:  
  \( \{01CD|1 < B < D\} \), containing \( (n - 3)(n - 4)/2 \) elements.

- **Class 3**:  
  \( \{0B1D|1 < D < B\} \), containing \( (n - 2)(n - 3)/2 \) elements.

- **Class 4**:  
  \( \{0BCD|1 < B, 1 < C < D, B \neq C, B \neq D\} \), containing \( (n - 2)(n - 3)(n - 4)/2 \) elements.

- **Class 5**:  
  \( \{ABCD|0 < A < B, A < C < D, B \neq C, B \neq D\} \), containing \( (n - 2)(n - 3)/2 \) elements.

For example, for the 5 node network with nodes \{\(A, B, C, D, E\}\) the following quads will yield linearly independent q-ranges:

- \((A, B, C, D)\)
- \((A, B, C, E)\)
- \((A, C, B, D)\)
- \((A, C, B, E)\)
- \((A, D, B, E)\)
Chapter 4

Error Sources of RIPS

Thus for a network with \( n = 5 \) there are \( 5(5 - 3)/2 = 5 \) linearly independent q-ranges, which corresponds to the number of quads determined above.

In the next section, the implementation of RIPS as given in [16], based on the above theorems will briefly be discussed.

4.4 Error Sources of RIPS

In section 3 of [16] the sources of errors as discussed in this section are listed. We propose that by negating the effect of the error sources or trying to at least minimise their effect, we should be able to improve on the accuracy and precision obtained by the experimental implementation of RIPS in [16].

**Carrier frequency inaccuracy** The difference between the nominal and actual carrier frequency of the transmitted signal can cause phase measurement errors as large as \((0.33 \%) (2\pi)\) for an average carrier frequency inaccuracy of 1 kHz [16].

**Carrier frequency drift and phase noise** Theorem 1 is valid only for fixed frequencies, thus any frequency drift will adversely affect the accuracy of the measured phase offset. Phase noise will also affect the measured phase offset, negatively impacting on the accuracy of the measured phase offset.

**Multipath effects** In Theorem 3 it was assumed that the radio wave propagates directly from the receiver to the transmitter for a time of \( d_{XY}/c \) s, but due to multipath effects, this is not always true. The multiple paths may also cause fluctuations in amplitude and phase due to their interfering with one another at the receiver.

**Antenna Orientation** It is possible that the propagation time of the signal may be influenced by the orientation of the antenna, thus affecting \( d_{XY}/c \) and the measured phase offset [16].
**RSSI measurement delay jitter** There is a delay between the signal arriving at the antenna of the receiver and the RSSI signal being delivered to the signal processing unit, causing a relative phase offset error at the receivers.

**RSSI Signal-to-Noise Ratio (SNR)** The better the SNR, the better the accuracy of the phase offset estimate at the receiver.

**Signal Processing error** The algorithm that estimates the phase offset of the received signal is not very accurate due to the fact that the phase is estimated from the RSSI signal and the complexity of the algorithm is limited by the computational power of the hardware.

**Time synchronisation error** The accuracy of the relative phase offset is influenced by the time difference between the two receivers starting to measure the absolute phase offset.

## 4.5 Implementation

The experimental implementation of RIPS done in [16] is now discussed, starting with a functional breakdown of the system in section 4.5.1. A discussion on the operational flow of the implementation is presented in section 4.5.2.

### 4.5.1 Functional Breakdown of Implementation

The implementation in [16] was done on a WSN. Figure 4.3 is a functional breakdown of a WSN.

As illustrated, the WSN used to implement RIPS can be broken down into the hardware and software of the motes and Base Station (BS) respectively.

A more detailed functional breakdown of the hardware used in the implemented system is given in figure 4.4. It is evident from the hardware functional breakdown that
RIPS is implemented using only the hardware of the WSN, no additional hardware is needed, as stated in section 3.4.2.

The implementation was done using MICA2 motes, with CC1000 radios, transmitting in the 433 MHz band [16]. According to [16] the CC1000 provides the following features required to successfully implement RIPS:

- The capability to transmit unmodulated sine waves in a wide frequency band (400 MHz - 460 MHz) in small steps (65 Hz). The wide frequency band is needed in order to make multiple measurements at different frequencies to determine the actual q-range, while the small frequency step size is necessary to satisfy the frequency separation requirement of Theorem 3.

- The frequency stability of the sine wave for a period less than 29 ms. This stability enables the measurement of multiple periods of the signal at the receivers,
enabling averaging to increase the SNR.

- Capturing the RSSI relatively accurately with a small delay jitter. The accurate capturing as well as small delay jitter reduces the error in the estimated relative phase offset.

- The ability to transmit at different power levels. The distance between the different transmitters and a receiver may cause the signal from the transmitter closest to the receiver to completely overpower the signal from the other transmitter, implying the power level of the closest transmitter needs to be decreased or the power level of the transmitter further away, needs to be increased.

The software functional breakdown is presented in two figures. Figure 4.5 is a functional breakdown of the software on the mote.

![Figure 4.5: Functional Breakdown of Software on the Motes](image)

The motes use TinyOS as operating system. Additional to the normal communication software on the motes, the following software is used to implement localization [16]:

- A driver enabling the CC1000 radio to transmit pure sine waves at certain frequencies.

- A radio engine that manage two tasks. The first is the coordination and synchronisation of nodes participating in a measurement. The second is the transmission
of the pure sine wave if the node acts as a transmitter, or alternatively the reception of the interference signal if the node acts as a receiver.

- A frequency tuning algorithm that ensures the difference between the transmission frequencies are in the range of 200 Hz to 800 Hz, to satisfy the sampling constraints due to the 9 kHz sampling rate of the RSSI signal and the 29 ms time constraint on the measurement.

- Signal processing software to estimate the frequency and phase from the RSSI Signal.

In figure 4.6 the functional breakdown of the software on the BS is given.

![Figure 4.6: Hardware Functional Breakdown of RIPS in a WSN](image)

Five main software components run on the BS, these are [16]:

- Software for communication.

- An algorithm for selecting groups of nodes to perform q-range measurements as well as scheduling to coordinate the activities of the transmitters and receivers.

- Software assisting with the frequency calibration process.

- An algorithm for determining the q-ranges estimation from multiple phase offset measurements.

- The algorithm performing the localization of the nodes from the q-range measurements.

In the next section the operational flow of RIPS is discussed.
4.5.2 Operational Flow of Implementation

In this section the operational flow of RIPS is discussed in order to develop a better understanding of how the system was implemented. According to [16] the implementation consists of the following steps:

1. A pair of transmitters are selected from a group of motes and their transmission times are scheduled.

2. The radios of the transmitters are calibrated to transmit at close frequencies.

3. Pure sine waves are transmitted by the motes identified in step 1. The transmissions are repeated at multiple frequencies.

4. The RSSI signal at two receiving nodes are analysed to estimate the frequency and phase of offset of the interference signal.

5. The q-range measurement, $d_{ABCD}$, is calculated using the relative phase offsets for each pair of receivers.

6. The location of nodes are determined from the calculated q-ranges using a localization algorithm.

We developed an operational flow diagram for the implementation from the above as well as the details provided in sections 4, 5 and 6 of [16]. The operational flow diagrams are presented using the format described in [28]. Figure 4.7 is the first level of the operational flow.

In the above figure, the second step (2.0) of the operational flow is the selection of a pair of receivers. According to section 4.5 of [16] the base station is responsible for the selection of transmitters. In their implementation the base station selects all possible pairs of transmitter and all the other nodes within their range acts as receivers. Once a pair of transmitters are selected a time synchronisation protocol is used to coordinate all the necessary measurements for the current pair of transmitters.
Synchronisation is started by a master node broadcasting a message which [16]:

- Identifies the other transmitting node.
- Specifies the type of measurement (calibration/tuning or ranging).
- Specify the transmit powers for each of the transmitters.
- Specifies a synchronisation point (a time instant) when the measurement must start.
- Contains a time stamp of when the message was sent.

The receiving nodes use the arrival timestamp to convert the synchronisation point to their local time. The synchronisation point is used to set up a timer determining when to start calibration, as well as transmission/measurement, at fixed time intervals after the synchronisation point. Each time a new set of transmitters is selected from the possible combinations, the operational flow repeats from 2.0 to 7.0.

In the third step of the operational flow (3.0 in figure 4.7) the transmitters used in the measurements are calibrated. According to [16] the difference between the actual and nominal frequency for the CC1000 radio can be as large as 2 kHz. Calibration is done to ensure that the frequency of the envelope is in the range of 200 Hz to 800 Hz. The envelope has to be restricted to this range due to the 9 kHz sampling frequency of the RSSI signal on the Mica2 platform as well as the time constraint for a measurement (29 ms) [16]. Calibration is done by determining the radio settings for transmitters.
corresponding to the same frequency [16]. Assume $f_1$ and $f_2$ are the actual signal frequencies. One transmitter is set to transmit a sine wave at frequencies 325 Hz apart,

$$f_1(i) = f_1 + i \cdot 325, \ i = -15, -14, \ldots, 14, 15$$ (4.8)

while the second transmits a fixed frequency, $f_2$. In [16] it was found that the maximum carrier frequency error is less than 2 kHz, thus the assumption is made that

$$|f_1 - f_2| < 2 \cdot 2 = 4 \text{ kHz}$$

A receiver analyses the frequency of the interference signal for each $i$ given by

$$|f_1(i) - f_2|$$ (4.9)

and determines for which $i$ the interference signal frequency is closest to 0 Hz. This information is send to the node transmitting the multiple frequencies, and is used to determine the settings for the radio to keep the interference signal in the required range [16].

After step 3.0 in figure 4.7 the operational flow divides into three parallel activities:

- The transmission of the sine waves (4.0).
- The measurement of the interference signal (5.0).
- Online processing of the measured RSSI signal (6.0).

A level 2 operational flow of 4.0 is presented in figure 4.8

The transmitter nodes wait for the scheduled transmission time, determined during synchronisation in 2.0, before transmitting the pure sine wave for a predetermined time.
Parallel to this (see figure 4.9), the receivers wait for the scheduled time to start measuring the interference signal. Synchronisation is very important in this step due to the fact that the system uses the relative phase offset between different nodes. For the relative phase offset to be accurate the measurements of the interference signal on the different nodes need to start at the same time. In the implementation of the experimental system [16] the synchronisation errors were in the microsecond range for time periods of less than 1 s. Once the measurements are started, the RSSI signal is sampled for a predetermined time.

The third parallel activity, 6.0, is the online RSSI processing, shown in figure 4.10.

According to [16] the limited communication bandwidth of the WSN necessitates the processing of the RSSI values on the motes. Standard signal processing techniques like Fourier analysis and autocorrelation can not be used due to the resources on the motes
limiting computational complexity. A signal processing algorithm was implemented on the motes that estimates the frequency and the phase of the RSSI signal. The algorithm consists of an online and post processing part. The algorithm is divided to eliminate the need for large buffers and to reduce the post processing time. Peak detection is done during online processing for 256 samples of the measured RSSI signal. The incoming signal is filtered using a moving average to increase the SNR. The first 24 samples (which is long enough to contain one period of the signal) are used to determine a minimum and maximum value for the signal. These maximum and minimum values are used to set a low as well as a high threshold. A peak is detected whenever the signal crosses the high threshold from low to high and then from high to low. The amplitude is used as a quality indicator of the measured RSSI signal [16].

During post processing (7.0) the peak indices calculated during online processing (6.0) are used to estimate the frequency and phase of the measured RSSI signal. The algorithm calculates the shortest period between two peaks and use it to determine the average period of the signal by calculating the average of all the periods between peaks that is not greater than 30 % of the shortest period. The frequency is then calculated as the inverse of this average period. The phase of the signal is estimated as the average phase of these peaks. The estimated amplitude, frequency and phase is sent to the base station after post processing [16].

After post processing one of three actions can be taken. As stated earlier on in this chapter, multiple q-range measurements at different frequencies are used to solve the ambiguity problem. If more measurements of the same q-range measurements need to be made at another frequency the process repeats from 3.0 as shown by 8.0. Multiple linearly independent q-range measurements are needed to localise the nodes. If a new q-range is to be measured, the process restarts from 2.0.

If enough q-range measurements have been made to localise the nodes in the WSN the system starts estimating the q-range measurements (10.0) from the data sent to the base station from the various nodes after post processing. The process of estimating the q-ranges from the ambiguous phase results are presented in section 4.6.
After the q-ranges have been estimated they are used to determine the location of the nodes in the WSN. Localization is discussed in section 4.7.

### 4.6 Solving q-range Ambiguity

In this section a method for solving the q-range ambiguity is presented as discussed in [16].

From the estimated phase data, for the set of frequencies $f_1, ..., f_k$, the q-ranges are calculated solving the equation

$$d_{ABCD} = \lambda_i n_i + \gamma_i = \lambda_j n_j + \gamma_j$$

(4.10)

where

- $\lambda_i = c / f_i$ is the wavelength,
- $\gamma_i = \lambda_i \frac{\vartheta_i}{2\pi}$ is the phase offset relative to the wavelength,
- $\vartheta_i$ is the measured phase offset, and
- $n_i$ is an integer.

For the implementation in [16], using a frequency separation of 5 MHz between multiple measurements for different wavelengths, yields a set of $\lambda_i$ with least common multiple larger than the possible domain of $d_{ABCD}$ at the frequency band of 433 MHz. This means only 3 different measurements are necessary to solve equation 4.10 assuming perfect phase estimation. Due to the fact that the phase estimation is not perfect, equation 4.10 is invalid and is reformulated as the inequality

$$|(\lambda_i n_i + \gamma_i) - (\lambda_j n_j + \gamma_j)| < \epsilon$$

(4.11)

where $\epsilon$ is a fraction of the wavelength determined by the accuracy of the phase measurement [16]. According to [16] inequality 4.11 can be solved and they then define the
\( d_{ABCD} \) solution as the mean of the individual \( d_i = \lambda_i n_i + \gamma_i \) values. It is further stated that if multiple solutions for \( d_{ABCD} \), differing by small multiple integers of the wavelength, is found, the solution with the minimum error value (calculated using equation 4.12) is used as the final \( d_{ABCD} \) estimate.

\[
\xi = \sqrt{\Sigma (d_{ABCD} - d_i)^2}
\] (4.12)

For a more detailed discussion on dealing with the q-range ambiguities see section 3.1 and 3.3 of [25].

### 4.7 Localization of Object from q-range Measurements

In this section two methods that can be used to localise the nodes are presented. The first uses a Genetic Algorithm (GA) and the second is a form of lateration using hyperbolas with a shared foci, which can only be used when three of the four node locations of a quad is known.

#### 4.7.1 Localization using Genetic Algorithms

In [16] the localization of the nodes is done using an optimisation method based on GAs, due to the fact that a large number of nonlinear equations need to be solved if a traditional method (like e.g. lateration) is used. For a set of unknown node locations and a set of q-range measurements, \( M \), an error function of a solution \( s \) is defined as

\[
\text{error}(s) = \frac{1}{n} \sqrt{\sum_{ABCD \in M} (d_{ABCD} - d_{ABCD}(s))^2}
\] (4.13)

where a solution is a placement of nodes and a node placement is represented by a vector of \((x,y,z)\) coordinates. For these sets and the error function (4.13) the GA is applied as follows [16]

1. An initial population of \textit{populationSize} random solutions is generated.
2. A random selection of \textit{subpopulationSize} solutions are selected from the population.

3. Each solution in the subset is evaluated using (4.13).

4. The subset is sorted according to error.

5. The worst 20\% of the subset is removed and replacements generated by applying genetic operators to parents selected from the best 20\%.

6. Go to step 2.

The genetic operators used on the parents are [16]:

1. \textit{Crossing Over}: each node position is inherited from one of two parents with even chance.

2. One of three \textit{Mutations}, each with equal chance:
   
   (a) One node moved by Gaussian random number with a small variance $\epsilon$.
   
   (b) One node moved to a random position.
   
   (c) All nodes are moved by a Gaussian random number with a small variance $\epsilon$.

The values of $\epsilon$ is set to the value of the error function, enabling bigger jumps for larger errors and fine tuning when the errors get smaller.

\subsection*{4.7.2 Localization using Hyperbolas}

This localization method for RIPS as presented in [26] can be used to determine the position of an unknown node analytically, if the position of three of the nodes in a quad is known. For example, if only the position of node $A$ in the quad $(A, B, C, D)$ is unknown, equation 4.4 can be rewritten in as
by moving all known quantities to the left. Using this quantity a t-range is defined as follows:

**Definition 7** [26] For the quad \((A, B, C, D)\) with only the position of \(A\) unknown, a **t-range** is defined as

\[
d_{ACD} = d_{AD} - d_{AC}
\]  

(4.14)

where \(d_{ACD} = d_{ABCD} - d_{BC} + d_{BD}\).

For multiple t-ranges the position of the unknown node can be determined as the intersection of two hyperbolas, sharing a foci.

The hyperbola \(h_{AB}\) is defined by its foci \(A, B\) and the distance \(R_{AB}\) where for any point \(X \in h_{AB}\), (4.15) is valid [26].

\[
|AX| - |AB| = R_{AB}
\]  

(4.15)

Similarly, the hyperbola \(h_{AC}\) is defined by its foci \(A, C\) and the distance \(R_{AC}\). The values \(R_{AB}\) and \(R_{AC}\) are the distances between the transverse vertices of the hyperbolas respectively [29]. The foci of the two hyperbolas are given by \(A(0,0), B(b,0)\) and \(C(c_x,c_y)\), where \(A(0,0)\) is the shared foci. The intersection of the two hyperbolas is the point \(X(x,y)\).

The following quantities are also defined [26]:

\[
A_{AB} = R_{AB}/2 \quad \text{and} \quad A_{AC} = R_{AC}/2
\]

\[
C_{AB} = b/2 \quad \text{and} \quad C_{AC} = c/2 = \sqrt{c_x^2 + c_y^2}/2
\]

\[
B_{AB} = \sqrt{b^2 - R_{AB}^2}/2 \quad \text{and} \quad B_{AC} = \sqrt{c^2 - R_{AC}^2}/2
\]

\[
D = \frac{c_y R_{AB}}{(bR_{AC} - c_x R_{AB})}
\]
Chapter 4  Localization of Object from q-range Measurements

\[ E = \frac{2(B_{AB}^2 R_{AC} - B_{AC}^2 R_{AB})}{(b R_{AC} - c X R_{AB})} \]

\[ F = \frac{b E - 2B_{AB}^2}{R_{AB}} \]

By substituting and simplifying (4.15) using the given coordinates and the above defined quantities the following equations are found [26]:

\[ \sqrt{x^2 + y^2 R_{AB}} = b x - 2B_{AB}^2 \]  \hspace{1cm} (4.16)

and

\[ \sqrt{x^2 + y^2 R_{AC}} = c x + c y - 2B_{AC}^2 \]  \hspace{1cm} (4.17)

By substituting (4.16) into (4.17) and simplifying using \(D\) and \(E\) it follows that [26]:

\[ x = D y + E \]  \hspace{1cm} (4.18)

By substituting (4.18) into (4.16) and simplifying, the quadratic equation in \(y\) follows [26]:

\[ (1 + D^2 - \frac{b^2 D^2}{R_{AB}^2}) y^2 + (2DE - 2\frac{bDE}{R_{AB}}) y + E^2 - F^2 = 0 \]  \hspace{1cm} (4.19)

The discriminant of (4.19) is given by [26]:

\[ Disc_y = 4(F^2 - E^2) + 4D^2(\frac{F^2 + b^2 E^2}{R_{AB}^2} - \frac{2bEF}{R_{AB}}) \]  \hspace{1cm} (4.20)

The intersection points of the two hyperbolas can be computed by solving for
\[ y_1 = \frac{-(2DE - 2\frac{bDF}{R_{AB}}) + \sqrt{Disc_y}}{2(1 + D^2 - \frac{b^2D^2}{R_{AB}})} \]  

and

\[ y_2 = \frac{-(2DE - 2\frac{bDF}{R_{AB}}) - \sqrt{Disc_y}}{2(1 + D^2 - \frac{b^2D^2}{R_{AB}})} \]

and then solving (4.18) for \( x_1 \) and \( x_2 \) respectively.

For (4.19) to be applicable all the coefficients must be finite and the coefficient of \( y^2 \) must be non-zero. For alternate methods to finding the solution if one of these conditions is not met, see section 2.1 of [26].

### 4.8 Summary

In the previous chapter it was concluded that RIPS should be used as the base for the concept system. In this chapter a detailed discussion on RIPS was presented from [16] as well as other related work [25], [26] and [27].

An overview of the basics of Radio Interferometric Positioning was given. Two nodes are used to emit sine waves with frequencies close to one another. Two other nodes are used to measure the interference wave caused by the two transmitting nodes. The interference signal (see figure 4.1) has a high frequency carrier given by

\[ f_c = \frac{f_A + f_B}{2} \]

and a low frequency envelope

\[ f_e = f_A - f_B \]

where \( f_A \) and \( f_B \) are the transmitting nodes.
Chapter 4 Summary

The relative phase offset of the envelope of the interference signal, measured at nodes C and D, is a function of the distances between the nodes (see figure 4.2) and the wavelength of the carrier, \( \lambda_c \). The position of the nodes in the network can be determined by making multiple measurements with different transmitter receiver combinations.

The theory substantiating the overview was then presented. It was shown that for a quad of nodes \((A, B, C, D)\) the distances between the nodes are related to one another by a q-range, defined as the linear combination

\[
d_{ABCD} = d_{AD} - d_{BD} + d_{BC} - d_{AC}
\]

The q-range value can be calculated from the relative phase offset between the two receiving nodes using equation

\[
d_{ABCD} \text{mod} \lambda = \phi_{ABCD} \frac{\lambda}{2\pi}
\]

and making measurements at multiple wavelengths to overcome the q-range ambiguity due to the mod \( \lambda \) in equation 4.8. It was further stated that the upper bound on the number of linearly independent q-range measurements that can be made in an \( n \) node network is

\[
n(n - 3)/2.
\]

The error sources as listed in [16] are briefly discussed in section 4.4.

The implementation of RIPS done in [16], using a WSN of MICA2 motes, running TinyOS operating system and with CC1000 radios, was discussed in section 4.5. A functional breakdown as well as the operational flow of the system is discussed in sections 4.5.1 and 4.5.2.

In section 4.6 a method for dealing with the q-range ambiguity is presented by solving
the inequality

\[ |(\lambda i n_i + \gamma_i) - (\lambda j n_j + \gamma_j)| < \varepsilon \]

where \( \varepsilon \) is a fraction of the wavelength determined by the accuracy of the phase measurement.

Finally in section 4.7 two methods that can be used to determine the location of the nodes are presented. The first method, discussed in section 4.7.1, is the method used in [16]. The method uses a GA to solve for the location of the nodes.

The second method (section 4.7.2), presented in [26], can be used to localise the nodes analytically if only one position in the quad \((A, B, C, D)\) is unknown, by finding the intersection of two hyperbolas (the unknown position) sharing the same foci.

