Applying a credit default swap valuation approach to price South African weather derivatives

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Dissertation submitted in the School of Economics of the North-West University (Potchefstroom Campus) in partial fulfillment of the requirements for the degree of 

*Master of Commerce* in Risk Management

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Potchefstroom  
November 2010
DEDICATION

To my son, Sebastian
ACKNOWLEDGEMENTS

Thank you God, for making it possible for me to submit this dissertation.

Thank you for the gift of writing and sharing, and for the gift of life.

I want to sincerely thank each and every individual who made valuable contributions towards my dissertation. Thank you for your valuable time, endless help, co-operation, insight and patience.

I also want to extend my thanks to:

- Prof. Paul Styger, for his wisdom, endless patience, guidance, suggestions, knowledge and his eagerness to explore new concepts.
- Dr. Gary van Vuuren, for his time, energetic manner and insight when I thought all was lost. Thank you for sharing your knowledge with me.
- My mother and father, for giving me a chance at a good education, for their love, their guidance, their non-stop encouragement and sacrifices.
- The School of Economics at the North West University for presenting me with an opportunity to further my studies.
- Heinie Nel from Bosman Wineries, for all the telephone conversations and information about grape cultivars and grape farming.
- Kobus Bothma from Conradie farming for the effort to compile information about recovery rates for grape crops.
- Enrico Malan for his insight into grape crop insurance, and for the critical numbers which he provided me with.
- For everyone else who was on the sidelines; thank you for your kind words and support.
The theoretical and practical work described in this dissertation was conducted partly whilst a full-time student at the North-West University and partly in the employment of Momentum Group Limited under the supervision of Dr Gary van Vuuren and Professor Paul Styger.

These studies represent the original work of the author and have not been submitted in any form to another University. Where use was made of the work of others, this has been duly acknowledged in the text.

Signed: ___________________________________________ Date: ______________________

Amelia Nadine Holemans
ABSTRACT

Most farmers in South Africa use standard insurance to protect their crops against natural disasters such as hail or strong winds. However, no South African insurance contracts exist to compensate for too much or too little rain (although floods are covered), or which will pay out if temperatures were too high or too low for a certain period of time for the relevant crop.

Weather derivatives – which farmers may employ to ensure crops against adverse temperatures – do exist, but these are mostly available in foreign markets in the form of Heating Degree Days contracts and Cooling Degree Day contracts and are used chiefly by energy companies. Some South African over-the-counter weather derivatives are available, but trading in these is rare and seldom used.

The goal of this dissertation is to establish a pricing equation for weather derivatives specifically for use in the South African market. This equation will be derived using a similar methodology to that employed for credit default swaps. The premium derived will be designed to compensate grape farmers from losses arising from two different climatic outcomes – in this case temperature and precipitation. These derivatives will be region and crop specific and the formulation will be sufficiently flexible as to allow for further climatic possibilities (which may be added at a later stage).

These weather derivative premiums will then be compared to standard crop insurance to establish economic viability of the products and recommendations will be made regarding their usage. The possibility of the simultaneous use of these derivatives and standard crop insurance for optimal crop coverage will also be explored and discussed.
Meeste boere in Suid-Afrika gebruik tans gewone oesversekering om hul oes te verseker teen natuurlike rampe soos hael of stromsterk winde. Daar bestaan egter geen versekeringskontrakte in Suid-Afrika wat die boer sal verskans indien dit te veel (vloede word wel ingesluit) of te min reën, of wat sal uitbetaal indien die temperature nie optimaal is vir sekere tydperke van die jaar vir `n sekere gewas nie.

Weerafgeleide instrumente- wat `n boer kan gebruik om sy oes te verseker teen ongewensde temperatuurafwykings- bestaan wel, maar hierdie instrumente kom meestal in die oorsese mark voor in die vorm van "Heating Degree Days" kontrakte en "Cooling Degree Days" kontrakte en word meestal gebruik deur energie maatskappye. Sekere Suid-Afrikaanse oor-die-toonbank weerafgeleide instrumente is wel beskikbaar, maar die handel in hierdie instrumente is baie kleinskaals en word min gebruik.

Die doel van hierdie verhandeling is om `n prysingsformule te vind vir weerafgeleide instrumente wat in die Suid-Afrikaanse mark gebruik sal kan word. Hierdie formula sal afgelei word deur `n gelyksoortige metodologie te gebruik as die van die kredietwanbetalings-ruilooreenkoms prysingsmetodologie. Die afgeleide premie sal so saamgestel wees dat dit druiweboere sal kan vergoed indien daar enige verliese is as gevolg van twee verskillende weer afwykings- in hierdie geval temperatuur en reënval. Hierdie weerafgeleide instrumente sal streek en gewas spesifiek wees en sal so saamgestel wees dat dit vir ander klimaat moontlikhede ook gebruik sal kan word (wat op `n later stadium bygevoeg kan word).

Die weerafgeleide instrumente se premies sal dan vergelyk word met die van gewone oesversekering om ekonomiese vatbaarheid van die produkte te bepaal en aanbevelings sal gemaak word in verband met die gebruik daarvan. Die moontlikheid van die gelyktydige gebruik van weerafgeleide instrumente en oesversekering vir optimale versekering sal ook ondersoek en bespreek word.
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1. **Background**

1.1 **Definition**

Weather conditions such as temperature or rainfall have enormous impact on businesses and farming activities. For example, a higher than average temperature in summer will increase the revenue of air conditioner makers and electric companies but will reduce the profit of department stores due to air conditioning costs. Too little rain may cause farmers to have a smaller than expected harvest which will result in a decline in revenue. Such variations in revenue or profit due to weather conditions are called weather risks. To hedge these risks and stabilise revenue, financial instruments called weather derivatives were developed in the market and will be explored.

Weather derivatives are relatively new financial instruments which provide financial security to participants with payoffs dependent on weather indices or weather events. These are in turn based on climatic factors and historical weather data. Weather derivative contracts provide participants with the ability to manage risks which arise from unpredictable weather changes (Cyr and Kusy, 2007:2) and they include combinations of instruments such as swaps, options and option collars in which the payoff depends upon a wide variety of underlying weather related variables such as temperature, precipitation, humidity, sunshine hours and temperature forecasts (Campbell and Diebold, 2003:3).

1.1.2 **Development of weather derivatives**

Weather became an underlying for derivatives in the United States (US) in 1997. Randall (2008:1-3) recognises seven factors that led to weather becoming a commodity:

- the convergence of the capital and the insurance markets in the 1990s,
- the insurance industry was going through a cyclical phase in 1997, and risk capital became available to hedge weather risks,
- the El Niño event of 1997-1998, where warm weather in the Northern US led to a decrease in gas sales, which led to reduced profits,
• the deregulation of the electricity markets in the US since 1996,

• energy companies became very keen to examine a new way to mitigate the risk imposed by weather, the most important being Enron,

• airport stations with good historical records will supply meteorological data, and with good analysis of the data it would be relatively easy to price a contract based on these data, as well as to find a probability of a specified event occurring and

• the rise of environmental markets, particularly air pollutants and the fear of climate change led companies into examining the effects of weather on their profits.

1.1.3 The first weather derivative deal

The first major weather derivative deal in the US took place in 1997 between Enron and Koch (Geyser, 2004:447). In the United Kingdom (UK), the first deal was sold by Enron to Scottish Hydropower in 1998. Other European deals followed, first in Germany and the Netherlands and later in Scandinavia. The Japanese and Australians followed and in 1999 the Chicago Mercantile Exchange (CME) started listing weather contracts. On February 28, 2002, Gensec Bank, a wholly owned subsidiary of the JSE listed financial services group Sanlam, and the US-based Aquila subsidiary of UtiliCorp United, one of the largest energy marketers and risk management companies in North-America, announced the formation of a strategic alliance to be the first to begin marketing weather derivative products in South Africa (Anon, 2002).

Because weather is unpredictable and ubiquitous, it influences almost everything. Rain can cause businesses to have a decline in profit (shoppers would rather stay home in rainy weather than go shopping), too much or too little rain can have a negative effect on farmers' harvests, depending on the stage of their crop, thunder storms are potentially dangerous to spectators and players alike of outdoor sporting events and so on. These and other considerations necessitate the management of weather risk through the use of weather derivatives.

1.2 Problem statement and objectives

This dissertation explores the problem of how to correctly price weather derivatives in South Africa using a credit default swap (CDS) methodology.
This work could also assist in the eventual construction of a South African "weather index" which could then be used to price other, bespoke, weather derivatives.

Weather derivatives constitute a relatively small market in South Africa currently (November 2010). By pricing these instruments correctly they can cover gaps which current weather insurance protocol does not cover. This can assist businesses, farmers, event coordinators and so on to hedge their companies or themselves against undesirable – and unpredictable – weather events.

This dissertation focuses on the use of weather derivatives in agriculture.

Most farmers use crop insurance to cover potential losses which may be experienced in crop failure. This type of insurance coverage is efficient when natural disasters like hail or a natural fire hits, but it is inefficient if an exceptionally warm summer is experienced when farmers were expecting a mild summer, or when it rains too much (or too little) for some crops. Weather derivatives can protect against losses for these events, but there exists no good platform in South Africa for farmers to freely use these kinds of derivatives.

Existing weather derivatives have only one underlying measure, so they can only be used to hedge against one underlying (e.g. temperature). This can be problematic as most farmers would wish to hedge themselves against multiple underlying measures. This dissertation focuses on a winery and grape farm in the Western Cape region which must be hedged against undesirable rainfall and temperature.

The goals of this dissertation are thus to describe, construct and price a new weather derivative to mitigate weather risks faced by grape farmers in the Western Cape. These farmers require unique weather conditions for the successful maturation and cultivation of their grapes so a financial derivative methodology is sought which may be used to successfully mitigate these weather risks. A credit default swap pricing methodology is used to this end. It is important to note that although a derivative instrument is constructed to mitigate the specific weather conditions faced by Western Cape grape farmers, in principle, the methodology employed in this dissertation may be applied to any weather conditions in order to mitigate potential losses.

In addition, this work could potentially contribute to the construction of a South African "weather index". This index could be used to price bespoke (and, if necessary, exchange traded) weather derivatives resulting in a more liquid and transparent market for these instruments.
1.3 Research design and procedure

The research design of this thesis followed the outline below:

**Pose research questions:** Broad questions were posed regarding the types of weather derivatives that are currently available in South Africa and globally, and how these weather derivatives can be adapted to assist farmers with crop coverage in the event of unfavourable weather events.

**Critical literature review:** A critical literature review ensued in which the evolution and uses of derivatives, credit derivatives and weather derivatives were discussed. Abundant literature exists to address the different models for the pricing of weather derivatives and these are explored and commented upon.

**Action research/data collection:** Data were used from original sources at all times. The South African weather bureau provided historical data for temperature and rainfall, and Bosman Wineries provided their own data regarding grape crops.

**Theory building/adapting/testing:** In this case, pursuing existing, well-established methodologies allows subtle, but significant, improvements to be made to the credit default swap pricing equation. Adjustments and additions to the existing CDS pricing formula were made to fit the needs of this dissertation’s problem statement. Developing new ideas requires much back-testing, validation and endorsement. Ultimately, the bulk of the results reported in this dissertation were from empirical testing: Historical data of weather stations near and in Wellington in the Western Cape are used as parameters for the equation, along with current market related statistics such as the yield curve of South Africa. The empirical study compares current weather insurance for grape crops with the calculated weather derivative premium and proposals follow on how to use this hedging tool.

**Conceptual development:** This research is intended to provide accurate, but highly practical, solutions for use by financial institutions and farmers. As a direct result, the primary source of analytical work was done in Microsoft Excel™ since this is the tool of choice for almost all financial institutions. While clearly not designed to perform the most advanced statistical or algebraic analysis, Microsoft Excel™ nevertheless performs adequately. These spreadsheet-based models use visual basic programming language (a flexible, functional and highly valuable desktop tool available to all quantitative analysts and risk managers alike) to develop macros for un-
dertaking onerous and repetitive computing tasks. Results (the premium payable on the weather derivatives) were compared with normal insurance quotes for farmers and were commented on.

1.4 Chapter Layout

The broad objective of this dissertation is to correctly price a weather derivative in South Africa.

The specific objectives are:

- to explain where derivatives have originated (Chapter 2),
- to explore the history and evolution of credit derivatives (Chapter 2),
- to explain a credit event (Chapter 2),
- to explain credit default swaps (as a basis for weather derivatives) (Chapter 2),
- to provide an insight into the origin and evolution of weather derivatives in the foreign market (Chapter 2),
- to explain different weather indices, structures and contracts (Chapter 3),
- to familiarise with the different models which have been used to price weather derivatives (Chapter 3),
- to understand weather evolution models (Chapter 4),
- to formulate a pricing equation which will price a weather derivatives with multiple underlying measures (Chapter 4),
- to test the equation in the empirical study (Chapter 5) and
- to price a weather derivative designed to hedge weather conditions prevalent in the Western Cape (wineries and grape farms) using historical weather data and a CDS methodology (Chapter 5), and
- to conclude the dissertation and give recommendations (Chapter 6).

1.5 Conclusion

Existing crop insurance is not adequate to cover losses made by farmers when the reason was not a severe hail or wind storm. This dissertation addresses that gap, and a credit default swap is adjusted to price a weather derivative where the farmer can hedge himself against temperature and rainfall fluctuation. This pricing result in a premium which must be paid monthly for a certain period until the cover period (usually the life cycle of a crop) terminates. A weather derivative
can be purchased with a maturity of six months (e.g.), which means that for months that farmers do not require coverage, there is no need to pay for insurance using weather derivatives.

The next chapter provides a comprehensive literature review of the derivative concept, its history and the evolution to credit derivatives.
CHAPTER 2

THE DERIVATIVE CONCEPT APPLIED TO WEATHER DERIVATIVES

2.1 Introduction

A derivative is a financial instrument of which the value is derived from the value of another asset, known as the underlying (Chisholm, 2004:1). Alan Greenspan,¹ noted that "by far the most significant event in finance during the past decade – up to 2005 – has been the extraordinary development and expansion of financial derivatives" (Hetamsaria and Kaul, 2005). These derivative structures improved the capability to differentiate risk and to distribute this risk to those investors whom would be willing and financially able to take it. This process of transferring risk has unquestionably helped to improve the national productivity growth as well as the standard of living in the US (Hetamsaria and Kaul, 2005).

The purpose of Chapter 2 is to provide an oversight of the history of derivatives. The essentials of credit derivatives (as an example of event driven derivatives) are also presented. This is necessary because weather derivatives are special case event derivatives that have evolved from the credit derivative methodology. This chapter also examines credit risk and further discusses the structures of different types of credit derivatives. A definition for weather derivatives is also defined, followed by an in depth explanation about the evolution of weather derivatives. Lastly, an overview is provided regarding the CME, the world’s leading weather derivative trading market.

2.2 The evolution of financial derivatives

A derivative is a bet between two parties: e.g., "X takes a bet of R50 with Y that the Bulls will win the Currie Cup ". The underlying asset is the winning or losing of the Bulls, and the R50 is the payout.

Financial derivatives are not new; they have been in existence in one form or another for centuries. A description of the first known option contract² is found in the work of Aristotle, a Greek philosopher. He tells of Thales, a poor philosopher from the nearby island of Miletus. Thales developed a "financial device, which involves a principle of universal application" (Siems, 1997).

¹ Previous chairman of the Board of Governors of the US Federal Reserve Bank.
² The right to buy or sell an asset
As a skilled forecaster, Thales predicted that the olive harvest would be exceptionally good the following autumn. Confident in his prediction, Thales agreed with olive-press owners to deposit what little money he owned with them to guarantee him exclusive use of their olive presses when the following autumn's harvest was ready. He negotiated low prices because the harvest was a future event and no one knew whether it would be plentiful or paltry.

Aristotle's story about Thales ends predictably: "When the harvest-time came, and many [presses] were wanted all at once, he let them out at any rate which he pleased, and made a quantity of money. He thus showed the world that philosophers can easily enjoy wealthy rewards, but that their ambition is of another sort" (Hutchins, 1952:453).

Thales exercised the first known option contract some 2 500 years ago as he was not obliged to exercise the options (unlike the olive merchants, who were). If the olive harvest had been poor, the option contracts would have expired unused. The loss to Thales would have been limited to the original price paid for the options. Since the olive crop was exceptionally good, he exercised the options and sold his claims on the olive presses at a high profit (Hutchins, 1952:453).

The establishment of the first exchange for derivative trading – the Royal Exchange, in London – was the next significant development for derivative trading (Chance, 1998). The Royal Exchange began operations in 1570 and permitted only forward contracting at the time. The next important development in the derivative market, and also the first futures contracts to be traded, occurred in Japan in 1650, where standardised contracts were traded in the Yodoya rice market in Osaka (Chance, 2008).

The first modern, organised futures market in North America was founded on April 3, 1848 by 83 merchants and was called the Chicago Board of Trade (CBOT, 2008:1). This was the next major event in the history of derivatives as the CBOT had a prime location on Lake Michigan, Chicago. Because of the location, the exchange could develop into a major centre for the storage, sale and distribution of Midwestern grain (Chance, 1998). CBOT Holdings and the CME Holdings Inc. announced on October 17, 2006, that a document had been signed which agreed to merge both organisations, which would then form the most diverse global derivatives exchange (CBOT, 2008:6). The CME continues to play a very important role in derivatives today: this is discussed later in this chapter.
2.3 Credit derivatives as a basis for weather derivatives

Financial derivatives create new contracts which derive their value from original contracts or assets. For example, stock market derivatives are contracts that are settled based on the movements of the stock prices, without transfer of the underlying stock. The contracts are based on certain events, in this case, stock market movements.

The development of credit derivatives is a logical extension of derivative trading in the market. Credit derivatives contracts (which involve contracts between two parties, again with an asset as the underlying and which are settled without transferring the asset (Kothari, 2009a:3)) allow the transfer of credit risk – i.e. the probability of credit event risk, such as default, repayment risk, etc., – to a counterparty. This section explores the mechanics underlying credit derivatives as they form the basis on which weather derivatives are constructed.

Credit derivatives are financial instruments that are used to manage, mitigate or hedge credit event risks in the financial sector. The key role of this group of financial instruments is to manage credit exposures, such as credit, default, foreign exchange or interest rate risks (Batten and Hogan, 2002:252).

Kothari (2009a:4-5, 10-11) clarifies a credit derivative as follows:

- a credit asset is the extension of credit in some form, normally a loan, instalment credit or financial lease contract,

- every credit asset is a bundle of risks and returns, implicating that every credit asset is acquired to make certain returns on the asset. The probability of not making the expected return to the holder as a result of delinquency, default losses, foreclosure, prepayment, interest rate movements, exchange rate movements etc motivates the existence of a credit derivative,

- the credit derivative concept was designed by credit institutions, such as banks, to diversify portfolio risks without diversifying the portfolio itself. The drive towards credit derivatives has been goaded by bankers' needs to meet their capital adequacy necessities,

- credit derivatives are entirely marketable contracts. The credit risk inherent in a portfolio can be securitised and sold in the capital market just like any other capital market security. Thus the purchase of such a security involves the purchase of a fragment of the risk inherent in the
portfolio. Buyers of such securities are buying a fraction of the risks and returns of a portfolio held by the originating bank, thus derivatives and securitisation together make credit risk a tradable commodity.

2.4 The history of credit derivatives

Kothari (2009b) lists significant milestones in the development of credit derivatives. Credit derivatives emerged in 1992 when the International Swaps and Derivatives Association (ISDA) first described a new exotic type of over-the-counter contract. In 1993, the first credit portfolio model was introduced by Moody’s KMV, another significant milestone for credit derivatives.

According to Ranciere (2002:4), the credit derivative market began in 1996. Financial institutions worried about their credit risk exposure, viewed credit derivatives as tools that could be used to manage these risks. Many companies viewed credit derivatives as a counterbalance to the loan securitisation market. Consequently, credit derivatives developed rapidly and independently and became key instruments used to hedge credit risks (Ranciere, 2002:4).

The credit derivative market emerged simultaneously with the Asian Crisis in the second half of 1997. The lack of standardised documentation slowed down the development, but it accelerated again in 1999 with the publication of ISDA’s credit derivative definitions (Ranciere, 2002:4).

In 2000, the total notional principal for outstanding credit derivatives contracts was US$800 billion, and by 2002, this amount had grown to US$2 trillion (Hull, 2005:449). In 2004, the notional value of credit derivatives was US$5 trillion, which increased to over US$20 trillion in 2006 (Pool and Mettler, 2007:30). LaCroix (2008) adds that, as of the beginning of March 2008, the credit derivative market had a notional value of US$45 trillion, an amount that was equivalent to total, global bank deposits. Today, the credit derivatives market is one of the most important markets in the financial world.

The growth in credit derivatives by the end of the 1990s occurred for three reasons (JP Morgan, 2008:7):

- new products continued to emerge from the traditional building blocks of options and futures, and were forming second and third generation derivatives which proved to be complex with path-dependent risks,
• derivatives were being expanded beyond the usual management of price or event risk\(^3\) - and were being used to manage portfolio risk, balance sheet growth, shareholder value and overall business performance and

• derivatives were being expanded beyond the normal interest rate-, currency-, commodity- and equity markets to new underlying risks such as catastrophe-, pollution-, electricity-, inflation- and credit risk.

For much of the history of finance, credit risk was one of the major elements of business risk for which no risk management products existed. For a loan portfolio manager, the management of risk involved merely diversifying the portfolio with financial manipulation in the secondary market. Reliance was placed on the purchasing of insurance and letters of credit or guarantees, but the separation of credit risk management from the assets with which those risks were associated was not considered. This made these credit risk strategies highly ineffective (JP Morgan 2008:7).

Credit risk is today better managed using credit derivatives. They allow companies to trade credit risk much in the same way as market risk. Banks and other financial institutions were once in the position where credit risk was assumed to be unavoidable, and all that these companies could do was to wait and see how this risk would unfold. Today, credit risk portfolios can be actively managed by retaining some of the credit risk and entering into credit derivative contracts to protect against the remainder of the risk imposed on the company (Hull, 2005:449).

### 2.5 Credit events

A credit default swap, a type of a credit derivative, is triggered by a credit event (Lehman Brothers International, 2001:61). Credit events are events which trigger the exercise of derivative obligations such as bankruptcy, insolvency, a downgrade of the company’s rating or the change in the credit spread exceeding a specified level (Bessis, 2002:726).

ISDA defines several standard credit events which may be employed in credit derivative transactions, including bankruptcy, obligation default/acceleration, failure to pay, repudiation, moratorium and restructuring: these are listed in the sections that follow (Kothari, 2008).

---

\(^3\) Price risk – the risk that the liquidation value of collateral can fluctuate as the market fluctuates (Bessis, 2002:510).  
Event risk – defined as the risk of an event that will trigger the default of a pre-determined financial obligation (Bessis, 2002:726).
2.5.1  **Bankruptcy or Insolvency**

A financial institution or company is bankrupt when it is declared insolvent or when it cannot pay its debts. Certain actions taken by the reference entity, such as a meeting where shareholders consider filing for a liquidation request, is also viewed as an act of bankruptcy, which can trigger a credit event.

2.5.2  **Obligation Default or Obligation /Acceleration**

Obligation default or obligation acceleration occurs when a relevant obligation becomes outstanding and payable as a result of a default by the reference entity before the actual time when this obligation would have been declared. The total amount of obligations should be greater than the default obligation.

It is important to note that "default" is used in such terms that are relevant to a specific contract or agreement.

2.5.3  **Failure to meet payment obligations**

Failure to pay occurs when the reference entity fails to make a payment under one or more obligations at a certain time due. In this case grace periods for payments are taken into account, mostly to prevent accidental triggering due to administrative errors.

2.5.4  **Repudiation or Moratorium**

Repudiation (or moratorium) deals with the scenario where the reference entity or any governmental authority disclaims, disaffirms, questions or challenges the validity of the relevant obligation.

2.5.5  **Restructuring**

Restructuring covers events which occur when any of the terms agreed upon by the reference entity and the holders of the specific obligation in question before entering the contract have become less favourable to the holders as what it would have been otherwise. These events can be a decrease in the principal amount, a decrease of the interest payable under the obligation, a delay of a payment or a change in the ranking in the priority of the payment. Restructuring will change the debt obligations of the reference entity.
Credit derivatives provide protection against any of the credit events that were discussed above to financial entities holding these derivatives. If any one of these pre-determined events were to occur, the event precipitates the settlement of the credit derivative, in which the buyer of a credit derivative has the right to settle the credit derivative, thus protecting the position against credit risk.

2.6 Types of credit derivatives

Credit derivatives are financial products that isolate credit risk from other forms of risk (like market or operational risk) of a certain asset and then transfers that risk from one party to another (Brandon and Fernandez, 2005:52). They offer protection against credit or default risk of credit risky financial instruments (Ranciere, 2002:3).

The payoff of a credit derivative is subject to the occurrence of a credit event, for example the failure of one participant to pay (Brandon and Fernandez, 2005:52), or any other event as explained in Section 2.5.

Credit derivatives have three distinguishing characteristics (US Federal Reserve, 1998:566):

- the transfer of the credit risk linked with a reference asset through the use of contingent payments that are based on default events on the prices of instruments before, at, and shortly after default,
- the periodic exchange of payments or the payment of a premium rather than the payment of fees which is usually the case with other off-balance-sheet credit products, like letters of credit and
- the use of an ISDA Master Agreement and the legal format of a derivatives contract.

The terminologies used when considering credit derivative transactions can be confusing. A buyer can be either a buyer of credit risk, or the buyer of credit protection, while the seller can either sell the credit risk or sell the credit protection.

In this section, the following convention is adopted to avoid any confusion. The buyer is considered as the buyer of credit protection and the seller of credit risk and is the party responsible for contingent payments. The buyer is also the beneficiary of the credit derivative transaction. The
seller is the seller of credit protection and the buyer of credit risk (and will usually receive a fee for this protection (Tavakoli, 2001:5)).

The US Federal Reserve (1998:566) defines three credit derivative types:

1. total-rate-of-return swaps\(^4\),
2. credit default swaps and
3. credit default notes\(^5\).

The next section defines only credit default swaps, as these are the simplest and most popular form of credit derivative that could form a basis for pricing (and understanding) weather derivatives.

### 2.6.1 Credit Default Swaps

The simplest form of a credit derivative, as well as the most popular credit derivative, is the credit default swap (CDS). CDSs form the basic building blocks of the credit derivatives market. By using a default swap the buyer is offered protection against the loss of the asset value that would normally follow a credit event (Ranciere, 2002:3). Hull (2006:507) describes a CDS as a contract that provides insurance against the default risk of a particular company. The reference entity is typically a corporate, bank or sovereign issuer (Lehman Brothers International, 2001:26) and a default by the company is known as the credit event.

The buyer of the insurance (or the CDS) obtains the right to sell the underlying asset of the reference entity if or when a credit event will occur. For example, if the CDS underlying is a bond, the buyer has the right to sell that bond for the face value\(^6\) when a credit event occurs. The total face value of the bonds that can be sold is known as the CDS's notional principal. The buyer of the CDS will make periodic payments to the seller until a credit event occurs, or otherwise until the maturity of the CDS. If a credit event occurs, the settlement is either the physical delivery of the bonds or a cash payment (Hull, 2006:507).

---

\(^4\) A swap where the return on an asset such as a bond is exchanged for LIBOR plus a spread. The return on the asset includes income such as coupons and the change value of the asset (Hull, 2005).

\(^5\) A credit default note is a security with an embedded credit default swap allowing the issuer to transfer a specific credit risk to credit investors (Hull, 2005).

\(^6\) The face value, also known as the par value of a bond, is the principal amount that the issuer will repay at maturity if the bond does not default (Hull, 2005).
In a CDS, one counterparty, in this case Bank A, agrees to make payments of x basis points of the notional amount, either per quarter or per year. In return, Bank A receives a payment (from Bank B) in the event of the default of a pre-specified reference asset. This set of occurrences is illustrated below in Figure 2.1.

**Figure 2.1: Credit Default Swap.**

![Credit Default Swap Diagram]


Since the payoff of a CDS is contingent on a default event, the word "swap" might be considered a misnomer, as the transaction actually more closely resembles an option (US Federal Reserve, 1998:567).

When credit default swaps are traded the following conventions are commonly followed (US Federal Reserve, 1998:568):

- the reference entities are generally public, investment-grade companies,
- trades are for senior, unsecured risk,
- five-year contracts are the most common, but one- and three-year contracts are also traded,
- U.S. trades usually only includes bankruptcy, failure to pay and modified restructuring (as defined under the May 11th, 2001 ISDA Restructuring Supplement) as credit events,
- European trades generally include standard restructuring credit events and
- trades become effective in three days (T+3, where T is the contract duration).
2.6.2 Credit Default Swap cash flows

Figure 2.2 sets out the details required for the pricing of a credit default swap trade and Figure 2.3 illustrates the contingent and fixed payouts.

**Figure 2.2:** Example of a Credit Default Swap.

<table>
<thead>
<tr>
<th>Default Swap details:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Currency:</td>
</tr>
<tr>
<td>Maturity:</td>
</tr>
<tr>
<td>Reference Entity:</td>
</tr>
<tr>
<td>Notional</td>
</tr>
<tr>
<td>Default Swap Spread:</td>
</tr>
<tr>
<td>Frequency:</td>
</tr>
<tr>
<td>Size of cash flow:</td>
</tr>
<tr>
<td>Payoff upon Default:</td>
</tr>
<tr>
<td>Recovery Rate:</td>
</tr>
<tr>
<td>Credit Event:</td>
</tr>
</tbody>
</table>


**Figure 2.3:** Cash flows on the Credit Default Swap in Figure 2.2.

The credit derivative in Figure 2.2 is a €50 million, 3 year default swap. The cost of the protection is 33 basis points per annum, paid quarterly. The size of each quarterly cash flow is €41,250. If default were to occur, the recovery rate is 50% of the face value – i.e. the protection buyer will receive €25 million from the protection seller (Lehman Brothers International, 2001:27-28).

Figure 2.3 depicts the cash flows when a default or credit event occurs, as well as the payout if default were not to occur. The credit protection buyer pays a quarterly cash flow of €41,250 in the case of no default until maturity of the swap. In the event of default, the credit protection buyer receives a payment of half of the notional amount – as specified in the contract – €25 million. The other 50% of the notional amount is recovered from the bond writer (from the 50% recovery rate).

2.7 Weather derivatives

Weather derivatives are an emerging class of financial instruments, designed to provide protection against adverse changes in temperature, precipitation, and the wide range of weather risks to which businesses are subjected (Ali, 2004:75). Weather derivatives are constructed using the occurrence of a weather event (be it rainfall for a specified period, or temperatures for a specified period, etc) as the trigger for contingent payments. This fairly new area of derivatives has opened up a completely new spectrum of previously unavailable possibilities to the market when managing risks.

2.7.1 Background

The Chicago Board of Trade (CBOT) introduced catastrophe futures in 1993, which permitted hedging against catastrophic weather conditions such as hurricanes. This allowed the freezing of projected loss ratios and underwriters could thereby lock in their profits. This could be done with more flexibility and it cost less than re-insurance. The catastrophe futures did not allow the underwriters to protect themselves against non-catastrophic weather conditions, such as the loss of revenue resulting from unexpected warm winters. A dire need for hedging against conditions like the El Niño aroused, and an over-the-counter\(^7\) (OTC) market in weather options emerged (Ray, 2004:295).

\(^7\) Over-the-counter markets are explained in detail in Section 2.8.
A significant factor which fuelled the growth of weather derivatives was energy market deregulation. Energy prices are highly correlated with the weather: energy producers want to set the prices in such a way that adverse weather will not cause a loss of trading revenue. By trading weather derivatives, energy companies have found a new way to hedge the risks that are unfavourably imposed upon them by the uncontrollable effect of the weather, such as declining energy sales (Alaton et al., 2002:1). This traceable factor implies that the unpredictability in weather conditions is the most significant factor that affects energy usage, but the effects of the inconsistency in weather patterns were always absorbed and managed within a monopoly-like environment. With the deregulation of the US energy market, the participants in energy delivery were even more exposed to weather variability, and this enabled them to see the new and significant risk that they had to take into account when thinking of their financial stability (Brockett et al., 2005:130).

The origins of weather derivatives can be traced back to the extreme El Niño climate of 1997—1998. An extremely warm winter in the northern hemisphere caused a decline in the revenues of many utilities (Ray, 2004:293-294). Thus, in 1997, major weather derivative deals took place between US energy companies: these were all over-the-counter (OTC) transactions which were privately negotiated between the parties and were structured as protection against the decline in income as a result of the deviation of average temperatures in specific regions (Climetrix, 2008). One of the deals consisted of a custom-made swap in which Enron agreed to pay Koch US$10 000 for each degree that the temperature would fall below the normal average temperature and Koch agreed to pay Enron US$10 000 for every degree that the temperature would be above normal, in the winter of 1997 – 1998 (Perin, 1999). In the UK, the first weather derivative deal was concluded in 1998 (Randalls, 2008:2).

Capital markets provided companies with limited capital by the use of contingent capital programmes in the event of a natural catastrophe and a loss of equity capital (Considine, 2000:1). This process provided capital that was repaid to the creditors or investors after the expiry of the contingent capital transaction and was purely for financing. This was followed by insurance se-

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8 The first weather derivative deal, actually took place in July 1996 when Aquila Energy structured a dual-commodity hedge for Consolidated Edison Company (Nicholls, 2008:45). The transaction involved Consolidated's purchase of electric power from Aquila for August 1996. The price of the power was agreed upon, but a weather clause was added to the contract stipulating that Aquila would pay Consolidated a rebate if August turned out to be cooler than expected (Nicholls, 2008:47).
curitisation, in other words, the securitisation of insurance risks as well as the transfer of these risks to the capital markets by the use of bonds and derivatives, which laid down the foundation for the convergence of the insurance and capital markets (Grandi and Müller, 1999:610). Up until the convergence point, which happened in the 1990s, insurance was separated from the capital markets. Companies wanted to insure themselves in the capital market, which allowed them to trade insurance-like products with more freedom, and thus the two markets became more and more entangled (Randalls, 2008:1).

During 1997 – 1998, the insurance industry entered a cyclical downward phase that brought on low premiums in the underwriting business. This meant that companies were in the position to release adequate amounts of risk capital to hedge weather risks (Considine, 2000:1). As the concept of weather derivatives had already been introduced, companies saw this as the opportunity to hedge themselves against this newly tradable risk type.

The CME introduced futures and options on weather that could be traded on the exchange in 1999 (CME, 2008a:12). These options and futures were adapted to average temperature indices in different locations, and were the first of its kind. Unlike the OTC weather derivatives, which were privately negotiated and individual agreements between two parties, the CME weather futures were standardised contracts that were traded freely on the open market. The trading would take place in an electronic, auction-like surrounding with ongoing negotiation of the prices, whilst keeping the prices totally transparent (CME, 2008a:3).

The first CME weather futures were only available for the weather of US cities, but it became very popular with international clients, whom quickly requested similar future contracts to hedge their own weather risk. In October 2003 the CME launched weather contracts on five European cities and in July 2004 on two Asia Pacific cities (CME, 2008b).

Until 2003, all the weather contracts were monthly contracts, but in 2003, the CME expanded their weather products to include contracts that could be traded seasonally, and enabled traders to hedge their risk for an entire season. Only two seasons, summer and winter, were acknowledged, and consisted of five consecutive months, which could be traded as one product (CME, 2008b).

Other exchanges that followed the CME by trading standardised weather derivative contracts are the London International Financial Futures and Options Exchange (LIFFE). They introduced weather futures for Western Europe in 2001 that allowed Europeans to hedge their weather risks.
The Helsinki Exchange followed in 2002, which allowed Scandinavian countries to hedge against their weather conditions (Ray, 2004:295), and the Intercontinental Exchange is another emerging exchange for weather contracts (Climetrix, 2008). The latest exchange that is now offering weather-related indices and derivatives is the Storm Exchange Inc. in New York (Marshall, 2007). Marshall (2007) reports that this exchange was formed in April 2006, and allows subscribers to choose from a series of customised indices of weather-related data. The company founder and chief executive officer David Riker said that the Storm Exchange focuses on mid-market companies which have between US$10 million and US$500 million in revenues per year, because the larger companies will most likely already have derivative deals and contracts with other companies. The company has already created more than 500 customised indices in five industries, which have temperature as the underlying. Their goal is to ultimately provide transparency around the pricing of their weather derivatives (Marshall, 2007).

The weather derivatives market is the fastest-growing derivative market today (CME, 2008a:2). The value of weather derivatives traded in 1998 was US$1.8 billion and grew to US$4.3 billion in 2001. In a survey of PricewaterhouseCoopers, PWC (2008) it was found that the notional value of weather derivatives in 2002 was US$4.2 billion, in 2003 US$4.7 billion and in 2004 US$9.7 billion. The notional value in 2005 was a record high of US$45.2 billion, which declined again to US$19.2 billion between April 2006 and March 2007. Cundy (2008) further reports that weather derivatives showed a dramatic leap of 39% in future and option volumes in the first quarter of 2008 compared with the previous year. After the first quarter of 2008 the weather derivatives market kept on expanding to an astonishing US$32.0 billion in notional value in June 2008 (Cundy, 2008).

2.7.2 Weather derivatives in South Africa

South Africa actively entered the weather derivatives market in February 2002. Gensec Bank and Aquila announced their development of a strategic association to be the first to begin marketing weather derivatives in South Africa (Anon, 2002). Gensec Bank is a subsidiary of Sanlam which is fully guaranteed by Gensec, with a capital base of approximately R5 billion, which specialises in providing derivative-based risk management products to the savings industry. Aquila is a subsidiary of the listed Kansas City based company UtiliCorp United, and is one of the world’s leading market makers of weather derivatives. Aquila is a provider of risk management services,
providing wholesale energy and risk management services in the UK, Germany and Scandinavia, and also a leading wholesaler of natural gas, power and coal in North America (Anon, 2002).

2.7.3 Influence of weather on the industry

Weather exerts an enormous impact upon businesses and market activities. The CME(2008a:2) estimated that almost 30% of the US economy is directly affected by the weather. The weather influences the choices that are made regarding going to an outdoor concert or sporting event, the choice to switch on a heater or an air-conditioner (influencing electricity consumption) and even transportation efficiency (delayed flights as a result of misty conditions or snowstorms). The effect that is has on the business sector is omnipresent and very expensive (Dischel, 2002).

Industries affected by the weather are similarly ubiquitous, including utilities (gas, electricity), energy vendors (oil, coal), agricultural sector (farmers, processors), entertainment industry (resort areas, casinos), travel industry (airlines, trains), sporting events (Soccer World Cup, Tri Nations Rugby, Super 14), manufacturers (recreational vehicles, weather gear) and insurers (underwriters, re-insurers) (Ray, 2004:294).

Weather unpredictability can affect different entities in different ways. Jewson and Brix (2008:3) give many examples e.g.:

- a gas supply company will sell less gas in a warm winter,
- a ski resort will attract fewer skiers when the snowfall is sparse,
- a clothing company will sell fewer summer clothes in a cold summer,
- an amusement park will have few visitors if it rains,
- a construction company can have delays when it is raining or is very cold,
- a hydroelectric power generation company will generate less electricity when there is less rainfall than usual and
- a fish farm can have difficulties in breeding fish if the sea temperature is too low.
Table 2.1 provides a comprehensive resource for weather risk professionals and is yet another good example to illustrate the links between weather indices and financial risks.

**Table 2.1: Illustrative links between weather indices and financial risks.**

<table>
<thead>
<tr>
<th>RISK HOLDER</th>
<th>WEATHER TYPE</th>
<th>RISK</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy industry</td>
<td>Temperature</td>
<td>Lower sales during warm winters or cool summers</td>
</tr>
<tr>
<td>Energy consumers</td>
<td>Temperature</td>
<td>Higher heating/cooling costs during cold winters and hot summers</td>
</tr>
<tr>
<td>Beverage producers</td>
<td>Temperature</td>
<td>Lower sales during cool summers</td>
</tr>
<tr>
<td>Building material companies</td>
<td>Temperature/snowfall</td>
<td>Lower sales during severe winters (construction sites shut down)</td>
</tr>
<tr>
<td>Construction companies</td>
<td>Temperature/snowfall</td>
<td>Delays in meeting schedules during periods of poor weather</td>
</tr>
<tr>
<td>Ski Resorts</td>
<td>Snowfall</td>
<td>Lower revenue during winters with below-average snowfall</td>
</tr>
<tr>
<td>Agricultural industry</td>
<td>Temperature/snowfall</td>
<td>Significant crop losses due to extreme temperatures or rainfall</td>
</tr>
<tr>
<td>Municipal governments</td>
<td>Snowfall</td>
<td>Higher snow removal costs during winters with above-average snowfall</td>
</tr>
<tr>
<td>Road salt companies</td>
<td>Snowfall</td>
<td>Lower revenues during low snowfall winters</td>
</tr>
<tr>
<td>Hydroelectric power generation</td>
<td>Precipitation</td>
<td>Lower revenue during periods of drought</td>
</tr>
</tbody>
</table>

Source: Climetrix, 2008.

All of these risks can be hedged by using weather derivatives.

**2.7.4 The unique aspects of weather risk**

Weather risk is unique: it has special characteristics that separate it from commodity price risk and other sources of risk (Cogen, 1998). Weather affects the costs and revenues resulting from changes in *volume* and not changes in *prices*. It is important to note that there are no physical markets in weather. A hot day in January cannot be stored until it is needed in July, and rainfall in Gauteng cannot be transported to the Free State. Weather risk is also localised, as there are few, if any, benchmark indices in weather that have commercial meaning to broad markets (Cogen, 1998). Weather is also completely beyond human control: it cannot be influenced, modified or manipulated by regulation, speculation, cartels, major market players or mass-market dy-
namics. When weather is involved everyone gets the same deal, because "Mother Nature does not bargain" (Cogen, 1998).