RIPS as discussed in this chapter forms the base of the concept system discussed in the next chapter.
Chapter 5

Conceptual Design

In this chapter the conceptual design of the system is presented. The chapter starts with the design considerations and the design decisions due to these considerations. The concept is then presented, first in a general format and then in more detail by using a functional breakdown and operational flow analysis.

“Intuition and concepts constitute...the elements of all our knowledge, so that neither concepts without an intuition in some way corresponding to them, nor intuition without concepts, can yield knowledge.”

–Immanuel Kant

5.1 Introduction

In chapter 3 we selected RIPS as a base for the proposed system. In chapter 4 RIPS was discussed in detail. In this chapter the design of our concept system is done, based on the information and ideas gained during the previous chapters.
Chapter 5 Design Considerations

The conceptual design is done taking into consideration the requirements of the system as well as the aspects of RIPS that influence the concept. We try to improve on the accuracy and precision obtained by RIPS by designing to either eliminate the error sources as discussed in section 4.4 or minimise their effect. This is possible due to the fact that the concept system is designed exclusively for tracking and localisation purposes.

It should further be noted that the conceptual design is done for the simplest scenario, tracking only one node through the RPVM. Once the concept is proven for this simplest scenario, it can be used as a base to build on. This might seem to contradict the second requirement as stated below, but as will be seen from the discussion after the system requirements, this is not the case.

This chapter address the final sub-objective from section 1.4.1 and is organised as follows. Section 5.2 discuss the design considerations. In section 5.3 the concept is presented. The chapter ends with a brief summary in section 5.4.

5.2 Design Considerations

In this section we discuss the design considerations influencing the conceptual design of the system, as well as the decisions made due to these considerations.

5.2.1 System Requirements

We start by revisiting the answers in section 3.2, for the questions from section 2.3. The system’s goal is to track spheres with a 6 cm diameter through a cylinder with height 2 m and radius 0.5 m. The spheres are tracked to determine their flow path through the cylinder, which is a scale model of the RPV of the PBMR, referred to as the RPV Model (RPVM). We base the requirements of the concept system on the objectives as given in section 1.1 and the criteria that will be used to validate the system as discussed
in section 1.3. The requirements are as follows:

1. The position of a specified sphere in the model shall be determined with an accuracy of 3 cm with 100 % precision. The definitions of accuracy and precision are used as defined in section 2.2.

2. The system shall determine the position of multiple spheres for each time frame between configuration changes of the spheres in the RPVM due to a sphere being removed.

3. The positions determined for the spheres shall be logged, and shall be used in post-processing to reconstruct the flowpath of the spheres.

4. The system shall be independent of the RPVM geometry, as long as the geometry does not exceed a volume of 2 m x 2 m x 2 m.

5. The system shall operate at room temperature.

The first requirement implies that the minimum acceptable accuracy of the system is 3 cm. This accuracy is based on the fact that if a minimum 3 cm accuracy is obtained, the sphere’s measured position will never differ more than half the diameter of the sphere, from its actual position.

The second requirement states that the position of multiple spheres need to be determined for each time frame between configuration changes of the spheres in the RPVM. This requirement is based on the fact that in the time between removing one sphere from the RPVM and removing the next, the configuration of the spheres will not change. We can thus say that although the spheres are not tracked simultaneously, we can determine their position during the same time frame $t_a$ (one sphere is removed) to $t_b$ (the next sphere is removed) as their positions stay the same for the entire period of time.

The third requirement states that the position of the spheres shall be logged as they move through the model. This is necessary as the goal of the system is to determine
the flowpath of the spheres through the model. If self-positioning is used, the system will need to either store the location data on the device, enabling us to download it once the sphere exits the model, or to send it to a base station using some form of wireless communication. We need to restrict the hardware on the mote to a minimum due to the size constraints of the sphere. We achieve this by using remote-positioning in the concept system, thus limiting the hardware on the mote exclusively to localisation.

The fourth requirement states that the system shall be independent of the reactor geometry. This is necessary for the system to achieve its goal of being used to experiment with multiple reactor geometries as discussed in section 1.1. The volume constraint on the different reactor geometries is necessary as the system will be designed to work in a confined space, selected as a cube with dimensions 2 m x 2 m x 2 m. Based on this requirement we decide to use a cartesian reference grid rather than a cylindrical reference grid in the concept system. Using a cartesian grid makes the system more adaptable to different RPVM geometries due to the fact that it is less intuitive to relate positions from non-cylindrical scenarios to cylindrical coordinates than relating the position of the spheres in the cylinder to a cartesian grid with origin somewhere outside the cylinder.

The final system requirement is for the system to operate at room temperature. This is necessary as the material used to construct the RPVM corresponds to the material used in the actual RPV in terms of elasticity and energy conservation of collisions at room temperature.

In chapter 3 this proposed system was characterised and classified using the method from chapter 2. A brief summary of the properties of the of the system as determined during characterisation is presented in table 5.1.

In the next section the aspects of RIPS influencing the design is discussed.
### Table 5.1: Properties of Concept System

<table>
<thead>
<tr>
<th>Property</th>
<th>System Characteristic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resolution, Accuracy &amp; Precision</td>
<td>Resolution smaller than 3 cm</td>
</tr>
<tr>
<td>Reference Grid</td>
<td>Absolute 3D Cartesian Grid</td>
</tr>
<tr>
<td>Physical Position &amp; Symbolic Location</td>
<td>Physical Position</td>
</tr>
<tr>
<td>Computational Power</td>
<td>Internal Limited space available</td>
</tr>
<tr>
<td></td>
<td>External Good</td>
</tr>
<tr>
<td>Position Computation</td>
<td>Remote Positioning Size of Cylinder, One object</td>
</tr>
<tr>
<td>Scale</td>
<td>No</td>
</tr>
<tr>
<td>Recognition</td>
<td>Time Unknown</td>
</tr>
<tr>
<td>Cost</td>
<td>Space Limited space on sphere</td>
</tr>
<tr>
<td></td>
<td>Capital Unknown</td>
</tr>
<tr>
<td>Limitations</td>
<td>No visual LOS</td>
</tr>
</tbody>
</table>

### 5.2.2 RIPS

In chapter 3 we concluded that RIPS is a suitable option to be used as a base for the concept system. As discussed in chapter 4, RIPS uses linear independent quads of nodes to determine the location of a node in a WSN. By making measurements for multiple quads the location of the nodes in the network can be solved relative to one another. If the position of anchor nodes relative to a grid are known, the position of the other nodes can also be determined relative to the grid.

For the concept system we define the following:

**Definition 8** A *measurement* is defined as the process of measuring the phase offset of an interference signal, caused by transmitters $X$ and $Y$ transmitting pure sine waves at close frequencies, at two receivers, $U$ and $V$, respectively, for the quad $(X, Y, U, V)$.

From definition 8 it follows that for any quad $(X, Y, U, V)$ in the concept system, nodes $X$ and $Y$ act as the transmitters and nodes $U$ and $V$ act as the receivers.

We further define
Chapter 5  
Design Considerations

**Definition 9** The mobile node is the node with an unknown position.

and

**Definition 10** An anchor node is a node of which the position is known.

We now consider the simplest scenario. We want to determine the location of a single mobile node, while the rest of the nodes in the network are anchor nodes. From the discussion of RIPS in chapter 4 we know that the time transmission started does not have an effect on the relative phase offset. In section 5.2.1 we said we need to limit the amount of hardware on the sphere due to the space constraint. Based on this we choose the mobile node as one of the transmitters as this reduces the amount of functionality (and thus hardware) needed on the sphere to only:

- A power source.
- A pure sine wave source.
- Transmission hardware.

We denote this mobile node as node $A$, with position given by the 3D coordinate $A(x_A, y_A, z_A)$. From the above it follows that we have to solve for 3 unknowns to determine mobile node $A$’s position.

From theorem 4 in chapter 4 we know the upperbound on the number of linearly independent q-ranges in a $n$ node network is (4.7)

$$\frac{n(n - 3)}{2}$$

Assuming the simplest solution, with $n = 4$ nodes, the minimum number of nodes required to make a single q-range measurement, the upperbound on the number of linear independent measurements that can be made is 2. For this scenario we have two equations and 3 unknowns, a system with infinitely many solutions. By using a
network with \( n = 5 \) nodes, the equation yields an upperbound of 5 linearly independent equations, enabling us to solve the 3 unknowns uniquely. Based on the above, the concept system consists of five nodes, denoted as \( A \) - the mobile node, and \( B,C,D, \) and \( E \) - the anchor nodes. To satisfy the requirement of theorem 3 that \( f_A > f_B \), node \( A \) will always transmit at a slightly higher frequency than the anchor node used as the other transmitter.

From the discussion on RIPS in chapter 4 we know that the q-range measurements are done using multiple frequencies to enable the solving of the q-range ambiguity. In the concept system we have far greater control over variables of the system and we are of the opinion that by making suitable choices regarding the frequencies as well as the position of the anchor nodes the possible solution range for each of the q-range measurements \( d_{XYUV} \) can be limited to exclude ambiguity. For example if interferometric positioning is possible in the reactive near-field of the antennas, the physical setup and wavelength can be chosen in such a way that the mobile node is never more than a single wavelength away from any of the anchor nodes, thus eliminating the q-range ambiguity. According to [30] the outer boundary of the reactive near-field is taken to be

\[
R < 0.62 \sqrt{\frac{D^3}{\lambda}} \tag{5.1}
\]

where \( D \) is the largest dimension of the antenna and \( \lambda \) is the wavelength. If interferometric positioning is not possible in the reactive near-field of the antennas and localisation has to be done in far-field (Fraunhofer region), the physical setup can be done to limit the mobile node to a position that is a fixed integer multiple of the wavelength away from all of the anchor nodes, thus limiting the possible solution range for the q-range measurements. According to [30] the far-field is the region with the inner boundary

\[
R > \frac{2D^2}{\lambda} \tag{5.2}
\]

For the concept system, we assume that the physical setup is done in such a way that there is no q-range ambiguity. This eliminates the need for making q-range measurements at multiple frequencies, again contributing to simpler functionality needed on the sphere.
For the RIPS system in [16], the hardware’s primary goal is communication between the motes, not the localisation of the nodes. Due to this, the system uses the RSSI signal of the nodes to measure the interference signal. The nodes have to perform the frequency and phase estimation due to the limited communication bandwidth of the WSN. The complexity of the signal processing techniques that can be used to estimate the frequency and phase of the measured RSSI signal is limited by the resources on the node. For the concept system the pair of receivers will always be two of the anchor nodes. We aim to improve the accuracy and precision obtained during the experimental implementation of RIPS as discussed in chapter 4 by designing the anchor nodes in such a way that the interference signal can be measured accurately by using a combination of oversampling and high resolution Analog to Digital Convertors (ADCs). To further improve accuracy and precision, the concept system doesn’t estimate phase and frequency on the anchor nodes, but all the measured data are sent to a Control PC via a wired communication channel for calculation of the frequency and phase. This approach eliminates the error sources due to the measurement of the RSSI signal as listed in section 4.4.

For the relative phase offset between two receiving nodes to be calculated accurately, the measurement of the interference signal at each of the receivers need to start at the same time. In the RIPS implementation as discussed in chapter 4 an elaborate time synchronisation algorithm is used and synchronisation of the start of the RSSI signal measurements are reported to be accurate to microsecond range. In the concept system the anchor nodes only contain the “front-end” of the receiver. The front end from each of the anchor nodes are connected to a Data Aquisition unit (DAQ) with wires of a similar length and type. Theoretically this will cause the same phase shift for both wires to the DAQ, which should cancel once the relative phase offset is calculated. A suitable DAQ will enable the sampling of the measurements at the same time. The measurements taken by the DAQ is then sent to the control PC.

In the concept system we use Fourier analysis to calculate the frequency and phase of the measured signals. We choose Fourier analysis as it enables us to determine the frequency of the envelope of the received signal, eliminating the effect of an inaccurate
For RIPS it was reported in [25] that interference due to multipath can adversely affect the accuracy of the system. The effect of multipath was the worst when the ground reflected signal had a small angle of incidence, causing a \(180^\circ\) or \(\pi\) rad phase shift of this reflected signal. The \(180^\circ\) phase shifted signal causes a big attenuation in the direct LOS signal. Due to the attenuation, the phase of the direct LOS signal is distorted by the other reflected signals, which would usually have an amplitude to small to affect the direct LOS signal. To minimise distortion due to multipath in the concept system we take the following measures:

- The physical setup of the system is done in such a way that the ground reflected signal will have a large angle of incidence and will travel a long distance compared to the distance the direct LOS signal travels.
- The anchor nodes’ antennas are designed to be directional, limiting unnecessary reflected signals.

Finally as discussed in chapter 4, the accuracy of RIPS is negatively affected by frequency drift. This implies that the shorter the measurement the better, as the frequency is stable for a small amount of time. On the other hand to improve the SNR of the measured signal, a longer measurement is better. In the RIPS implementation in [16] it was found that the best compromise is to make a measurement of 29 ms, which is short enough not to be affected by frequency drift and long enough to have a positive effect on the SNR of the measured RSSI signal. In the concept system we address this problem by using sine wave generators that are stable for relatively long periods of time compared to the period of the envelope frequency. This should enable us to measure multiple periods of the envelope signal, improving the SNR of the measured signal.

We find the best compromise for the concept system by experimenting with simulations as well as the prototype implementation of the concept system.

In the next section the concept is presented.
5.3 Concept

In this section the concept system is presented based on the system requirements and design considerations as discussed in the previous section. An illustration of the system components as well as the physical setup of the concept system is given in figure 5.1.

![Figure 5.1: Setup of Concept System](image)

The concept system uses five nodes, $A$, $B$, $C$, $D$, and $E$ to implement interferometric positioning. Node $A$ denotes the mobile node being tracked inside the RPVM. Node $A$ always acts as a transmitter to simplify the functionality needed on the sphere. Note that although not incorporated in the design as discussed here, the system can be used to track multiple node $A$s, by switching from one to the other until all the spheres being tracked are localised for the current configuration of the spheres in the RPVM. This can be done by, for example, using timers on the nodes to switch the transmitter of the various nodes on and off according to a predetermined schedule.
The anchor nodes are placed at specifically chosen, known locations. The choice of location is done to:

- Limit the possible solution space for the q-range measurement, thus eliminating the q-range ambiguity.
- Cause the angle of incidence of the ground reflected wave to be large as well as causing a large difference between the distance the wave travels to the ground and back to the anchor nodes relative to the distance the direct LOS signal travels.

The anchor nodes can act as transmitters as well as receivers in order to perform measurements for the five linear independent q-range measurements made by the system.

The anchor nodes are connected to a DAQ with the ability to sample two or more analog signals simultaneously. The DAQ is in turn connected to a control PC. The control PC uses Fourier analysis of the sampled envelope signals to determine the absolute phase offset at each of the receivers for each of the measurements. By subtracting the absolute phase offsets at the receivers from each other, for each of the measurements, the relative phase offsets can be calculated. These relative phase offsets are used to determine the q-ranges for each of the measurements.

Using the classes from theorem 4, and the set of nodes \( \{A, B, C, D, E\} \) the linear independent quads that can be used for making q-range measurements are:

\[
(A, B, C, D) \\
(A, B, C, E) \\
(A, C, B, D) \\
(A, C, B, E) \\
(A, D, B, E)
\]

Note that according to definition 8, node \( A \) always act as a transmitter, as we intended it to. To satisfy the requirement of theorem 3 that \( f_A > f_B \), the mobile node always transmits at the higher frequency.
Making measurements for these five quads yield the q-ranges $d_{ABCD}$, $d_{ABCE}$, $d_{ACBD}$, $d_{ACBE}$, and $d_{ADBE}$. From equation (4.4) the following set of linear independent equations follows:

\[
\begin{align*}
\quad d_{ABCD} &= d_{AD} - d_{BD} + d_{BC} - d_{AC} \\
\quad d_{ABCE} &= d_{AE} - d_{BE} + d_{BC} - d_{AC} \\
\quad d_{ACBD} &= d_{AD} - d_{CD} + d_{BC} - d_{AB} \\
\quad d_{ACBE} &= d_{AE} - d_{CE} + d_{BC} - d_{AB} \\
\quad d_{ADBE} &= d_{AE} - d_{DE} + d_{BD} - d_{AE}
\end{align*}
\]

(5.3)

We represent the 3D position coordinate of node $U$ by $U(x_U, y_U, z_U)$. If we rewrite the first equation of set (5.3) in terms of distance equations for the nodes position coordinates it follows that

\[
\begin{align*}
\quad d_{ABCD} &= \sqrt{(x_A - x_D)^2 + (y_A - y_D)^2 + (z_A - z_D)^2} \\
&\quad - \sqrt{(x_A - x_C)^2 + (y_A - y_C)^2 + (z_A - z_C)^2} - d_{BD} + d_{BC}
\end{align*}
\]

(5.4)

Similar results follow for the rest of the equations in the set above, thus we have a system of five equations with three unknowns. This system can be solved for $A(x_A, y_A, z_A)$ by using a numeric solver, implying that due to the specific circumstances of this system it is not necessary to use a more complicated method for estimating object position as discussed in sections 4.7.1 and 4.7.2

The calculated position coordinates of node $A$ are logged, before the configuration of the spheres in the RPVM is changed by removing a sphere from the model. The system then repeats until mobile node $A$ is removed from the system. The flowpath of the sphere can be reconstructed from the logged position coordinates.
5.3.1 Functional Breakdown of Concept

In this section a functional breakdown of the system is presented based on the previous sections.

The concept system consist of five main functional units as illustrated in figure 5.2. Each of these functional blocks are discussed in the following sub sections.

Mobile Node Functional Breakdown

A functional breakdown of the mobile node is shown in figure 5.3.

As stated in section 5.2 we keep the functionality of the node to a minimum by using the hardware exclusively for localisation and implementing remote-positioning. The power source of the device is a compromise between size and battery life. The source needs to be as small as possible due to the space constraint of the sphere while also providing enough power to complete the process of tracking the sphere through the RPVM.
The pure sine wave generator is the source of one of the two signals used to cause the interference signal. The frequency stability of the generator affects the accuracy of the system. The more stable the frequency, the better the accuracy of the system.

The final functional component is the transmit hardware, which is divided into two parts. A power amplifier, used to amplify the sine wave from the generator before transmission. The second part is the antenna, used to transmit the amplified sine wave. The antenna needs to be small enough to fit inside the sphere and should not be seriously affected due to orientation, as we cannot control the orientation of the sphere.

Anchor Node Functional Breakdown

Figure 5.4 shows the functional breakdown of the anchor node.

![Diagram of Anchor Node Functional Breakdown](image)

Figure 5.4: Functional Breakdown of Anchor Node

The anchor nodes are not inside the RPVM as is the case with the mobile node, and thus an external power source is used to power the device.

The anchor nodes have a control interface that is used to control the basic functionality of the node e.g. switching the node on or off and switching between transmit and receive modes, according to the measurement being made.

The anchor nodes need to provide the DAQ with the received signal when in receive mode. This is done via the communication interface to the DAQ.

The transmit hardware of the anchor node is further divided as illustrated in figure 5.5. The pure sine wave generator is the same as the one used on the mobile node, but is set to a slightly lower frequency to cause the interference signal.
Chapter 5

Concept

Figure 5.5: Functional Breakdown of Anchor Node Transmit Hardware

The power amplifier used is a digitally controlled amplifier to adjust for the distances between the transmitters and receivers. This is done to prevent one of the signals totally obscuring the other signal due to attenuation over distance.

The antenna used for the anchor node is directional to prevent unnecessary reflections that could lead to multipath and is designed according to the transmit frequency of the node.

The receiver hardware of the anchor node is further divided as illustrated in figure 5.6

Figure 5.6: Functional Breakdown of Anchor Node Receive Hardware

The antenna used is the same as used for transmission as described for the transmit hardware breakdown. The received interference signal is amplified using a Low Noise Amplifier (LNA).

An envelope decoder is used to extract the envelope of the signal from the amplified interference signal. This is done by first rectifying the signal and then filtering it. Rectification is necessary for the envelope frequency component to be present in the
frequency spectrum of the signal due to the fact that the energy of the envelope for the positive and negative cycle is equal and thus no frequency component for the envelope is present in the signal’s Fourier transform before rectification (see figure 4.1).

A Low-Pass Filter (LPF) is used to remove the high frequency carrier component of the signal. This is necessary to prevent the signal from aliasing when it is sampled. The filter should have the minimum phase distortion to prevent the filter adversely affecting the accuracy of the system.

**DAQ Functional Breakdown**

A functional breakdown of the DAQ is shown in figure 5.7.

![Functional Breakdown of DAQ](image)

The DAQ is powered from an external power source.

The Digital Input Output (IO) of the DAQ is used to control the anchor nodes via their control interfaces. The digital IO can be either a logic high or low.

The analog inputs of the DAQ are connected to the communication interfaces of each of the anchor nodes. It receives the envelope signal as decoded by the anchor nodes and samples the received signal. Three aspects of the sampling at the analog inputs are important for the DAQ:

- Simultaneous sampling on all the analog input channels.
- Sufficiently high sampling rate.
- High resolution.
As discussed in section 5.2.2 the analog inputs of the DAQ have to be able to sample simultaneously to be able to accurately calculate the relative phase offset of the envelope between the two receiving nodes.

To improve the accuracy with which the phase offset can be calculated the DAQ has to be able to sample the envelope signal at a sufficiently high rate. The minimum sampling frequency needed to avoid aliasing is the Nyquist frequency, which is double the frequency of the highest frequency component in the signal [31]. According to [32] it is the trend to use oversampling (using a sampling frequency much higher than the Nyquist frequency) as there are multiple benefits. For the concept system the main benefits are the simplification of the LPF needed on the anchor nodes and a better SNR. The simplification of the LPF implies that the roll-off of the filter can be less steep, thus the phase distortion due to the filter is reduced.

When the signal is sampled it is quantised by assigning it to one of \( 2^B \) values, where \( B \) is the number of bits of the ADC. The quantisation step size of the ADC can be calculated by the formula

\[
q = \frac{V_{fs}}{2^B - 1}
\]

where \( V_{fs} \) is the full scale voltage range of the signal. The maximum quantisation error that can be made is half the quantisation step size [32]. Thus by using an ADC with a high resolution, the size of the quantisation error is minimised.

The DAQ has an interface through which it is connected to the control PC, allowing it to communicate with the control PC.

The DAQ contains some form of control software which is accessed from the PC for setting up the sampling procedure.

**Control PC Functional Breakdown**

The functional breakdown of the control PC is illustrated in figure 5.8.
The control PC is a normal PC with the necessary hardware to communicate with the DAQ enabling it to:

- Control the DAQ.
- Download the sampled signals from the DAQ.

Software to control and communicate with the DAQ is installed on the control PC. The software needed to determine the location of the mobile node as well as software used to create a log of all the positions from which the flowpath can be reconstructed operates on the control PC.