The difference between weather derivatives and conventional derivatives is that there is "no original, negotiable underlying or price of an underlying that normally forms the basis of any derivative" (Müller and Grandi, 2000:274). The underlyings employed by standard financial derivatives are negotiable objects such as shares, bonds, exchange rates or currencies. The underlying of weather derivatives is based on data (such as rainfall and temperature). The purpose of weather derivatives, therefore, cannot be to hedge the price of the underlying, because different facets of the weather cannot be linked to a monetary value or price. Instead, weather derivatives are suitable for other objectives, such as hedging other risks brought on by the changes in the weather (Müller and Grandi, 2000:274).

2.7.5 Are weather derivatives the same as normal insurance products?

A weather derivative contract is a contract between two parties that specifies the payment between the parties and which depends on certain meteorological conditions during the length of the contract (Geyser, 2004:445).

Insurance companies traditionally offer protection against certain weather events, usually to the extent of the damage inflicted on the suffered (Mahal, 2001:325). Weather related, this insurance is usually against extensive rain damage or replacements of roofs, in the scenario of houses damaged by for example a hurricane or tornado. These "catastrophic events" are covered by standard insurance contracts whereas a weather derivative will hedge a person or company against a certain risk, resulting from a small weather event, like unfavourable temperature changes.

A key difference between insurance and weather derivatives is that, in the case of insurance, the person has to submit a claim to show that there has been a loss, in order to receive a payout. With weather derivatives, the payout is automatic upon exercise. Another difference between the two is that insurance products in general have been taxed. This increased the price of these products, while in the over-the-counter markets no taxation was imposed upon weather derivatives. Insurance is usually sold for the protection of tangible or intangible assets, including property, commodities, vehicles and life insurance. Derivatives, on the other hand, are used to mitigate a risk, and are thus a form of risk management (Mahal, 2001:325).
The standard insurance contract has a few shortcomings. Richards et al (2004:1005) give the example of a farmer who claims a loss under an insurance contract: Firstly, he or she must prove that an actual loss occurred on his or her farm. To adjust crop losses is pricey to manage and it contains a certain element of subjectivity that seldom pleases farmers. Secondly, insurance is usually intended to cover events that do not happen often and are usually high-loss events (hence also "catastrophic events"). Insurance does not cover events that occur frequently, even if they are limited-loss events. Thirdly, alternative tools that pay out based on an objective measure of the weather can be a preferable substitute, because some crop insurance products are based on individual firm losses that are subject to moral hazard problems.

Insurance companies provide protection against the consequences of extreme climatic events and natural disasters, such as hurricanes, tornadoes and earthquakes, while weather derivatives provide protection against more moderate climatic changes, such as changes in temperature (Ali, 2004:75).

Weather derivatives do not replace insurance contracts, due to the following significant differences (Geyser, 2004:445):

- insurance contracts insure the person/company paying the premium against high risk, low probability events, while weather derivatives cover low risk, high probability events,
- weather insurance’s payout is a once-off lump sum that can or cannot be proportional to the degree of the event that took place, thus lacking flexibility. Weather derivatives' payout is designed in such a way that it is proportional to the magnitude of the event that took place,
- insurance will (normally) pay out if there has been proof of damage or loss. Weather derivatives only require that a predetermined index value must be passed,
- the traditional insurance on catastrophic events is relatively expensive whereas weather derivatives in comparison are not so costly and
- unlike normal insurance, it is possible to monitor the performance of the hedge during the life of the contract. Towards the end of the contract, the contract holder might wish to release himself from the derivative, and this is possible, as there will always be a price at which he/she can sell or buy back the contract, seeing that the contract is a traded security.
2.8 Over-the-Counter (OTC) trading and exchange trading

Weather derivative trades can be executed in a number of ways. Most of the time the primary market trades is OTC, which means that the trade takes place between two counterparties (Jewson and Brix, 2008:7). Trading of the secondary market can also be done by voice-brokers, who operate as intermediates and will persuade participants in the market to execute deals. These intermediates do not trade themselves, and the deals made by them are also known as OTC trading. These trades are event-driven (Jewson and Brix, 2008:7).

Another part of the secondary market is traded on exchanges, such as the CME. These exchanges list weather derivatives for different countries and locations, which is based on temperatures for every month of the year. This index driven weather derivatives ensure transparency by providing freely available prices on the internet and also eliminate credit risk since trading is undertaken on an exchange such as the CME, rather than with another counterparty. Margin payments are also made on a daily basis by the exchange (Jewson and Brix, 2008:7). Indexed based weather derivatives are usually futures: these are covered in more detail in Chapter 3.

South Africa still has a long way to go in the development of weather derivatives. Currently weather derivatives are only available over-the-counter in South Africa, and most of these contracts are based on weather events influencing agriculture, thus focusing more on farmers.

2.9 The Chicago Mercantile Exchange (CME)

Most of the following information is the CME's own views, facts, figures and developmental information about weather derivatives. Any additional information from different sources is indicated. This section provides an overview of the history of the CME, explains CME weather derivatives and contracts and provides the advantages that exchange traded weather derivatives have over the OTC traded weather derivatives.

2.9.1 Background of the CME

The CME was founded in 1898 and is currently the world’s largest and most diverse exchange for options and futures trading. The CME Clearing House handles more than 92% of all the futures and options traded in the US. The CME executes almost 800 million contracts per year, worth more than US$460 trillion. US$45 billion of collateral deposits are held daily to support
the transactions: between US$1.5 billion and US$6 billion of funds are moved to and from mar-
ket participants each day.

The products used cover the major market segments including interest rates, equities, foreign ex-
change, commodities and alternative investment products. CME weather derivatives fall under
the alternative investment products category, together with CME economic derivatives and CME
ethanol futures. These non-traditional investment products enable customers to distribute and
thus manage their risks in an improved way.

Before weather derivatives came into play, the only coverage that companies had for weather
risk events were insurance for catastrophic damages, which included all the major weather disas-
ters and events like hurricanes, tornadoes and hailstorms. A gap was identified for the protection
against non-catastrophic problems that businesses experienced, because of the fluctuating
weather conditions. CME weather products filled that gap.

2.9.2 CME weather derivatives

CME weather products are futures and options (explained in detail in Chapter 3) which have a
temperature-based index. These contracts can be monthly or seasonal and are traded in the fol-
lowing cities: US (Atlanta, Baltimore, Boston, Chicago, Cincinnati, Dallas, Des Moines, Detroit,
Houston, Kansas City, Las Vegas, Minneapolis, New York, Philadelphia, Portland, Sacramento,
Salt Lake City and Tuscon), Europe (Amsterdam, Berlin, Essen, London, Madrid, Paris, Rome
and Stockholm) and Asia-Pacific (Osaka, Tokyo) cities. The newest derivative that the CME in-
roduced was CME Frost Days Futures, which are traded exclusively on the CME Globex plat-
form. This derivative was designed to manage the significant risk that is posed by frost, and is
currently traded in Amsterdam (Netherlands).

CME weather derivatives were the first to be traded on the open market in an electronic envi-
ronment as well as on the CME trading floor, which gave the contracts complete price transpar-
ency.

2.9.3 CME weather contracts

CME weather contracts are concluded between two parties and are legally binding. The contract
is settled in cash based on an index value at the end of the contract after being closed at the clear-
inghouse. The indices that are mostly used are heating degree days (HDDs) and cooling degree
days (CDDs). The workings of these indices are explained in Chapter 3. These indices can be monthly or seasonal and are determined by the Earth Satellite (EarthSat) Corporation, an international company in geographic information technologies. EarthSat’s temperature data are provided by the National Climate Data centre (NCDC). European and Asian weather companies calculate the indices for the European and Asian contracts.

2.9.4 The advantages of CME weather derivatives

CME weather derivatives have four advantages, namely market integrity, price transparency, liquidity and accessibility that the OTC market does not offer. The CME clearing house ensures the integrity of each transaction. It uses risk management techniques to ensure that both sides of the contract is able to carry out their obligation.

CME weather futures are traded electronically on the CME Globex platform, with complete price transparency showing the top five bids and offers. All traders have equal access to the best bids and offers. The trading floor is used to trade the weather options. The CME weather market is supplied with continual prices from global market makers. It is an automated trading system, which gives real-time, dealable prices and this allows the CME to provide outstanding market liquidity as well as an active trading venue for a diverse set of people and organisations. In the event of an unanticipated weather occurrence, it is critical that anyone must be able to enter the CME weather derivatives at any time. The CME guarantees easy accessibility for various locations around the world. Trading on the CME Globex is available almost 24 hours a day, which is more than any other exchange in the world. Their customers can access the trading platform from 740 direct connections in 27 countries, and through telecommunication hubs located in eight countries.

2.10 Conclusion

This chapter explained how weather derivatives fit into the derivative network. Credit derivatives were the first kind of derivative that could be "triggered" by a certain event. These events were called credit events. Just as credit derivatives are based on an event, weather derivatives were formed to decrease the risk imposed on companies by unfavourable weather events. Some exchanges similar to the CME also have indexed based weather derivatives, like futures, which are different from the ordinary OTC trading weather derivatives.
A background on credit and weather derivatives were given, and by looking at the CME in the US, the importance to expand the current OTC weather derivative market in South Africa was emphasised.

The next chapter will explain how weather indices work and will look at the different weather derivatives which are available to hedge an unfavourable weather event.
CHAPTER 3

DEFINING WEATHER MEASURES AND DERIVATIVE STRUCTURES

3.1 Introduction

The previous chapter explained the concept of weather derivatives and introduced aspects of weather risk. This chapter explains the form and function of weather derivatives.

There are a wide variety of weather derivatives, which are usually structured as swaps, futures, and options (both calls and puts based on different underlying weather indices) (Alaton et al., 2002:3). It is important for a hedger or speculator to understand how each of these derivatives functions so as to make prudent risk management and investment decisions. Section 3.2 focuses on weather measures – the underlying "asset" used for weather derivatives. Section 3.3 explores different types of weather derivative contracts and provides a case study of a rainfall derivative trade in Malawi.

3.2 Weather measures

The construction of a weather derivative – as with any derivative – requires an underlying asset (Hull, 2006:759). The underlying asset of a weather derivative is some variable that defines an aspect of the weather (e.g. temperature, wind speed, rainfall, snowfall, number of sunlit hours, a heating degree day (HDD), a cooling degree day (CDD), etc., (Garman et al., 2000:1). The weather variable used depends on contract specifications.

The following section introduces and defines weather indices and explores HDDs and CDDs in greater detail.

3.2.1 Weather indices

According to the CME (2008a:3), weather indices can record monthly or seasonal weather parameters determined by the Earth Satellite (EarthSat) Corporation (an international company involved with geographic information technologies). EarthSat’s temperature data are provided by the National Climate Data Centre (NCDC). The indices for the European and Asian contracts are calculated by European and Asian weather companies. In the winter months in the US and European cities, weather contracts are based on HDD values, i.e. days in which energy was expended
for heating. For the summer months CDD values are used, i.e. days in which energy was used for air conditioning. In Europe, CME weather contracts use Cumulative Average Temperature (CAT) as a base (CME, 2008a:3).

Up until the fourth quarter of 2005, seasonal contracts were for five consecutive months and excluded the seasonal swing months of April and October. The CME then introduced monthly and seasonal strip contracts, which provided customers with the flexibility to select from two to six months per contract (CME, 2008a:4). CME rules state that months must be consecutive and in the same season (November to April for winter and May to October for summer) (CME, 2008b:4). If monthly and seasonal strips were to apply in South Africa a summer strip would be from November to April and a winter strip would be from May to October. Customers would have the flexibility to choose how many consecutive months they preferred for the contract.

3.2.2 HDDs and CDDs

Between 98 – 99% of all weather derivatives are based on temperature (Garman et al., 2000:1). A degree-day compares the outdoor temperature to a standard of 65ºF (18ºC). For the rest of the discussion ºF is used, as this is the reference measure of the CME.

For a specific weather station, \( T_{i}^{\text{max}} \) and \( T_{i}^{\text{min}} \) are the maximum and minimum temperatures for a specific day \( i \). The temperature for day \( i \) is defined as the average of the maximum temperature and the minimum temperature, or

\[
T_{i} = \frac{T_{i}^{\text{max}} + T_{i}^{\text{min}}}{2}
\]  

(Alaton et al., 2002:3).

The base temperature is pre-specified, and is usually 65ºF (sometimes 75ºF in warmer climates) (Garman et al., 2000:2). An HDD measures the coldness of the daily temperature if it is compared to 65ºF, and a CDD measures the warmth of the daily temperature if it is compared to 65ºF (Brockett et al., 2005:129). Thus cold days are measured in HDDs and warm days in CDDs. HDDs are thus used to measure the average temperature for the winter half-year, and CDDs are

---

9 Seasonal strip contracts: A customer may wish to buy a strip of CDDs or HDDs contracts spanning over a specific time. Each contract has a specified strike for each month. In the OTC market, it is common for options to be written on a multi-month period with a single strike over the entire period. The benefit of buying a strip of monthly contracts is that the strip can be broken apart more readily than a single, longer-term contract. The simplest approach is to purchase a strip of call or put options over the range of months that are of interest to the user (Considine, 2000:2).
used to measure the average temperature for the summer half-year (Müller and Grandi, 2000:277).

The determination of a HDD and a CDD is as follows:

For a single day:

\[
\text{Daily HDD} = \text{Max} \left( 0, \text{base temperature} - \text{daily average temperature} \right) \quad (3.2)
\]

\[
\text{Daily CDD} = \text{Max} \left( 0, \text{daily average temperature} - \text{base temperature} \right) \quad (3.3)
\]

where the base temperature is set to 65°F (Garman et al., 2000:2).

If the right hand side of Equation 3.2 < 0, the daily HDD = 0. If the right hand side of Equation 3.3 < 0, the daily CDD = 0.

For cumulative days the following equations are used:

\[
\text{HDDs} = \sum_{i=1}^{N} \text{max}(0, 65^\circ F - T_i) \quad (3.4)
\]

\[
\text{CDDs} = \sum_{i=1}^{N} \text{max}(0, T_i - 65^\circ F) \quad (3.5)
\]

where the subscript \( s \) indicates that the measures are cumulative (Zeng, 2000:2077).

Figure 3.1 is a fictional illustration of the measurement of the temperature for 12 consecutive days from a weather station. The dashed line represents the 65°F pre-specified temperature, and the heights of the bars are the actual average temperature measured on each day.
Figure 3.1  The determination of HDDs and CDDs.

The CDD is measured from the dashed line upwards towards the top end of each bar which is above 65°F and the HDD is measured from the dashed line downwards toward the top end of each bar below 65°F. Figure 3.1 shows that the CDD for Day 1 is 20°F (85°F – 65°F). The CDD for Day 2 is 10°F (75°F – 65°F). Day 3 is 15°F and so on. On days 4, 7, 8, 9 and 10 there are no CDDs, but HDDs, as temperatures then are less than the base temperature of 65°F. The HDDs for Days 4 and 7 are then 10°F (65°F – 55°F), for Day 8 the HDDs = 20°F (65°F – 45°F), and so on.

When HDDs and CDDs are used in weather contracts they are usually employed in cumulative form, cumulated over a period of time (e.g. a month or a season). The calculations in Tables 3.1 and 3.2 provide an example for daily HDD and CDD and the cumulative HDD/CDD for a week:

Table 3.1:  Daily and cumulative HDDs measured over one week.

<table>
<thead>
<tr>
<th>CALCULATION OF HDD</th>
<th>DATA</th>
<th>TOTAL HDDs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Day</td>
<td>50</td>
<td>48 55 67 61</td>
</tr>
<tr>
<td>Average daily temperature</td>
<td>15 17 10 0 4 14 16</td>
<td>76</td>
</tr>
</tbody>
</table>

Table 3.2: Daily and cumulative CDD measured over one week.

<table>
<thead>
<tr>
<th>Day</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>TOTAL CDDs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average daily temperature</td>
<td>76</td>
<td>66</td>
<td>64</td>
<td>60</td>
<td>68</td>
<td>70</td>
<td>74</td>
<td>29</td>
</tr>
</tbody>
</table>

Source: Garman et al., 2000:2.

The total (or cumulative) CDDs are the simple sum of the CDD for every day in the sample.

Statistics are also available for indices other than HDDs and CDDs: some contracts use growing degree days, average temperature, maximum or minimum temperature, precipitation (such as rainfall or snowfall), humidity and even sunshine as the underlying asset (Campbell and Diebold, 2003:1).

The next section defines a simple weather derivative contract and elaborates upon the different contracts that are available for hedging purposes.

### 3.3 Weather derivative structures and contracts

Weather derivatives usually take the form of futures, options (calls or puts) and swaps which are based on the different underlying weather indices (Katz, 2008). According to Climetrix (2008), standard derivative structures on which weather derivatives are based are usually puts, calls, swaps, collars, straddles and strangles.

Six common types of products used in weather risk management market include:

- weather calls,
- weather puts,
- weather swaps,
- weather collars,
- weather futures,
- weather binary options.
3.3.1 Standard weather derivative structures

Each contract must comprise certain characteristics:

- the length or period of the contract, with a start and end date,
- a weather measurement station,
- a weather variable, which is measured at a measurement station as specified in the contract, over the contract period,
- an index, which will aggregate the weather variable over the length of the contract,
- a pay-off function, equation or model, which will convert the index into the settlement value and
- a premium that is paid to the seller by the buyer at the start of the contract – only applicable for some kind of contracts (Jewson and Brix, 2008:4).

In addition, the following need to be in place:

- a measurement agency which will measure and evaluate the weather variable,
- a settlement agency, which will have the final say over the values of the index according to their own algorithms,
- a back-up station, in case the main station is out of order and
- a time period over which the settlement will take place (Jewson and Brix, 2008:4).

The list of contracts that are being traded is wide-ranging and is constantly evolving (Garman et al., 2000:3). When pricing a weather contract the most important features of weather derivative structures are (Climetrix, 2008):

- the strike price – the price at which an option is due to be exercised at some time in the future, or when the contract starts to pay out,
- the tick size – the payout amount per unit increase in the index beyond the strike price and
- the limit – the maximum payout of the contract.

The following seven parameters are used continuously throughout the examples which follow (Zeng, 2000:2076):
• contract type,
• contract period,
• an official weather station from which the meteorological data are obtained,
• a definition of the weather index \( W \) underlying the contract,
• the strike price \( S \),
• the tick size \( k \) or constant payment \( P_0 \) for a linear or binary payment scheme and
• the premium paid.

The various contracts can now be examined separately.

### 3.3.2 Weather call and put options

Ray (2004:299) stipulates that all weather options are European in style, meaning contracts can only be exercised after the full length of the contract, i.e. at contract maturity. This is in opposition to American style options that may be exercised at any time during the contract but before maturity.

The holder of a **weather call option** has the right, but not the obligation, to buy the contract at the striking price (Zeng, 2000:2075 and Ray, 2004:299). The buyer and the seller have to agree on a contract period, as well as the weather index \( W \) that will serve as the basis or the underlying asset of the contract. As an example, \( W \) may represent the total rainfall during the length of the contract. At the start of the contract, the buyer pays a premium to the seller and at the end of the contract, if \( W \) is bigger than the pre-negotiated threshold or strike price \( S \), the buyer receives a payout \( P = k(W - S) \) from the seller.

When the payment is structured as binary,\(^{10}\) a fixed amount \( P_0 \) is received if \( W \) is bigger than \( S \), or otherwise no payment is received (Zeng, 2000:2075-2076).

The holder of a **weather put option** has the right, but not the obligation, to sell the contract at the striking price (Ray, 2004:299). A *put* contract works in the same way as the call contract, except that the buyer receives a payout of \( P = k(S - W) \) from the seller when \( W \) is less than the pre-

---

\(^{10}\) See binary or digital options, Section 3.3.
negotiated threshold or strike price \( S \). When the payment is structured as binary, a fixed amount 
\( P_0 \) is received if \( S \) is bigger than \( W \), or otherwise no payment is received (Zeng, 2000:2076).

In the case where temperature is the underlying, a dollar amount is associated with every degree 
day when the payoff is calculated (Garman et al., 2000:3).

**Figure 3.2: Example of a call option.**

Consider a CDDs call option with a strike of 1 000 CDDs which will pay US$3 000 per degree-
day. The cumulative CDDs must be determined over the length of the contract, and the strike of 
a 1 000 CDDs should then be subtracted from this. If this value is \( > 0 \), it should be multiplied by 
the pre-determined US$3 000. Thus the payoff for this option is: (Garman et al., 2000:3)

\[
Call \ Payoff = \$3000 \times Max(0, CDD_{t,T} - 1000)
\]

where

\( CDD_{t,T} = \) the cumulative CDDs over the life of the contract.

In general, Equations 3.6 and 3.7 are used to represent the payoffs of weather put and calls: 
(Garman et al., 2000:3)

\[
Call \ Payoff = p(\$/DD) \times Max(0, W_{t,T} - S)
\]  \hspace{1cm} (3.6)

\[
Put \ Payoff = p(\$/DD) \times Max(0, S - W_{t,T})
\]  \hspace{1cm} (3.7)

where

\( p(\$/DD) = \) the per degree day payoff,

\( W_{t,T} = \) the underlying (for example a HDD/CDD) and

\( S = \) the strike price, in terms of the associated underlying measure.

An investor who has purchased a *call option* will receive the payoff if the recorded HDDs or 
CDDs are greater than the strike price \( S \). An investor who has purchased a *put option* will receive 
the payoff if the HDDs or CDDs are lower than the strike price \( S \) (Garman et al., 2000:3).
Companies and individuals such as farmers (who wish to use put and call options to manage their risks against unfavourable weather circumstances) should understand how to use put and call options to their favour.

Table 3.3 gives a good example of how to use put and call options to mitigate risk, when the underlying is temperature:

Table 3.3: A system for temperature options.

<table>
<thead>
<tr>
<th>OPTION</th>
<th>PROTECTS AGAINST:</th>
<th>EXERCISE WHEN:</th>
<th>PAYOUT:</th>
</tr>
</thead>
<tbody>
<tr>
<td>HDD call</td>
<td>Very cold winters</td>
<td>HDD &gt; strike price</td>
<td>tick size × (HDD – strike price)</td>
</tr>
<tr>
<td>HDD put</td>
<td>Very warm winters</td>
<td>HDD &lt; strike price</td>
<td>tick size × (strike price – HDD)</td>
</tr>
<tr>
<td>CDD call</td>
<td>Very hot summers</td>
<td>CDD &gt; strike price</td>
<td>tick size × (CDD – strike price)</td>
</tr>
<tr>
<td>CDD put</td>
<td>Very cool summers</td>
<td>CDD &lt; strike price</td>
<td>tick size × (strike price – CDD)</td>
</tr>
</tbody>
</table>

Source: Müller and Grandi, 2000:278.

Table 3.3 can be used by energy companies who wish to mitigate against very warm or very cool winters and summers. In the case of a warm winter, the energy company will have to exercise their HDDs put options if the HDDs are smaller than the strike price. The payout is the tick size, as pre-negotiated, multiplied by the difference between the HDDs and the strike price. When a cold winter is expected, the weather call option is exercised if the HDDs are above the strike price, and the payout is the tick size multiplied by the difference between the strike price and the HDDs. In the case of a cool summer, the energy company will exercise their CDDs put options if the CDDs are smaller than the strike price. The payout is the tick size, as pre-negotiated, multiplied by the difference between the strike price and the CDDs. When a hot summer is expected, the weather call option is exercised if the CDDs are above the strike price, and the payout is the tick size multiplied by the difference between the strike price and the CDDs. This way the company is compensated for the loss that would have been incurred if they did not take out the options: For example less energy would have been used for heaters in the warm winters and less energy would have been used for air-conditioning in the cool summers, and the company's revenue would have declined.
Figure 3.3 and Table 3.4 illustrate the purpose and technique used for a weather option hedge to protect a company against the risk of a decline in revenue due to the weather:

**Figure 3.3: Example of a CDD put option for the summer half-year.**

Assume that an energy company wishes to protect itself against a decline in revenue in the event of the declining demand of energy, because of a very cool summer, where little use is made of air-conditioning systems.

**Table 3.4: Specifications of the CDD weather put option.**

<table>
<thead>
<tr>
<th>City</th>
<th>Start</th>
<th>End</th>
<th>Deal</th>
<th>Strike</th>
<th>Tick size (R)</th>
<th>Limit (R)</th>
<th>Price (R)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jhb, Gauteng</td>
<td>1-Nov-07</td>
<td>30-Apr-08</td>
<td>CDD put</td>
<td>985</td>
<td>15 000</td>
<td>3 million</td>
<td>335 000</td>
</tr>
</tbody>
</table>

Figure 3.3 and Table 3.4 demonstrate that the energy company will buy a CDD put option for the summer half-year (a long put). The specifications of this contract are as follows:

- the contract is a summer strip put option, which starts on November 1st, 2007 and ends on April 30th, 2008,
- the strike value is set at 985 CDDs, the tick size is R15 000, the floor is R3m and the price of the put option is R335 000,
- the strike value of 985 CDDs are used as a basis, and corresponds to an average temperature, which is calculated over 180 days, namely 70.47ºF = \([985/180] + 65ºF\),
- the break-even point \(X_{cdd}^{-}\) can be determined by using the option price and the tick size, by following the linear relationship,

\[
\text{Tick Size} \times (\text{Strike Value} - \text{Break-Even Point}) = \text{Option Price}
\]

thus

\[
\text{Tick Size} \times (\text{Strike Value} - \text{Break-Even Point}) - \text{Option Price} = 0
\]

\[
15000 \times (985 - X_{cdd}^{-}) - 335000 = 0
\]

\(^{11}\) The strike price is measured in CDDs, and determines the exercising of the option.
\[ 985 - X_{CDD}^* = \frac{335000}{15000} = 22.33 \]

\[ X_{CDD}^* = 985 - 22.33 = 962.66 \quad \text{and} \]

- by using this break-even point of 962.67°F, a corresponding average temperature can be calculated: \([962.67/180] + 65°F = 70.34°F\).

This value should be interpreted as follows:

For the buyer to obtain a positive payout from this option, the average temperature during the length of the contract should not exceed a maximum of 70.34°F. If the average temperature is greater than 70.34°F, the option should be exercised by the buyer up to an average temperature of 70.47°F. This is the amortisation of the optimal premium. If the average temperature is greater than 70.47°F the option expires and the expenditure is the option price, namely R335 000.

Based on this, the payoff of a long put option contract can be formulated as follows (Müller and Grandi (2000:280)):

\[
\text{Payoff Long Put} = \min(L - OP, k \times \max(0, S - CDD)) - OP
\]

where

\[ CDD = \sum_{i=1}^{180} (T(t) - 65) \quad \forall T(t) > 65, \]

\[ L = \text{limit (floor)}, \]

\[ OP = \text{option price}, \]

\[ k = \text{tick price} \]

\[ S = \text{strike value}. \]

As an example, assume a total of 800 CDDs is measured over the length of the contract. This implies that:

\[ \text{Payoff Long Put} = \min(3000000 - 335000, 15000 \times \max(0, 985 - 800) - 335000) \]

\[ = 15000 \times (985 - 800) - 335000 = R2440000 \]

Figure 3.4 illustrates these results.
3.3.3 **Weather swaps**

A *swap* is an agreement to exchange cash flows in the future according to a pre-arranged formula (Hull, 2005:522). Garcia and Sturzenegger (2001:57) explain a *swap* as a privately negotiated financial contract in which two participating parties agree to exchange specific, pre-determined, price risk exposure over a period of time. It is also defined as a combination of a call sold by party A to B and a put sold by party B to A (Zeng, 2000:2076). The swap requires no premium payment at the beginning of the contract, and at the end of the contract, party A will make a payment of \( P = k(W - S) \) to party B. If \( P \) is negative, the payment is actually made by B to A. In the case of a swap, the strike \( S \) is selected in such a way that the put and the call command the same premium (Zeng, 2000:2076).

The section above represents a simplified explanation of a weather swap: it is important to remember that there are no standardised swap transactions. Most swap transactions comprise an exchange of periodic payments between two parties. One party pays a fixed price and the other a variable price. All the specific terms of the swap agreement such as the weather index, floating price reference, the terms of the contract and the quantity to be hedged are set by the two parties. These can vary just as the needs of the parties vary from one contract to the next. A swap can be
customised by volume, timing, location and seasonality and is settled in cash against a pre-negotiated market price index (Garcia and Sturzenegger, 2001:57).

For most weather swaps, there is only one due date when the cash-flows may be swapped, as opposed to interest rate swaps, which usually have several swap dates. Weather swaps transact with no initial exchange of money and are therefore "zero cost" contracts (Faure, 2006:14-15). This zero-cost feature of swaps is sometimes attractive to speculators, because this allows them to build a risk portfolio and to assume risk positions, without the preliminary expenditure of capital (Dischel and Barrieu, 2002:26). For a weather swap, if the real index value is higher than the swap level, the seller of the swap makes a payment to the buyer of the swap. If the real index value is lower than the swap level, the buyer of the swap makes a payment to the seller of the swap (Faure, 2006:15).

Figure 3.5: Example of a weather swap.

A simple example is an electricity provider whom enters into a CDD swap to ensure that his revenue flow is protected from an abnormally cool summer. If the number of CDDs for the predetermined summer period exceeds the stipulated limit, the electricity provider makes a payment. If the number of CDDs is less than the pre-determined limit, the electricity provider receives a payment, which helps to maintain the average company revenue.

The amount of payments made is determined by the difference between the actual CDDs and the stipulated pre-determined limit (Faure, 2006:15).

A general equation for a swap payoff is as follows: (Garman et al., 2000:4)

\[
Payoff = \min \left[ p(\$/DD) \times \max [0, W_{\text{tr}} - S], h \right] - \min \left[ p(\$/DD) \times \max [0, S - W_{\text{tr}}], h \right]
\]

(3.9)

where

\[ p(\$/DD) = \text{per degree day payoff,} \]

\[ W_{\text{tr}} = \text{underlying (for example HDD/CDD),} \]

\[ S = \text{strike, in terms of the associated underlying measure and} \]

---

\[ ^{12} \text{Speculators: A speculator wishes to take a position in the market, and will bet that a price of an asset will go up or down. This is opposed to hedgers whom wish to avoid exposure to adverse movements in the price of an asset (Hull, 2006:11,756).} \]
\( h = \) maximum payoff denominated in dollars.

**Figure 3.6:** Graphical presentation of a swap with CDDs as the underlying asset.

An investor who has purchased the swap as illustrated in Figure 3.6 receives payment if the CDDs is greater than the strike, or the swap level, and will make a payment if the actual CDDs is lower than the swap level.

### 3.3.4 Collars

Geyser (2004:447) explains that a **collar** is a modified version of a swap. The parties involved have an agreement only to make payments when the underlying \( W \) moves outside a pre-negotiated upper and lower level. A collar, like a swap, is a combination of a call sold by party A to B, and a put sold by party B to A. The difference is that the put and call have different strike levels (Geyser, 2004:447).

Collars protect the buyer from extreme market movements, by constraining price movements within a defined range (Garcia and Sturzenegger, 2001:59). A collar involves financing the purchase of a put (call) with a particular strike and the selling of a call (put) with a different strike (Garman *et al.*, 2000:4). There is usually no cash premium paid for a collar, thus they are also called costless collars (Garcia and Sturzenegger, 2001:59). The payoff is calculated as follows (Garman *et al.*, 2000:4):

\[
Payoff = \min[p(\$/DD) \times \max[0, W_{ij} - S_1], h] - \min[p(\$/DD) \times \max[0, S_2 - W_{ij}], h]\]

where
$p(s/DD) =$ per degree day payoff, \\
$w_{\alpha\gamma} =$ underlying (for example HDD/CDD), \\
$s_{1,2} =$ Strike 1 and Strike 2, in terms of the associated underlying measure and \\
$h =$ maximum payoff denominated in dollars.

**Figure 3.7: Example of a CDD collar.**

Consider a CDD collar with a short put strike, $S_1$ of 900 CDDs and a long call strike, $S_2$ of 1100 CDDs which will pay R2 500 per degree-day. The cap\(^{13}\) value is pre-determined at R1 000 000. The length of the contract is five consecutive months, and the cumulative CDDs are calculated at 1 250. The payoff diagram for this transaction is indicated in Figure 3.8.

The equation for this CDD collar is:

$$\text{Payoff} = \text{Min}[R2500 \times \text{Max}(0, CDD_{\alpha\gamma} - 900), R1000000] - \text{Min}[R2500 \times \text{Max}(0, 1100 - CDD_{\alpha\gamma}), R1000000]$$

where

$CDD_{\alpha\gamma} =$ the cumulative CDDs over the life of the contract.

First the cumulative CDDs must be determined over the length of the contract. In this case it is already given (1 250). The put strike $S_1$ (R900) must be subtracted from the cumulative CDDs, and the maximum of this value and 0 must be determined. In this case the maximum value is 350. This value is then multiplied by the pre-negotiated R2 500. If this value is smaller than the cap value of R1 000 000 it should be used, otherwise the cap will serve its purpose and the value of R1 000 000 is exercised. In this case the smaller value is used.

$$1250 - 900 = 350$$

$$\text{Max}[0,350] = 350$$

$$350 \times R2\ 500 = R875\ 000$$

$$\text{Min}[R875\ 000, R1\ 000\ 000] = R875\ 000$$

The same needs to be undertaken for the call strike $S_2$, except the cumulative CDDs should now be subtracted from $S_2$, and the maximum between zero and this value must be determined. In

---

\(^{13}\) A cap value is the upper limit of which can be paid (Hull, 2005).
this case the maximum value is zero and when multiplied by the pre-negotiated R2 500 it remains zero. If this value is smaller than the cap value of R1 000 000 it should be exercised, otherwise the cap serves its purpose and the value of R1 000 000 is used. In this case 0 is used.

\[
1110 - 1250 = -150
\]

\[
\text{Max}[0, -150] = -150
\]

\[
0 \times R2 \ 500 = 0
\]

\[
\text{Min}[0, R1 \ 000 \ 000] = R0
\]

The last step is to subtract the second value from the second to find the payoff value, thus:

\[
\text{Payoff} = R875 \ 000 - R0 = R875 \ 000.
\]

**Figure 3.8:** Graph of payoff of collar in Figure 3.7.

Any payoff value may be read from the graph in Figure 3.8. In this example, the 1 250 CDDs provide a payout of R875 000. The cap is set at R1 000 000 and if any value moves beyond this level, 0 is assumed in further calculations, as indicated in the example.

### 3.3.5 Digital or binary options

*Binary options* are options which have sporadic payoffs which pay a pre-negotiated amount if a certain index value (e.g. a certain temperature or degree day level) is reached or 0 (Garcia and Sturzenegger, 2001:59). A *cash-or-nothing call* is an example of a binary option and pays out a fixed amount if the underlying value is above the strike value at the end of the contract, and pays out nothing if the underlying price is below the strike price. A *cash-or-nothing put* is similar to
the cash-or-nothing call, and pays out a fixed amount if the underlying value is below the strike value and 0 if the underlying value is above the strike value (Garcia and Sturzenegger, 2001:59-60).

Equations 3.11 and 3.12 are used for digital or binary options (Zeng, 2000:2077):

\[
\text{Payoff}_{\text{put}} = P_0 \quad \text{if} \quad W - S < 0; \quad \text{Payoff}_{\text{put}} = 0 \quad \text{if} \quad W - S \geq 0
\] (3.11)

\[
\text{Payoff}_{\text{call}} = P_0 \quad \text{if} \quad W - S > 0; \quad \text{Payoff}_{\text{call}} = 0 \quad \text{if} \quad W - S \leq 0
\] (3.12)

where

\[P_0\] = the pre-negotiated fixed amount,

\[W\] = the underlying and

\[S\] = the strike value, in terms of the associated underlying measure.

Figure 3.9: Example of a binary option with snowfall as the underlying.

A snow blower retailer promises his customers a substantial refund if the total snowfall for the coming winter is less than a certain threshold.

As a result, the company faces a substantial liability if the snow level falls below the threshold for that winter. Using a binary option, the company can transfer this risk by the purchase of a snowfall binary put option with a strike value that is equal to the threshold in the refund contract (Zeng, 2000:2077).

Assume each customer pays US$30 every two weeks for the snow blower's services. The snow blower retailer promises a US$10 refund per customer for each week, if the snowfall is less than 4 inches in that specific week.

The snow blowing company takes out binary put options for each week (number of options = number of customers) and pays a premium of US$2 for each option, which in return pays out US$12 per option if the snowfall is less than the threshold of 4 inches snow.

The following are fictional data for snowfall in Cleveland, Ohio for a hypothetical December:

<table>
<thead>
<tr>
<th>Week 1</th>
<th>Week 2</th>
<th>Week 3</th>
<th>Week 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>9 inches</td>
<td>10 inches</td>
<td>3 inches</td>
<td>2 inches</td>
</tr>
</tbody>
</table>
In weeks one and two, the company receives the income from customers and put options pay 0. In weeks 3 and 4, the binary options pays out the pre-negotiated amount of US$12 per option and the company may use this to reimburse their customers as promised. US$10 is allocated to each customer and the remaining US$2 is used to compensate for the premium paid in the beginning to take out the options.

Figure 3.10 shows a graph of the example in Figure 3.9. The threshold indicates that the option pays either 0 or the pre-negotiated amount.

**Figure 3.10: Graph of payoff of figure 3.9.**

Source: Author.

### 3.3.6 Weather Future Contracts

A *futures contract* is one which obligates the holder to buy or sell an asset at a predetermined delivery price during a specified future time period (Hull, 2006:749). In the case of *weather future contracts* the underlying is not a physical asset, but a weather index, as opposed to options and swaps which are triggered by certain events. Standardised future contracts written on temperature indices have been traded on the CME since October 2003 (Benth and Šaltytė-Benth, 2005:53) and this trading on futures which are written on temperature indices are offered to several US and European cities. The underlying that is used for weather futures are the number of HDDs or CDDs and in Europe future contracts are written on the cumulative average temperature (CAT) over a season (Benth and Šaltytė-Benth, 2005:75).
Figure 3.11: Example of how to hedge with weather futures.

For this example 18°C is used instead of 65°F as the comparable degree-day temperature to place it in a South African context.

ABC Entertainment Ltd., an amusement park in Gauteng, wishes to hedge the risk of an unusually cool March. March is one of the most profitable months of the year, because it is towards the end of the summer and the park still receives many foreign tourists. The company is, however, aware that should the temperatures be unusually cool for the month, many tourists will rather do something else than go to the park which will severely affect the company's revenues.

Assuming that the daily average temperature in March in Johannesburg is 28°C, the contract is written on 31 days and the tick price is set at R160 ($20 × R8 – estimated exchange rate). The company believes that March is 5°C cooler than the average historical temperature for March. The marketing manager calculates that this will reduce revenues by R500 000 and the financial advisors propose that the company should enter into twenty-five March futures contracts to sell a CDD Index.

The average CDDs will then be: \[ 28°C - 18°C = 10°C \]

The index value is: \[ 10°C × 31 \text{ days} = 310 \]

The index cash value is: \[ 310 × R160 = R49 600 \]

Suppose that March is indeed 5°C cooler than normal. The index value is: \[ 5°C × 31 \text{ days} = 155 \]

The index cash value is: \[ 155 × R160 = R24 800 \]

The company would cancel its contracts in April by buying them back. Since the company bought each contract at R24 800 and sold each contract for R49 600 each, it realised a gain of R24 800 per contract and consequently R620 000 in total for the 25 contracts. This is more than enough to cover the estimated loss of R500 000 by the market manager.

Should March turn out to be unusually warm, ABC Entertainment Ltd realises futures losses, but these losses would be offset by gains in the income generated by the amusement park. Either way, the company has locked in its normal March revenue.

Adapted from: Faure, 2006:15-16
For a summary of the transaction which was explained in Figure 3.11 see Table 3.5.

### Table 3.5: Hedging with weather futures.

<table>
<thead>
<tr>
<th>SPOT MARKET</th>
<th>FUTURES MARKET</th>
<th>RESULT</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>February:</strong> ABC Entertainment Ltd wishes to hedge against a possible cool March</td>
<td><strong>February:</strong> ABC Entertainment Ltd sells March contracts currently priced at R49 600 each</td>
<td>Sell at R49 600&lt;br&gt;Buy at R24 800&lt;br&gt;&lt;strong&gt;Gain:&lt;/strong&gt; 24 800 per contract</td>
</tr>
<tr>
<td><strong>April:</strong> ABC Entertainment suffers a decline in revenue in March due to cool weather</td>
<td><strong>April:</strong> ABC Entertainment Ltd cancels futures position by buying back March contracts, now priced at R24 800 each</td>
<td><strong>Net result:</strong> Futures gains offset spot losses</td>
</tr>
</tbody>
</table>

Considering international markets, the CME settlement price of the underlying weather index, measured in HDDs or CDDs, is equal to the value of the relevant month's or season's HDDs or CDDs multiplied by the tick price of US$20. The tick price was first set to US$100, but later reduced to US$20 (CME, 2008c).

The trading of weather future contracts makes it possible for businesses to protect against losses which are caused by unexpected changes in weather conditions.