**Software Functional Breakdown**

The functional breakdown of the software used in the concept system is shown in figure 5.9

Software enabling communication between the DAQ and the control PC is installed on the control PC. This software communicates with the control software embedded on the DAQ. The control software on the DAQ controls the sampling process as well as the digital IO used to control the anchor nodes.

The localisation software installed on the control PC is divided into two functional units.
The first functional unit calculates the q-ranges for the measurements made, by calculating the Fourier transform of the measured envelope signals. From the Magnitude data of the Fourier transform the exact frequency of the envelope signal at a node can be determined. Using this identified envelope frequency, the phase offset of the interference signal at a node can be determined from the phase data of the Fourier transform. These absolute phase offsets are then used to calculate the relative phase offset between the pairs of receivers used in the respective measurements. The combination of position data of the anchor nodes, the carrier frequency of the interference signal, and the relative phase offsets are then used to calculate the q-ranges.

The second functional unit uses the calculated q-ranges to determine the position of the mobile node by solving the set of equations (5.3) for the three unknowns representing the 3D coordinates of the mobile node, \( A(x_A, y_A, z_A) \). This position coordinate is then logged with a unique timestamp.

In the next section the operational flow of the concept system is presented based on the functions of each of the system components (illustrated in figure 5.2) as discussed in this section.

### 5.3.2 Operational Flow of Concept

The operational flow of the concept system is illustrated in figure 5.10.
The mobile node starts transmitting (2.0) before entering the RPVM and transmit throughout the entire tracking process.

Parallel to this a range of different operations are executed. The first parallel operation is the selection of a quad for which the current q-range measurement is done.

The operational flow of the quad selection (3.0) is shown in figure 5.11.

In section 5.2.2 we established that q-range measurements for five linear independent quads need to be made. The control PC determines which q-range measurement is to be made and switches the digital IO on the DAQ to place one of the nodes in transmit mode and two others in receive mode, based on the quad used for the current q-range measurement.
Once the transmitter and receivers are selected they start to transmit and receive respectively for a predetermined time (4.0). We determine this time by taking the longest possible (to improve the SNR of the signal) measurement before the frequency drift of the transmitters start to negatively affect the accuracy and precision of the system. The average time before frequency drift starts will have to be determined empirically. Simultaneously the DAQ samples the signals it receives from the anchor nodes acting as receivers for the measurement. The quantised samples are sent to the control PC. This operational flow for the sampling is shown in figure 5.12

Figure 5.12: Operational Flow of Concept System - Level Two, 5.0

The sample data received from the DAQ are used by the control PC to calculate the q-range. The operational flow diagram for the q-range calculations are shown in figure 5.13.

Figure 5.13: Operational Flow of Concept System - Level Two, 6.0

The control PC uses the measured envelope signals from the DAQ to calculate the Fourier transform for both the signals respectively. From the Fourier transform the frequency and absolute phase offsets of the respective signals are determined.

Once the absolute phase offsets are known, the control PC calculates the q-range by using the relative phase offset (difference between the absolute phase offsets) and the known frequency of the carrier. The q-range is saved for future use in the localisation phase, 7.0.
Once the q-range is calculated the control PC determines whether all five of the linear independent measurements are done. If all the measurements are not done, the process restarts from 3.0 for the next quad in the sequence. If the five linear independent measurements are complete, the control PC starts with the localisation of the mobile node’s current position (7.0).

The localisation of the node’s current position is done by using the five q-range measurements to solve the set of equations presented at the beginning of this section using a non-linear numerical solver. The control PC then saves the calculated location coordinates of the mobile node.

After the localisation is done, a sphere is removed from the of the RPVM. If the removed sphere is the mobile node A, the tracking is complete (10.0) and the flowpath of the sphere can be reconstructed from the saved location data. If the removed sphere is not the mobile node A, the system repeats from 3.0.

5.4 Summary

In this chapter the conceptual design of the system is presented. The design is done based on the requirements with RIPS as the base. The aim of the design is to improve the accuracy and precision obtained by RIPS through the use of hardware designed exclusively for tracking and localisation.

The concept consists of:

- The sphere being tracked through the RPVM, denoted as node A. This node acts as a transmitter that keeps transmitting throughout the tracking process.

- Four anchor nodes, denoted by A, B, C, D, and E. The anchor nodes can act as both transmitters or receivers.

- A DAQ that can sample analog channels simultaneously. The DAQ performs two
functions, controlling the anchor nodes’ mode and sampling the envelope signals from the anchor nodes acting as receivers.

- A control PC that controls the tracking process, calculates the q-range measurements, performs localisation calculations and logs the position of the sphere for each localisation.

The tracking is done as follows:

- The transmitter of the mobile node is activated and the sphere placed in the RPVM.

- The five linear independent q-range measurements are made.

- From the q-range measurements the location of the mobile node for the current configuration of the spheres in the RPVM is determined.

- The location of the mobile node is logged.

- This process is repeated until the mobile node is removed from the RPVM.

- The logged locations are then used to reconstruct the flowpath of the mobile node through the RPVM.

In the next chapter a mathematical model for the conceptual design is derived and used to simulate the system.
Chapter 6

Mathematical Model & Simulation

In this chapter the derivation of the mathematical model is done based on the conceptual design of the system that was presented in chapter 5. The algorithms used for the implementation of the model in simulation are then discussed. Finally the model as well as the simulation is verified and validated.

6.1 Introduction

In chapter 5 the conceptual design of the proposed system was presented, based on radio interferometric positioning. In this chapter an ideal, deterministic mathematical model of the system is derived from the conceptual design. The model is then implemented in simulation software that allows experimentation with the concept system parameters in order to determine whether or not the required minimum accuracy of 3 cm can be obtained. If the model fails to obtain the accuracy for ideal circumstances it implies the system will fail to achieve the accuracy for non-ideal circumstances. It is however important to note that the converse of this is not necessarily the case.
This chapter addresses the first and second sub-objectives as stated in section 1.4.2 and is organised as follows. In section 6.2 the mathematical model is derived from the conceptual design of the system. In section 6.3 the implementation algorithms, based on the mathematical model, are presented.

Algorithms that are used to implement the model in simulation software, are presented based on the mathematical model. In section 6.4.2 the mathematical model is validated, the simulation implementation based on the algorithms is verified, and the operational validation of the simulation implementation is done. The chapter ends with a summary of the work done.

### 6.2 Derivation of the Mathematical Model

In this section we derive an ideal mathematical model for the concept system. From chapter 5 we know that the concept system consists of the set of five nodes \( \{A, B, C, D, E\} \). Node \( A \) is the mobile node and always acts as a transmitter. The signal transmitted by \( A \) is given by

\[
s_A(t) = a_A \cos(2\pi f_A (t - t_A))
\]  

(6.1)

where

- \( a_A \) is the amplitude of the transmitted signal,
- \( f_A \) is the frequency node \( A \) transmits at and
- \( t_A \) is the elapsed time since the node started generating a waveform.

The other nodes in the set act as either a transmitters or receivers, depending on the current q-range measurement. We denote the other transmitting node as node \( X \), where \( X \) can be any of the nodes \( B, C, D, \) or \( E \).
The signal transmitted by \( X \) is given by

\[
s_X(t) = a_X \cos(2\pi f_X(t - t_X))
\]

(6.2)

As discussed in chapter 5, the frequency of the mobile node is always slightly higher than that of the transmitting anchor node, \( f_A > f_X \).

We denote the receiving nodes as nodes \( U \) and \( V \), where \( U \) and \( V \) can be any of the anchor nodes, depending on the current q-range measurement. According to [16] the interference signal at a receiver node can be mathematically represented as the sum of the two transmitted signals, thus the interference signal received at node \( U \) is

\[
I_U(t) = a_{AU} \cos(2\pi f_A t + \phi_{AU}) + a_{XU} \cos(2\pi f_X t + \phi_{XU})
\]

(6.3)

where

- \( \phi_{AU} = -2\pi f_A(t_A + \frac{d_{AU}}{c}) \) and \( \phi_{XU} = -2\pi f_X(t_X + \frac{d_{XU}}{c}) \) are the phase offsets of the two component signals.
- \( d_{AU} \) and \( d_{XU} \) are the distance between nodes \( A \) and \( U \), and \( X \) and \( U \), respectively.
- \( c \) is the speed of light.
- \( a_{AU} \) and \( a_{XU} \) is the reduced amplitude, due to path loss during propagation from the transmitting (first subscript) to the receiving (second subscript) node.

The carrier frequency of this interference signal is given by

\[
f_c = \frac{f_A + f_X}{2}
\]

(6.4)
Chapter 6 Derivation of the Mathematical Model

The envelope or beat frequency is given by

\[ f_e = f_A - f_X \]  \hspace{1cm} (6.5)

A similar equation follows for interference signal received at node \( V \). The interference signal is illustrated in figure 6.1

![Interference Signal](image)

Figure 6.1: Interference Signal at Receiver

To generate figures 6.1 to 6.5 the following parameters were used:

- \( f_A = 10 \) kHz
- \( f_X = 9.9 \) kHz
- \( a_{AU} = 2 \)
- \( a_{XU} = 3 \)
Chapter 6  Derivation of the Mathematical Model

- \( \phi_{AU} = 45^\circ \)
- \( \phi_{XU} = 30^\circ \)

The magnitude and phase plots of the interference signal, generated by computing the Fast Fourier Transform (FFT) of the interference signal is given in figure 6.2

![Interference Signal FFT](image)

**Figure 6.2: FFT of Interference Signal at Receiver**

It can be seen that the frequency component of the envelope is not present in the frequency spectra of the interference signal. We expected the component to be absent based on the discussion in 5.

The interference signal received at node \( U \) is halfwave rectified. The rectified signal is given by the equation:

\[
R_U(t) = \begin{cases} 
  a_{AU}\cos(2\pi f_A t + \phi_{AU}) + a_{XU}\cos(2\pi f_X t + \phi_{XU}), & \forall I_U(t) > 0 \\
  0, & \forall I_U(t) < 0 
\end{cases} \quad (6.6)
\]
A similar equation follows for the rectified signal at node $V$. The rectified signal is shown in figure 6.3.

![Rectified Signal](image)

**Figure 6.3: Rectified Signal at Receiver**

The magnitude and phase plots of the rectified signal is generated by computing its FFT with the results illustrated in figure 6.4.

From the magnitude plot it is easily verified that the rectification of the signal causes the envelope signal frequency ($f_e$) to be present.

It can also be seen that the phase offset of the envelope frequency is given by:

$$\varphi_U = \varphi_{AU} - \varphi_{XU}$$  \hspace{1cm} (6.7)

The phase information needed is thus contained in the envelope frequency, as expected from the discussion on RIPS. To simplify the sampling equipment needed, the signal is filtered using a LPF that has a cut-off frequency $f_{cut} \ll f_c$ and $f_e < f_{cut}$. 
The magnitude and phase plot of the filtered signal is shown in figure 6.5. The filter applied to the signal is a third order Butterworth LPF with $f_{\text{cut}} = 150$ Hz. From the magnitude plot it is clear that the filter removed most of the unwanted frequency components. The phase plot shows that the phase offset of the envelope was changed due to the filtering.

Filtering can be represented in the frequency plane by the multiplication of the input function with the transfer function of the filter. For the concept system that is

$$F_U(\omega) = R_U(\omega) \cdot H_{\text{filter}}(\omega)$$

$$= |R_U(\omega)| \cdot |H_{\text{filter}}(\omega)| \cdot e^{j(\phi_U - \epsilon_{\text{filter}})} \quad (6.8)$$
where

- $F_U(\omega)$ is the frequency domain representation of the rectified and filtered interference signal at node $U$ and
- $\varepsilon_{\text{filter}} = \arctan\left(\frac{\omega}{\omega_c}\right)$ for a Butterworth filter according to [31].

From (6.8) it follows that we can expect the phase shift due to filtering as seen in figure 6.5.

Based on the above discussion we formulate proposition 1.
**Proposition 1** Assume two nodes A and X transmit pure sine waves at two close frequencies \( f_A > f_X \) respectively and the interference signal received at node U due to the transmissions from nodes A and X is given by

\[
I_U(t) = a_{AU} \cos(2\pi f_A t - \varphi_{AU}) + a_{XU} \cos(2\pi f_X t - \varphi_{XU})
\]

Then the envelope of the interference signal, with frequency \( f_e = f_A - f_X \) can be obtained by the halfwave rectification and low-pass filtering of the signal with the cut off frequency of the filter \( f_{cut} \ll f_c \) and \( f_e < f_{cut} \). The phase offset of the envelope signal is given by

\[
\varphi_U = \varphi_{AU} - \varphi_{XU} - \varepsilon_{filter}
\]  

(6.9)

where \( \varepsilon_{filter} \) is the phase shift due to filtering.

Using (6.5) we can expand the phase offset of the envelope (6.9) as follows:

\[
\varphi_U = \varphi_{AU} - \varphi_{XU} - \varepsilon_{filter}
= -2\pi f_A (t_A + \frac{d_{AU}}{c}) + 2\pi f_X (t_X + \frac{d_{XU}}{c}) - \varepsilon_{filter}
\]  

(6.10)

A similar equation is valid for node V.

Using equation (6.10) we can calculate the relative phase offset of the envelope for receiving nodes U and V as

\[
\varphi_U - \varphi_V = (\varphi_{AU} - \varphi_{XU} - \varepsilon_{filter}) - (\varphi_{AV} - \varphi_{XV} - \varepsilon_{filter})
= \left( -2\pi f_A (t_A + \frac{d_{AU}}{c}) + 2\pi f_X (t_X + \frac{d_{XU}}{c}) - \varepsilon_{filter} \right)
- \left( -2\pi f_A (t_A + \frac{d_{AV}}{c}) + 2\pi f_X (t_X + \frac{d_{XV}}{c}) - \varepsilon_{filter} \right)
= \frac{2\pi f_A}{c} (d_{AV} - d_{AU}) + \frac{2\pi f_X}{c} (d_{XU} - d_{XV})
\]  

(6.11)

Equation (6.11) can be rewritten as

\[
\varphi_U - \varphi_V = 2\pi \left( \frac{d_{AV} - d_{AU}}{c/f_A} - \frac{d_{XU} - d_{XV}}{c/f_X} \right)
\]  

(6.12)
which corresponds to the relative phase offset of RIPS as given in theorem 2.

Let $\delta = f_c/2$. By using $f_c$ and $\delta$, (6.12) can be rewritten as

$$\phi_U - \phi_V = 2\pi \frac{d_{AV} - d_{AU} + d_{XU} - d_{XV}}{c/f_c} + 2\pi \frac{d_{AV} - d_{AU} - d_{XU} + d_{XV}}{c/\delta}$$  \hspace{1cm} (6.13)

which is the same as equation (8) from [16]. By using the same method as [16], if $\delta$ is sufficiently small, the denominator, $c/\delta$, of the second term becomes sufficiently large relative to the distances involved in the q-range measurement and thus this second term can be disregarded, simplifying (6.13) to

$$\phi_U - \phi_V = 2\pi \frac{d_{AV} - d_{AU} + d_{XU} - d_{XV}}{c/f_c}$$  \hspace{1cm} (6.14)

By letting the relative phase offset between receivers $U$ and $V$ for the q-range measurement of the quad $(A, X, U, V)$ be denoted as

$$\varphi_{AXUV} = \phi_U - \phi_V$$  \hspace{1cm} (6.15)

it follows that

$$d_{AXUV} = \varphi_{AXUV} \frac{\lambda_c}{2\pi}$$  \hspace{1cm} (6.16)

At this point it should also be taken into consideration that the relative phase offset can be on any one of $n$ different wavelengths, thus (6.16) becomes

$$d_{AXUV} = \varphi_{AXUV} \cdot \frac{\lambda_c}{2\pi} + n\lambda_c, \quad n \in \mathbb{Z}$$

From the above discussion we formulate proposition 2.
Proposition 2 Assume two nodes $A$ and $X$ transmit pure sine waves at two close frequencies $f_A > f_X$ respectively and two nodes $U$ and $V$ measure the envelope of the interference signal. If $f_e$ is sufficiently small, causing $2c / f_e$ to be sufficiently large compared to the distances $d_{AU}, d_{AV}, d_{XU},$ and $d_{XV}$, the q-range is given by

$$d_{AXUV} = \varphi_{AXUV} \cdot \frac{\lambda_e}{2\pi} + n\lambda_e, \quad n \in \mathbb{Z}. \quad (6.17)$$

By using the classes from theorem 4 for the set of five nodes $\{A, B, C, D, E\}$, we find the set of linear independent equations

$$

d_{ABCD} = d_{AD} - d_{BD} + d_{BC} - d_{AC} \\

d_{ABCE} = d_{AE} - d_{BE} + d_{BC} - d_{AC} \\

d_{ACBD} = d_{AD} - d_{CD} + d_{BC} - d_{AB} \\

d_{ACBE} = d_{AE} - d_{CE} + d_{BC} - d_{AB} \\

d_{ADBE} = d_{AE} - d_{DE} + d_{BD} - d_{AE}
$$

(6.18)

Rewriting the generic form $d_{AXUV}$ of the q-range by using distance equations and denoting each node $X$’s position as $X(x_X, y_X, z_X)$ it follows that

$$d_{AXUV} = \sqrt{(x_A - x_V)^2 + (y_A - y_V)^2 + (z_A - z_V)^2}$$

$$- \sqrt{(x_A - x_U)^2 + (y_A - y_U)^2 + (z_A - z_U)^2} - d_{BD} + d_{BC} \quad (6.19)$$

where the q-range value can be computed using (6.17). Thus for (6.19) the only unknowns are the position coordinates of $A$.

Using the set of five linear independent equations from (6.18), the three unknowns, $x_A, y_A,$ and $z_A$ can be computed.
Chapter 6  
Simulation Algorithms

6.3 Simulation Algorithms

In this section the algorithms, based on the mathematical model derived in section 6.2, used to implement the simulation, are presented. The simulations are implemented using MATLAB. We start with the algorithm for making a single q-range measurement in section 6.3.1.

6.3.1 Single q-range Measurement

The algorithms used to perform a single q-range measurement are discussed in this section. We start with the input parameters. Then we discuss the first algorithm, used for the calculation of the variables needed to simulate a q-range measurement. We then discuss the algorithms used for the calculation of the phase offset at a node $U$ due to the signals transmitted by nodes $A$ and $X$. Finally we discuss the algorithm for calculating the q-range from the relative phase offsets between receiving nodes $U$ and $V$.

Input Parameters

The algorithm requires the specification of the following parameters:

- Actual Position of the mobile node in 3D cartesian coordinates - $x_A$, $y_A$, and $z_A$ in meter (m).
- Positions of the anchor nodes in 3D cartesian coordinates in meter (m).
- The amplitude values $a_{XU}$, $\forall X \neq U \in \{A, B, C, D, E\}$.
- The transmitter frequencies:
  - The frequency at which the mobile node transmits - $f_A$ in Hertz (Hz).
  - The frequency at which the anchor node transmits - $f_X$ in Hertz (Hz).
• The amplitudes

• The time elapsed since the pure sine wave generator on each of the nodes started generating the sine wave for transmission, \( t_X \) in seconds (s), for each of the nodes respectively.

• The filter parameters:
  – The \( s \) coefficients of the numerator as a vector from the highest order coefficient to the lowest.
  – The \( s \) coefficients of the denominator as a vector from the highest order coefficient to the lowest.

• The sampling frequency - \( f_s \) in Hertz (Hz).

• The sampling duration - \( T_d \) in seconds (s).

• The quantisation parameters of the ADC:
  – Bits - \( B \)
  – Full scale voltage range - \( V_{fs} \) in volts (V).

Calculation of Variables

The variables needed to do the q-range calculations are computed using the input parameters.

The algorithm starts by calculating the distances between all of the nodes using distance equations, for example, the distance between nodes \( U \) and \( V \) can be calculated as

\[
d_{UV} = \sqrt{(x_U - x_V)^2 + (y_U - y_V)^2 + (z_U - z_V)^2}
\]  
(6.20)

From the transmitter frequencies the carrier frequency, \( f_c \), as well as the envelope frequency, \( f_e \), are calculated using equations (6.4) and (6.5) respectively.
Next the actual phase shift between all the nodes, due to the time elapsed since generation of the wave started, as well as the distance the wave propagates are calculated. For example the phase shift in the signal from node $U$ to node $V$ is calculated as:

$$\varphi_{UV} = -2\pi f_U (t_U + \frac{d_{UV}}{c})$$  \hspace{1cm} (6.21)$$

Due to the fact that the simulation is implemented on a computer, the interference signal used is not a truly continuous signal. To minimise the effect of this non-continuous signal, the interference signal is generated using a generation frequency, $f_g$, that is much higher than the theoretical highest frequency component of the interference signal, $f_A$. The algorithm computes the generation frequency as:

$$f_g = 100 \cdot f_c$$  \hspace{1cm} (6.22)$$

Using the period of $f_g$, denoted as $T_g$, the generation time series is calculated as

$$t_g[i + 1] = t_g[i] + T_g, \quad \forall i = 0, \ldots, n - 1$$  \hspace{1cm} (6.23)$$

a vector with length $n_g = \text{round}(\frac{T_d}{T_g})$ starting at $t_g[0] = 0$ and ending at $t_g[n] = T_d$. The operator $\text{round}(x)$ rounds the answer to the nearest integer.

The time series for the sampled signal is generated using $T_s$, the period of the sampling frequency, $f_s$, as

$$t_s[i + 1] = t_s[i] + T_s, \quad \forall i = 0, \ldots, n - 1$$  \hspace{1cm} (6.24)$$

a vector with length $n_s = \text{round}(\frac{T_d}{T_s})$ starting at $t_s[0] = 0$ and ending at $t_s[n] = T_d$.

The algorithm then calculates the sampling ratio, $SR$, of the generating frequency to the sampling frequency, which is later used in the downsampling of the filtered signal.

The wavelength of the carrier frequency is calculated as

$$\lambda_c = \frac{c}{f_c}$$  \hspace{1cm} (6.25)$$
Finally the numerator and denominator of the LPF are converted from the s-domain to the z-domain.