#### 3.3.7 World Bank weather derivative

A weather derivative trade finalised on October 1, 2008 was intended to hedge the Malawian government against the financial risk of severe or catastrophic drought (Pengelly, 2008:17). The pilot trade of roughly US$3 million in notional size was expected to be the first in a series intermediated by the institution, aimed at enabling developing countries to hedge against weather risk. The deal was part financed by the UK Department for International Development. The trade was linked to an index based on the average daily rainfall between October and April across 23 individual weather stations in Malawi. The water requirement satisfaction index was weighted with a bias towards regions that were more important for producing maize, the country's staple crop. According to Pengelly (2008:17), Julie Dana, Washington-based technical specialist in the World Bank's agricultural and rural development commodity risk management group explained that this model in question has been used by the government since 1992 to do predictions on maize production, and that this model has been adapted slightly to use as the basis for an index-
based derivative product. Malawi’s rainy season is typically between October and April, with the harvest running until July. Any payout from the weather derivative would have come by April or May. Dana continued to say that the payout from this instrument would provide contingency financing to the government in a new way, because of the timely and predictable payout that would be available in the process. The next step for the World Bank is to investigate the possibility of using any payout from the weather derivatives contract to purchase a call option on maize to lock in future prices.

3.4 Conclusion

This chapter explained the working of weather derivative contracts. The most important concept is the difference between weather derivatives driven by events and weather derivatives driven by indices. Weather derivatives which are triggered by an event serve as OTC market derivatives and weather derivatives like futures have indices as their underlying.

The next chapter focuses on how weather derivatives are priced by examining the pricing models used internationally.
CHAPTER 4

THE PRICING OF WEATHER DERIVATIVES IN FOREIGN AND DOMESTIC MARKETS

4.1 Introduction

In Chapter 2, credit derivatives were discussed as the base from which weather derivatives are derived, followed by weather derivatives in general. In Chapter 3, different weather derivative contracts were discussed. The next step is to derive a price for a specific weather derivative contract. This is the focus of this chapter.

Knowing what to pay for a weather derivative is a significant issue to buyers and sellers of contracts. The price may be linked to either a weather event or to an index. Campbell and Diebold (2003:6) explain that when modelling weather risk, there are two alternative statistical methods which can be used. One is to estimate the distribution of the weather event directly, parametrically or non-parametrically. The other is to develop a daily model of the underlying weather variable from which the relevant weather index may then be derived.

The most common weather derivative pricing models is discussed in this chapter. Most pricing models used in foreign markets are based on a HDD index or a CDD index. These indices – as well as weather derivatives that have rainfall as the underlying – are employed efficiently in the South African market. The next section focuses on different methods and principles when pricing a weather derivative. Section 4.3 will focus on the pricing of event driven derivatives.

4.2 Pricing methods and principles

This section explains four different pricing models used in foreign markets to price weather derivative, namely the Black-Scholes pricing model, the Historical Data or Burn Analysis Model, the Monte Carlo Based Simulations model and Mean Reverting models. Before this can be done, a quick revision of the standard option pricing theory is necessary before investigating its extensions later in this chapter.
4.2.1 Understanding the weather evolution models

Predicting the weather is a hazardous task because multiple variables dictate the characteristics of the weather. By looking at the past, previous information can be obtained about possible behaviour of the weather which can be assumed as regular behaviour, because changes in the weather seem to follow a cyclical pattern with some variability (Garcia and Sturzenegger, 2001:17).

Geometric Brownian Motion (GBM) represents a process by which a variable $X_i$ moves with drift $\mu$ in a period $dt$ with variance $\sigma^2$ around the drift. Such a process is represented by stochastic differential Equation 4.1, (Tindall, 2006:16):

$$dX_i = \mu dt + \sigma dW_i$$

where

$\mu =$ drift,

$\sigma =$ volatility,

$t =$ time and

$W_i =$ representing the GBM process (or a Wiener process).

This may be satisfactory for modelling biological processes, but GBM is the process that is generally used to model financial variables such as share and commodity prices. GBM is used because simple log functions do not permit negative parameter values. Such values are essential when modelling asset or derivative returns (which are, in turn, used to model prices) (Tindall, 2006:16).

GBM will also be used in the discussion regarding weather derivatives. Garcia and Sturzenegger (2001:17) defined the probability space in which the derivative operates as:

$$\text{Probability Space} = (\Omega, F_t, P)$$

where

$\Omega =$ the set of states of nature,

$F_t =$ the filtration of information available at time $t$ and
$P =$ the statistical probability measure.

Within this probability space GBM is then represented by the following stochastic differential equation (Garcia and Sturzenegger, 2001:17):

$$\frac{dT_t}{T_t} = \mu(T_t, t)dt + \sigma(T_t, t)dW_t$$  \hspace{1cm} (4.3)

where

$\mu(T_t, t) =$ the drift represented by a mean-reverting process which captures seasonal cyclical patterns,

$\sigma(T_t, t) =$ volatility parameter,

$t =$ time,

$T_t =$ temperature and

$W_t =$ the GBM (or Wiener) process, where $W_t$ is normally distributed with variance $t$, thus $W_t \sim N(0, t)$ (Swart and Venter, 2006:114).

Special attention should be paid to the volatility parameter (Garcia and Sturzenegger, 2001:18). A time-window frame needs to be established in order to calculate the volatility. To do this it is important that there are no missing data otherwise the volatility parameter is biased. The consistency of historical patterns is also important, because it is repeated in the future making it a reliable parameter.

The next step is to construct a riskless portfolio which should also yield a risk-free rate. This will give the partial differential equation (PDE) for a call option (Garcia and Sturzenegger, 2001:18):

$$-rF + rF_S S_t + F_r + \frac{1}{2} F_{SS} \sigma^2 S_t^2 = 0, \quad 0 \leq S_t, \quad 0 \leq t \leq T$$  \hspace{1cm} (4.4)

with the corresponding boundary condition given by:

$$F(T) = \max[S_t - K, 0]$$  \hspace{1cm} (4.5)

where

$F =$ payoff of the option contract (using the Fundamental Theorem of finance),
S = asset price, 
\( t \) = time, 
\( r \) = interest rate, 
\( \sigma \) = instantaneous standard deviation or volatility, 
T = length of the contract and 
K = strike price.

Garcia and Sturzenegger (2001:18-19) stress that the construction of such a riskless portfolio is possible with cases of share options or interest-sensitive securities when the underlying is traded. However, when working with weather derivatives the same argument can no longer hold as weather is not traded: there is no concrete underlying such as a share or a treasury bill. Weather options are written on cumulative HDDs and CDDs so an exotic option should be considered, namely an Asian-type option. Asian options are options whose payoff depends on the average price of the underlying instrument during a specified period, rather than the price of the underlying asset on the maturity date (Hull, 2005:509).

Assuming that the price driven under the risk-adjusted probability \( Q \) is described by (Eydeland and Geman, 1999:11-12):

\[
dS(t) = rS(t)dt + \sigma S(t)dW(t)
\]

and also assuming that the number of values which are being computed is sufficiently large as to allow the representation of the average \( A(t) \) over the interval \([0,T]\) by the integral:

\[
I(t) = \frac{1}{T} \int_0^t S(u)du
\]

thus the valuation formula of an Asian call option at time \( t \) can be expressed as:

\[
C(t) = E^Q[e^{-r(T-t)} \max(I(t) - k, 0)]
\]

where (Garcia and Sturzenegger, 2001:19; Eydeland and Geman, 1999:12)

\( C(t) \) = value of the Asian call option, 
\( I(t) \) = the spot price as per definition of an Asian Option (the value of the integral),
\[ Q = \text{risk adjusted probability}, \]
\[ S = \text{asset price}, \]
\[ t = \text{time}, \]
\[ r = \text{risk-free rate}, \]
\[ \sigma = \text{instantaneous standard deviation or volatility}, \]
\[ T = \text{length of the contract and} \]
\[ k = \text{strike price}. \]

Let \( V \) denote the value of an Asian option and let \( S \) denote the price of the underlying asset. The PDE shown in Equation 4.9 must be solved with appropriate boundary conditions (Garcia and Sturzenegger, 2001:19):

\[ \frac{\partial V}{\partial t} + \frac{\sigma^2 S^2}{2} \frac{\partial^2 V}{\partial S^2} + rS \frac{\partial V}{\partial S} + S \frac{\partial V}{\partial t} - rV = 0 \]  
\[ (4.9) \]

Tindall (2006:14) argued that the Black-Scholes Pricing Model used the same PDE as Equation 4.9 in the famous hedging method to derive a partial differential equation – which is no longer a stochastic equation – for the dynamics of the option price based upon the GBM of Equation 4.3. The next section explains the Black-Scholes pricing model.

### 4.2.2 The Black-Scholes pricing model

Black and Scholes developed this model in 1973 to price put and call options (Black and Scholes, 1973:637:654), and it remains in wide use today. The following ideal conditions were assumed in the equation’s construction (Black and Scholes, 1973:640):

- the asset price follows a lognormal random walk,
- the risk-free interest rate and the asset volatility are known functions of time over the life of the option,
- there are no transaction costs,
- the underlying asset pays no dividends during the life of the option,
- there are no arbitrage possibilities so that all risk-free portfolios must earn the risk-free rate,
• trading takes place continuously and

• short selling is permitted and the assets are divisible.

Following from Equation 4.9, Jewson and Zervos (2003:1) explained that the Black-Scholes partial differential equation adapted for weather derivatives is:

\[
\frac{\partial V}{\partial t} + \frac{1}{2} \sigma_s^2 S \frac{\partial^2 V}{\partial S^2} + rS \frac{\partial V}{\partial S} - rV = 0
\]

(4.10)

where

\( V \) = payoff of the option contract,

\( S \) = asset price,

\( t \) = time,

\( r \) = interest rate and

\( \sigma_s \) = instantaneous standard deviation or volatility.

Equation 4.10 corresponds to the Black-Scholes equation for weather swaps trading with premium \( S \). The only difference from the actual Black-Scholes equation is the coefficient \( \sigma_s^2 \).

Following the same hedging procedure as used in deriving the Black-Scholes equation, the dynamics of options on a futures contract are given by Equation 4.11:

\[
\frac{\partial V}{\partial t} + \frac{1}{2} \sigma_s^2 S \frac{\partial^2 V}{\partial S^2} - rV = 0
\]

(4.11)

where

\( V \) = payoff of the option contract,

\( S \) = asset price,

\( t \) = time,

\( r \) = interest rate and

\( \sigma \) = instantaneous standard deviation or volatility.
Note that the third term on the left hand side of Equation 4.10 has been removed to arrive at Equation 4.11. This is the same equation that would be used to calculate the price of an option on a dividend-paying share in which the dividend yield was equal to the risk free rate and can thus could be used for weather derivatives.

For ease of understanding and using the Black-Scholes pricing model a standard notation was introduced by Tindall (2006:18). The option price on a futures contract can thus be written as:

\[ V(y,t) = e^{-rt}BS(y,t,r,\sigma) \]  

(4.12)

where

\( V(y,t) \) = option price,

\( BS() \) = standard Black-Scholes Pricing formula,

\( y \) = asset price,

\( t \) = time,

\( r \) = interest rate and

\( \sigma \) = instantaneous standard deviation or volatility.

The Black-Scholes model, however, is based upon assumptions (explained above) which do not apply sensibly to weather derivatives (Garman et al., 2000:6). The Black-Scholes pricing methodology is based on continuous hedging (Nelken, 2000:3). This works well when pricing options on currencies, shares, commodities or other fungible assets that can be traded in the spot market. When considering weather derivatives, the underlying is not traded (Mahal, 2001:329). This makes the Black-Scholes an ineffective pricing methodology.

One of the main assumptions behind the Black-Scholes model is that the underlying asset of the contract always follows a random walk without mean reversion. In the case of weather derivatives based on temperature the underlying is HDDs or CDDs. This implies that the temperature could wander off to any level whatsoever (Garman et al., 2000:6), which is an unrealistic scenario. Figure 4.1 shows different simulated daily temperature values for three consecutive months, assuming no mean reversion. The simulated temperatures differ significantly from the expected temperatures, and the variability also increases with time. It is clear that the random walk with no mean reversion simulated temperatures are entirely unrealistic, since towards the
end of the simulation temperatures reached as high as 70°C and as low as minus 30°C for the same day of the year.

**Figure 4.1:** Random walk processes without mean reversion.

![Graph showing temperature over months](image)

Garman *et al.* (2000:7) also assert that the Black-Scholes formulation is inadequate for weather derivatives for the following reasons:

- weather does not "walk" like a random walk of an asset price which can in principle wander off to infinity or zero, it will not apply. Instead, variables such as temperature tend to remain within relatively narrow bands because of the mean-reverting tendency of the variable, constantly reverting to historical level,

- weather is not "random" like an asset price in its random walk process. Because of weather’s inherent nature, it is approximately predictable in the short run and approximately random (around historical averages) in the long run. This implies that short-dated weather derivatives may behave fundamentally different than their long-dated counterparts,

- weather derivatives usually provide an average underlying over a period of time, whereas Black-Scholes option payoffs are determined by the value of the underlying asset exactly at
the maturity of the contract. Thus, weather derivative payoffs behave as average price options or Asian options, rather than pure Black-Scholes options,

- many weather derivatives are capped or floored in the payoff, whereas the standard Black-Scholes option does not provide for this and

- the underlying variables of precipitation or temperature cannot be free of economy risk aversion factors, whereas the Black-Scholes pricing model are set free of these risks.

For all of these reasons, the use of the Black-Scholes Pricing Model will not be effective in the pricing of weather derivatives. The next section explores the use of the Burn Analysis Method to price a weather derivative.

4.2.3 Simulations based on historical data - The Burn Analysis method

This is the most popular method used among practitioners because of the ease of use of this method (Richards et al., 2004:1008-1009). The Burn Analysis Method is a typical actuarial approach which has been adopted to price a contingency in which no assumptions are necessary regarding the nature of the process on which the contingency relies (Tindall, 2006:18). This approach is commonly used in the insurance industry. It employs a simulation using historical information to estimate uncertain weather related payments (Katz, 2008). The Burn Analysis Method is used, usually by insurance companies, to price products which have not been previously sold or where there are no data available for comparable products. A question is asked: "How much would have been paid out or what would the return from the contract have been if a similar product has been sold or purchased every year for the past x years?" (Mahal, 2000:330, Tindall, 2006:18). Mahal (2000:330) explains that there are six steps in the Burn Analysis Process:

1. the historical weather data must be collected,

2. the collected data must be converted into HDDs and CDDs,

3. the converted data must be checked for errors and corrections must be made accordingly,

4. the amount that the weather derivative would have paid out every year, or on specific occasions or events in the past, must be determined

5. the average of the payouts found in the previous step must be estimated and
6. the average amount found in the previous step must be discounted back to the settlement date of the weather derivative.

The most difficult steps in the Burn Analysis Method are the first and third steps. Collecting historical data can be tricky. While there are internet sites with downloadable historical weather for the US, obtaining historical weather data for South Africa is difficult and expensive. Even when data are available, there are still some missing data, gaps and errors. The historical data must be "scrubbed" before they can be used for pricing (Nelken, 2000:4).

The correction of step three presents its own problems (Nelken, 2000:4-5):

- the period in question can be a leap year, for example, there are more days in the period from November 1st, 2007 to March, 31st, 2008, than there are in the corresponding period the year before. This is because of the extra day on February 28th 2008,
- the weather station may have had to be moved as a result of construction, or may have been moved from the sun to the shade,
- it is difficult to know how many years’ worth of historical data should be used,
- many cities exhibit the "urban island effect ", in which, as a result of heavy industrial activity, construction and pollution, the weather gradually grows warmer over time. In some urban centres it is even possible to detect warming trends in the weather. These trends must be accounted for when pricing the weather option and
- sometimes extreme weather patterns such as El Niño or La Niña\(^{14}\) occur. The pricing of an option in an El Niño year is different from the pricing in a year that it does not occur.

\(^{14}\) El Niño or La Niña-Southern Oscillation, or ENSO, is a quasiperiodic climate pattern that occurs across the tropical Pacific Ocean with on average five year intervals. It is characterized by variations in the temperature of the surface of the tropical eastern Pacific Ocean—warming or cooling known as El Niño and La Niña respectively—and air surface pressure in the tropical western Pacific—the Southern Oscillation. The two variations are coupled: the warm oceanic phase, El Niño, accompanies high air surface pressure in the western Pacific, while the cold phase, La Niña, accompanies low air surface pressure in the western Pacific. ENSO causes extreme weather (such as floods and droughts) in many regions of the world. Developing countries dependent upon agriculture and fishing, particularly those bordering the Pacific Ocean, are the most affected (Anon, 2005: 12-19).
These problems can be resolved sensibly: for example, weather trends can be corrected by observing the average HDDs for the past 10 or 20 years and then comparing these with the corresponding trailing 10 or 20 years average of the HDDs for each of the years in the past (Nelken, 2000:5). After this comparison it is sensible to make a linear shift in the data relating to the year in question. For example, assume that (Nelken, 2000:5):

- for the year 1998, in the period from November 1st, 1998, to March 1st, 1999, the HDDs count was 5 050,
- the average of the HDDs counts for the ten years from 1999 to 2008 was 5 000 and
- the average of the HDDs counts for the 10 years preceding and including 1998 was 5 020.

When these numbers are compared, it is found that the HDDs have dropped from 5 020 to 5 000. This is consistent with the warming of the weather. In this case it is reasonable to shift the data relating to 1998. A linear shift is:

\[
\text{Shifted HDDs} = \text{Observed HDDs} + \text{last average} - \text{previous average}
\]

\[
= 5 050 + (5 000 - 5 020)
\]

\[
= 5 030.
\]

Garman et al. (2000:7) found the Burn Analysis Model very simple to implement, but objected about the historical data that are used, and states that the model would be more useful if temperature forecasts were also incorporated in the model. From a statistical point of view, the Burn Analysis Method does not allow the incorporation of trends or any probabilistic modelling for the future (Geman and Leonardi, 2005:64). From a financial and economic standpoint, this method should thus not be legitimate. This leads to the Monte Carlo simulations model which does incorporate future forecasting in the equation and it differs from the Burn Analysis Method in that certain assumptions must be made regarding the dynamics of the underlying variable (Tindall, 2006:18).

The next section explores the Monte Carlo simulations model as a possibility for the pricing of weather derivatives.
4.2.4 The Monte Carlo simulation model

The Monte Carlo simulation model is a technique which uses random number generation for specific statistical scenarios to determine the expected value of $E[g(X(t))]$ (Alaton et al., 2002:17), using the equation

$$E[g(X(t))] = \frac{1}{N} \sum_{i=1}^{N} f(\overline{X}(t, \psi_i))$$  \hspace{1cm} (4.13)

where

- $g$ = a real-valued function,
- $X$ = the solution for the outcomes of the specific scenario,
- $\overline{X}$ = an approximation of $X$ (used if the exact solution of $X$ is not available),
- $t$ = time and
- $\psi_i$ = series of calculation points.

The advantage of the Monte Carlo simulation model is the simplicity of the implementation of this technique, as well as the knowledge of the approximation error (room for error is already accounted for) (Geman and Leonardi, 2005:67). The method generates random numbers for a specific event which has been statistically constructed for a certain scenario (Tindall, 2006:18). The model involves running a series of simulations which are based on a statistically derived model and then calculating the expected return from these simulations. In the case of weather derivatives, Garman et al., (2000:8) explain that the Monte Carlo simulation is a computer-based method which generates random numbers for each scenario and can be used to statistically construct weather scenarios. This simulation provides a flexible way to price different weather derivatives structures. The various types of averaging periods such as those based on CDDs and HDDs can easily be specified by this method.

Zanders Partners (2009:2) provide a detailed explanation of how to use the Monte Carlo Simulation to forecast future series.

First, a model based on historical series must be created, thereafter Monte Carlo simulations should be generated. To value a weather derivative, it is essential to estimate the value of the underlying. Since the sum of HDDs and CDDs depends directly upon the temperature, the expected
payoff can be estimated using a forecast of the mean daily temperatures in the contract period. The average temperature can be estimated using the seasonal temperature pattern, the trend of the temperature and the simulated temperatures as variables in the model.

After creating the model based on historical series, it is possible to forecast a future series – a series of average temperatures for each day for a certain period of time – of the underlying variable (temperature) using a Monte Carlo simulation (Zanders Partners, 2009:3).

The following expectation value must be determined:

\[ \hat{E}(g(y)) = g(y)f(y)dy = \bar{g} \]  \hspace{1cm} (4.14)

where

\( g(y) = \) random function and

\( f(y) = \) probability density.

A number \( n \) of independent sample values should be drawn from the probability density to generate an estimate for \( g \). For example, if \( n=10 000 \) then 10 000 scenarios are generated for the temperature series in a specified future period.

The Law of large numbers allows \( \bar{g} \) to be estimated by (Zanders Partners, 2009:3):

\[ g = \frac{1}{n} \sum_{i=1}^{n} g(y_i) \]  \hspace{1cm} (4.15)

This is the expected payoff for the weather derivative in question.

Tindall (2006:18) provides the same formula for the Monte Carlo Simulation as Alaton et al. (2002:17) and explains that this formula gives the arithmetic average of all the simulation outcomes:

\[ E[f(X_i)] = \frac{1}{N} \sum_{i=1}^{N} f(X(t,\psi_i)) \]  \hspace{1cm} (4.16)

Where

\( X = \) all the simulation outcomes,

\( t = \) time and

\( \psi_i = \) series of calculation points.
The simulation of the temperature trajectories can occur in one of two ways (Alaton et al., 2002:17): The simulation may commence today using today’s observed temperature as the initial value, or the simulation may be started at a future date close to the first day of the period of interest, with the expected mean temperature for that day as the initial value.

If the contract is already operational, simulations should begin on the current date. If the contract period is far enough ahead in time it is not necessary to begin simulations on today’s date. This is because the temperature in the near future will not affect the temperature much during the contract period. After some time in the simulation process, the temperature will no longer be dependent on the initial value anymore and the variance will have reached its equilibrium or stable value.

For Monte Carlo based simulations, it is important to select the right random process for temperature (Garman et al., 2000:8). As explained in the Black-Scholes model, temperature is mean reverting (temperature data indicate that the temperature tends to revert to normal, or long run average values, within 2 to 3 days). Mean reversion has important consequences for the selection of a suitable start point for the Monte Carlo simulation process (Tindall, 2006:18-19). If the interest is in the short-term then the simulations will have to start from the present day values because any current deviation from the mean will have an impact on the process in the short-term.

The next section explores mean reverting models.

4.2.5 Mean reverting models

The key difference to modelling derivatives with weather as the underlying when it is compared to general financial variables like share prices, is that most weather components show some degree of mean reversion. A mean reverting process is one in which the drift component of the stochastic differential Equation 4.1 always acts in a direction that opposes the current displacement from the mean process, in much the same way as a spring will act on a weight (Tindall, 2006:15). A model with mean reverting properties can be converted to a Mean Reverting Model especially designed to price weather derivatives (Nelken, 2000:7).

Mean reverting models have been used for a long time to model interest rates. As with interest rates, weather also has the characteristic that it is unlikely that the rainfall or temperature is ten times higher next year than it is this year. The most important difference between interest rate
derivative models and weather derivative models is the calibration process. Interest rate derivative models are calibrated to the market prices of liquid instruments whereas weather derivative models are calibrated to past or historical data (Nelken, 2000:9).

Nelken (2000:8) considers the simple Gaussian Model to illustrate the working of the Mean Reverting Model. The differential equation describing the model is given by:

$$dr = a \times (b - r)dt + vdz \tag{4.17}$$

where

$r = $ the continuously compounded instantaneous interest rate,

$dr = $ the instantaneous change in $r$,

$dt = $ an infinitesimally small unit of time,

$b = $ the mean interest rate,

$(b - r) = $ the "pull to the mean",

$a = $ the speed of mean reversion,

$v = $ the volatility and

$dz = $ a Weiner process based on a normal distribution with a mean of zero and a standard deviation of 1.

An explanation of this equation is based on interest rate derivatives. Later in this section it is applied to weather derivatives.

In this model, the assumption is made that interest rates will converge to some long-term mean $b$. The instantaneous interest rate $r$, changes by an amount equal to $dr$. If $r > b$, then the contribution $(b - r)$ is negative. This will tend to pull interest rates to a lower level. In the same way, if $r < b$, the term $(b - r)$ is positive, which will tend to pull the interest rate higher.

The term $(b - r)$ is the "pull to the mean ", or the strength to which the interest rate is pulled to the mean. This is then multiplied by $a$, the speed of the mean reversion, which is one addition in the sum representing $dr$. In addition, there is a random component to the short-term interest rate, represented by the product of the volatility with the Weiner process $v dz$, which can be negative or positive.
Before deriving at Nelken's mean reverting model for weather derivatives, two more models which employ mean reverting properties are discussed.

The average daily temperature varies by season, but tends to revert to a long-run average which is most likely moving slowly upward with the accumulation of carbon dioxide in the atmosphere (Richards et al., 2004:1008). It has been found that day to day temperatures are not entirely random as weather tends to be more like the previous day, or the next day, or "warm spells" and "cold snaps". Temperature also tends to be more volatile in the winter than in the summer. These factors contributed to the mean reverting GBM model with lognormal jumps and time-varying volatility developed by Richards et al. (2004:1008):

\[
d\omega_t = \kappa(\alpha_\omega(\omega_t, t) - \lambda \phi - \omega_t)dt + h_\omega(t)dz + \phi dq
\]

where

\( \omega \) = average daily temperature,

\( \alpha_\omega \) = the instantaneous mean of the process,

\( \kappa \) = the rate of mean reversion,

\( h_\omega \) = standard deviation,

\( dz \) = Wiener process,

\( dq \) = Poisson process: Discreet jumps occur according to \( q \)

\[
dq = \begin{cases} 
0 & \text{with probability } 1 - \lambda dt \\
1 & \text{with probability } \lambda dt
\end{cases}
\]

\( \lambda \) = the arrival rate of the Poisson process and

\( \phi \) = random percentage shock with a lognormal distribution.

To accommodate seasonal fluctuations in the level and the volatility of temperature the mean and the variance of Equation 4.18 are specified as functions of temperature and time by Richards et al., (2004:1008).

Merton (1976:125-144) extended the Black-Scholes Model to allow for discontinuous share returns. Merton adjusted the GBM process to allow for discontinuous jumps caused by new infor-
mation. In the case of weather derivatives this new information can be events such as hurricanes, or other rare events. The jump-process emerges as a well-argued and logical model for the process of temperature. Stewart (2002:11) applies Merton’s jump-diffusion process and models it as follows:

\[
\frac{dS}{S} = (\mu - jk)dt + \sigma dz + dq
\]

(4.19)

where

\( S(t) \) = share price process,

\( \sigma \) = instantaneous standard deviation, conditional on no rare weather events,

\( \mu \) = instantaneous expected change in the temperature,

\( z \) = Wiener process,

\( q \) = independent Poisson process,

\( j \) = the mean number of arrivals per unit time and

\( k \) = average jump size measurement as a proportional increase in the underlying.

Vasicek (1977:177-188) used an Ornstein-Uhlenbeck or mean reverting process for interest rates to price a zero bond. This is the same model as the simple Gaussian model explained by Nelken (2000:8) earlier in this section.

Consider the two models explained by Richards et al., and Stewart, and recall that the weather derivative model is based on those models which incorporate interest rates, with a few exceptions namely:

- the weather changes with the season, which signify that it is allowed that the mean of the weather can vary,
- the parameter representing the mean \( b \), is replaced with \( b(i) \), which represents the mean for day number \( i \),
- the mean reversion rate is also allowed to vary. The parameter \( a \), which represents the mean reversion rate, changes over time. The mean reversion rate for day \( i \) is represented by \( a(i) \) and
weather, by its very nature, exhibits 'seasonality'. Assume that it is now spring and that the 
temperature today is exactly equal to its long-term mean. It can be expected that the tem-
perature tomorrow is slightly warmer than it is today. This shows that there is a natural 
‘drift’ in the weather.

The differential equation describing the weather model is thus given by (Nelken, 2000:8):

\[ dr = a(i) \times (b(i) - r) dt + v dz \]  

(4.20)

where

\( r \) = the average daily temperature,

\( dr \) = the instantaneous change in \( r \),

\( dt \) = an infinitesimally small unit of time,

\( b(i) \) = the mean of the temperature of day \( i \),

\( b-r \) = the "pull to the mean",

\( a(i) \) = the speed of mean reversion for day \( i \),

\( v \) = the volatility and

\( dz \) = a Weiner process based on a normal distribution with a mean of zero and a standard devia-
tion of 1.

Interest rate derivative models are calibrated to the market prices of liquid instruments and 
weather derivative models are calibrated to past data. In South Africa an active and liquid market 
for weather derivatives does not yet exist. This is where historical weather data come into use. In 
the calibration process the set of model parameters that are taken into account is those that have 
the highest probability of generating the past weather patterns. To achieve this result the maxi-
mum likelihood has to be estimated. This can be done when the observed data are the result of a 
stochastic process, thus where the parameters for which the probability of having generated the 
observed data are maximal.

To explain the maximum likelihood technique, consider flipping a coin. Assume the coin is 
flipped a thousand times and the result from that are 900 heads and 100 tails. If this is a fair coin, 
this is a highly unlikely outcome as the coin should have a 50/50 chance of landing on heads or
tails, thus with an outcome of ±500 for each side. Either way, the possibility does exist that the coin is not even and has a much higher chance of showing heads than tails. The maximum likelihood technique would choose the second explanation, because it is more likely that the coin was unfair (Nelken, 2000:12).

The principle of maximum likelihood estimation provides a means of choosing an asymptotically efficient estimator for a parameter or a set of parameters (Dunis and Karalis, 2003:7). Consider a random sample of \( n \) observations from a normal distribution. The probability function for each observation is

\[
f(x_i) = (2\pi)^{-1/2} \times (\sigma^2)^{-1/2} \times e^{-\frac{1}{2} \left( \frac{(x_i - \mu)^2}{\sigma^2} \right)}
\]  

(4.21)

where

\( \mu = \) mean and

\( \sigma^2 = \) variance.

Since the observations are independent, their joint density can be described as:

\[
f(x_1, x_2, ..., x_n \mid \mu, \sigma^2) = \prod_{i=1}^{n} f(x_i) = (2\pi)^{-n/2} \times (\sigma^2)^{-n/2} \times e^{-\frac{1}{2} \sum_{i=1}^{n} \left( \frac{(x_i - \mu)^2}{\sigma^2} \right)}
\]  

(4.22)

Equation 4.22 provides the possibility of observing a specific observation in a given sample. The focus is now shifted to the values of the mean and variance which makes the sample most probable, thus the maximum likelihood estimates (MLE) of the mean and the variance. The log function is monotonically increasing and is also easier to work with, thus the MLE is calculated by maximising the natural logarithm of the likelihood function:

\[
\ln L(\mu, \sigma^2 \mid x_1, x_2, ..., x_n) = -\frac{n}{2} \times \ln(2\pi) - \frac{n}{2} \times \ln(\sigma^2) - \frac{1}{2} \sum_{i=1}^{n} \left( \frac{(x_i - \mu)^2}{\sigma^2} \right)
\]  

(4.23)

This translates into finding solutions for the following first order derivatives:

\[
\frac{\partial \ln L}{\partial \mu} = \frac{1}{\sigma^2} \times \sum_{i=1}^{n} (x_i - \mu) = 0
\]

and
\[
\frac{\partial \ln L}{\partial \sigma^2} = -\frac{n}{2\sigma^2} + \frac{1}{2\sigma^2} \times \sum_{i=1}^{n} (x_i - \mu)^2 = 0
\]

(4.24)

Thus, the maximum likelihood estimates are:

\[
\hat{\mu}_{ML} = \frac{1}{n} \sum_{i=1}^{n} x_i = \bar{x}_n \\
\hat{\sigma}^2_{ML} = \frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x}_n)^2
\]

and

(4.25)

The Mean Reverting weather derivative model thus works as follows (Nelken, 2000:12):

- it calibrates the model to the observed past data using a maximum likelihood technique as explained above,
- once the model parameters are determined, weather sequences are generated using a Monte Carlo process,
- the random sequences drive a mean reverting model, similar to models used to price interest rate derivatives,
- many of these sequences are simulated, with each representing a possible future weather pattern,
- for each weather sequence, the payout of the option is determined,
- the average payout of the weather option under the different scenarios is deemed to be the expected payout of the option and
- the fair value price is then obtained by taking the present value of the expected payout.

The models used in foreign markets are mostly based on temperature, which is a continuous variable. The main problem of the modelling of weather derivatives in South Africa is to find a model that is based on the occurrence of a specific event. In South Africa most weather derivatives will have rainfall or temperature as the underlying, as farmers will want to hedge themselves against drought or against floods, as well as against too high or too low temperatures, depending on the crop.

This section revised the standard option pricing model and explained four different pricing models for temperature based weather derivatives namely the Black-Scholes pricing model, the Historical Data or Burn Analysis model, the Monte Carlo Based Simulations model and Mean Re-
verting models. Except for the Black-Scholes Pricing models, all of these models are effective when pricing a weather derivative.

The next section discusses an appropriate weather model to price an event driven weather derivative.

4.3 Pricing event driven derivatives

A credit default swap (CDS) was explained in Chapter 2 as an event driven derivative. This derivative is the simplest and most popular form of a credit derivative (Ranciere, 2002:3) and is "triggered" by a credit event (Lehman Brothers International, 2001:61). This section derives the pricing equation for (and then values) a credit default swap, which will then be modified to price a weather derivative based on a specific event.

4.3.1 Pricing a Credit Default Swap

The assumptions made when pricing the credit default swap below are (Hull and White, 2000:13):

- the notional principle is set at US$1,
- the default events, interest rates and recovery rates are mutually independent and
- the claim in the event of a default is the face value plus accrued interest.

Hull and White (2000:13) define the following before deriving the formula:

\[ T = \text{life of a credit default swap in years}, \]
\[ p_t = \text{risk-neutral default probability at time } t, \]
\[ R = \text{expected recovery rate on the reference obligation in a risk-neutral world, independent of the time of default}, \]
\[ u(t) = \text{present value of payments at the rate of US$1 per year on the payment dates between time zero and time } t, \]
\[ e(t) = \text{present value of a payment at time } t \text{ equal to } t - t^* \text{ where } t^* \text{ is the payment date immediately preceding time is the payment date immediately preceding time } t, \]

\[ ^{15} \text{See Chapter 2 for details on default events.} \]
\[ v(t) = \text{present value of US}\$1 \text{ received at time } t, \]

\[ w = \text{total payment per year made by the CDS buyer,} \]

\[ s = \text{value of } w \text{ that causes the CDS to have a zero value,} \]

\[ \pi = \text{the risk-neutral probability of no credit event during the life of the swap and} \]

\[ A(t) = \text{accrued interest on the reference obligation at time } t \text{ as a percent of the face value.} \]

The value of \( \pi \) is given as one minus the probability that a credit event will occur by time \( T \). This may be calculated from the risk-neutral default probability:

\[ \pi = 1 - \sum_{i=1}^{n} p_i \quad (4.26) \]

The payments last until a credit event occurs, or until time \( T \), whichever comes first. Therefore the present value of the payments is:

\[ w \sum_{i=1}^{n} [u(t_i) + e(t_i)]p_i + w\pi u(T) \quad (4.27) \]

If a credit event is to occur, the risk-neutral expected value of the reference obligation as a percent of its face value is:

\[ [1 + A(t_i)] \hat{R} \quad (4.28) \]

Following form the risk-neutral expected payoff from the CDS:

\[ 1 - [1 + A(t_i)] \hat{R} = 1 - \hat{R} - A(t_i) \hat{R} \quad (4.29) \]

And the present value of the expected payoff from the CDS will then be:

\[ \sum_{i=1}^{n} [1 - \hat{R} - A(t_i) \hat{R}]p_i v(t_i) \quad (4.30) \]

The value of the CDS to the buyer is the present value of the expected payoff minus the present value of the payments made by the buyer:

\[ \sum_{i=1}^{n} [1 - \hat{R} - A(t_i) \hat{R}]p_i v(t_i) - w \sum_{i=1}^{n} [u(t_i) + e(t_i)]p_i + w\pi u(T) \quad (4.31) \]

The CDS spread is that value of \( w \) which makes Equation 4.31 = 0, i.e.:
Equation 4.32 prices a CDS, which has the same characteristics as a weather derivative, because it is event driven. Unfortunately this equation cannot be used in this form for pricing a weather derivative: some changes must be made.

A typical CDS contract usually specifies two potential cash flow streams – a fixed leg (or the buyer of the protection) and the contingent leg (or the seller of the protection). The buyer of the protection will make a series of fixed, periodic payments until maturity or until the reference credit defaults (or until a specific event occurs), and the seller of the protection will make a payment if the reference credit defaults (or if the event occurs). The amount of a contingent payment is usually the notional amount multiplied by \((1 - R)\), where \(R\) is the recovery rate, as a percentage of the notional. Thus, the value of a CDS contract to the protection buyer at any given point in time is the difference between the present value of the contingent leg, which the protection buyer expects to receive, and that of the fixed leg, which he expects to pay (Nomura, 2004:7):

\[
\text{Value of CDS (to the protection buyer)} = \text{PV[contingent leg]} - \text{PV[fixed premium leg]}
\]  

(4.33)

In order to calculate these values one needs information about the default probability of the reference credit, the recovery rate in the case of default and the risk-free discount factors, such as the yield curve.

For the fixed leg a periodic payment must be calculated as the annual CDS premium \(S\), multiplied by the accrual days \(d_i\) (expressed as a fraction of a year) between payment dates. This payment will only be made if the reference credit has not defaulted by the payment date. Thus, the survival probability \(q\) (the probability that the reference credit has not defaulted on the payment date) must be taken into account at time \(t\). The payment must then be discounted back to get the present value. Let the discount factor be \(D(t_i)\). Summing up present values for these payments gives Equation 4.34 (Nomura, 2004:7)

\[
\sum_{i=1}^{N} D(t_i)q(t_i)Sd_i
\]  

(4.34)

The other part of the fixed leg is the accrued premium paid to date of default when the default event happens between periodic payment dates. The accrued payment may be approximated by
assuming that default, if it occurs, occurs at the middle of the interval between consecutive payment dates. Thus, when the reference entity defaults between payment date \( t_{i-1} \) and \( t_i \), the accrued payment amount is \( Sd_i/2 \) (Nomura, 2004:7). This accrued payment has to be adjusted by the probability that the default occurs in this time interval. In other words, the reference credit survived through payment date \( t_{i-1} \) but not through the next payment date \( t_i \). This probability is given by (Nomura, 2004:8)

\[
\{q(t_{i-1}) - q(t_i)\} \quad (4.35)
\]

For a particular interval, the expected accrued premium payment is

\[
\{q(t_{i-1}) - q(t_i)\}Sd_i/2 \quad (4.36)
\]

and the present value of all the expected accrued payments will then be given by

\[
\sum_{i=1}^{N} D(t_i)\{q(t_{i-1}) - q(t_i)\}Sd_i \frac{d_i}{2} \quad (4.37)
\]

The sum of Equations 4.34 and 4.37 gives the present value of the fixed leg (Nomura, 2004:8):

\[
PV[Fixed \ Leg] = \sum_{i=1}^{N} D(t_i)q(t_i)Sd_i + \sum_{i=1}^{N} D(t_i)\{q(t_{i-1}) - q(t_i)\}Sd_i \frac{d_i}{2} \quad (4.38)
\]

Next, the present value of the contingent leg must be computed. Assuming that the reference entity defaults between payment date \( t_{i-1} \) and payment date \( t_i \), the protection buyer receives the contingent payment of \((1 - R)\) where \( R \) is the recovery rate. This payment is made only if the reference credit defaults, and therefore has to be adjusted by \(\{q(t_{i-1}) - q(t_i)\}\), the probability that the default actually occurs in this time period. Discounting each expected payment to its present value and summing over the term of the contract gives (Nomura, 2004:8):

\[
PV[Contingent \ Leg] = (1 - R)\sum_{i=1}^{N} D(t_i)\{q(t_{i-1}) - q(t_i)\} \quad (4.39)
\]

Combining Equations 4.38 and 4.39 provides the equation for valuing a CDS transaction.

When two parties enter a CDS trade, the CDS spread is set so that the value of the swap transaction is 0. Thus the value of the fixed leg = the value of the contingent leg. Equation 4.40 thus holds (Nomura, 2004:8):
Rearranging gives, $S$, the annual premium (Nomura, 2004:8):

\[
S = \frac{(1-R)\sum_{i=1}^{N} D(t_i)(q(t_{i-1})-q(t_i))}{\sum_{i=1}^{N} D(t_i)q(t_i)d_i + \sum_{i=1}^{N} D(t_i)(q(t_{i-1})-q(t_i)) \frac{d_i}{2}}
\] (4.41)

To apply this formula to the pricing of weather derivatives, the denominator and the numerator in Equation 4.41 is explained separately and the numerator expanded to accommodate a weather derivative.

The denominator remains the same as in Equation 4.41, because it involves annual premium payments (which remain, no matter what the reference entity) and probabilities of default. These default probabilities are calculated using historical weather data.

The numerator is slightly different than stated in Equation 4.41. This is because a single recovery rate is used in the pricing of a CDS (as performed in Equation 4.41). However, when working with weather derivatives, there are many probabilities of various disasters happening and various recovery rates should be taken into account when those events occur. The $(1 - R)$ term is outside the summation in Equation 4.41 because it is a single value. For weather derivatives a matrix of recovery rates is required and the numerator in Equation 4.41 changes to:

\[
\sum_{m=1}^{g} \sum_{i=1}^{N} (1-R)(q(t_{i-1,m})-q(t_{i,m}))D(t_i)
\] (4.42)

where $g$ is the maximum summation variable indicating the number of possible disasters or events – or the number of cells in the matrix of recovery rates.