For the pseudocode of the above, see algorithm 1

**Algorithm 1 Calculate Variables**

1. calculate $d_{UV}$, $\forall$ $U \neq V \in \{A, B, C, D, E\}$
2. $f_c \leftarrow \frac{f_A + f_X}{2}$
3. $f_e \leftarrow f_A - f_X$
4. calculate $\varphi_{UV} \leftarrow -2\pi f_U(t_U + \frac{d_{UV}}{c})$, $\forall$ $U \neq V \in \{A, B, C, D, E\}$
5. $f_g \leftarrow 100f_c$
6. $n_g \leftarrow \frac{T_d}{T_g}$
7. $t_g[0] \leftarrow 0$
8. for $i \leftarrow 0$ to $n_g - 1$ do
9. $t_g[i + 1] \leftarrow t_g[i] + T_g$
10. end for
11. $n_s \leftarrow \frac{T_d}{T_s}$
12. $t_s[0] \leftarrow 0$
13. for $i \leftarrow 0$ to $n_s - 1$ do
14. $t_s[i + 1] \leftarrow t_s[i] + T_s$
15. end for
16. $SR \leftarrow \frac{f_g}{f_s}$
17. $\lambda_c \leftarrow \frac{c}{f_c}$
18. $s - \text{numerator } Z_s$, $z - \text{numerator}$
19. $s - \text{denominator } Z_s$, $z - \text{denominator}$

**Calculation of Phase Offset at Node U**

In this section the algorithms for the calculation of the absolute phase offset of the envelope signal at node $U$ due to the pure sine wave transmissions from nodes $A$ and $X$ are presented.
The interference signal at node $C$ due to transmissions from $A$ and $X$ are given by (6.5) and calculated by substituting the generation time series, $t_G$, into $t$ as shown in algorithm 2.

**Algorithm 2 Calculate Interference Signal**

1: $\text{for } i \leftarrow 0 \text{ to } n_G - 1 \text{ do}$
2: $I_U[i] \leftarrow a_{AU} \cos(2\pi f_A t_G[i] - \varphi_{AU}) + a_{XU} \cos(2\pi f_X t_G[i] - \varphi_{XU})$
3: $\text{end for}$

The signal is then halfwave rectified by using equation (6.6), see algorithm 3

**Algorithm 3 Calculate Rectified Signal**

1: $\text{for } i \leftarrow 0 \text{ to } n_G - 1 \text{ do}$
2: $\text{if } I_U[i] > 0 \text{ then}$
3: $R_U[i] \leftarrow I_U[i]$
4: $\text{else}$
5: $R_U[i] \leftarrow 0$
6: $\text{end if}$
7: $\text{end for}$

The rectified signal is filtered and then sampled, by using build-in MATLAB filter and downsample functions and then quantised, to represent the sampled envelope signal at node $U, E_U$ (see algorithm 4).

**Algorithm 4 Calculate Measured Envelope Signal**

1: $\text{filter } R_U \quad \triangleright \text{Uses MATLAB filter function}$

\[ F_U \leftarrow \text{filtered } R_U \quad \triangleright \text{Assign filtered array to } F_U \]

2: $\text{downsample } F_U \quad \triangleright \text{Uses MATLAB downsample function}$

\[ F^*_U \leftarrow \text{downsampled } F_U \quad \triangleright \text{Assign downsampling array to } F^*_U \]

3: $\text{for } i \leftarrow 0 \text{ to } n_s - 1 \text{ do}$
4: $E_U[i] \leftarrow \text{QUANTISE}(V_{f_s}, B, F^*_U[i])$
5: $\text{end for}$

In algorithm 4 num and den are the z-transformed numerator and denominator of the
LPF’s transfer function and the function `QUANTISE(voltageRange, Bits, Value)` is presented in algorithm 5.

The number of quantisation levels is calculated using [32]

\[ N_q = 2^B \] (6.26)

and the quantisation step size, using (5.5). The algorithm uses a loop to determine in which interval the value of the filtered envelope signal is, and returns the quantised value of the envelope as the product of the interval number with the quantisation step size.

**Algorithm 5 Quantisation Function**

1. **function** `QUANTISE(VoltageRange, Bits, Value)`
2. \[ N_q \leftarrow 2^B \] \hspace{1cm} \triangleright \text{Number of quantisation levels}
3. \[ q \leftarrow \frac{V_{fs}}{2^B - 1} \] \hspace{1cm} \triangleright \text{Quantisation step size}
4. \[ i \leftarrow 0 \]
5. \hspace{1cm} **while** not \((i \cdot q \leq \text{Value})\) and \((\text{Value} < (i+1) \cdot q)\) **do**
6. \hspace{1cm} \[ i \leftarrow i + 1 \]
7. \hspace{1cm} **end while**
8. \[ \text{return } i \cdot q \] \hspace{1cm} \triangleright \text{Return quantised value}
9. **end function**

Note from algorithm 5 that the quantisation function will only work for a rectified signal due to the fact that conversion is done from 0 V.

To determine the absolute phase offset, Fourier analysis of the quantised envelope signal is done using its Discrete Fourier Transform (DFT). The \(n_s\) point DFT is calculated using MATLAB’s FFT algorithm from the signal processing toolbox. The FFT consists of complex values. To extract the phase component, the MATLAB angle command is used. This yields an array, \(\Theta_{UL}\) containing \(n_s\) phase values, corresponding to different frequencies.
The step size of the frequency between two points in the array can be calculated as

\[ f_{step} = \frac{f_s}{n_s} \]  \hspace{1cm} (6.27)

The index of the envelope frequency in the phase array can thus be calculated as

\[ k = \left( \frac{f_e}{f_{step}} \right) + 1 \]  \hspace{1cm} (6.28)

The one added to (6.28) is necessary due to the fact that MATLAB indices work from 1, not from 0.

The measured absolute phase offset of the envelope is given by

\[ \vartheta_U = \Theta_U[k] \]  \hspace{1cm} (6.29)

Note that in the simulations, \( \varphi \) is used to denote the actual phase offsets, as calculated from the known distances, and \( \vartheta \) is used to denote the estimated phase offsets, as computed from the DFT. The error in the actual phase offset and the estimated phase offset is given by

\[ \varepsilon_{phase} = \varphi - \vartheta \]  \hspace{1cm} (6.30)

Algorithm 6 shows the calculation of the absolute phase offset from the quantised envelope.

Using the above algorithms the absolute phase offset of the envelope signal at node \( V \) due to the pure sine wave transmissions from nodes \( A \) and \( X \) can also be determined. These absolute phase offsets are used by the algorithms in the following section.
Algorithm 6 Calculate Absolute Phase Offset from $E_U$

1: compute DFT of $E_U$
   \[ E_U^{DFT} \leftarrow \text{DFT of } E_U \] ▶ Uses MATLAB fft function

2: compute Phase of $E_U^{DFT}$
   \[ \Theta_U \leftarrow \text{phase of } E_U^{DFT} \] ▶ Uses MATLAB angle function

3: $f_{\text{step}} \leftarrow \frac{f_s}{n_s}$ ▶ Frequency step size

4: $k \leftarrow \left(\frac{f_e}{f_{\text{step}}}\right) + 1$ ▶ Index of $f_e$ in phase array

5: $\vartheta_{U} \leftarrow \Theta_U[k]$ ▶ Absolute phase offset of $E_U$

q-range Value Calculation

By using the algorithms of the above section, the absolute phase offset of the interference signal at nodes $U$ and $V$ can be obtained. Using (6.15) the relative phase offset of nodes $U$ and $V$ can be obtained from these absolute phase offsets. To simplify the simulation of the system we assume that the q-range ambiguity can be solved, as stated in chapter 5, and all simulations will be done with distances limiting the relative phase shift to less than one wavelength. This implies that the q-range value, $d_{XYUV}$ can be calculated using (6.16). Algorithm 7 shows the calculation of the $d_{XYUV}$.

Algorithm 7 Calculate q-range value, $d_{AXUV}$

1: $\vartheta_{AXUV} \leftarrow \vartheta_U - \vartheta_V$ ▶ Calculation of relative phase offset for quad $(A, X, U, V)$

2: $d_{XYUV} \leftarrow \vartheta_{AXUV} \cdot \frac{\lambda_c}{2\pi}$

By using the above algorithms to simulate the measurement of the five linear independent q-ranges from equation (6.18), the position of mobile node $A$ can be calculated, as discussed in the section

6.3.2 Localization of Node $A$

The simulation of the localization for node $A$ is implemented using the algorithms presented in section 6.3.1 to calculate the q-range values for all of the linear indepen-
dent q-ranges from (6.18). By expanding the set of equations using (6.19) we find a set of five non-linear equations with three unknowns (the 3D position coordinates of $A$). MATLAB’s numerical solver function is used to solve the set of equations for the three unknowns, yielding the location of the mobile node $A$. Algorithm 8 shows the implementation of the localization simulation.

**Algorithm 8 Localization of Node $A$**

1: calculate simulation variables \(\Delta\) Calculation done using Algorithm 1  
2: for all \(d_{XYUV} \in (6.18)\) do  
3: \hspace{1em} for all \(U \in \{U, V\}\) of current quad do  
4: \hspace{2em} calculate \(I_U\) \(\Delta\) Calculation done using Algorithm 2  
5: \hspace{2em} calculate \(R_U\) \(\Delta\) Calculation done using Algorithm 3  
6: \hspace{2em} calculate \(E_U\) \(\Delta\) Calculation done using Algorithm 4  
7: \hspace{2em} calculate \(\theta_U\) \(\Delta\) Calculation done using Algorithm 6  
8: \hspace{1em} end for  
9: \hspace{1em} calculate \(d_{XYUV}\) \(\Delta\) Calculation done using Algorithm 7  
10: end for  
11: solve (6.18) \(\Delta\) Uses MATLAB fsolve function

### 6.4 Verification and Validation of Mathematical Model and Simulation

In this section the verification and validation of the mathematical model and its corresponding simulation is done. According to [12] model verification is defined as “ensuring that the computer program of the computerised model and its implementation are correct”. They further define model validation as substantiating “that a computerised model within its domain of applicability possesses a satisfactory range of accuracy consistent with the intended application of the model”.

We start by defining the intended application of the model from the previous chapters. The model is intended to validate the concept system by using it to determine whether
Chapter 6 Verification and Validation of Mathematical Model and Simulation

or not the concept system is able to achieve an accuracy of at least 3 cm. The model is implemented for ideal circumstances, as the failure of the model to achieve the desired accuracy for ideal circumstances implies the system will fail to achieve the desired accuracy for non-ideal circumstances.

According to [12], model verification and validation can be related to the modelling process by:

- Conceptual model validation.
- Computerised model verification.
- Operational validation.

Conceptual model validation is done by determining that the theories and assumptions underlying the conceptual model are correct and that the structure, logic and, mathematical and causal relationships are reasonable for the intended application [12].

Computerised model verification ensures the implementation of the conceptual model is correct [12]. Use of a simulation results in fewer errors than implementation using a general programming language, and reduces verification mainly to ensuring that the model is programmed correctly [12].

Operational validity is done to determine whether the model’s output behaviour is accurate enough for its intended purpose [12].

In the following sections each of the above will be done for the mathematical model and simulation.

6.4.1 Conceptual Model Validation

We start the concept model validation by determining whether the theorems and assumptions underlying the model are correct.
During the derivation of the concept model, we use the Fourier analysis of the signal during the various stages of manipulation (from the interference signal to the envelope signal) to prove the phase offset of the envelope signal is the difference between the phase offsets of the two sine waves comprising the interference signal. According to [31] sufficient conditions for the existence of a Fourier transform of a signal is:

1. On a finite interval
   (a) The signal is bounded.
   (b) The has a finite number of maxima and minima.
   (c) The signal has a finite number of discontinuities.

2. The signal is absolutely integrable.

From the above it can be concluded that a Fourier transform for the signal (in all its various forms) exists, and Fourier Analysis can thus be used.

Theorem 4 is used to determine the set of five linear independent equations for the concept model. The definition of a quad as used by the theorem corresponds to the definition of a quad as used in the model. Further the concept model contains \( n = 5 \geq 3 \), satisfying the requirement of the theorem that \( n \geq 3 \). The theorem can thus be used to determine the set of linear equations used in the model.

From the above we can conclude that the theorems used in the model are valid and applied correctly.

We now consider the assumptions made in the model. The first assumption is that the waves propagate at the speed of light, \( c \). If we take into account that the intended purpose of the model is the validation of the concept system for ideal circumstances, this assumption holds. The second assumption made is that the second term of (6.13) can be disregarded. For the model, the distances used should be less than 2 m maximum between all of the nodes. If we consider that for ideal circumstances the frequency of the envelope can be kept as small as needed, the assumption is valid. We can thus con-
clude that the assumptions made for the model are valid if the intended application of the model is considered.

The second part of the validation of the model is to determine whether the representation of the structure, logic and, mathematical and causal relationships are reasonable for the intended application of the system.

The mathematical relationships as expressed in (6.1) and (6.2) are the standard representation of pure sine waves, and are thus reasonable. The mathematical representation of the interference signal is cited from [16], a peer reviewed, published conference article. The noise term is dropped from the equation, consistent with the intended application of the model, which is implemented for ideal circumstances. It is further reasonable to assume for ideal circumstances that the phase shifts in the interference signal will be due to the time the waves have been generated as well as the distances the waves propagated. Equations (6.5) and (6.4) are used from [16]. An ideal rectified wave contains only the positive parts of the input wave and (6.6) is thus reasonable considering the intended application of the model. The filtering of the interference signal is modeled by the multiplication of the frequency representation of the rectified signal with the transfer function of a Butterworth LPF. This is valid as the output of a system is the equivalent to the multiplication of the frequency domain input function with the frequency domain transfer function of the system [31]. The Butterworth filter transfer function is used as given in [33], and can thus be assumed to be valid. The mathematical relationships for equations (6.10) to (6.18) are based on the above mentioned valid equations and are consistent with the equations from the published work on RIPS as given in chapter 4. The expansion of the q-range formulas in (6.19) is done using the distance formula, which is valid for 3D cartesian space.

The structure and logic followed is consistent with the published work on RIPS as discussed in chapter 4.

From the discussion in this section it is concluded that the theorems and assumptions made are correct and the representation of the structure, logic and, mathematical and
causal relationships are reasonable for the intended purpose of the model. The conceptual model is thus successfully validated according to requirements as stated in [12]. In the next section the verification of the computerised model is done.

### 6.4.2 Computerised Model Verification

As stated previously, the verification of the computerised model ensures the programming and implementation of the model are correct. We now verify each of the algorithms in section 6.3.

#### Algorithm 1 Verification

We verify algorithm 1 by entering the input parameters and comparing the results to the expected results calculated using the applicable formulas. The input parameters are presented in table 6.1.

<table>
<thead>
<tr>
<th>Input Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>((x_A, y_A, z_A))</td>
<td>((0.65, 0.77, 0.37))</td>
</tr>
<tr>
<td>((x_B, y_B, z_B))</td>
<td>((0.4, 1.7, 0.9))</td>
</tr>
<tr>
<td>((x_C, y_C, z_C))</td>
<td>((1.8, 0.5, 1.7))</td>
</tr>
<tr>
<td>((x_D, y_D, z_D))</td>
<td>((1.6, 1.5, 1.3))</td>
</tr>
<tr>
<td>((x_E, y_E, z_E))</td>
<td>((0.9, 0.4, 0.4))</td>
</tr>
<tr>
<td>(a_{XU}, \forall X \neq U \in {A, B, C, D, E})</td>
<td>1</td>
</tr>
<tr>
<td>(f_X)</td>
<td>15 MHz</td>
</tr>
<tr>
<td>(t_X, \forall X \in {A, B, C, D, E})</td>
<td>0 s</td>
</tr>
<tr>
<td>Numerator Coefficients</td>
<td>([2.481 \cdot 10^{17}])</td>
</tr>
<tr>
<td>Denominator Coefficients</td>
<td>([1; 1.257 \cdot 10^{6}; 7.896 \cdot 10^{11}; 2.481 \cdot 10^{17}])</td>
</tr>
<tr>
<td>(f_s)</td>
<td>20 kHz</td>
</tr>
<tr>
<td>(T_d)</td>
<td>0.005 s</td>
</tr>
<tr>
<td>(B)</td>
<td>4</td>
</tr>
<tr>
<td>(V_{fs})</td>
<td>1 V</td>
</tr>
</tbody>
</table>
The results obtained from the simulation is shown in figure 6.6.

![Figure 6.6: Screenshot of Results obtained for algorithm 1](image)

The transformation of the numerator and denominator of the filter from the s to the z domain is done using a built-in MATLAB function and should thus be correct. The rest of the results correspond to the expected results, thus we can assume that the implementation of algorithm 1 is correct.

Algorithm 2 Verification

As with the verification of algorithm 1, we again use the input parameters of table 6.1 and check the result to verify whether the algorithm and its implementation is correct.
Figure 6.7 is the plot of the interference signal at node C due to transmissions from nodes A and B calculated using the implementation of algorithm 2.

The plot corresponds to what we expect, based on plots from the work on RIPS. The sampling duration, $T_d$ from table 6.1 is equal to 10 periods of the envelope frequency. Figure 6.7 can easily be seen to contain 10 envelope periods. We thus conclude that the interference signal calculated using algorithm 2 is correct. As the same algorithm is used for all the interference signal calculations, it implies that if the interference signal at node C for transmissions from nodes A and B is correct, all the other combinations calculated using the implementation of algorithm 2 will also be correct.

**Algorithm 3 Verification**

Using the interference signal from the verification of algorithm 2 as input, algorithm 3 is used to calculate the corresponding rectified signal at node C due to transmissions from nodes A and B. The plot of the calculated rectified signal is shown in figure 6.8.
Figure 6.8: Plot of Rectification Signal Obtained at Node C due to Transmission of Nodes A and B

The plot corresponds to the expected result, as it is almost an exact replica of the interference signal, the only exception being that only the positive values of the signal is present, as expected when implementing (6.6). We again contend that if the algorithm is successful in calculating the rectified signal for the specific case of the interference signal at C due to transmissions by nodes A and B, it will be able to correctly calculate the rectified signal for all the other combinations of transmitters and receivers.

**Algorithms 4 and 5 Verification**

The rectified signal calculated using the implementation of algorithm 3 in the previous section is used as the input to algorithm 4 to calculate the envelope signal at node C due to the transmission of nodes A and B. The plot of the filtered and downsampled signal ($F_U^s$, line 2 in algorithm 4) is shown in figure 6.9.

As expected, the high frequency component in figure 6.8 is not present in the envelope signal, due to the low pass filtering of the signal. The rough edges (see the maximum
Figure 6.9: Plot of Envelope Signal Obtained at Node C due to Transmission of Nodes A and B

point for each of the periods) can be explained by the downsampling of the signal, and is thus expected. To verify the frequency of the envelope signal we calculate and plot the magnitude of the FFT of envelope signal in figure 6.10, using the MATLAB fft function.

The frequency plot contains two major components, the DC or 0 Hz component, that can be expected due to the rectification of the signal and the 2 kHz envelope frequency. The magnitude spectrum confirms that algorithm 4 calculated \( F^*_U \) correctly.

Algorithm 5 then quantise the \( F^*_U \) function in line 4 of algorithm 4. The implementation calculated the following values for the number of quantisation levels and the quantisation step size:

- \( N_q = 16 \)
Chapter 6 Verification and Validation of Mathematical Model and Simulation

Figure 6.10: FFT Magnitude Plot of Envelope Signal Obtained at Node C due to Transmission of Nodes A and B

- $q = 0.0667 \text{ V}$

Both of which is correct in terms of (6.26) and (5.5).

The quantised signal as calculated by the implementation of algorithm 5 is plotted in figure 6.11, in green, with the measured sample from figure 6.9 plotted in blue.

A scatter plot of the quantised values is shown in figure 6.12.

From these values it is clear that the quantised signal has values at integer multiples of $q$. For example, at $x = 0.00075$ the value of $y = 0.1333 \approx 2q$ and at $x = 0.00085$ the value of $y = 0.2 \approx 3$ and so forth. The small difference between the calculated value and the actual multiple integer of $q$ is due to rounding errors.

From the above it is concluded that algorithms 4 and 5 are implemented correctly.

Algorithm 6 Verification

The quantised envelope signal from the previous section is used as input to the implementation of algorithm 6. The magnitude and phase plots of the DFT of the signal as
calculated using the MATLAB fft and angle function is shown in figure 6.13.

The values for the variables used in the calculation of the absolute phase offsets are:

- \( f_{step} = 200 \text{ Hz} \)
- \( k = 11 \)

These are correct if compared to calculations done using (6.28) and (6.28). The absolute phase offset is calculated as

\[
\theta_U = 3.5323^\circ
\]

which corresponds to value indicated in figure 6.13.

The difference between the actual phase offset, \( \varphi_U \), and the calculated phase offset, \( \theta_U \), was expected from (6.9). From the above it is concluded that the algorithm successfully calculated the measured phase offset.
Chapter 6  Verification and Validation of Mathematical Model and Simulation

Algorithms 7 and 8 Verification

For the verification of the last two algorithms, we use a new set of input parameters, given in table 6.2, and test the overall implementation.

The transfer function of the LPF is that of a 3rd order linear phase filter with \( f_{\text{stop}} = 500 \text{ kHz} \).

The simulation variables, calculated using the implementation of algorithm 1 is given in table 6.3.

The calculated measured \( \theta_U, \forall U \in \{U, V\} \) of the quads used in localization, computed using the combination of algorithms 3 to 6 is presented in table 6.4. The sub-subscript denotes the two transmitting nodes for which the absolute phase offset at the receiver is made.

Using the implementation of algorithm 7 the relative phase offset, as well as the q-range measurement for each of the measured quads are presented in table 6.5.
By using the above q-range measurements to solve for the system of equations using the MATLAB fsolve function, with starting point \((0.5,0.5,0.5)\), the location of A is calculated as:

\[
\begin{align*}
A(x_A, y_A, z_A) &= (0.8603, 0.7803, 0.8696)
\end{align*}
\]

Which corresponds to the input parameters for the placement of the nodes. From this we conclude that the implementation of algorithms 7 and 8 is implemented correctly.

### 6.4.3 Operational Validity of Model and Simulation

As defined earlier in the chapter, the operational validation of the system is done by determining whether the output behaviour of the system is accurate enough for the intended purpose of the model. The results from the verification of the algorithms in section 6.4.2, specifically the results obtained during the verification of algorithms 7 and 8, proves that the output behaviour of the system can be used to determine the
response of the concept system under ideal conditions. It can thus be concluded that the system output behaviour is accurate enough for the intended purpose of the model.