The new numerator sums over the recovery rate as a variable as well as over time. The recovery rate variables are each associated with a default/survival probability.\textsuperscript{16} Using the new numerator in Equation 4.41, the formula for the pricing of a weather derivative is:

\textsuperscript{16} See Chapter 5 for the explanation and use of default and survival probabilities.
\[ S = \frac{\sum_{m=1}^{N} \sum_{i=1}^{L} (1-R)(q(t_{i-1,m}) - q(t_{i,m}))D(t_i)}{\sum_{i=1}^{N} D(t_i)q(t_i)d_i + \sum_{i=1}^{N} D(t_i)(q(t_{i-1}) - q(t_i)) \frac{d_i}{2}} \]  

(4.43)

4.3.2 Summary

Using the technical approach of CDS spread pricing, the price of a weather derivative may be determined. The numerator of the CDS pricing formula was altered to fit the demand of various recovery rates when working with a weather derivative and the summation was changed to sum over a variable recovery rate as well as over time. The different recovery rates are explained in Chapter 5.

4.4 Conclusion

The most common weather derivative pricing models were discussed in this chapter. Section 4.2 explained four different pricing models for weather derivative pricing, namely the Black-Scholes pricing model, the Historical Data or Burn Analysis model, the Monte Carlo simulations model and Mean Reverting models. Excepting the Black-Scholes pricing model, all of these were deemed fit for the pricing of a weather derivative.

Section 4.3 focused on the pricing of an event driven derivative, and a CDS was priced (Equation 4.32). This equation was then modified to price an event driven weather derivative in Equation 4.43. This equation is used in Chapter 5 to explain the application of weather derivatives in the South African market.
CHAPTER 5

EMPIRICAL STUDY: PRICING A WEATHER DERIVATIVE FOR A WESTERN CAPE WINERY

5.1 Introduction

In Chapter 4 the pricing of a weather derivative was explained and an equation derived (Equation 4.43). Any farmer may make use of a weather derivative to hedge against bad weather, be it for sunflowers, fruit or vegetables. If a crop is involved and weather can influence the quality of the crop, a weather derivative can be used to hedge against that risk.

The empirical study in this chapter was based on a wine farm in Wellington in the Western Cape. This farm grows their own grapes which are then used to produce wines. Many different wines produced on the farm originate from different grape cultivars. For this study, the Chardonnay cultivar was chosen to be hedged against unfavourable weather events.

Before pricing can be carried out, a number of factors must be considered. The region must be specified, as the historical weather data used in the calculations are region-specific. Recovery rates must be available for every unfavourable weather event that may occur for the crop in question. It is also important to acknowledge the different stages of growing grapes and the different optimal weather requirements that accompany each stage. Every stage is also characterised by different recovery rates and default probabilities.

Chapter 5 explains how the data were gathered and the way they were applied in the empirical study. The different stages of the Chardonnay cultivar’s life cycle were explored as well as which weather requirements should be met at each stage of the cycle.

A description of how weather prediction or forecasting was undertaken is provided using historical weather data. Recovery rates are used when a weather derivative is priced and the specific recovery rates for the Chardonnay cultivar are explained. A premium for a weather derivative is derived and this is compared to the normal grape crop insurance used by farmers.
5.2 Data requirements

Normally a weather derivative has only one underlying, e.g. temperature, precipitation, wind or snow. In pricing examples of foreign markets only temperature is used and in rare cases rainfall (Chapter 4). In this empirical study there are two underlying factors: temperature and rainfall. Combining the two provides different recovery rates for each scenario (Table 5.1) and therefore a matrix must be constructed to indicate which scenario is required when pricing is undertaken.

Table 5.1 shows the criteria matrix for specific rainfall and specific temperature events. The horizontal labels indicate possible rainfall intervals, and the vertical labels indicate possible temperature intervals. Together, in matrix format, address (2, 3) in Table 5.1, for example, refers to an event in which rainfall is between 5mm and less than 7.5mm and the temperature range between 10°C and less than 15°C. This matrix shows all possible combination for temperature and rainfall, forming the crux of pricing calculations. Recovery rates – which are also required for pricing – are also required in matrix format.

The South African Weather Service provided historic data for daily temperatures in Wellington. The Department of Water and Forestry provided historical data for daily rainfall in Wellington. The time period of data used in this study is from March 1963 to August 2009. These data were used to determine the probabilities of the combined weather events to happen again as they appear in Table 5.1. The intervals for the rainfall and temperature were chosen by Bothma (2010) when recovery rates were allocated.

Table 5.1: Criteria matrix for specific rainfall and specific temperature events.

| Temperature | Rainfall
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>[5°C; 10°C)</td>
<td>[0mm; 2.5mm)</td>
</tr>
<tr>
<td>[10°C; 15°C)</td>
<td>2.1</td>
</tr>
<tr>
<td>[15°C; 20°C)</td>
<td>3.1</td>
</tr>
<tr>
<td>[20°C; 25°C)</td>
<td>4.1</td>
</tr>
<tr>
<td>[25°C; 30°C)</td>
<td>5.1</td>
</tr>
<tr>
<td>[30°C; 35°C)</td>
<td>6.1</td>
</tr>
<tr>
<td>[35°C; 40°C)</td>
<td>7.1</td>
</tr>
<tr>
<td>&gt;40°C</td>
<td>8.1</td>
</tr>
</tbody>
</table>

Source: Author.
5.3 The six critical stages in the life cycle of the Chardonnay cultivar

To price the weather derivative correctly, the critical growing stages of the Chardonnay cultivar must be taken into account. There are certain time periods when the temperature or rain or both must fall in specific intervals for optimal growth. There are six stages that may be classified as critical when growing Chardonnay cultivars namely pruning, bud burst, shoot cutting, flowering and fruit set, veraison and harvesting (Nel, 2009). An explanation of each stage follows.

5.3.1 Explanation of the six stages

5.3.1.1 Pruning

Pruning is a critical stage which involves cutting back the grapevines, usually during the dormant season (when no growth takes place). Pruning helps to properly maintain the vines to produce a good crop of high-quality grapes. It strengthens the vines, improves grape quality and makes the grapes easier to harvest. In the pruning stage the temperature and rainfall do not affect the harvest much (Eichhorn and Lorenz, 1984:295). The pruning stage for Chardonnay takes place from August 1st to August 10th.

5.3.1.2 Bud burst / bud break

The grape begins its annual growth cycle in the spring with bud break. In the northern hemisphere this stage is around March, while in the southern hemisphere it begins around September. If the vine has been pruned at the end of the winter, the start of this cycle is signalled by the bleeding of the vine, in which water is expelled from the cuts remaining after pruning. After this, buds swell on the vine and eventually shoots grow from the buds (Eichhorn and Lorenz, 1984:296). The bud bursting for the Chardonnay cultivar in Wellington takes place from September 10th to September 20th. It is important that the average daily temperature stays between 25ºC and 28ºC and that there is no (or almost no) rainfall between these dates. Undesired temperature and rainfall causes the buds to break immaturely.

5.3.1.3 Cutting of the shoots

After bud burst, shoots grow from the buds. Most of the time shoots do not grow in equal lengths necessitating cutting them into equal lengths. Cutting of shoots usually occurs in the northern hemisphere wine regions around April and for the southern hemisphere regions in October. This stage for the Chardonnay cultivar will take place from October 10th to October 20th and during
this stage little or no rain and warm temperatures are most favourable. This is the stage prior to flowering and fruit set in which moist conditions can cause downy mildew – a fungi which will affect the crop’s yield.

5.3.1.4 Flowering and Fruit Set

Depending on the average temperatures, 40 – 80 days after bud break the process of flowering begins. Flowering occurs in the Northern Hemisphere wine regions generally around May and for the Southern Hemisphere regions around November. Small flower clusters appear on the tips of the young shoots and a few weeks after the initial cluster appears, flowers grow in size and individual flowers become visible. It is during this stage that the pollination and fertilisation of the grapevine takes place with the resulting product being a grape berry. Following fertilisation, the fruit forms (Eichhorn and Lorenz, 1984:296).

The fruit set stage follows the flowering almost immediately, when the fertilised flower begins to develop a seed and a grape berry to protect the seed. This stage is critical for wine production since it determines the potential crop yield (Eichhorn and Lorenz, 1984:296). During this stage warm, sunny weather is essential with no rain and low humidity: clusters are susceptible to downy mildew, a disease caused by a type of fungi which lowers crop yield considerably. Adverse weather (cold, wind and rain) severely affects the flowering process, causing many flowers to remain unfertilised. Flowering and fruit set for the Chardonnay cultivar occurs between November 1st to November 14th.

5.3.1.5 Veraison

Veraison means "the onset of ripening or the “change of colour of the grape berries”. In the veraison stage berries soften and change colour as they ripen. This occurs when grapes are roughly half their final size. This stage signals the start of the ripening process and takes place around 40 – 50 days after the fruit set. In the northern hemisphere this is around the end of July and for the southern hemisphere it is in January. During this stage the colours of the grapes alter from red to black or yellow to green, depending on the cultivar. Within six days of the start of veraison, berries grow dramatically as they accumulate glucose and fructose and acids form (Eichhorn and Lorenz, 1984:297).
The onset of veraison does not occur uniformly among all berries. Typically berries and clusters most exposed to warmth on the outer extents of the canopy undergo veraison first with berries and clusters closer to the trunk and under the canopy shade undergoing it last. Some factors that can control the onset of veraison are limited water and/or canopy management with warm temperatures which will create a high "leaf to fruit" ratio which in turn encourage veraison. During this stage little or no rain with warm temperatures is essential. The veraison stage for the Chardonnay cultivar occurs between December 28th and January 10th.

5.3.1.6 Harvesting

Harvesting sees the grapes removed from the vine and transported to the winery to begin the wine making process. In the northern hemisphere this is generally between September and October while in the southern hemisphere between February and April. The time of harvest depends on a variety of factors, most notably the subjective determination of ripeness. Cooler weather is recommended for this stage and very low rainfall. Rainfall below 25mm for the period is acceptable during this stage (Nel, 2009). If rainfall and temperatures are too high, grey rot fungi destroys the grape crop completely. The harvesting and beginning of the wine making process for the Chardonnay cultivar takes place between February 1st and February 14th.

5.3.2 Optimal temperature and rainfall for the Chardonnay cultivar for each stage

Table 5.2 summarises the temperature and rainfall requirements for each stage in the Chardonnay cultivar growth cycle.

Table 5.2: Stages, time periods, rainfall and temperature for optimal growth.

<table>
<thead>
<tr>
<th>STAGE</th>
<th>CRITICAL:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>PERIOD</td>
</tr>
<tr>
<td>1 - Pruning</td>
<td>1 Aug – 10 Aug</td>
</tr>
<tr>
<td>3 - Cutting shoots</td>
<td>10 Oct – 20 Oct</td>
</tr>
<tr>
<td>4 - Flowering and Fruit set</td>
<td>1 Nov – 14 Nov</td>
</tr>
<tr>
<td>5 - Veraison</td>
<td>28 Dec – 10 Jan</td>
</tr>
<tr>
<td>6 - Harvesting</td>
<td>1 Feb – 14 Feb</td>
</tr>
</tbody>
</table>

Source: Nel, 2009.
The critical time periods (the exact dates) are used in the weather event forecasting, which are explained in the next section.

5.4 Weather event forecasting

The information supplied in Table 5.2 was used together with the collected historical weather data to determine what the temperature and rainfall per criteria as stated in Section 5.2 (see Table 5.1) was for each specified stage in the Chardonnay cultivar's life cycle. This was accomplished by counting the number of times a weather event occurred per criteria, per critical time period, over the period of 1963 – 2009 and expressing this as a percentage of the total observations. Assuming all else remaining equal, this provides an estimate of the probability of each event occurring in the future.

5.4.1 Number of events per criteria

Each stage of the Chardonnay cultivar has its own table for every calculation. For the number of events per criteria calculation, then, there are six tables, representing stages 1 – 6 of the life cycle of the cultivar. Tables 5.3 – 5.8 show the number of times that specified events occurred since 1963 for each growth stage. For example, Table 5.3 represents stage one (pruning), and the event that it rained between 0 and 2.5mm, while the temperature was between 20°C and 25°C occurred 83 times since 1963. These tables show which events occurred most frequently and which least frequently. Using these data, prediction tables or tables of probabilities (see Section 5.4.2) may be constructed and used in Equation 4.43.
Table 5.3: Number of events per criteria for Stage 1.

<table>
<thead>
<tr>
<th>Rainfall Temp</th>
<th>[0mm; 2.5mm)</th>
<th>[2.5mm; 5mm)</th>
<th>[5mm; 7.5mm)</th>
<th>[7.5mm; 10mm)</th>
<th>&gt; 10mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>[5°C; 10°C]</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>[10°C; 15°C]</td>
<td>41</td>
<td>8</td>
<td>12</td>
<td>6</td>
<td>46</td>
</tr>
<tr>
<td>[15°C; 20°C]</td>
<td>162</td>
<td>15</td>
<td>13</td>
<td>5</td>
<td>25</td>
</tr>
<tr>
<td>[20°C; 25°C]</td>
<td>83</td>
<td>2</td>
<td>0</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>[25°C; 30°C]</td>
<td>32</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>[30°C; 35°C]</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>[35°C; 40°C]</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>&gt;40°C</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Source: Author.

Note – the table above represents the full range of rainfall and temperature possibilities for the period of the 1st of August to the 10th of August. The data span some 46 years (1963 – 2009) so 46 × 10 = 460. This principle applies to Tables 5.4 through 5.8 as well.

Table 5.4: Number of events per criteria for Stage 2.

<table>
<thead>
<tr>
<th>Rainfall Temp</th>
<th>[0mm; 2.5mm)</th>
<th>[2.5mm; 5mm)</th>
<th>[5mm; 7.5mm)</th>
<th>[7.5mm; 10mm)</th>
<th>&gt; 10mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>[5°C; 10°C]</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>[10°C; 15°C]</td>
<td>13</td>
<td>4</td>
<td>5</td>
<td>1</td>
<td>16</td>
</tr>
<tr>
<td>[15°C; 20°C]</td>
<td>130</td>
<td>15</td>
<td>11</td>
<td>10</td>
<td>18</td>
</tr>
<tr>
<td>[20°C; 25°C]</td>
<td>158</td>
<td>1</td>
<td>3</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>[25°C; 30°C]</td>
<td>81</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>[30°C; 35°C]</td>
<td>24</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>[35°C; 40°C]</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>&gt;40°C</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Source: Author.
### Table 5.5: Number of events per criteria for Stage 3.

<table>
<thead>
<tr>
<th>Rainfall Temp</th>
<th>[0mm; 2.5mm)</th>
<th>[2.5mm; 5mm)</th>
<th>[5mm; 7.5mm)</th>
<th>[7.5mm; 10mm)</th>
<th>&gt; 10mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>[5°C; 10°C]</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>[10°C; 15°C]</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>[15°C; 20°C]</td>
<td>61</td>
<td>12</td>
<td>2</td>
<td>9</td>
<td>13</td>
</tr>
<tr>
<td>[20°C; 25°C]</td>
<td>162</td>
<td>4</td>
<td>4</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>[25°C; 30°C]</td>
<td>136</td>
<td>2</td>
<td>2</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>[30°C; 35°C]</td>
<td>71</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>[35°C; 40°C]</td>
<td>8</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>&gt;40°C</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
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</tr>
</tbody>
</table>

Source: Author.

### Table 5.6: Number of events per criteria for Stage 4.

<table>
<thead>
<tr>
<th>Rainfall Temp</th>
<th>[0mm; 2.5mm)</th>
<th>[2.5mm; 5mm)</th>
<th>[5mm; 7.5mm)</th>
<th>[7.5mm; 10mm)</th>
<th>&gt; 10mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>[5°C; 10°C]</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>[10°C; 15°C]</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>[15°C; 20°C]</td>
<td>18</td>
<td>4</td>
<td>4</td>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>[20°C; 25°C]</td>
<td>165</td>
<td>7</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>[25°C; 30°C]</td>
<td>221</td>
<td>2</td>
<td>3</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>[30°C; 35°C]</td>
<td>133</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>[35°C; 40°C]</td>
<td>28</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>&gt;40°C</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Source: Author.
Table 5.7: Number of events per criteria for Stage 5.

<table>
<thead>
<tr>
<th>Rainfall Temp</th>
<th>[0mm; 2.5mm)</th>
<th>[2.5mm; 5mm)</th>
<th>[5mm; 7.5mm)</th>
<th>[7.5mm; 10mm)</th>
<th>&gt; 10mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>[5°C; 10°C]</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>[10°C; 15°C]</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>[15°C; 20°C]</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>[20°C; 25°C]</td>
<td>40</td>
<td>11</td>
<td>2</td>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>[25°C; 30°C]</td>
<td>215</td>
<td>1</td>
<td>5</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>[30°C; 35°C]</td>
<td>224</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>[35°C; 40°C]</td>
<td>101</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>&gt;40°C</td>
<td>8</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Source: Author

Table 5.8: Number of events per criteria for Stage 6.

<table>
<thead>
<tr>
<th>Rainfall Temp</th>
<th>[0mm; 2.5mm)</th>
<th>[2.5mm; 5mm)</th>
<th>[5mm; 7.5mm)</th>
<th>[7.5mm; 10mm)</th>
<th>&gt; 10mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>[5°C; 10°C]</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>[10°C; 15°C]</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>[15°C; 20°C]</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>[20°C; 25°C]</td>
<td>26</td>
<td>2</td>
<td>2</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>[25°C; 30°C]</td>
<td>126</td>
<td>4</td>
<td>3</td>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>[30°C; 35°C]</td>
<td>190</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>[35°C; 40°C]</td>
<td>98</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>&gt;40°C</td>
<td>12</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Source: Author.
5.4.2 The probability of each event occurring in the future for each time period

The number of events which occurred for each criterion for each stage was calculated in the previous section. The probability of each event occurring in the future (assuming all else remains equal) for each critical time period may now be calculated, by dividing each matrix component by the number of total days in the sample. The matrix components are thus converted to percentages representing probabilities of the events occurring in the future.

This can be more clearly seen by considering Table 5.8. The number of times that the temperature was between 25 and 30°C and the rainfall was > 10mm = 7 (matrix address (5, 5)). There were 470 days in the sample period matrix. Hence, 7/470 = 1.4% (address (5, 5) in Table 5.14. Percentages were rounded to integer percents for easier reading. In the model exact percentages were used.

Tables 5.9 – 5.14 show the probabilities of each event occurring for each stage using data from Tables 5.3 – 5.8.

Table 5.9: Probability of event occurring for each criterion in Stage 1.

<table>
<thead>
<tr>
<th>Rainfall Temp</th>
<th>Probability of event occurring for the time period 1 Aug – 10 Aug</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[0mm; 2.5mm)</td>
</tr>
<tr>
<td>[5°C; 10°C]</td>
<td>0%</td>
</tr>
<tr>
<td>[10°C; 15°C]</td>
<td>9%</td>
</tr>
<tr>
<td>[15°C; 20°C]</td>
<td>35%</td>
</tr>
<tr>
<td>[20°C; 25°C]</td>
<td>18%</td>
</tr>
<tr>
<td>[25°C; 30°C]</td>
<td>7%</td>
</tr>
<tr>
<td>[30°C; 35°C]</td>
<td>0%</td>
</tr>
<tr>
<td>[35°C; 40°C]</td>
<td>0%</td>
</tr>
<tr>
<td>&gt;40°C</td>
<td>0%</td>
</tr>
</tbody>
</table>

Source: Author.
Table 5.10: Probability of event occurring for each criterion in Stage 2.

<table>
<thead>
<tr>
<th>Rainfall Temp</th>
<th>[0mm; 2.5mm)</th>
<th>[2.5mm; 5mm)</th>
<th>[5mm; 7.5mm)</th>
<th>[7.5mm; 10mm)</th>
<th>&gt; 10mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>[5°C; 10°C)</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>[10°C; 15°C)</td>
<td>3%</td>
<td>1%</td>
<td>1%</td>
<td>0%</td>
<td>3%</td>
</tr>
<tr>
<td>[15°C; 20°C)</td>
<td>26%</td>
<td>3%</td>
<td>2%</td>
<td>2%</td>
<td>4%</td>
</tr>
<tr>
<td>[20°C; 25°C)</td>
<td>32%</td>
<td>0%</td>
<td>1%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>[25°C; 30°C)</td>
<td>16%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>[30°C; 35°C)</td>
<td>5%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>[35°C; 40°C)</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>&gt;40°C</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
</tbody>
</table>

Source: Author.

Table 5.11: Probability of event occurring for each criterion in Stage 3.

<table>
<thead>
<tr>
<th>Rainfall Temp</th>
<th>[0mm; 2.5mm)</th>
<th>[2.5mm; 5mm)</th>
<th>[5mm; 7.5mm)</th>
<th>[7.5mm; 10mm)</th>
<th>&gt; 10mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>[5°C; 10°C)</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>[10°C; 15°C)</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>[15°C; 20°C)</td>
<td>12%</td>
<td>2%</td>
<td>0%</td>
<td>2%</td>
<td>3%</td>
</tr>
<tr>
<td>[20°C; 25°C)</td>
<td>32%</td>
<td>1%</td>
<td>1%</td>
<td>0%</td>
<td>1%</td>
</tr>
<tr>
<td>[25°C; 30°C)</td>
<td>27%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>[30°C; 35°C)</td>
<td>14%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>[35°C; 40°C)</td>
<td>2%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>&gt;40°C</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
</tbody>
</table>

Source: Author.
Table 5.12: Probability of event occurring for each criterion in Stage 4.