### 6.5 Summary

In this chapter the derivation of the ideal deterministic mathematical model of the system is done from the conceptual design. The model is based on two propositions. The first (proposition 1) states that the phase offset of the envelope of the interference signal at a node $U$ due to the transmission of pure sine waves by nodes $A$ and $X$, is the difference of the phase offsets of each of the transmitted sine signals, as well as an error introduced by filtering the signal. Equation 6.5 gives the representation of the interference signal as

$$I_U(t) = a_{AU} \cos(2\pi f_A t + \varphi_{AU}) + a_{XU} \cos(2\pi f_X t + \varphi_{XU})$$
### Chapter 6 Summary

#### Table 6.3: Calculated Variables - Verification Test of Algorithms 7 and 8

<table>
<thead>
<tr>
<th>Variable</th>
<th>Calculated Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d_{AB}$</td>
<td>0.9407</td>
</tr>
<tr>
<td>$d_{AC}$</td>
<td>1.2849</td>
</tr>
<tr>
<td>$d_{AD}$</td>
<td>0.9426</td>
</tr>
<tr>
<td>$d_{AE}$</td>
<td>0.6057</td>
</tr>
<tr>
<td>$d_{BC}$</td>
<td>1.9519</td>
</tr>
<tr>
<td>$d_{BD}$</td>
<td>1.1183</td>
</tr>
<tr>
<td>$d_{BE}$</td>
<td>1.3928</td>
</tr>
<tr>
<td>$d_{CD}$</td>
<td>1.0539</td>
</tr>
<tr>
<td>$d_{CE}$</td>
<td>1.5843</td>
</tr>
<tr>
<td>$d_{DE}$</td>
<td>1.4442</td>
</tr>
<tr>
<td>$f_c$</td>
<td>94.999 MHz</td>
</tr>
<tr>
<td>$f_e$</td>
<td>2 kHz</td>
</tr>
<tr>
<td>$\varphi_{AB}$</td>
<td>1.8717 rad</td>
</tr>
<tr>
<td>$\varphi_{AC}$</td>
<td>2.5565 rad</td>
</tr>
<tr>
<td>$\varphi_{AD}$</td>
<td>1.8755 rad</td>
</tr>
<tr>
<td>$\varphi_{AE}$</td>
<td>1.2052 rad</td>
</tr>
<tr>
<td>$\varphi_{BC}$</td>
<td>3.8836 rad</td>
</tr>
<tr>
<td>$\varphi_{BD}$</td>
<td>2.225 rad</td>
</tr>
<tr>
<td>$\varphi_{BE}$</td>
<td>2.7712 rad</td>
</tr>
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<td>$\varphi_{CD}$</td>
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<td>$\varphi_{CE}$</td>
<td>3.1552 rad</td>
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<td>$\varphi_{DE}$</td>
<td>2.8734 rad</td>
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<td>47499500</td>
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<tr>
<td>$n_s$</td>
<td>2500</td>
</tr>
<tr>
<td>$SR$</td>
<td>19000</td>
</tr>
<tr>
<td>$\lambda_c$</td>
<td>3.1579 m</td>
</tr>
<tr>
<td>z-numerator Coefficients</td>
<td>$[4.5198 \times 10^{-12}; 1.3556 \times 10^{-11}; 1.3564 \times 10^{-11}; 4.5173 \times 10^{-12}]$</td>
</tr>
<tr>
<td>z-denominator Coefficients</td>
<td>$[1; -2.9994; 2.9987; -0.9994]$</td>
</tr>
</tbody>
</table>

The absolute phase offset is then given by (6.9)

$$\varphi_U = \varphi_{AU} - \varphi_{XU} - \varepsilon_{filter}.$$  

The envelope signal can be obtained from the interference signal by rectifying, filtering, and sampling the interference signal.
Table 6.4: Measured $\theta_{UI}$ - Verification Test of Algorithms 7 and 8

<table>
<thead>
<tr>
<th>Variable</th>
<th>Measured Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_{BAC}$</td>
<td>2.0043 rad</td>
</tr>
<tr>
<td>$\theta_{BAD}$</td>
<td>0.3454 rad</td>
</tr>
<tr>
<td>$\theta_{CAB}$</td>
<td>1.32 rad</td>
</tr>
<tr>
<td>$\theta_{DAB}$</td>
<td>0.3416 rad</td>
</tr>
<tr>
<td>$\theta_{DAC}$</td>
<td>0.2131 rad</td>
</tr>
<tr>
<td>$\theta_{EAB}$</td>
<td>1.5588 rad</td>
</tr>
<tr>
<td>$\theta_{EAC}$</td>
<td>1.9394 rad</td>
</tr>
<tr>
<td>$\theta_{EAD}$</td>
<td>1.6609 rad</td>
</tr>
</tbody>
</table>

Table 6.5: Quad Measurements - Verification Test of Algorithms 7 and 8

<table>
<thead>
<tr>
<th>Quad</th>
<th>Relative Phase Offset (rad)</th>
<th>q-range</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(A, B, C, D)$</td>
<td>0.9784</td>
<td>0.4918</td>
</tr>
<tr>
<td>$(A, B, C, E)$</td>
<td>-0.2388</td>
<td>-0.12</td>
</tr>
<tr>
<td>$(A, C, B, D)$</td>
<td>1.7912</td>
<td>0.9002</td>
</tr>
<tr>
<td>$(A, C, B, E)$</td>
<td>0.0649</td>
<td>0.0326</td>
</tr>
<tr>
<td>$(A, D, B, E)$</td>
<td>-1.3155</td>
<td>-0.6611</td>
</tr>
</tbody>
</table>

The second proposition (proposition 2) states that the q-range can be calculated from the relative phase offset between two receivers using (6.17)

$$d_{AXUV} = \varphi_{AXUV} \cdot \frac{\lambda_c}{2\pi} + n\lambda_c, \quad n \in \mathbb{Z}.$$  

Using the set of linear independent q-ranges from (6.18) the location of $A(x_A, y_A, z_A)$ can be solved.

In section 6.3 algorithms used to implement the model in simulation software are discussed. Finally the model and the simulation are verified and validated using the method presented in [12]. It is concluded that the mathematical model is valid, the various algorithms and their implementations are verified, and it is determined that the output behaviour of the model can be used to experiment with the response of the system under ideal circumstances.

In the next chapter the verified and validated mathematical model and implemented simulation are used to experiment with the concept system.
Chapter 7

Simulation: Tests and Results

In this chapter the simulation implementation of the mathematical model of the system is used to determine the effect of various parameters on the accuracy of the system. The results of these tests are then used to make recommendations concerning the setup of the concept system. Finally the recommended setup is used in the simulation of the system for 20 random positions. The result of this simulation validates the concept system.

7.1 Introduction

In the previous chapter we derived an ideal deterministic mathematical model of the concept system from the conceptual design in chapter 5. The model as well as its simulation implementation was then verified and validated. In this chapter we use the model to simulate the concept system for different scenarios affecting the accuracy of the system. The simulation results enable us to make recommendations on how to set up the concept system to maximise accuracy. Using these recommendations as the input parameters to the model, we simulate the system for a set of 20 random positions within the bounds of the cylinder. The results obtained from this simulation enable us
to determine whether the concept system is a viable solution for tracking the spheres through the RPV model.

This chapter address the third and final sub-objective as stated in section 1.4.2 and is organised as follows. In section 7.2 the simulation are used to determine the effect of the placement of the anchor nodes on the accuracy of the system. In section 7.3 simulations are done to determine the effect of phase ambiguity on the accuracy of the system. The simulations presented in section 7.4 are used to determine the effect of the carrier and envelope frequency on the accuracy of the system. The effect of transmitting unsynchronised sine waves as well as the effect of the differing amplitudes of the two sine waves at the receiver are investigated in sections 7.5 and 7.6. In section 7.7 the effect of the sampling frequency on the accuracy of the system is determined. In sections 7.8 and 7.9 the effect of the duration the interference signal is sampled and the quantisation error introduced by sampling respectively on the accuracy of the system are investigated. Finally in section 7.10 the system is simulated for a set of 20 random mobile node positions.

### 7.2 Node Placement Simulations

In these simulations we experiment with the placement of the anchor nodes to determine the effect on the accuracy of the system. From the previous chapters we know that the RPV model is a cylinder with height 2 m and radius 0.5 m. Based on this we decide to use a 3D cartesian grid of 2 m x 2 m x 2m for the simulation. In this grid the maximum distance is the distance diagonally from one corner of the cube to another as illustrated in figure 7.1. It follows from the measurements of the cube that $d_{\text{max}} = 3.464$ m.

To prevent q-range ambiguity we choose $f_A = 20$ MHz and $f_X = 19.998$ MHz implying $f_c = 19.999$ MHz with wavelength, $\lambda_c = 15.008$ m, which is much larger than $d_{\text{max}}$. We further limit the effect of other parameters on the system by choosing $B = 18$, limiting the effect of quantisation due to the quantisation error, setting
Chapter 7

Node Placement Simulations

Figure 7.1: Grid Volume and Maximum distance

\[ t_U = 0, \forall U \in \{A, B, C, D, E\}, \text{ setting } a_{XU} = 1, \forall X \neq U \in \{A, B, C, D, E\}, \text{ and the sampling parameters are chosen as } f_s = 500 \text{ kHz} \gg f_e = 2 \text{ kHz and } T_d = 0.005s. \]

We place the centre of the bottom circle of the cylinder at the centre of the square in the X-Y plane, coordinates \((1, 1, 0)\), as illustrated in figure 7.2.

Figure 7.2: Position of Cylinder
We now investigate the effect of the anchor nodes on the system using two different mobile node positions. The two mobile node locations are $A_1(0.8, 0.6, 1.3)$ and $A_2(0.86, 0.78, 0.87)$.

### 7.2.1 Anchor Node Positions Test 1

The first simulation test is done with the nodes all at the same height, and on the corner edges of the cube. The height is chosen as 1 m as this will cause a large angle of incidence for the ground reflection and increase the length of the reflected path relative to the direct LOS signal in the physical implementation of the system. The positions are illustrated in figure 7.3, where the gray nodes represent the anchor spheres, the red sphere is $A_1$ and the green sphere is $A_2$.

![Figure 7.3: Positions of Anchor Nodes for First Anchor Location Simulation](image)

The positions of the anchor nodes are presented in table 7.1

The results of the simulation is presented in table 7.2

For the calculation of $A_2$’s position, the numerical solver fails to converge.
### Table 7.1: Anchor Nodes Position - Test 1

<table>
<thead>
<tr>
<th>Node</th>
<th>Position</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>(0, 0, 1)</td>
</tr>
<tr>
<td>C</td>
<td>(0, 2, 1)</td>
</tr>
<tr>
<td>D</td>
<td>(2, 2, 1)</td>
</tr>
<tr>
<td>E</td>
<td>(2, 0, 1)</td>
</tr>
</tbody>
</table>

### Table 7.2: Results of Anchor Nodes Position - Test 1

<table>
<thead>
<tr>
<th>Mobile Node</th>
<th>Actual</th>
<th>Calculated</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>(0.8, 0.6, 1.3)</td>
<td>(0.8048, 0.6083, 0.9427)</td>
<td>(0.0048, 0.0083, 0.3573)</td>
</tr>
<tr>
<td>$A_2$</td>
<td>(0.86, 0.78, 0.87)</td>
<td>Solving Failed</td>
<td></td>
</tr>
</tbody>
</table>

#### 7.2.2 Anchor Node Position Test 2

In the second simulation we move the anchor nodes to the positions as shown in figure 7.4 to determine whether the anchor nodes’ positions resulted in the solver failing to localise $A_2$.

![Figure 7.4: Positions of Anchor Nodes for Second Anchor Location Simulation](image)

Figure 7.4: Positions of Anchor Nodes for Second Anchor Location Simulation
The positions are presented in table 7.3

<table>
<thead>
<tr>
<th>Node</th>
<th>Position</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>(0, 1, 1)</td>
</tr>
<tr>
<td>C</td>
<td>(1, 2, 1)</td>
</tr>
<tr>
<td>D</td>
<td>(2, 1, 1)</td>
</tr>
<tr>
<td>E</td>
<td>(1, 0, 1)</td>
</tr>
</tbody>
</table>

The simulation results are presented in table 7.4

<table>
<thead>
<tr>
<th>Mobile Node</th>
<th>Position (m)</th>
<th>Actual</th>
<th>Calculated</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>A₁</td>
<td>(0.8, 0.6, 1.3)</td>
<td>(0.8082, 0.619, 0.9999)</td>
<td>(0.0082, 0.019, 0.3001)</td>
<td></td>
</tr>
<tr>
<td>A₂</td>
<td>(0.86, 0.78, 0.87)</td>
<td>(0.8604, 0.7805, 0.9077)</td>
<td>(0.0004, 0.0005, 0.0377)</td>
<td></td>
</tr>
</tbody>
</table>

### 7.2.3 Anchor Node Position Test 3

The results of the second test show that the errors in the calculated z coordinates are large compared to the other errors. When examining the physical setup for the above two tests, all the anchor position coordinates change, except for the z-coordinates, which stay the same (they are all on the level z = 1). In this test we determine whether the accuracy can be improved by changing the z-coordinates of the anchor nodes to different values. The positions of the anchor nodes are shown in figure 7.5.

The positions are presented in table 7.5.

The simulation results are presented in table 7.6

Although the accuracy of the x and y coordinates are worse, the accuracy of the z coordinates are better.
Figure 7.5: Positions of Anchor Nodes for Third Anchor Location Simulation

<table>
<thead>
<tr>
<th>Node</th>
<th>Position (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>(0, 1, 2)</td>
</tr>
<tr>
<td>C</td>
<td>(1, 1, 1.5)</td>
</tr>
<tr>
<td>D</td>
<td>(2, 1, 0.5)</td>
</tr>
<tr>
<td>E</td>
<td>(1, 0, 0)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mobile Node</th>
<th>Actual (m)</th>
<th>Calculated (m)</th>
<th>Error (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>(0.8, 0.6, 1.3)</td>
<td>(0.7835, 0.6289, 1.2767)</td>
<td>(0.0165, 0.0289, 0.0233)</td>
</tr>
<tr>
<td>$A_2$</td>
<td>(0.86, 0.78, 0.87)</td>
<td>(0.8589, 0.7826, 0.8678)</td>
<td>(0.0011, 0.0026, 0.0022)</td>
</tr>
</tbody>
</table>

### 7.2.4 Anchor Node Position Test 4

The results obtained in Test 3 (section 7.2.3) seems to imply that the more distributed the anchor nodes are along an axis, the better the localization accuracy of the coordi-
nates along that axis will be. We intuitively divide each of the axes into four and dis-
tribute the nodes into all of these regions while simultaneously trying to avoid sym-
metry between the nodes’ positions. Figures 7.6, 7.7, and 7.8 shows how the anchor
nodes are divided into these regions.

Figure 7.6: Distribution of Anchor Nodes along z-axis

The positions of the anchor nodes are presented in table 7.7.

<table>
<thead>
<tr>
<th>Node</th>
<th>Position</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>(0.4, 1.7, 0.9)</td>
</tr>
<tr>
<td>C</td>
<td>(1.8, 0.5, 1.7)</td>
</tr>
<tr>
<td>D</td>
<td>(1.6, 1.5, 1.3)</td>
</tr>
<tr>
<td>E</td>
<td>(0.9, 0.4, 0.4)</td>
</tr>
</tbody>
</table>

The simulation results are presented in table 7.8.

The results obtained for this test are all accurate to withing 2 mm, a definitive im-
provement on the results obtained during the previous three tests. In the next section
a summary of the results for the anchor nodes positions tests is presented.
Chapter 7

Node Placement Simulations

Figure 7.7: Distribution of Anchor Nodes along x-axis

<table>
<thead>
<tr>
<th>Mobile Node</th>
<th>Actual</th>
<th>Calculated</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>$(0.8, 0.6, 1.3)$</td>
<td>$(0.7979, 0.598, 1.3016)$</td>
<td>$(0.0021, 0.002, 0.0016)$</td>
</tr>
<tr>
<td>$A_2$</td>
<td>$(0.86, 0.78, 0.87)$</td>
<td>$(0.8581, 0.7786, 0.8709)$</td>
<td>$(0.0019, 0.0014, 0.0009)$</td>
</tr>
</tbody>
</table>

### 7.2.5 Summary of Node Placement Simulation Results

In this section (7.2) we experimented with the effect of the position of the anchor nodes on the accuracy of the system. The input parameters were chosen to negate their influence on the results as far as possible, to determine the effect of the anchor nodes’ positions. The parameters are given in table 7.9.

The positions of the anchor nodes as used in each of the tests are presented in table 7.10.

Table 7.11 gives a summary of the difference between the actual and measured posi-
Chapter 7

Node Placement Simulations

Figure 7.8: Distribution of Anchor Nodes along y-axis

<table>
<thead>
<tr>
<th>Input Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_{XU}$, $\forall X \neq U \in {A, B, C, D, E}$</td>
<td>1</td>
</tr>
<tr>
<td>$f_A$</td>
<td>20 MHz</td>
</tr>
<tr>
<td>$f_X$</td>
<td>19.998 MHz</td>
</tr>
<tr>
<td>$t_X$, $\forall X \in {A, B, C, D, E}$</td>
<td>0 s</td>
</tr>
<tr>
<td>Numerator Coefficients</td>
<td>$[3.101 \cdot 10^{19}]$</td>
</tr>
<tr>
<td>Denominator Coefficients</td>
<td></td>
</tr>
<tr>
<td>$f_s$</td>
<td>500 kHz</td>
</tr>
<tr>
<td>$T_d$</td>
<td>0.005 s</td>
</tr>
<tr>
<td>$B$</td>
<td>18</td>
</tr>
<tr>
<td>$V_{fs}$</td>
<td>2 V</td>
</tr>
</tbody>
</table>

From the results in table 7.11 it is easily verified that the distributed anchor positions used during the fourth simulation, yields the most accurate results. Finding an optimal
Table 7.10: Anchor Nodes Positions - All Tests

<table>
<thead>
<tr>
<th>Test</th>
<th>Node B</th>
<th>Node C</th>
<th>Node D</th>
<th>Node E</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(0, 0, 1)</td>
<td>(0, 2, 1)</td>
<td>(2, 2, 1)</td>
<td>(2, 0, 1)</td>
</tr>
<tr>
<td>2</td>
<td>(0, 1, 1)</td>
<td>(1, 2, 1)</td>
<td>(2, 1, 1)</td>
<td>(1, 0, 1)</td>
</tr>
<tr>
<td>3</td>
<td>(0, 1, 2)</td>
<td>(1, 1, 1.5)</td>
<td>(2, 1, 0.4)</td>
<td>(1, 0, 0)</td>
</tr>
<tr>
<td>4</td>
<td>(0.4, 1.7, 0.9)</td>
<td>(1.8, 0.5, 1.7)</td>
<td>(1.6, 1.5, 1.3)</td>
<td>(0.9, 0.4, 0.4)</td>
</tr>
</tbody>
</table>

Table 7.11: Anchor Nodes Positions - Error Results for All Tests

<table>
<thead>
<tr>
<th>Mobile Node</th>
<th>Test 1</th>
<th>Test 2</th>
<th>Test 3</th>
<th>Test 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>(4.8, 8.3, 215.3)</td>
<td>(8.2, 1.9, 300.1)</td>
<td>(16.5, 28.9, 23.3)</td>
<td>(2.1, 2, 1.6)</td>
</tr>
<tr>
<td>$A_2$</td>
<td>No Result</td>
<td>(0.4, 0.5, 37.7)</td>
<td>(1.1, 2.6, 2.2)</td>
<td>(1.9, 1.4, 0.9)</td>
</tr>
</tbody>
</table>

Mathematical solution for the positions of the anchor nodes, to increase the accuracy of the system, provides an interesting research question outside the scope of this dissertation. Based on this result we will use these anchor positions (see 7.7 or 7.10) for the rest of the simulations in this chapter.

### 7.3 Phase Ambiguity Simulations

In this section we investigate the effect of phase ambiguity on the accuracy of the system, due to the propagation of the wave being more than one wavelength. As was the case with the anchor node position simulations, we choose the input parameters to negate their effect. The input parameters are presented in table 7.12.

We use the three mobile node locations given in table 7.13. The maximum distance between two nodes in this system is $d_{\text{max}} = 2.01$ m. According to [25] the value of any of the q-ranges can be in the range

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Table 7.12: Input Parameters - Phase Ambiguity Simulations

<table>
<thead>
<tr>
<th>Input Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mobile Node Positions</td>
<td>See table 7.13</td>
</tr>
<tr>
<td>Anchor Node Positions</td>
<td>See table 7.7</td>
</tr>
<tr>
<td>$a_{XU}$, $\forall$ $X \neq U \in { A, B, C, D, E }$</td>
<td>1</td>
</tr>
<tr>
<td>$t_X$, $\forall$ $X \in { A, B, C, D, E }$</td>
<td>0 s</td>
</tr>
<tr>
<td>Numerator Coefficients</td>
<td>$[3.101 \cdot 10^{19}]$</td>
</tr>
<tr>
<td>Denominator Coefficients</td>
<td>$[1 ; 5.971 \cdot 10^6 ; 2.197 \cdot 10^{13} ; 3.101 \cdot 10^{19}]$</td>
</tr>
<tr>
<td>$f_s$</td>
<td>500 kHz</td>
</tr>
<tr>
<td>$T_d$</td>
<td>0.005 s</td>
</tr>
<tr>
<td>$B$</td>
<td>18</td>
</tr>
<tr>
<td>$V_{fs}$</td>
<td>2 V</td>
</tr>
</tbody>
</table>

$$-2d_{max} \leq d_{XYUV} \leq 2d_{max}$$

(7.1)

thus ambiguity may theoretically occur for wavelengths $\lambda_c \leq 2d_{max} = 4.02$ m.

Table 7.13: Mobile Node Positions - Phase ambiguity simulations

<table>
<thead>
<tr>
<th>Node</th>
<th>Position</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>(0.8, 0.6, 1.3)</td>
</tr>
<tr>
<td>$A_2$</td>
<td>(0.86, 0.78, 0.87)</td>
</tr>
<tr>
<td>$A_3$</td>
<td>(1.33, 0.5, 0.63)</td>
</tr>
</tbody>
</table>

We run the simulations for the three different wavelengths shown in table 7.14, chosen specifically to vary from much larger than $2d_{max}$ to just larger than $2d_{max}$ to smaller than $2d_{max}$.

Table 7.14: Wavelengths - Phase ambiguity simulations

<table>
<thead>
<tr>
<th>Test</th>
<th>$f_c$ (MHz)</th>
<th>$\lambda_c$ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>19.999</td>
<td>15.008</td>
</tr>
<tr>
<td>2</td>
<td>69.999</td>
<td>4.2858</td>
</tr>
<tr>
<td>3</td>
<td>144.999</td>
<td>2.069</td>
</tr>
</tbody>
</table>

The simulation results for the three tests are presented in the following sections.
7.3.1 Phase Ambiguity Test 1

For the first simulation we have $\lambda_c = 15.008 \text{ m} \gg 2d_{\text{max}} = 4.02 \text{ m}$. Based on this we expect no phase ambiguity and the results should thus correspond to the results obtained in test 4 of the anchor node positions. The results are presented in table 7.15.

<table>
<thead>
<tr>
<th>Mobile Node</th>
<th>Position (m)</th>
<th>Actual</th>
<th>Calculated</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(0.8 , 0.6 , 1.3)</td>
<td>(0.7979 , 0.598 , 1.3016)</td>
<td>(0.0021 , 0.002 , 0.0016)</td>
</tr>
<tr>
<td>$A_1$</td>
<td></td>
<td>(0.86 , 0.78 , 0.87)</td>
<td>(0.8581 , 0.7786 , 0.8709)</td>
<td>(0.0019 , 0.0014 , 0.0009)</td>
</tr>
<tr>
<td>$A_2$</td>
<td></td>
<td>(1.33 , 0.5 , 0.63)</td>
<td>(1.3287 , 0.498 , 0.631)</td>
<td>(0.0013 , 0.002 , 0.001)</td>
</tr>
</tbody>
</table>

As expected the results corresponds to those obtained in test four in section 7.2

7.3.2 Phase Ambiguity Test 2

For the second simulation we have $\lambda_c = 4.2858 \text{ m} \approx 2d_{\text{max}} = 4.02 \text{ m}$. Based on this there should theoretically be no ambiguity. The results of the simulation are presented in table 7.16.

<table>
<thead>
<tr>
<th>Mobile Node</th>
<th>Position (m)</th>
<th>Actual</th>
<th>Calculated</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(0.8 , 0.6 , 1.3)</td>
<td>(0.7986 , 0.5987 , 1.3009)</td>
<td>(0.0014 , 0.0013 , 0.0009)</td>
</tr>
<tr>
<td>$A_1$</td>
<td></td>
<td>(0.86 , 0.78 , 0.87)</td>
<td>(0.8589 , 0.7792 , 0.8706)</td>
<td>(0.0011 , 0.0008 , 0.0006)</td>
</tr>
<tr>
<td>$A_2$</td>
<td></td>
<td>(1.33 , 0.5 , 0.63)</td>
<td>(1.3286 , 0.4989 , 0.6313)</td>
<td>(0.0014 , 0.0011 , 0.0013)</td>
</tr>
</tbody>
</table>

As expected no large (relative to that expected) localization errors were found, thus we can assume that no phase ambiguity occurred.
7.3.3 Phase Ambiguity Test 3

For the third simulation we have $d_{\text{max}} < \lambda_c = 2.069 \text{ m} < 2d_{\text{max}} = 4.02 \text{ m}$. Based on this, ambiguity may theoretically occur. If ambiguity occurs we can assume it is due to $\lambda_c < 2d_{\text{max}}$ and not due to $d_{\text{max}}$, as the maximum distance between the two nodes are still smaller than the wavelength. The results of the simulation are presented in table 7.17.

<table>
<thead>
<tr>
<th>Mobile Node</th>
<th>Actual (m)</th>
<th>Calculated (m)</th>
<th>Error (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>(0.8, 0.6, 1.3)</td>
<td>(0.8001, 0.5996, 1.2994)</td>
<td>(0.0001, 0.0004, 0.0006)</td>
</tr>
<tr>
<td>$A_2$</td>
<td>(0.86, 0.78, 0.87)</td>
<td>(0.7896, 0.9041, 0.936)</td>
<td>(0.0704, 0.1241, 0.066)</td>
</tr>
<tr>
<td>$A_3$</td>
<td>(1.33, 0.5, 0.63)</td>
<td>(0.6939, 1.3593, 1.348)</td>
<td>(0.6361, 0.8593, 0.718)</td>
</tr>
</tbody>
</table>

The localization error for position $A_1$ is the best result we have obtained thus far and may possibly be attributed to the higher carrier frequency, as is alluded to in the discussion on RIPS in [16]. The localization errors for positions $A_2$ and $A_3$ are larger than expected when compared to the results obtained in the previous two simulations, and we thus assume that the errors are due to phase ambiguity.

7.3.4 Summary of Phase Ambiguity Simulation Results

In this section (7.3) we tested the effect of the wavelength on the accuracy of the system, in terms of phase ambiguity, by using the wavelengths in table 7.14, the anchor nodes positions in table 7.7, and the mobile node positions in table 7.13. The localization error for the location of each of the mobile nodes, simulated for the three different wavelengths are presented in table 7.18.

From the results it can be seen that phase ambiguity occurs when the $\lambda_c < 2d_{\text{max}}$. It also seems that the accuracy of the localization increases as the carrier frequency,
Table 7.18: Phase Ambiguity Simulations - Error Results for all Tests

<table>
<thead>
<tr>
<th>Mobile Node</th>
<th>Test 1</th>
<th>Test 2</th>
<th>Test 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>(2.1, 2, 1.6)</td>
<td>(1.4, 1.3 0.9)</td>
<td>(0.1, 0.4, 0.6)</td>
</tr>
<tr>
<td>$A_2$</td>
<td>(1.9, 1.4, 0.9)</td>
<td>(1.1, 0.8, 0.6)</td>
<td>(70.4, 124.1, 66)</td>
</tr>
<tr>
<td>$A_3$</td>
<td>(1.3, 2, 1)</td>
<td>(1.4, 1.1, 1.3)</td>
<td>(636.1, 859.3, 718)</td>
</tr>
</tbody>
</table>

$f_c$, increases. Based on these results, the carrier frequency for the system used in the following simulations is kept below 70 MHz.

7.4 Frequency Simulations

In this section we experiment with the effect of the carrier ($f_c$) and envelope ($f_e$) frequency on the accuracy of the system. As with the previous simulations we choose the input parameters to minimise their effect. The input parameters are given in table 7.19.

For the first three simulations we keep the envelope frequency constant at $f_e = 20$ kHz while increasing the frequency of the carrier. In the other three simulations we keep the carrier frequency constant at the frequency for which the simulation achieved the best accuracy, while varying the envelope frequency. We force the carrier and envelope frequencies to the desired values by our choice of $f_A$ and $f_X$ using equations (6.4) and (6.5). We choose the sampling time as 10 envelope frequency periods, to eliminate the effect of different lengths of the interference signal being sampled when the envelope frequency varies.

7.4.1 Frequency Test 1

In the first simulation we use the frequencies given in table 7.20

The results for the simulation are given in table 7.21
Table 7.19: Input Parameters - Frequency Simulations

<table>
<thead>
<tr>
<th>Input Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mobile Node Positions</td>
<td>See table 7.13</td>
</tr>
<tr>
<td>Anchor Node Positions</td>
<td>See table 7.7</td>
</tr>
<tr>
<td>( a_{XU}, \forall X \neq U \in {A, B, C, D, E} )</td>
<td>1</td>
</tr>
<tr>
<td>( t_X, \forall X \in {A, B, C, D, E} )</td>
<td>0 s</td>
</tr>
<tr>
<td>Numerator Coefficients</td>
<td>([3.101 \cdot 10^{19}])</td>
</tr>
<tr>
<td>Denominator Coefficients</td>
<td>([1 ; 5.971 \cdot 10^6 ; 2.197 \cdot 10^{13} ; 3.101 \cdot 10^{19}])</td>
</tr>
<tr>
<td>( f_s )</td>
<td>500 kHz</td>
</tr>
<tr>
<td>( T_d )</td>
<td>10( T_e ) s</td>
</tr>
<tr>
<td>( B )</td>
<td>18</td>
</tr>
<tr>
<td>( V_{fs} )</td>
<td>2 V</td>
</tr>
</tbody>
</table>

Table 7.20: Frequency Values - Frequency Simulations Test 1

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_A )</td>
<td>15 MHz</td>
</tr>
<tr>
<td>( f_X )</td>
<td>14.98 MHz</td>
</tr>
<tr>
<td>( f_c )</td>
<td>14.99 MHz</td>
</tr>
<tr>
<td>( f_e )</td>
<td>20 kHz</td>
</tr>
</tbody>
</table>

Table 7.21: Results of Frequency Simulation - Test 1

<table>
<thead>
<tr>
<th>Mobile Node</th>
<th>Actual (m)</th>
<th>Calculated (m)</th>
<th>Error (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_1 )</td>
<td>(0.8 , 0.6 , 1.3)</td>
<td>(0.778 , 0.5775 , 1.3172)</td>
<td>(0.022 , 0.0225 , 0.0172)</td>
</tr>
<tr>
<td>( A_2 )</td>
<td>(0.86 , 0.78 , 0.87)</td>
<td>(0.8397 , 0.7647 , 0.8789)</td>
<td>(0.0203 , 0.0153 , 0.0089)</td>
</tr>
<tr>
<td>( A_3 )</td>
<td>(1.33 , 0.5 , 0.63)</td>
<td>(1.3165 , 0.4791 , 0.6401)</td>
<td>(0.0135 , 0.0209 , 0.0101)</td>
</tr>
</tbody>
</table>

The errors are large relative to the results of some of the previous simulation results, obtained using a carrier frequency of \( f_c = 19.999 \) MHz.
Chapter 7

7.4.2 Frequency Test 2

In the second simulation we use a higher carrier frequency, the frequencies are given in table 7.22.

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_A$</td>
<td>35 MHz</td>
</tr>
<tr>
<td>$f_X$</td>
<td>34.98 MHz</td>
</tr>
<tr>
<td>$f_c$</td>
<td>34.99 MHz</td>
</tr>
<tr>
<td>$f_e$</td>
<td>20 kHz</td>
</tr>
</tbody>
</table>

The results for the simulation are given in table 7.23

<table>
<thead>
<tr>
<th>Mobile Node</th>
<th>Actual</th>
<th>Calculated</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>(0.8, 0.6, 1.3)</td>
<td>(0.7786, 0.5786, 1.3169)</td>
<td>(0.0214, 0.0214, 0.0169)</td>
</tr>
<tr>
<td>$A_2$</td>
<td>(0.86, 0.78, 0.87)</td>
<td>(0.8413, 0.7662, 0.8781)</td>
<td>(0.0187, 0.0138, 0.0081)</td>
</tr>
<tr>
<td>$A_3$</td>
<td>(1.33, 0.5, 0.63)</td>
<td>(1.3159, 0.4804, 0.6418)</td>
<td>(0.0141, 0.0196, 0.0118)</td>
</tr>
</tbody>
</table>

The results do not differ much from those obtained in simulation one, with most of the errors for nodes $A_1$ and $A_2$ being slightly smaller and the errors for the $x$ and $z$ coordinates of node $A_3$ getting slightly bigger.

7.4.3 Frequency Test 3

In the third simulation we use the highest carrier frequency possible before running the risk of phase ambiguity as determined in section 7.3. The frequencies are given in table 7.24

The results for the simulation are given in table 7.25
Table 7.24: Frequency Values - Frequency Simulations Test 3

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_A$</td>
<td>70 MHz</td>
</tr>
<tr>
<td>$f_X$</td>
<td>69.98 MHz</td>
</tr>
<tr>
<td>$f_c$</td>
<td>69.99 MHz</td>
</tr>
<tr>
<td>$f_e$</td>
<td>20 kHz</td>
</tr>
</tbody>
</table>

Table 7.25: Results of Frequency Simulation - Test 3

<table>
<thead>
<tr>
<th>Mobile Node</th>
<th>Actual</th>
<th>Calculated</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>(0.8, 0.6, 1.3)</td>
<td>(0.7839, 0.5847, 1.3106)</td>
<td>(0.0161, 0.0153, 0.0106)</td>
</tr>
<tr>
<td>$A_2$</td>
<td>(0.86, 0.78, 0.87)</td>
<td>(0.8468, 0.7702, 0.8772)</td>
<td>(0.0132, 0.0098, 0.0072)</td>
</tr>
<tr>
<td>$A_3$</td>
<td>(1.33, 0.5, 0.63)</td>
<td>(1.3144, 0.4871, 0.6439)</td>
<td>(0.0156, 0.0129, 0.0139)</td>
</tr>
</tbody>
</table>

Again we see only a small difference, with most of the errors shrinking slightly. Based on this small accuracy improvement for higher carrier frequencies, we use $f_c = 70$ MHz for the following simulations.

7.4.4 Frequency Test 4

In this simulation we keep the carrier frequency constant and vary the envelope frequency. The frequencies used in the simulation are given in table 7.26.

Table 7.26: Frequency Values - Frequency Simulations Test 4

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_A$</td>
<td>70 MHz</td>
</tr>
<tr>
<td>$f_X$</td>
<td>69.9 MHz</td>
</tr>
<tr>
<td>$f_c$</td>
<td>69.95 MHz</td>
</tr>
<tr>
<td>$f_e$</td>
<td>100 kHz</td>
</tr>
</tbody>
</table>

The results for the simulation are given in table 7.27

Compared with the results of the previous simulation (section 7.4.3), performed with
the same carrier frequency, the errors for this simulation are very large. Based on this result, we run the next simulation with a small envelope frequency.

### 7.4.5 Frequency Test 5

This simulation is done with a very low envelope frequency compared to the others used in the simulations thus far. The frequencies used are given in table 7.28.

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_A$</td>
<td>70 MHz</td>
</tr>
<tr>
<td>$f_X$</td>
<td>69.998 MHz</td>
</tr>
<tr>
<td>$f_c$</td>
<td>69.999 MHz</td>
</tr>
<tr>
<td>$f_e$</td>
<td>2 kHz</td>
</tr>
</tbody>
</table>

The results for the simulation are given in table 7.29.

<table>
<thead>
<tr>
<th>Mobile Node</th>
<th>Actual</th>
<th>Calculated</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>(0.8, 0.6, 1.3)</td>
<td>(0.7986, 0.5987, 1.3009)</td>
<td>(0.0014, 0.0013, 0.0009)</td>
</tr>
<tr>
<td>$A_2$</td>
<td>(0.86, 0.78, 0.87)</td>
<td>(0.8589, 0.7792, 0.8706)</td>
<td>(0.0011, 0.0008, 0.0006)</td>
</tr>
<tr>
<td>$A_3$</td>
<td>(1.33, 0.5, 0.63)</td>
<td>(1.3286, 0.4989, 0.6313)</td>
<td>(0.0014, 0.0011, 0.0013)</td>
</tr>
</tbody>
</table>

It is clear from the results in table 7.29 that the errors are smaller for the lower envelope frequency.
7.4.6 Summary of Frequency Simulation Results

In this section we tested the effect of changing the carrier and envelope frequency on the accuracy of the system. The simulations were done using the input parameters in table 7.19. In the first three tests the envelope frequency was kept constant, \( f_e = 20 \text{ kHz} \). The difference between the actual and calculated position of the nodes for the three tests are given in 7.30.

<table>
<thead>
<tr>
<th>Mobile Node</th>
<th>Test 1 ((f_c = 14.99 \text{ MHz}))</th>
<th>Test 2 ((f_c = 34.99 \text{ MHz}))</th>
<th>Test 3 ((f_c = 69.99 \text{ MHz}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_1 )</td>
<td>(22, 22.5, 17.2)</td>
<td>(21.4, 21.4, 16.9)</td>
<td>(16.1, 15.3, 10.6)</td>
</tr>
<tr>
<td>( A_2 )</td>
<td>(20.3, 15.3, 8.9)</td>
<td>(18.7, 13.8, 8.1)</td>
<td>(13.2, 9.8, 7.2)</td>
</tr>
<tr>
<td>( A_3 )</td>
<td>(13.5, 20.9, 10.1)</td>
<td>(14.1, 19.6, 11.8)</td>
<td>(15.6, 12.9, 13.9)</td>
</tr>
</tbody>
</table>

For tests tree to five the envelope frequency varies while the carrier frequency is kept constant at \( f_c = 69.99 \text{ MHz} \). Note that test 3 is common to both tables.

<table>
<thead>
<tr>
<th>Mobile Node</th>
<th>Test 3 ((f_e = 20 \text{ kHz}))</th>
<th>Test 4 ((f_e = 100 \text{ kHz}))</th>
<th>Test 5 ((f_e = 2 \text{ kHz}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_1 )</td>
<td>(16.1, 15.3, 10.6)</td>
<td>(104.4, 117.1, 53.1)</td>
<td>(1.4, 1.3, 0.9)</td>
</tr>
<tr>
<td>( A_2 )</td>
<td>(13.2, 9.8, 7.2)</td>
<td>(104.1, 85.4, 63.9)</td>
<td>(1.1, 0.8, 0.6)</td>
</tr>
<tr>
<td>( A_3 )</td>
<td>(15.6, 12.9, 13.9)</td>
<td>(113.7, 87.9, 95.1)</td>
<td>(1.4, 1.1, 1.3)</td>
</tr>
</tbody>
</table>

From the results in tables 7.30 and 7.31 we conclude that a higher carrier frequency can slightly improve the accuracy of the system while lower envelope frequency will result in significant accuracy improvements. Based on these results we use \( f_c = 69.999 \text{ MHz} \) and \( f_e = 2 \text{ kHz} \) for the remainder of the simulations.
Chapter 7  Asynchronous Transmission Simulations

7.5 Asynchronous Transmission Simulations

In this section we use simulations to determine whether the use of the relative phase offsets in the localization offsets eliminate the need for synchronisation of the transmitting nodes, as expected theoretically. We perform two tests with input parameters provide in table 7.32.

<table>
<thead>
<tr>
<th>Table 7.32: Input Parameters - Asynchronous Transmission Simulations</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Input Parameter</strong></td>
</tr>
<tr>
<td>Mobile Node Positions</td>
</tr>
<tr>
<td>Anchor Node Positions</td>
</tr>
<tr>
<td>( f_A )</td>
</tr>
<tr>
<td>( f_X )</td>
</tr>
<tr>
<td>( a_{XU}, \ \forall \ X \neq U \in { A, B, C, D, E } )</td>
</tr>
<tr>
<td>Numerator Coefficients</td>
</tr>
<tr>
<td>Denominator Coefficients</td>
</tr>
<tr>
<td>( f_s )</td>
</tr>
<tr>
<td>( T_d )</td>
</tr>
<tr>
<td>( B )</td>
</tr>
<tr>
<td>( V_{fs} )</td>
</tr>
</tbody>
</table>

During the first test, all the anchor nodes are set to \( t_x = 0 \) s and \( t_A = 0.4 \) s. For the second simulation we set \( t_A = 0.4 \) s and \( t_C = 0.002 \) s and the rest to \( t_x = 0 \) s. A summary of the error results obtained for the simulations are given in table 7.33.

<table>
<thead>
<tr>
<th>Table 7.33: Asynchronous Simulations - Error Results for Simulations</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mobile Node</strong></td>
</tr>
<tr>
<td>( A_1 )</td>
</tr>
<tr>
<td>( A_2 )</td>
</tr>
<tr>
<td>( A_3 )</td>
</tr>
</tbody>
</table>

From the results it is clear that exactly the same results were obtained for both the tests. The results also correspond to the results obtained in test 5 in section 7.4, where all the time synchronisation values were set to zero. Based on these results we conclude that
Chapter 7 Varying Amplitudes Simulations

the use of the relative phase offset eliminates the effect of asynchronous transmission, thus we will set \( t_X = 0 \), \( \forall X \in \{A, B, C, D, E\} \) for the remaining simulations.

7.6 Varying Amplitudes Simulations

In the physical system, the amplitudes of the two components of the interference signal will vary due to a variety of factors, including the distance the signals travel to the receiver node. In these simulations we determine what the effect of these varying amplitudes are on the accuracy of the system. The simulations are performed using the input parameters in table 7.34.

<table>
<thead>
<tr>
<th>Input Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mobile Node Positions</td>
<td>( A_1 ) and ( A_2 )</td>
</tr>
<tr>
<td>Anchor Node Positions</td>
<td>See table 7.7</td>
</tr>
<tr>
<td>( f_A )</td>
<td>70 MHz</td>
</tr>
<tr>
<td>( f_X )</td>
<td>69.998 MHz</td>
</tr>
<tr>
<td>( t_X, \forall X \in {A, B, C, D, E} )</td>
<td>0 s</td>
</tr>
<tr>
<td>Numerator Coefficients</td>
<td>([3.101 \cdot 10^{19}])</td>
</tr>
<tr>
<td>Denominator Coefficients</td>
<td>([1; 5.971 \cdot 10^6; 2.197 \cdot 10^{13}; 3.101 \cdot 10^{19}])</td>
</tr>
<tr>
<td>( f_s )</td>
<td>500 kHz</td>
</tr>
<tr>
<td>( T_d )</td>
<td>(10T_e) s</td>
</tr>
<tr>
<td>( B )</td>
<td>18</td>
</tr>
<tr>
<td>( V_{fs} )</td>
<td>2 V</td>
</tr>
</tbody>
</table>

Two simulations are done, one with random values, \( 0 < a_{XUI} \leq 1 \), given in table 7.35 and one with amplitudes as given in table 7.36. Note that we do not use the free space loss model to calculate attenuation based on the distance the wave propagates, due to the fact that the simulations are done for the near-field region of the antennas, and thus the free space loss formula is not valid [30].

A summary of the results obtained are presented in table 7.37

The errors for the results obtained are larger than those obtained in previous sections.
Table 7.35: Amplitude Values - Amplitude Simulation 1

<table>
<thead>
<tr>
<th>Amplitude</th>
<th>Value (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_{AB}$</td>
<td>0.96</td>
</tr>
<tr>
<td>$a_{AC}$</td>
<td>0.04</td>
</tr>
<tr>
<td>$a_{AD}$</td>
<td>0.32</td>
</tr>
<tr>
<td>$a_{AE}$</td>
<td>0.6</td>
</tr>
<tr>
<td>$a_{BC}$</td>
<td>0.28</td>
</tr>
<tr>
<td>$a_{BD}$</td>
<td>0.56</td>
</tr>
<tr>
<td>$a_{BE}$</td>
<td>0.64</td>
</tr>
<tr>
<td>$a_{CD}$</td>
<td>0.8</td>
</tr>
<tr>
<td>$a_{CE}$</td>
<td>0.88</td>
</tr>
<tr>
<td>$a_{DE}$</td>
<td>0.12</td>
</tr>
</tbody>
</table>

Table 7.36: Amplitude Values - Amplitude Simulation 2

<table>
<thead>
<tr>
<th>Amplitude</th>
<th>Value (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_{AB}$</td>
<td>1</td>
</tr>
<tr>
<td>$a_{AC}$</td>
<td>1</td>
</tr>
<tr>
<td>$a_{AD}$</td>
<td>1</td>
</tr>
<tr>
<td>$a_{AE}$</td>
<td>1</td>
</tr>
<tr>
<td>$a_{BC}$</td>
<td>0.1</td>
</tr>
<tr>
<td>$a_{BD}$</td>
<td>0.1</td>
</tr>
<tr>
<td>$a_{BE}$</td>
<td>0.1</td>
</tr>
<tr>
<td>$a_{CD}$</td>
<td>0.05</td>
</tr>
<tr>
<td>$a_{CE}$</td>
<td>0.05</td>
</tr>
<tr>
<td>$a_{DE}$</td>
<td>0.001</td>
</tr>
</tbody>
</table>

Table 7.37: Amplitude Simulations - Error Results for Simulations

<table>
<thead>
<tr>
<th>Mobile Node</th>
<th>Test 1</th>
<th>Test 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>(4.3, 5.4, 4.9)</td>
<td>(1.4, 21.8, 2.2)</td>
</tr>
<tr>
<td>$A_2$</td>
<td>(2.6, 2.4, 0.5)</td>
<td>(0.7, 22.7, 4.3)</td>
</tr>
</tbody>
</table>

with the same input parameters. Thus the difference in amplitude affects the accuracy of the system. We propose that for the physical implementation this can be eliminated by using variable power amplifiers on the anchor nodes, to compensate for the difference at the received node, by transmitting with more/less power as required by the
situation. Based on this compensation we will not include different amplitudes in the rest of the simulations.