<table>
<thead>
<tr>
<th>Rainfall Temp</th>
<th>[0mm; 2.5mm)</th>
<th>[2.5mm; 5mm)</th>
<th>[5mm; 7.5mm)</th>
<th>[7.5mm; 10mm)</th>
<th>&gt; 10mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>[5°C; 10°C]</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>[10°C; 15°C]</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>[15°C; 20°C]</td>
<td>3%</td>
<td>1%</td>
<td>1%</td>
<td>0%</td>
<td>1%</td>
</tr>
<tr>
<td>[20°C; 25°C]</td>
<td>27%</td>
<td>1%</td>
<td>0%</td>
<td>0%</td>
<td>1%</td>
</tr>
<tr>
<td>[25°C; 30°C]</td>
<td>36%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>[30°C; 35°C]</td>
<td>22%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>[35°C; 40°C]</td>
<td>5%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>&gt;40°C</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
</tbody>
</table>

Source: Author.

Table 5.13: Probability of event occurring for each criterion in Stage 5.

<table>
<thead>
<tr>
<th>Rainfall Temp</th>
<th>[0mm; 2.5mm)</th>
<th>[2.5mm; 5mm)</th>
<th>[5mm; 7.5mm)</th>
<th>[7.5mm; 10mm)</th>
<th>&gt; 10mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>[5°C; 10°C]</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>[10°C; 15°C]</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>[15°C; 20°C]</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>[20°C; 25°C]</td>
<td>6%</td>
<td>2%</td>
<td>0%</td>
<td>0%</td>
<td>1%</td>
</tr>
<tr>
<td>[25°C; 30°C]</td>
<td>35%</td>
<td>0%</td>
<td>1%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>[30°C; 35°C]</td>
<td>36%</td>
<td>1%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>[35°C; 40°C]</td>
<td>16%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>&gt;40°C</td>
<td>1%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
</tbody>
</table>

Source: Author.
Table 5.14: Probability of event occurring for each criterion in Stage 6.

<table>
<thead>
<tr>
<th>Rainfall Temp</th>
<th>[0mm; 2.5mm)</th>
<th>[2.5mm; 5mm)</th>
<th>[5mm; 7.5mm)</th>
<th>[7.5mm; 10mm)</th>
<th>&gt; 10mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>[5°C; 10°C)</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>[10°C; 15°C)</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>[15°C; 20°C)</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>[20°C; 25°C)</td>
<td>5%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>[25°C; 30°C)</td>
<td>26%</td>
<td>1%</td>
<td>1%</td>
<td>0%</td>
<td>1%</td>
</tr>
<tr>
<td>[30°C; 35°C)</td>
<td>39%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>[35°C; 40°C)</td>
<td>20%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>&gt;40°C</td>
<td>2%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
</tbody>
</table>

Source: Author.

This section showed the number of events per criteria, which aided the calculation of the probability of an event per criteria. The next step is to determine the recovery rates of the Chardonnay crop given the occurrence of specific events.

5.5 Recovery rates

The recovery rate is the percentage of the crop that may be saved if an unfavourable weather event occurs. Each criterion at each stage has different recovery rates: these are shown in Tables 5.15 – 5.21.

As an example, consider Table 5.16 which represents Stage 2 (or the bud burst stage of the Chardonnay cultivar). If the temperature were between 10 and 15°C and if the rainfall was more than 10mm for the period under consideration, 80% of the crop could be recovered. These data were procured from empirical, historical loss rates generously provided by Bothma (2010). Wine farmers will wish to hedge themselves against this 20% crop loss. This may be accomplished using weather derivatives: insurance companies do not cover small fluctuations in weather such as temperature changes or rainfall fluctuation.
Table 5.15:  Recovery rates for Stage 1.

<table>
<thead>
<tr>
<th>Rainfall Temp</th>
<th>[0mm; 2.5mm)</th>
<th>[2.5mm; 5mm)</th>
<th>[5mm; 7.5mm)</th>
<th>[7.5mm; 10mm)</th>
<th>&gt; 10mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>[5°C; 10°C]</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>[10°C; 15°C]</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>[15°C; 20°C]</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>[20°C; 25°C]</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>[25°C; 30°C]</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>[30°C; 35°C]</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>[35°C; 40°C]</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>&gt;40°C</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
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</tr>
</tbody>
</table>


Table 5.16:  Recovery rates for Stage 2.

<table>
<thead>
<tr>
<th>Rainfall Temp</th>
<th>[0mm; 2.5mm)</th>
<th>[2.5mm; 5mm)</th>
<th>[5mm; 7.5mm)</th>
<th>[7.5mm; 10mm)</th>
<th>&gt; 10mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>[5°C; 10°C]</td>
<td>85%</td>
<td>85%</td>
<td>85%</td>
<td>85%</td>
<td>80%</td>
</tr>
<tr>
<td>[10°C; 15°C]</td>
<td>85%</td>
<td>85%</td>
<td>85%</td>
<td>85%</td>
<td>80%</td>
</tr>
<tr>
<td>[15°C; 20°C]</td>
<td>90%</td>
<td>90%</td>
<td>90%</td>
<td>90%</td>
<td>85%</td>
</tr>
<tr>
<td>[20°C; 25°C]</td>
<td>95%</td>
<td>95%</td>
<td>90%</td>
<td>90%</td>
<td>85%</td>
</tr>
<tr>
<td>[25°C; 30°C]</td>
<td>100%</td>
<td>100%</td>
<td>95%</td>
<td>95%</td>
<td>85%</td>
</tr>
<tr>
<td>[30°C; 35°C]</td>
<td>100%</td>
<td>100%</td>
<td>95%</td>
<td>95%</td>
<td>85%</td>
</tr>
<tr>
<td>[35°C; 40°C]</td>
<td>100%</td>
<td>100%</td>
<td>95%</td>
<td>95%</td>
<td>85%</td>
</tr>
<tr>
<td>&gt;40°C</td>
<td>100%</td>
<td>100%</td>
<td>95%</td>
<td>95%</td>
<td>85%</td>
</tr>
</tbody>
</table>

Table 5.17: Recovery rates for Stage 3.

<table>
<thead>
<tr>
<th>Rainfall Temp</th>
<th>[0mm; 2.5mm)</th>
<th>[2.5mm; 5mm)</th>
<th>[5mm; 7.5mm)</th>
<th>[7.5mm; 10mm)</th>
<th>&gt; 10mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>[5°C; 10°C]</td>
<td>90%</td>
<td>90%</td>
<td>90%</td>
<td>85%</td>
<td>85%</td>
</tr>
<tr>
<td>[10°C; 15°C]</td>
<td>90%</td>
<td>90%</td>
<td>90%</td>
<td>85%</td>
<td>85%</td>
</tr>
<tr>
<td>[15°C; 20°C]</td>
<td>95%</td>
<td>95%</td>
<td>95%</td>
<td>90%</td>
<td>90%</td>
</tr>
<tr>
<td>[20°C; 25°C]</td>
<td>100%</td>
<td>100%</td>
<td>95%</td>
<td>90%</td>
<td>90%</td>
</tr>
<tr>
<td>[25°C; 30°C]</td>
<td>100%</td>
<td>100%</td>
<td>95%</td>
<td>90%</td>
<td>90%</td>
</tr>
<tr>
<td>[30°C; 35°C]</td>
<td>100%</td>
<td>100%</td>
<td>95%</td>
<td>90%</td>
<td>90%</td>
</tr>
<tr>
<td>[35°C; 40°C]</td>
<td>100%</td>
<td>100%</td>
<td>95%</td>
<td>90%</td>
<td>90%</td>
</tr>
<tr>
<td>&gt;40°C</td>
<td>100%</td>
<td>100%</td>
<td>95%</td>
<td>90%</td>
<td>90%</td>
</tr>
</tbody>
</table>


Table 5.18: Recovery rates for Stage 4.

<table>
<thead>
<tr>
<th>Rainfall Temp</th>
<th>[0mm; 2.5mm)</th>
<th>[2.5mm; 5mm)</th>
<th>[5mm; 7.5mm)</th>
<th>[7.5mm; 10mm)</th>
<th>&gt; 10mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>[5°C; 10°C]</td>
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<td>80%</td>
<td>75%</td>
<td>75%</td>
<td>70%</td>
</tr>
<tr>
<td>[10°C; 15°C]</td>
<td>90%</td>
<td>90%</td>
<td>80%</td>
<td>80%</td>
<td>75%</td>
</tr>
<tr>
<td>[15°C; 20°C]</td>
<td>90%</td>
<td>90%</td>
<td>85%</td>
<td>85%</td>
<td>75%</td>
</tr>
<tr>
<td>[20°C; 25°C]</td>
<td>95%</td>
<td>95%</td>
<td>85%</td>
<td>85%</td>
<td>80%</td>
</tr>
<tr>
<td>[25°C; 30°C]</td>
<td>95%</td>
<td>95%</td>
<td>90%</td>
<td>90%</td>
<td>80%</td>
</tr>
<tr>
<td>[30°C; 35°C]</td>
<td>95%</td>
<td>95%</td>
<td>90%</td>
<td>90%</td>
<td>80%</td>
</tr>
<tr>
<td>[35°C; 40°C]</td>
<td>95%</td>
<td>95%</td>
<td>90%</td>
<td>90%</td>
<td>80%</td>
</tr>
<tr>
<td>&gt;40°C</td>
<td>95%</td>
<td>95%</td>
<td>90%</td>
<td>90%</td>
<td>80%</td>
</tr>
</tbody>
</table>

Table 5.19: Recovery rates for Stage 5.

<table>
<thead>
<tr>
<th>Rainfall Temp</th>
<th>[0mm; 2.5mm)</th>
<th>[2.5mm; 5mm)</th>
<th>[5mm; 7.5mm)</th>
<th>[7.5mm; 10mm)</th>
<th>&gt; 10mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>[5°C; 10°C)</td>
<td>85%</td>
<td>85%</td>
<td>85%</td>
<td>80%</td>
<td>75%</td>
</tr>
<tr>
<td>[10°C; 15°C)</td>
<td>90%</td>
<td>90%</td>
<td>90%</td>
<td>85%</td>
<td>80%</td>
</tr>
<tr>
<td>[15°C; 20°C)</td>
<td>95%</td>
<td>95%</td>
<td>95%</td>
<td>95%</td>
<td>95%</td>
</tr>
<tr>
<td>[20°C; 25°C)</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>95%</td>
<td>90%</td>
</tr>
<tr>
<td>[25°C; 30°C)</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>95%</td>
<td>90%</td>
</tr>
<tr>
<td>[30°C; 35°C)</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>95%</td>
<td>90%</td>
</tr>
<tr>
<td>[35°C; 40°C)</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>95%</td>
<td>90%</td>
</tr>
<tr>
<td>&gt;40°C</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>95%</td>
<td>90%</td>
</tr>
</tbody>
</table>


Table 5.20: Recovery rates for Stage 6.

<table>
<thead>
<tr>
<th>Rainfall Temp</th>
<th>[0mm; 2.5mm)</th>
<th>[2.5mm; 5mm)</th>
<th>[5mm; 7.5mm)</th>
<th>[7.5mm; 10mm)</th>
<th>&gt; 10mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>[5°C; 10°C)</td>
<td>100%</td>
<td>100%</td>
<td>95%</td>
<td>90%</td>
<td>85%</td>
</tr>
<tr>
<td>[10°C; 15°C)</td>
<td>100%</td>
<td>100%</td>
<td>95%</td>
<td>90%</td>
<td>85%</td>
</tr>
<tr>
<td>[15°C; 20°C)</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>95%</td>
<td>90%</td>
</tr>
<tr>
<td>[20°C; 25°C)</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>95%</td>
<td>90%</td>
</tr>
<tr>
<td>[25°C; 30°C)</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>95%</td>
<td>90%</td>
</tr>
<tr>
<td>[30°C; 35°C)</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>95%</td>
<td>90%</td>
</tr>
<tr>
<td>[35°C; 40°C)</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>95%</td>
<td>90%</td>
</tr>
<tr>
<td>&gt;40°C</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>95%</td>
<td>90%</td>
</tr>
</tbody>
</table>


These recovery rates, together with the probabilities are now used in Equation 4.43 to evaluate a premium for the weather derivative.

As deduced from Tables 5.9 – 5.14, Stage 1 is not affected by unfavourable weather and farmers will choose to not take out a weather derivative for that time period. However, all the other stages are affected by unfavourable weather and by hedging with a weather derivative these losses may be mitigated.
5.6 Pricing weather derivatives for the Wellington Chardonnay cultivar

Recall Equation 4.4.3 for pricing of a weather derivative:

\[
S = \frac{\sum_{i=1}^{N} \sum_{m \in \mathbb{I}} (1-R)(q(t_{i,m}) - q(t_{1,m}))D(t_i)}{\sum_{i=1}^{N} D(t_i)q(t_i)d_i + \sum_{i=1}^{N} D(t_i)(q(t_{i-1}) - q(t_i)) \frac{d_i}{2}}
\]  

(4.43)

As explained in Chapter 4, recovery rates, survival probabilities (calculated from the default probabilities) and accrual days between payments are used in Equation 4.43 to calculate the monthly premium. These payments are then discounted back by using the yield curve to obtain present values.

The calculation proceeds as follows.

Using Equation 4.43, consider the numerator. From Table 5.15, the recovery rate for scenario (1, 1) is 100%, thus the loss is (100% – 100%) = 0%. The next component is the marginal probability of the event occurring – this may be obtained from Table 5.9. Assuming 100% of the crop at the start of this process, with a 0% chance of a (1, 1) event occurring (i.e. rainfall of 0 – 2.5mm and temperature between 5 and 10°C), the marginal probability between the start of the process and the (1, 1) event is 100% – 100% = 0% probability that the event will occur. The \(D(t_i)\) term (Equation 4.34) is obtained from the prevailing yield curve.

Remaining with the numerator, the double summation now requires the next matrix address (or scenario) to be addressed. Now consider address (2, 1). From Table 5.15, the recovery rate for scenario (2, 1) is also 100%, hence the loss is again (100% – 100%) = 0%. The marginal probability of this event occurring (obtained from Table 5.9) is 9%. Since 100% of the crop was present at the start of this process, a 9% chance of a (2, 1) event occurring (i.e. rainfall of 0 – 2.5mm and temperature between 10 and 15°C) means that the marginal probability between the start of the process and the (2, 1) event is 100% – 91% = 9% probability that the event will occur. Again, the \(D(t_i)\) term (Equation 4.34) is obtained from the prevailing yield curve.

This process is repeated for each matrix component in Tables 5.9 to 5.20 and the sum product, discounted by \(D(t_i)\), determined.
To determine the denominator, the same logic is applied (see Equation 4.43). The extra term in the denominator, $d_i$, is known since this is the time period over which the scenario could occur (in most cases, 10 days).

### 5.6.1 Weather derivative price compared to normal crop insurance

#### 5.6.1.1 Weather derivative price

Using Equation 4.43, the premium is R353. This means that a farmer can take out a weather derivative for R353 per month per ton where the notional is R4200, the amount that the farmer will receive per ton of Chardonnay grapes. The notional of R4200 is how much the winery in Wellington receives per ton of Chardonnay grapes (Nel, 2009).

#### 5.6.1.2 Insurance price

For a farmer to take out normal grape crop insurance, the following applies:

- a farm is made up out of blocks,
- each block is assumed to be 1Ha in size (for this example) and
- each block requires separate insurance.

Assume Block A is 1Ha and the farmer expects 12 ton of grapes @ R4200 per ton. For normal insurance the farmer will pay, according to Equation 5.1 (Malan, 2010):

$$\text{Size of block in Ha} \times \text{Expected ton per block} \times \text{Amount per ton}$$

which will then be $1\text{Ha} \times 12\text{ton} \times \text{R}4200 = \text{R}50\ 400$ per ton per Ha.

This amount needs to be multiplied with a region/district specific tariff. According to Malan (2010) each region or district has its own tariff value. The set tariff for Wellington is 3.5% (Malan, 2010). Thus $\text{R}50\ 400$ (from Equation 5.1) $\times 3.5\% = \text{R}1\ 764$. This premium is payable per season. Most seasons last six months so this premium should be divided by 6 to obtain a monthly premium, i.e. $\text{R}1\ 764/6 = \text{R}294$.

The amount that a farmer will pay per ton per Ha is thus R294 per month. This insurance only covers the farmer against hail and wind damages.
5.6.1.3 *Comparison between the weather derivative premium and the insurance premium*

Comparing the two premiums, the weather derivative premium of R353 per month is 20% more than the R294 per month for ordinary crop insurance. However, the farmer is covered (using the weather derivative) for the smallest unfavourable weather changes, be they rainfall or temperature changes.

The R353 is payable if the farmer wishes to hedge against an unfavourable weather event in *any* of the six different stages of the development of the grapes. However, it is possible for the farmer to be covered for only *one stage* of the development of the grapes. This weather derivative may then be combined with normal crop insurance for optimal risk cover.

Assume the farmer desires normal grape crop *insurance*, but that he also wishes to hedge against the crop against unfavourable weather events only during November (the flowering and fruit-set stage). The weather derivative price (Equation 4.43) for November events only = R11. This is calculated by changing the payout leg of Equation 4.43 for all other months *but* November to 0. This R11 premium combined with the R294 for the normal insurance = a monthly premium of R305. Hence, for R305 per month the farmer is covered for hail and wind damages throughout the grape growth stage, with an additional cover for the month of November against any unfavourable temperature and rainfall changes.

This procedure is easily followed for *any* combination of the above scenarios.

**5.7 Conclusion**

A farmer may employ weather derivatives to hedge against unfavourable weather events. The amount payable per month correlates with the amount payable for insurance. Although it is slightly higher than the insurance, there need not be a catastrophic event for the farmer to be compensated for losses. Detrimental temperature or rainfall conditions measured at different stages of the crop cycle and using the empirical recovery rates, results in a payout by the weather derivative and compensation for the farmer.
CHAPTER 6

CONCLUSION AND RECOMMENDATIONS

6.1 Summary and conclusions

This dissertation explored weather derivatives as relatively new financial tools to provide parties financial security via payoffs dependent on weather indices or specific, predefined weather events based upon climatic factors and historical weather data.

Chapter 2 traced the origin of derivatives and subsequently discussed the evolution and history of credit derivatives. A credit event was defined which formed the main building block of the pricing of weather derivatives. Credit default swaps were explained: their pricing methodology forming the base from which a weather derivative was derived in the next chapters. Weather derivatives in foreign markets were also discussed.

Chapter 3 focused on different indices, structures and contracts currently available in foreign markets for weather derivatives.

Chapter 4 explained the evolution and methodology of credit default swaps, as well as the different models for pricing weather derivatives. These models proved useful, but inadequate, for pricing South African weather derivatives. The CDS pricing model of Nomura (2004) was used as a prototype in the formulation weather derivative pricing and an equation was constructed for this purpose.

Chapter 5 provided an empirical study regarding the methodologies discussed in Chapter 4 and explained the dependence of weather derivatives upon recovery rates (which differ from product to product and are region-specific).

A winery in Wellington in the Western Cape was chosen for the case study, and a weather derivative monthly premium was determined for the Chardonnay cultivar. Standard grape crop insurance was explained and premiums of the weather derivative and insurance cover compared.

6.2 Recommendations

The market for trading weather derivatives in South Africa is very small. This dissertation priced a weather derivative in Wellington for a specific grape cultivar.
The research discussed in this dissertation could be extended by exploring different agricultural products (in different regions) and exploring the pricing for these disparate weather conditions. If pricing is undertaken in all regions for different crops, transparency of prices will encourage farmers to make use of weather derivatives in conjunction with normal crop insurance, or on its own.

In addition, recovery rates for different weather events could be established per agricultural product per region. This may further encourage the pricing of weather derivatives in South Africa for any product in any region. This may also encourage the transition of weather derivatives from over-the-counter based products to those with underlying indices. This development can only make trading of such financial instruments easier and more transparent.

6.3 Contribution

The contribution of this dissertation was a credit default swap pricing methodology which may be adjusted to price a highly specific weather derivative.

If the market for weather derivatives in South Africa accelerates because of the flexibility and transparency of the methodology described in this dissertation, this work could contribute to the eventual construction of a South African "weather index". Such an index could then, in turn, be used to price bespoke and exchange traded weather derivatives, creating a liquid, transparent market for these instruments. These developments will greatly assist farmers and other market participants to hedge against unfavourable weather conditions.

6.4 Final statement

If weather derivatives are explained to farmers as a hedging tool, and farmers accept these, the use of weather derivatives could revolutionise the crop insurance paradigm – at least in South Africa. Farmers may choose to enter into weather derivative contracts on one – or multiple – weather events, for a single month or for the entire season. These derivatives may be used in conjunction with normal crop insurance for optimal weather risk cover.
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