## 7.7 Sampling Frequency Simulations

In this section we determine the effect of the sampling frequency on the accuracy of the system, through simulation. We use the input parameters from table 7.38.

<table>
<thead>
<tr>
<th>Input Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mobile Node Positions</td>
<td>See table 7.13</td>
</tr>
<tr>
<td>Anchor Node Positions</td>
<td>See table 7.7</td>
</tr>
<tr>
<td>( f_A )</td>
<td>70 MHz</td>
</tr>
<tr>
<td>( f_X )</td>
<td>69.998 MHz</td>
</tr>
<tr>
<td>( t_X, \forall X \in {A, B, C, D, E} )</td>
<td>0 s</td>
</tr>
<tr>
<td>Numerator Coefficients</td>
<td>[3.101 \cdot 10^{19}]</td>
</tr>
<tr>
<td>Denominator Coefficients</td>
<td>[1; 5.971 \cdot 10^6; 2.197 \cdot 10^{13}; 3.101 \cdot 10^{19}]</td>
</tr>
<tr>
<td>( T_d )</td>
<td>(10T_e) s</td>
</tr>
<tr>
<td>( B )</td>
<td>14</td>
</tr>
<tr>
<td>( V_{fs} )</td>
<td>2 V</td>
</tr>
</tbody>
</table>

We simulate the system using three different sampling frequencies, 10 kHz, 100 kHz and, 500 kHz.

### 7.7.1 Sampling Frequency Test 1

The first of the sampling frequency simulations is done with a low sampling frequency, only five times the envelope frequency. The results for the simulation are presented in table 7.39.

The error in the calculated position is large, compared to previous results, with the same input parameters, but higher sampling frequency.
Table 7.39: Results of Sampling Frequency Simulation - Test 1

<table>
<thead>
<tr>
<th>Mobile Node</th>
<th>Actual</th>
<th>Calculated</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>(0.8 , 0.6 , 1.3)</td>
<td>(0.7099 , 0.5245 , 1.3725)</td>
<td>(0.0901 , 0.0755 , 0.0725)</td>
</tr>
<tr>
<td>A2</td>
<td>(0.86 , 0.78 , 0.87)</td>
<td>(0.8012 , 0.7535 , 0.8795)</td>
<td>(0.0588 , 0.0265 , 0.0095)</td>
</tr>
<tr>
<td>A3</td>
<td>(1.33 , 0.5 , 0.63)</td>
<td>(1.2654 , 0.4234 , 0.6716)</td>
<td>(0.0646 , 0.0766 , 0.0416)</td>
</tr>
</tbody>
</table>

7.7.2 Sampling Frequency Test 2

The second sampling frequency simulation is done with a higher sampling frequency than the previous, 50 times the envelope frequency. The results for the simulation are presented in Table 7.40.

Table 7.40: Results of Sampling Frequency Simulation - Test 2

<table>
<thead>
<tr>
<th>Mobile Node</th>
<th>Actual</th>
<th>Calculated</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>(0.8 , 0.6 , 1.3)</td>
<td>(0.7908 , 0.5913 , 1.3059)</td>
<td>(0.0092 , 0.0087 , 0.0059)</td>
</tr>
<tr>
<td>A2</td>
<td>(0.86 , 0.78 , 0.87)</td>
<td>(0.8525 , 0.7744 , 0.8742)</td>
<td>(0.0075 , 0.0056 , 0.0042)</td>
</tr>
<tr>
<td>A3</td>
<td>(1.33 , 0.5 , 0.63)</td>
<td>(1.3206 , 0.4926 , 0.6386)</td>
<td>(0.0094 , 0.0074 , 0.0086)</td>
</tr>
</tbody>
</table>

A definitive increase in accuracy can be seen from the results in test 1 to the results in test 2, thus we expect another increase in accuracy in test 3.

7.7.3 Sampling Frequency Test 3

The third sampling frequency test is done with a sampling frequency of 250 times the envelope frequency. The results are presented in Table 7.41.

As expected the accuracy increased again from the results obtained during test 2. A summary on the results is presented in the next section.
### Table 7.41: Results of Sampling Frequency Simulation - Test 3

<table>
<thead>
<tr>
<th>Mobile Node</th>
<th>Actual (m)</th>
<th>Calculated (m)</th>
<th>Error (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>$(0.8, 0.6, 1.3)$</td>
<td>$(0.7986, 0.5987, 1.3009)$</td>
<td>$(0.0014, 0.0013, 0.0009)$</td>
</tr>
<tr>
<td>$A_2$</td>
<td>$(0.86, 0.78, 0.87)$</td>
<td>$(0.8589, 0.7792, 0.8706)$</td>
<td>$(0.0011, 0.0008, 0.0006)$</td>
</tr>
<tr>
<td>$A_3$</td>
<td>$(1.33, 0.5, 0.63)$</td>
<td>$(1.3281, 0.4985, 0.6319)$</td>
<td>$(0.0019, 0.0015, 0.0019)$</td>
</tr>
</tbody>
</table>

#### 7.7.4 Summary of Sampling Frequency Simulation Results

In this section we experimented with the effect of the sampling frequency on the accuracy of the system. A summary on the difference between the actual and calculated positions for each of the tests are presented in table 7.42.

### Table 7.42: Sampling Frequency Simulations - Error Results for all Tests

<table>
<thead>
<tr>
<th>Mobile Node</th>
<th>Test 1 ($f_s = 10$ kHz)</th>
<th>Test 2 ($f_s = 100$ kHz)</th>
<th>Test 3 ($f_s = 500$ kHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>$(90.1, 75.5, 7.5)$</td>
<td>$(9.2, 8.7, 5.9)$</td>
<td>$(1.4, 1.3, 0.9)$</td>
</tr>
<tr>
<td>$A_2$</td>
<td>$(58.8, 26.5, 9.5)$</td>
<td>$(7.5, 5.6, 4.2)$</td>
<td>$(1.1, 0.8, 0.6)$</td>
</tr>
<tr>
<td>$A_3$</td>
<td>$(64.6, 76.6, 41.6)$</td>
<td>$(9.4, 7.4, 8.6)$</td>
<td>$(1.9, 1.5, 1.9)$</td>
</tr>
</tbody>
</table>

From the results, it is clear that the sampling frequency has a definitive effect on the accuracy of the system, with the accuracy that can be obtained reducing from centimetres to millimetres as the sampling frequency is increased. Based on these results, the following simulations will use a sampling frequency of $f_s = 500$ kHz.

#### 7.8 Sampling Duration Simulations

In this section we investigate the effect of the sampling duration on the accuracy of the system. The input parameters for these simulations are given in table 7.43.

We run the simulation with three different sampling durations, 1 envelope period, 5
Chapter 7

Quantisation Simulations

Table 7.43: Input Parameters - Sampling Duration Simulations

<table>
<thead>
<tr>
<th>Input Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mobile Node Positions</td>
<td>See table 7.13</td>
</tr>
<tr>
<td>Anchor Node Positions</td>
<td>See table 7.7</td>
</tr>
<tr>
<td>$f_A$</td>
<td>70 MHz</td>
</tr>
<tr>
<td>$f_X$</td>
<td>69.998 MHz</td>
</tr>
<tr>
<td>$t_X, \forall X \in {A, B, C, D, E}$</td>
<td>0 s</td>
</tr>
<tr>
<td>Numerator Coefficients</td>
<td>$[3.101 \cdot 10^{19}]$</td>
</tr>
<tr>
<td>Denominator Coefficients</td>
<td>$[1 ; ; ; 5.971 \cdot 10^6 ; ; 2.197 \cdot 10^{13} ; ; 3.101 \cdot 10^{19}]$</td>
</tr>
<tr>
<td>$f_s$</td>
<td>500 kHz</td>
</tr>
<tr>
<td>$B$</td>
<td>14</td>
</tr>
<tr>
<td>$V_{fs}$</td>
<td>2 V</td>
</tr>
</tbody>
</table>

Envelope periods, and 10 envelope periods. The summary of the simulation results, showing the difference between the actual and calculated positions for each of the mobile node locations are given in table 7.44.

Table 7.44: Sampling Duration Simulations - Error Results for all Tests

<table>
<thead>
<tr>
<th>Mobile Node</th>
<th>Test 1 ($T_d = 1T_e$ s)</th>
<th>Test 2 ($T_d = 5T_e$ s)</th>
<th>Test 3 ($T_d = 10T_e$ s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>(14.9, 13.9, 9.3)</td>
<td>(2.9, 2.7, 1.8)</td>
<td>(1.4, 1.3, 0.9)</td>
</tr>
<tr>
<td>$A_2$</td>
<td>(11.9, 8.8, 6.7)</td>
<td>(2.4, 1.8, 1.4)</td>
<td>(1.1, 0.8, 0.6)</td>
</tr>
<tr>
<td>$A_3$</td>
<td>(15.2, 11.7, 13.8)</td>
<td>(3, 2.3, 2.8)</td>
<td>(1.4, 1.1, 1.3)</td>
</tr>
</tbody>
</table>

From the results it is clear that a longer sampling duration improves the localization accuracy. This can be expected from the fact that using a longer sample, will increase the SNR. In the simulated system the noise is mainly due to the rectification, sampling, and filtering of the signal.

7.9 Quantisation Simulations

In this section we determine the effect of the of quantisation on the accuracy of the system. We use the input parameters in table 7.45.
We simulate the system for quantisation bits, 4, 8 and, 16. A summary of the results are presented in table 7.46.

It should also be noted that a simulation done with $B = 2$ failed completely, with the simulation not even able to extract the absolute phase offset.

From the results it seems that the error due to quantisation doesn’t impact the overall accuracy of the system, which is counter intuitive, as we expected the quantisation error to have an effect on the system.
7.10 Extended Simulation

Using the parameters as determined in the previous sections, we simulate the system for a set of 20 random positions within the volume bound by the cylinder. The input parameters used in the simulation are given in table 7.47.

<table>
<thead>
<tr>
<th>Input Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mobile Node Positions</td>
<td>Set of 20 random positions from the volume bound by the cylinder</td>
</tr>
<tr>
<td>Anchor Node Positions</td>
<td>See table 7.7</td>
</tr>
<tr>
<td>$f_A$</td>
<td>70 MHz</td>
</tr>
<tr>
<td>$f_X$</td>
<td>69.998 MHz</td>
</tr>
<tr>
<td>$a_{XU}$, $\forall X \neq U \in {A, B, C, D, E}$</td>
<td>1 V</td>
</tr>
<tr>
<td>$t_X$, $\forall X \in {A, B, C, D, E}$</td>
<td>0 s</td>
</tr>
<tr>
<td>Numerator Coefficients</td>
<td>$[3.101 \cdot 10^{19}]$</td>
</tr>
<tr>
<td>Denominator Coefficients</td>
<td>$[1 ; 5.971 \cdot 10^{6} ; 2.197 \cdot 10^{13} ; 3.101 \cdot 10^{19}]$</td>
</tr>
<tr>
<td>$f_s$</td>
<td>500 kHz</td>
</tr>
<tr>
<td>$T_d$</td>
<td>$10T_e$ s</td>
</tr>
<tr>
<td>$B$</td>
<td>14</td>
</tr>
<tr>
<td>$V_{fs}$</td>
<td>2 V</td>
</tr>
</tbody>
</table>

From the simulation results we calculated the mean value, the standard deviation, the minimum error and, the maximum error, for each of the coordinates. The results are presented in table 7.48.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>1.87 1.42 1.585</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>2.04 1.729 1.585</td>
</tr>
<tr>
<td>Minimum</td>
<td>0.1 0 0.1</td>
</tr>
<tr>
<td>Maximum</td>
<td>6.5 5.9 4.9</td>
</tr>
</tbody>
</table>

We can see from the results in table 7.48 that the system yielded mean errors of less than 2 mm for localization of coordinates on each of the axes. It can further be seen
that the maximum localization error was 6.5 mm (for one of the x coordinates).

## 7.11 Conclusion

In this chapter we followed an empirical investigation on the effect of various parameters of the concept system on the accuracy of the system. These tests were done using the verified and validated simulation implementation of the mathematical model of the concept system. Based on the results of the simulations, we recommend the input parameters given in table 7.49.

<table>
<thead>
<tr>
<th>Table 7.49: Input Parameters - Extended Simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input Parameter</td>
</tr>
<tr>
<td>Anchor Node Positions</td>
</tr>
<tr>
<td>$f_A$</td>
</tr>
<tr>
<td>$f_X$</td>
</tr>
<tr>
<td>$a_{XI}$, $\forall X \neq U \in {A, B, C, D, E}$</td>
</tr>
<tr>
<td>$f_s$</td>
</tr>
<tr>
<td>$T_d$</td>
</tr>
<tr>
<td>$B$</td>
</tr>
<tr>
<td>$V_{fs}$</td>
</tr>
</tbody>
</table>

The positions of the anchor nodes as given in table 7.7, were chosen by distributing the nodes into the zones illustrated in figures 7.6 to 7.8. Based on the simulation results presented in table 7.10 we conclude that the accuracy of the system improves when the nodes are distributed along all the axes, with as little symmetry as possible.

The results obtained from the simulations on the carrier and envelope frequency as presented in tables 7.30 and 7.31 implies that the accuracy of the system depends greatly on the envelope frequency. From the results obtained it is clear that the accuracy of the system increases as the envelope frequency decreases. The increase in accuracy due to a higher carrier frequency is less pronounced than that of the envelope frequency selection.
From the simulations done in section 7.3 it is concluded that phase ambiguity may occur for wavelengths smaller than two times the maximum distance between any of the nodes. From the physical setup of the system we know that the maximum distance will always be between a pair of anchor nodes if we keep to the recommended convention of distributing the nodes as discussed in section 7.2. Based on the choice of node positions and considering the conclusions on the accuracy due to the envelope and carrier frequency as well as the effect of phase ambiguity, we recommend the frequencies for $f_A$ and $f_X$ as given in table 7.49, as this choice achieves a low envelope frequency and a high carrier frequency, while avoiding the ambiguity that can result if the carrier frequency is too high.

The results presented in section 7.6 implies the system will achieve more accurate results if the amplitudes of the two signals comprising the interference signals is close to one another. We recommend that for the physical implementation of the system a form of gain control is used on the anchor nodes, that can compensate for the difference in the amplitudes due to distances.

From results obtained for the simulation of the sampling process as discussed in sections 7.7, 7.8 and 7.9, we conclude that oversampling the signal will increase the accuracy of the system. The amount of oversampling used in the simulations and recommended in table 7.49, is 500 kHz. It is further concluded from the results in table 7.44 that the longer the duration of the sampling period, the better the accuracy will be. The results for the effect of the quantisation on the accuracy of the system we conclude that a higher resolution will result in better accuracy, although the effect of a low resolution is not as bad as was expected.

Finally, from the result of the extended simulation we can conclude that the concept system is able to localise the mobile node with a high enough accuracy to be a viable solution to track the spheres through the RPV model.
Chapter 8

Conclusions and Recommendations

We start this chapter by revisiting the problem statement as well as the sub-objectives discussed in chapter 1, and explaining how each of the objectives were addressed. We then present conclusions based on the work done in the dissertation. Finally we end with recommendations and future work.

8.1 Overview of Work

In chapter 1 the problem statement for the work done in this dissertation is given as:

The goal of this research is to develop a concept system for accurately tracking objects in a confined space and testing this concept system for viability.

From this problem statement we identified the following issues to be addressed:

- The development of a concept system,
- The derivation and simulation implementation of the concept system.
Chapter 8

Overview of Work

- The validation of the concept system using the results obtained from simulation.

We addressed the first issue in chapters 2 to 5.

In chapter 2 we presented a literature survey on the basic aspects of tracking systems. These basic aspects provided useful insight into how tracking systems work. Based on these aspects we developed a method for the characterisation and classification of tracking systems into a general form.

We used this characterisation and classification method to achieve two different, although related, goals. The first was to characterise and classify the proposed system according to the requirements presented in chapter 1. We then used the characterisation and classification results for the proposed system to identify existing solutions that could serve as an analysis for the development of a system that would meet our unique requirements. The most promising systems were then characterised and classified using the method. The results of the characterisation and classification of the systems were presented in chapter 3. Based on these results we selected RIPS, a localization system used in WSN, to be used as the base for our proposed system.

In chapter 4 we presented an in depth discussion of RIPS, based on various publications. The discussion includes an overview on interferometric positioning, the theorems the system is based on, the possible sources of error impacting the accuracy of the system, as well as the implementation of the experimental system. We derived a functional breakdown of the system as well as an operation flow diagram for the system from the discussion to enable us to better understand the working of the system as well as provide a starting point for the conceptual design.

Based on the information and ideas gained in chapters 2 to 4 we developed the concept system in chapter 5. Although the system is based on radio interferometric positioning, similar to RIPS, there is one major difference. The implementation of the localization in RIPS is done using the resources available in a WSN, limiting the complexity of the computations that can be done to perform localization, as well as limiting control over the physical aspects involved in the setup of the system. For the concept system
we have control over the hardware implementation as well as the physical setup of the system. We use this design freedom to improve the accuracy and precision of the concept system by designing the system in order to eliminate most of the sources of error as discussed in section 4.4.

In chapter 6 we addressed the second issue. We started by deriving an ideal, deterministic mathematical model from the concept system. The model is derived for ideal circumstances (for example the effect of multipath is not included). The reasoning behind deriving an ideal model is that if the system fails to obtain the necessary accuracy for ideal circumstances, it won’t be able to achieve the required accuracy for non-ideal circumstances. We then present the algorithms used to implement the simulation of the model. We end the chapter with the validation and verification of the mathematical model and simulation of the model, based on the guidelines presented in [12].

In chapter 7 we followed an empirical investigation to determine the effect of various parameters on the accuracy of the concept system. We use the results obtained to recommend a set of parameters to be used with the concept system in order to increase the accuracy that can be obtained. Using these parameters we performed a simulation to localise 20 randomly selected positions from the volume bound by the cylinder. The results indicate that the concept solution is viable, as the maximum localization error for the set of nodes is 6.5 mm, which is much smaller than the required accuracy of 3 cm. It should however be kept in mind that although this result validates the concept system, it doesn’t prove that the system will obtain the 3 cm accuracy for the physical implementation, as this simulation was done for ideal circumstances.

8.2 Contributions made in this Dissertation

In this dissertation the following new contributions were made.

In chapter 2 we proposed a method that can be used for the characterisation and classification of tracking and localisation systems. We based this system on literature re-
Regarding the main aspects affecting localisation and tracking systems. The method can be used to characterise and classify systems from different application domains to a general form. This enables the comparison of systems from different application domains. The method can also be used to characterise proposed systems, as we did in chapter 3 for our proposed system. We presented this method as a full conference paper at SATNAC 2009, at the Royal Swazi spa, with the title *Method for Characterization and Classification of Localization/Tracking Systems* [34]. See appendix A.

In chapter 6 an ideal, deterministic, generic model of radio interferometric tracking was derived along with the algorithms that can be used to implement the model as a simulation. The model and simulation was validated and verified using the guidelines presented in [12]. This model can be used to experiment with a generic radio interferometric system (under ideal circumstances), enabling designers and researchers to determine the viability of the system for their application. The model can also be used to determine the effect of a specific setup on the accuracy of the system.

### 8.3 Concluding Remarks

**On the method for the characterisation and classification of tracking systems**

During the course of the literature study we found that although tracking and localization systems are becoming more widespread, with especially GPS being used globally, it is hard to compare systems with each other, due to the diversity in the applications of these systems. We found a few surveys and taxonomies, but these were confined to specific application domains, and thus couldn’t be used to characterise and classify systems outside of the domain. Based on our need to compare different systems from different application domains with one another we developed a characterisation and classification system based on the basic aspects of tracking systems identified during our literature study. The method is presented in chapter 2.
On the Conceptual Design

The decision to base the conceptual design on the simple case of tracking only a single sphere through the RPV model was done to keep the design from becoming unnecessary cluttered due to non-critical parts of the system. The inclusion of a mechanism to switch between nodes using for example a timer system, with nodes transmitting according to a predetermined time schedule can easily be implemented, as fine grain synchronisation is not needed, the only requirement being that the one mobile node stops transmitting before another starts.

The design of the concept system was done with the goal of negating or at least minimising the effect of the error sources of RIPS in order to achieve better accuracy and precision, while keeping the hardware needed on the mobile node (sphere) down to a minimum, due to the limited size available on the sphere.

We assumed for the sake of simplicity that the phase ambiguity of the system can be solved by one of two methods:

- Tracking in the reactive near field of the antennas or
- doing the physical setup of the system to limit the position of the object to an integer multiple of the wavelength.

If one or both of these methods fail, elaborate schemes have been developed in the literature on RIPS to solve the phase ambiguity due to wavelength, but it will lead to the hardware on the mobile and anchor nodes having to be redesigned to transmit at multiple frequencies.

Finally we opted not to use one of the two methods for localization presented at the end of chapter 4 as the set of equations can easily be solved using a numerical solver. This option is preferred as most mathematical software packages come with an optimised numerical solver and should perform localization well enough. This is later confirmed in chapter 7 where it can be seen from the results that the solver only fails to converge
for localization errors that are already outside the 3 cm requirement of the system.

**On the mathematical model and simulation implementation**

It is again emphasized that the mathematical model derived from the conceptual design is done for ideal circumstances. This is acceptable enough to prove that the system is viable, as failure to achieve the required 3 cm accuracy for ideal circumstances implies that the system will fail to achieve the required accuracy for non-ideal circumstances. Additional tests using a prototype (as will be discussed later) have to be performed before it can be absolutely certain that the system will work when physically implemented.

**On the simulation tests and results**

The simulation results on the effect of the placement of anchor nodes show that for the nodes distributed along the axes as illustrated in figures 7.6 to 7.8 the highest accuracy is achieved by the system. This presents an interesting mathematical optimisation problem, to find the best position for the anchor nodes. The problem is however outside the scope of this dissertation.

The results of the phase ambiguity simulations illustrates that contrary to intuition, phase ambiguity can occur for carrier wavelength smaller than two times the maximum distance, as stated in [25], not smaller than the maximum distance. This is due to the q-range measurement being a linear combination of the distances in the quad.

The results of the simulations done with different envelope and carrier frequencies indicate that the accuracy of the system improves for smaller envelope frequencies. We propose that this could be due to the fact that the approximation made from equation (6.13) to (6.14) becomes closer to being correct as the envelope frequency decreases.

Finally, the results obtained in the extended simulation run show that the concept
Chapter 8 Recommendations and Future Work

The results obtained by simulating the concept system in chapter 7 confirms the viability of the system. A prototype of the system now needs to be implemented to determine whether the system will be able to achieve the required accuracy.

We recommend that the next step in the testing of the system be to implement a modular prototype system. The modules used in this prototype should be based on the functional breakdown done in chapter 5, with each of the modules containing only a single component of the system. For example, for the receiver hardware the modular prototype will have four different components, each on a separate Printed Circuit Board (PCB). These components can be interconnected using matched SMA connectors for example. By using this approach the hardware can be tested separately to determine the effect of each of the blocks on the system. Further, if one of the blocks doesn’t perform as needed, it can easily be replaced, without having to do a total redesign of the system.

Due to the volatile nature of radio propagation we recommend that the hardware first be tested by emulating the channel. The channel needed for radio interferometry to work can be emulated by using the following hardware:

- Power dividers to split the signal from the transmitters.
- Phase shifting blocks or time delay blocks to emulate the phase shift of the wave
due to its propagation from one node to another.

- Variable attenuator blocks, to emulate the power loss due to propagation.

- Linear summer blocks to emulate the interference of the wave at the receiving nodes.

By characterising each of the channel emulating blocks, the effect of the channel can be identified and ignored, thus enabling accurate testing of the effect of the hardware on the accuracy of the system.

Test data obtained from testing the prototype with channel emulation, can be used to reach a verdict on the viability of the system to reach the 3 cm accuracy required. Further tests can then be done with the prototype system for an experimental setup without the channel emulation to determine whether the system can achieve the required accuracy for an actual setup. If the system proves to be viable the prototype can be used to design the final system for implementation in the model of the RPV.

Finally two other interesting research questions also arise from the work done in this dissertation.

The first question concerns the implementation of the system in the reactive near field of the antennas. Will such an implementation work and what will the effect be on the accuracy of the system if it does.

The second concerns effect of the placement of the anchor nodes on the accuracy of the system. Two questions can be asked in this regard. The first is, why does the placement of the nodes have such a big effect on the accuracy of the system and the second is how the optimal placement of the nodes can be determined.
Appendix A

Conference Contribution from Dissertation
Abstract—Due to the large variety of different tracking and localization systems intended for different applications from different disciplines, researchers and developers are faced with a problem when comparing these systems as they are not presented in a general form. This paper discuss and expands on the important aspects of two of the most helpful papers in the field in order to present a method for characterizing and classifying systems into a general form, enabling easier comparison of systems as well as making better design choices.

Index Terms—Localization, Tracking, Classification

I. INTRODUCTION

In the world today we get into contact with tracking and localization systems almost on a daily basis. The system that is almost certainly the most well known is the Global Positioning System or GPS. In addition to GPS there exists a plethora of other tracking/localization systems that are not as well known. These include systems that vary from determining the position of nodes and users in a network, creating the opportunity for location aware applications, to tracking the movement of a persons head in augmented and virtual reality applications. Due to this variety of different applications, these tracking systems are found in a multitude of different environments and disciplines. This makes it difficult for researchers and developers to compare these systems and make informed decisions as to the best options when designing a system, as results usually differ from discipline to discipline and application to application.

We are currently in the process of developing a system to track objects in a specific environment and thus needed a framework for comparing different systems from all different application domains, but found that taxonomies and related work are mostly limited to specific application domains, for example [1] provides a survey on visual tracking methods while [2] provides information on localization and tracking in WSNs. From a literature survey we identified [3] and [4], although also aimed at specific application domains, as articles that describe the main aspects of tracking and localization systems in a general enough way to be applicable to systems from different application domains. Thus, this paper is based mainly on previous work done in [3] and [4]. It combines the aspects presented in the articles respectively and expands on them and how they interact. We also discuss other aspects we identified as important from a survey of tracking/localization systems. We then use these aspects and their relationships to propose a method of characterizing and classifying tracking/localization systems in a general form. Once classified in this general form it is easier to compare different systems with one another as well as make informed choices with regards to implementing new systems.

The rest of the document is organized as follows. In Section II definitions of ambiguous terms from the field of tracking/localization will be defined as they are used in this article. In section III a discussion on what to keep in mind when analyzing and comparing tracking/localization systems will be given. In Section IV the aspects and their relationships are described, followed by the proposed method for the classification of tracking/localization systems.

II. DEFINITIONS OF AMBIGUOUS TERMS

A brief definition of terms as used in this paper will now be given.

Localization refers to the process of determining the position of an object while the object is stationary.

Tracking refers to the process of determining the position of an object while it is moving. Tracking is normally not as accurate as localization, as it requires a finite amount of time to determine the position of an object and the position of the object will change in this time. Note that in this paper the term Tracking will be used to refer to both Tracking and Localization.

Accuracy defines the error of the measured position relative to the actual position of the object and is given as a distance in meters. Precision is a measure of how often the stated accuracy can be obtained and is usually expressed as a percentage.

III. INITIAL CONSIDERATIONS WHEN ANALYZING TRACKING SYSTEMS

We have found that when analyzing a tracking system it is helpful to start by asking three important questions. These are What, Why, and Where.

What is being tracked? This could be people, nodes in a network, cars, and other objects. It is important to know what is being tracked as this imposes limitations on the system and determines certain properties of the system. For example, if
a car is being tracked, as the case may well be in a GPS system, power for the device can be obtained from the vehicle. If the object being tracked is a tag in a crate, issues may arise concerning the power source of the tag. What is being tracked also influences a variety of other factors for example size and computational power. All this needs to be kept in mind when analyzing and comparing tracking systems.

Why is the object being tracked? This question also affects the properties, principles and physical phenomena used in the system. For example, if a system tracks the position of devices in a network to determine what computer is nearest what printer or projector or other peripheral device, many properties such as the reference grid and resolution of the system are affected.

Where is the object that is being tracked? This refers to the environment in which the object is being tracked. This is important as it has a big effect on the physical phenomena used in the tracking system. For example, a magnetic tracking system cannot be used in an environment with lots of magnetic interference or with lots of reflective materials as the results obtained would be adversely affected. Keeping these three questions in mind will simplify the classification process as it will help with the identification and understanding of the systems requirements.

IV. CLASSIFICATION OF TRACKING SYSTEMS

To enable researchers and developers to better understand and compare different tracking systems from different engineering fields, it is necessary to define a set of guidelines that can be used to characterize and classify these systems into a general form. From research done, it seems that there are four important aspects to consider when analysing tracking/localization systems, these are Properties, Principles, Location Computing Techniques, and Physical Phenomenon. The properties as well as the location computing techniques described here are based mostly on [3]. The principles are based on [4]. Note however, that although the aspects described here draw strongly on the above mentioned articles, it does differ in certain areas. A discussion of the aspects and their relationship towards each other will now be given, followed by the proposed method for classification.

A. Properties

According to [3] their properties deal with a set of issues that arise when characterizing tracking systems and are normally not related to technology and techniques used in the system. We however found that in some instances, some properties may be affected by aspects related to the technology and techniques used when the relationships as described in this paper are taken into consideration. For example the limitations property may be influenced by physical phenomena used. The properties that are recommended for characterizing tracking systems are Resolution, Accuracy and Precision (Accuracy and Precision), Reference Grid (Absolute and Symbolic Location), Physical Position and Symbolic Location, Computational Power (Localized Location Computation), Scale, Recognition, Cost, and Limitations (Note that the properties in brackets correspond to those used in [3]).

1) Resolution, Accuracy and Precision: Resolution, accuracy and precision are very important characteristics of a tracking system. In order to function correctly a system needs to provide accurate results consistently as stated in [3]. Accuracy is a measure of the difference in the measured position of the object being tracked relative to the actual position of the object. Precision is a measure of how often this accuracy can be achieved and is usually expressed as a percentage. For example a system may yield an accuracy of 10 cm for 95% of the measurements made. Resolution is defined as the minimum accuracy of the system, which can be obtained with an acceptable precision, in order for the tracking system to achieve its goal.

2) Reference Grid: In order for the position information yielded by a tracking system to be of any use, it has to be given in terms of a reference grid. For example, the Cartesian grid as used in math and physics, or the grid used for GPS coordinates given in terms of latitude, longitude and altitude.

Reference grids are mostly used in one of two ways. Position information can be given in terms of a fixed shared reference grid, like the one used by GPS systems. These systems are said to give an Absolute location. Alternatively position information can be given relative to the object tracking other objects or relative to the object being tracked. In this case the zero position of the grid is not fixed to a specific point. These systems are said to give Relative location. An example of such a system is a system you can use inside a building, like a certain room or corridor and can thus be classified as a system providing symbolic location.

Usually a system that provides a physical position has a higher resolution than a system providing symbolic location. If a physical position system has a high enough resolution it may be used to convert the physical position to a symbolic location. For example, if a warehouse is divided into certain 20 m x 20 m zones a physical position system with a resolution of 1 m can be used to determine the symbolic location of an item.

4) Computational Power: The computational power property refers to the computational resources available for implementing the system. For example the RIPS tracking system [6] is used for determining the position of nodes in a wireless sensor network without adding any additional hardware to the nodes. This imposes certain limitations on the complexity of the computations that can be done by the system.

The two main factors to keep in mind when identifying
this property are the computational power of the object being tracked, as well as the computational power of the support hardware or external tracking hardware. These two factors influence aspects like where the location computation of the object can be done e.g. on the object self, or on the external hardware. It also influences other properties such as the accuracy and precision that can be obtained.

5) Scale: The scale of a system is affected mainly by two factors: The area covered by the system and the number of objects that can be tracked within infrastructure and time constraints. For example, GPS can be used by an unlimited amount of receivers and covers the whole world, while a system that tracks objects using RFID tags may only be able to deal with one object at a time and in a small space like a room [3]. The scalability of a system is thus influenced by how easy it is to increase the infrastructure and the cost thereof.

6) Recognition: In some tracking system the object being tracked needs to be identified. Recognition is usually achieved by designating each object being tracked or classes of objects of the same sort with a Globally Unique Identifier or GUID [3]. An example of a system that uses recognition is the Active Bat system [7].

7) Cost: According to [3] the cost of a tracking system can be assessed according to three different factors, time, space, and capital.

Time cost refers to the amount of time the installation and setup of the system requires as well as the amount of time it will take to administrate the system.

Space cost refers to the physical dimensions of the tracking system (form factor and size) as well as the amount of infrastructure that needs to be installed.

Capital cost refers to all financial costs of the system like the manufacturing and installation price of the system.

8) Limitations: The limitations of a tracking system refers to what the system cannot do. These limitations are mainly influenced by the environment and can differ from size constraints to the effect of the environment on the physical phenomena that can be used.

Note that the properties discussed above may influence one another. For example, if a tracking system is to be implemented in a limited space the limitation property of the system is influenced by the cost property (space cost).

B. Principles

Principles describe the basic method a system uses to track objects [4] and are thus one of the most important aspects to consider when analyzing and comparing tracking systems. It determines the core working of the tracking system and is therefore closely linked to the physical phenomena used to implement the system, which in turn is largely influenced by the environment the system is used in.

The principles as discussed here are based on [4]. Note that although [4] focus on principles used in tracking systems for augmented reality applications, it was found that the principles are applicable to most tracking systems.

The principles that will be discussed are Time of Flight (ToF), Spatial Scan, Inertial Sensing, Mechanical Linkage, Phase Difference Sensing, and Direct Field Sensing.

It is also important to note that many systems are based on a combination of these principles and are then referred to as hybrid systems.

1) Time of Flight (ToF): Systems based on the ToF principle determine the distance between two points by measuring the travelling time of a wave from one point to the other. Thus the ToF principle is always implemented using the physical phenomenon of propagation. In order for the principle to be used accurately the propagation speed of the wave used should be constant (or as close to constant as possible) in the medium the tracking is implemented in.

Most ToF systems are implemented using sound waves; usually in the ultrasound range (greater than 20 kHz, normally 40 kHz) as these waves cannot be heard by humans. According to [4] other waves used to implement ToF systems are light waves, using for example pulsed infrared diodes as well as electromagnetic waves.

As ToF systems yield distances and from the physical setup angles can be obtained, it is normally used with triangulation techniques to determine position.

2) Spatial Scan: Spatial scan systems use optical tracking methods, usually for the recognition of known features and their positions, from which angles and distances can be computed. Another form of optical tracking is by measuring the time between a light beam passing one sensor and then another. In [4], spatial scan systems are divided into two categories, outside-in and inside-out. Outside-in systems comprise external hardware looking for features on the object being tracked.

Inside-out systems use hardware on the object being tracked to identify features or reference points on/in the surroundings.

It should be noted that a main drawback of spatial scan systems is that a direct line of sight is necessary for the system to operate, implying that environment plays a big role in the viability of using a spatial scan system.

3) Inertial Sensing: Inertial sensing uses the physical phenomena of inertia to determine the orientation and acceleration of the object being tracked. It is usually implemented using accelerometers. From the acceleration data the distance the object has moved can be obtained by double integration over time.

A main drawback of these systems is the fact that the inertial sensors used in the implementation suffer from drift [8] and thus need to be calibrated very often.

4) Mechanical Linkage: Mechanical linkage systems are systems where the objects being tracked are physically linked to each other in some mechanical way. These mechanical links usually comprise some sort of arms that can rotate and extend. From the angles and distances the location of the tracked object as well as its orientation can then be determined.

It was found that mechanical linkage systems are not that common in modern tracking systems.

5) Phase Difference Sensing: Systems based on the phase difference sensing principle use the phase shift of a signal travelling through space to determine distance, as the phase shift is a function of the distance the signal has travelled. According to [4], these systems usually measure the phase of an incoming signal and compare it to a signal of the same
frequency on a fixed reference. The implementation of RIPS
[6] is a very good example of an innovative use of phase
difference sensing, implementing it with an interference signal.

Phase difference systems can achieve high resolutions, in
the order of centimetres [8]. According to [4], systems based
on phase difference sensing can obtain a higher accuracy than
ToF based systems due to their ability to generate high data
rates.

6) Direct Field Sensing: Systems based on direct field
sensing use measurements taken directly from some field
e.g. a magnetic field or gravitational field to determine an
object’s distance from another and can in some cases also
detect its orientation. These systems are usually implemented
using the physical phenomena of magnetic coupling by cre-
ating an orthogonal field. Magnetic trackers are inexpensive,
lightweight and compact; as a result they are widely used in
the augmented and virtual reality realms for tracking body and
head movement. Other magnetic phenomena may also be used
as well as gravitation. An example of a position and orientation
tracking system based on magnetic field sensing is discussed
in [9].

C. Location Computing Techniques

The implementation of a tracking system using one or
a combination of the principles mentioned above usually
provides some sort of measurement e.g. the distance from one
object to another, but normally the position of the object is
still unknown.

Location computing techniques are used to determine the
position of the object(s) being tracked according to the refer-
ence grid used by the system.

The three principle techniques identified in [3] are Triang-
ulation, Scene Analysis, and Proximity.

1) Triangulation: The triangulation technique is based on
the properties of triangles and uses distance and angle mea-
surements to compute the position of an object [3]. This
technique can be divided into two categories, Lateration and
Angulation.

Lateration uses only distance measurements from known
reference points to determine an object’s location [3]. In two
dimensions three non-collinear measurements are needed to
locate an objects position. The position is determined by
finding the intersection of the three circles with the reference
point as centre and the distance from the reference point to
the object as radius (see figure 1).

This technique can also be used to obtain three dimensional
position of an object if the distance from four reference points
to the object is known (the distances have to be non-coplanar),
by the intersection of the four spheres with the reference
point as centre and radius the distance from the object to the
reference point.

The amount of known ranges required may be reduced by
domain specific knowledge; for example in the Active Bat
System [7] measurements are made from an array of receivers
in the ceiling of a building, three dimensional position can
thus be determined from only three distance measurements as
one of the two points of intersection (the one above the array
of receivers) can be ignored [3].

Angulation uses a combination of angles and distances
between reference points and the object to determine its
position. Two dimensional angulation uses two angles and the
distance between the reference points to determine the position
of the object (see figure 2).

Three dimensional angulation requires one length measure-
ment, one azimuth measurement, and two angles to determine
the location of an object [3].

The triangulation techniques as described above, can be
used with all systems that yield distance and angle informa-
tion. Systems that give these results are usually based on ToF,
Mechanical Linkage, and Phase Difference Sensing. Some
beam scanning systems based on the Spatial Scan principle
could also yield the results required to use triangulation
techniques.

2) Scene Analysis: According to [3] scene analysis uses
features in a scene to estimate the position of the observer.
The scenes are usually simplified to make feature recognition
and comparison easier. There are two main forms of Scene
Analysis, static and differential. Static scene analysis com-
pares the features from the observed scene to a dataset. The
dataset contains the location from where specific features were
recognised. A features match in the dataset thus yields the
approximate location of the observer.

Differential scene analysis uses the change in the scene to

\[ \theta_1 \]
\[ \theta_2 \]
\[ d_1 \]
\[ d_2 \]
\[ d_3 \]
compute movement, as changes in the scene corresponds to movement of the observer. The position of known features enables the observer to determine its position relative to the feature [3].

This technique is used with optical spatial scan systems, as they yield scene results that can be used as a way to implement scene analysis with feature recognition and comparison.

3) Proximity: According to [3] systems using the proximity technique determines an object’s location near a known location using a physical phenomenon with limited range. The methods usually used for the implementation of this technique are Physical Contact, Monitoring wireless access points, and Observing automatic ID systems. It was also found that another common method for implementing proximity systems is by using beacons, placed in predetermined zones (like rooms in a building). Examples of systems using this technique are the Active Badge system [5] as well as the Cricket location support system [10].

Using physical contact to determine an objects position can be done by using pressure sensors or touch sensors [3]. As soon as physical contact is made it can be assumed that the object is near the location of the object that contact was made with.

By monitoring wireless cellular access points it can be determined when the object being tracked is in range of one or more of the access points. The position of the object is then known with accuracy the size of the area serviced by the access point [3].

Automatic ID systems include systems like credit card point of sales terminals and land line telephone records. By determining the location of the credit card point the location of the person using it can be tracked.

D. Physical Phenomena

We have found that the physical phenomena used to implement a tracking system, although closely related to the principle used, is an important aspect in system selection. It is influenced by the environment and is the factor determining in which environments the system will be able to function. We divided physical phenomena into four main categories: Propagation, Optic, Inertia and Magnetic.

1) Propagation: Propagation refers to all the physical phenomena using the propagation of waves through space to determine position. The distance a wave travelled through space is a function of the travelling time as well as the propagation speed of the wave and can thus be used to determine distance. The phase offset as a result of the travelling time of the wave is a function of the travelling time as well as the propagation speed of the wave and can thus be used to determine distance. The phase offset as a result of the travelling time of the wave can also be used to determine distance travelled.

2) Optic: Optic phenomena are used to implement spatial scan systems. Systems using these phenomena require a clear line of sight to or from the object.

3) Inertia: Inertia refers to the physical phenomenon as described by Newtons first law of motion. It is usually implemented with gyroscopes or accelerometers. The main drawback of systems using these technologies is that they suffer from drift [8] and thus need to be calibrated often to ensure accurate results. This physical phenomenon is used to implement the inertial sensing principle.

4) Magnetic: Magnetic phenomena are mainly used in the form of magnetic field sensing (magnetic coupling) where magnetic fields radiated by a source (usually comprised of three coils place perpendicular to each other to create an orthogonal field) induce a flux in a receiver. The flux is a function of the distance and orientation between the source and the receiver [4].

Systems can also use magnetic phenomena to measure its orientation with respect to a known magnetic field, for example the earths magnetic field.

E. Relationship between the main aspects

All the main aspects of tracking systems as discussed are related to each other. The principle that is used affects the location technique that can be used. The physical phenomenon used is in turn directly linked to the principle used. The properties of the system affect the choice of principle, location technique and physical phenomenon used by a system. Table I gives an indication of the aspects affected by the different properties of a system.

<table>
<thead>
<tr>
<th>Property</th>
<th>Principles</th>
<th>Location Techniques</th>
<th>Physical</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resolution, Accuracy &amp; Precision</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>Reference Grid</td>
<td>x</td>
<td>x</td>
<td></td>
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<tr>
<td>Physical Position &amp; Symbolic Location</td>
<td>x</td>
<td>x</td>
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<tr>
<td>Computational Power</td>
<td>x</td>
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<td>Scale</td>
<td>x</td>
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<td>Recognition</td>
<td>x</td>
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<td>Cost</td>
<td>x</td>
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<td>x</td>
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<tr>
<td>Limitations</td>
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</table>

F. Proposed Classification Method

We propose that the following method be followed when classifying a tracking system:

1) Determine the system requirements: The system requirements can be determined by answering the three questions in section III.

2) Characterize the system according to the system properties: Characterize the system according to the properties using the requirements.

3) Identify the most important properties: Identify the properties most important to the success of the system. For example, if it is important that the system has a high accuracy and is intended to work in a highly reflective environment, the most important properties are the Resolution, Accuracy & Precision and Limitations properties.

4) Determine the Principle, Location Technique and Physical Phenomenon: Once the most important properties affecting the system have been identified, they can be used to determine the best principle, location technique and physical phenomenon. For example, if the most important property of the system is recognition, it would be best to first decide on a physical phenomenon that can be used to implement GUID.
The next step would then be to determine which principle and location technique can be used with the physical phenomenon.

G. Example of characterizing and classifying a system

As an example, the system we are currently working on will be classified according to the proposed method.

We start of by answering the three questions from section III.

What  Spheres with a diameter of 6 cm are to be tracked.
Why  The spheres need to be tracked to determine their flow paths.
Where  The spheres to be tracked flow through a ceramic cylinder with radius 0.5 m and height 2 m.

We now characterize the system according to the properties.

In order to track the spheres to determine their flow the position of a sphere needs to be determined accurately enough to avoid ambiguity, thus the resolution of the system needs to be at least 3 cm.

As the flow paths need to be determined for different spheres in the same space an absolute reference grid providing a physical location need to be used.

The system needs to be very accurate, thus a system with good computational power will be needed as not to compromise accuracy.

The scale of the system is limited to the size of the cylinder (radius 0.5 m and height 2 m) and only a single object needs to be tracked at a specific time instance.

As only one object will be tracked at a specific time the system wont need recognition capability.

Our main cost concerns are the space cost, as the space on the sphere is limited and the capital cost due to the computational power needed.

The most important limitation on the system is related to the effect of the environment on the physical phenomena that can be used. Due to the environment the system will not be able to use optic phenomena.

For this system to achieve its intended results the resolution must be within the constraints (3 cm), thus making the resolution property the most important.

Based on this as well as the limitations due to the environment we decided to implement the system using the phase difference sensing principle. This principle is based on the physical phenomenon of propagation. This implementation should yield the best results, if compared to results obtained by other systems using the same principle and physical phenomenon.

The implementation will yield distance data. To determine position from this data, the the lateration location computing technique will be used.

V. Conclusion

In this paper we discussed different aspects that can be used to characterize and classify tracking systems and proposed a method for the characterization and classification of tracking systems using these aspects. This method should enable researchers and developers to better compare different systems from the multitude of applications and disciplines and make decisions regarding the implementation of new systems easier.

We recommend that future work includes a proposed method for presenting results in a general form, for its specific classification. This will be a further help with the comparison of different systems.

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