Chapter 2

Background

Permanent magnet electric machines as well as their high speed application and challenges are discussed in this chapter. The main heat transfer mechanisms found in electrical machines: conduction and convection are introduced. The modelling and solution techniques of thermal systems are discussed, with the focus on electric machines. Losses found in high speed PMSMs are discussed, being the heat sources. Finally, a summary of the literature points to the necessity and originality of the proposed work.

2.1 Permanent magnet machines

Permanent magnet machines have become one of the serious contenders in the world of electric machines since the availability of high flux density permanent magnets (PMs) constructed from rare earth materials like Sm-Co and Nd-Fe-B. Figure 2.1 shows where the two most common radial flux PM machines: permanent magnet brushless direct current machines (PMBLDM) and permanent magnet synchronous machines (PMSMs), fit into the electric machine world. In PM machines, the rotor flux is established by the PM. Rotation requires an alternating stator field, which is trapezoidal for the PMBLDMs and sinusoidal for the PMSMs [17]. Surface mounted PMs refers to when the PMs are placed on the outside of the rotor and in the case of interior magnet positions, the PMs are embedded in the rotor.

PM machines have the highest efficiency, high power density, high torque-to-inertia ratio and very good dynamic capability [7, 9], when compared to other electric machines. They are best suited for minimal pulsating torque [8], especially the PMSM which has a sinusoidal back-emf. The high efficiency is a result of a reduction in rotor losses: no rotor current flow is required to establish the rotor magnetic field as with induction machines or other synchronous machines. The brushless operation of PM machines also reduces maintenance cost [6]. Higher power density results in a smaller rotor outer radius, which is important for high speed operation since this reduces the surface speed and centrifugal force acting on the rotor. Eddy current loss on the rotor can be reduced by using segmented magnets at low speed and a highly conductive shielding cylinder at high speeds [10]. These are some of the advantages of PM machines.
Although most of the advantages of PM machines stem from the use of PMs to establish the rotor field, most of its disadvantages are also due to the PMs. Rare earth materials as well as the manufacturing processes involved are expensive [19]. Reluctance forces can cause large torque ripples in slotted PM machines due to spacial reluctance differences. This can also cause vibration and thus audible noise. The characteristics of a PM are largely influenced by its temperature; an increase in temperature results in a reduction of residual flux density and coercivity. Demagnetization occurs when the PM is heated above its Curie temperature.

PM machines have been used successfully in various applications. For energy generation, these machines have been used in wave [20], wind [12, 21] and vehicle [19, 22, 23] generation. Servo drives are another application PM machines are well suited for [17, 24, 25] and also air condition compressor drives [26]. High speed applications include energy storage [13], aircraft fuel pumps [15], disk-drive spindles [27] and automotive applications [14], to name a few. Research on combining induction machines and PMSMs to create line-start PM machines has also received attention [28, 29]. This results in a machine with high efficiency that does not require a variable frequency drive to start.

### 2.2 High speed electric machines

High speed electric machines are found in direct drive generators, turbochargers and machining spindles [30]. Turbo-molecular pumps used in the semiconductor industry, main fuel pumps for aircraft [31] and blowers for gas transport [32] are some of the other applications.
Friction welding units and vacuum pumps also require high speed drives.

The mechanical power ($P_m$) delivered by an electric machine can be determined using

$$P_m = T\omega$$

(2.1)

where $T$ is the torque and $\omega$ is the rotational speed [33]. From this relation it is clear that if two machines have the same mechanical output power, but one has a larger rotational speed, the torque of that machine must decrease by the same factor. Since the torque of an electric machine is related to its physical size, a reduction in torque also results in a reduction in physical size. That is why high speed machines are physically smaller than their low speed counterparts. A smaller physical size results in a smaller surface area for removing the heat due to losses. Heat removal of high speed machines is commonly done using water jacket cooling [34] or forced air cooling [35].

There are many challenges in high speed electric machines, ranging from control, power electronics, bearings and mechanical vibration. The machine’s maximum speed is always limited by the maximum temperature of the components. Efficient cooling of high speed machines is very important to ensure reliable, sustainable operation. Surface frictional loss is significant in high speed machines and can be lowered by operating in a decompressed environment. This also reduces rotor heat removal through convection. The high centrifugal forces associated with high speed rotation place higher importance on the rotor design. In the case of PM machines, the mechanical properties of PMs can be counteracted by using a containment cylinder or an interior magnet design. When a conductive material is used for the containment cylinder, eddy currents will be induced in the cylinder, further increasing rotor losses [36].

It is clear from the preceding two sections that PMSMs are a very good choice for high speed machines. In this case, thermal modelling of the whole machine, especially the PM, is needed to ensure reliable operation. The next two sections will discuss the two main heat transfer mechanisms found in electric machines: conduction and convection.

### 2.3 Conduction

The roots of thermal modelling are based on the work by Joseph Fourier published in 1822 which described the conduction rate equation, now called Fourier’s law. This law is based on observations of heat flow. From experiments it is clear that there exists a material property (thermal conductivity $k$) which relates the temperature difference over a solid to the heat flow through the solid. Fourier’s law in one direction is

$$q_x'' = \frac{q_x}{A} = -k\frac{dT}{dx}$$

(2.2)

where $q_x''$ is the heat flux, $q_x$ is the heat rate, $A$ is the cross sectional area, $k$ is the thermal conductivity and $T$ is the temperature [3]. The minus sign is due to the assumption that heat flows in the direction of decreasing temperature. Heat flux is a vector quantity, flowing in the normal direction of the cross sectional area in this one-dimensional case. When expanding
Fourier’s law to include three dimensions it can be written as shown in (2.3), where \( T(x, y, z) \) is the temperature field.

\[
q'' = k \nabla T = -k \left( i \frac{\partial T}{\partial x} + j \frac{\partial T}{\partial y} + k \frac{\partial T}{\partial z} \right)
\]  (2.3)

The conservation of energy is an important pillar of heat transfer. For a control volume, the rate of thermal energy flowing into the volume (\( \dot{E}_{in} \)), plus the rate of energy generation inside the volume (\( \dot{E}_g \)) must equal the rate of energy flowing out of the volume (\( \dot{E}_{out} \)) plus the rate of energy storage in the volume (\( \dot{E}_{st} \)), or

\[
\dot{E}_{in} + \dot{E}_g - \dot{E}_{out} = \dot{E}_{st}.
\]  (2.4)

Applying the conservation of heat on a differential control volume and using Fourier’s law, the heat diffusion equation can be derived, in Cartesian coordinates:

\[
\frac{\partial}{\partial x} \left( k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( k \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left( k \frac{\partial T}{\partial z} \right) + \dot{q} = \rho c \frac{\partial T}{\partial t}.
\]  (2.5)

where \( \dot{q} \) is the rate of heat generation per unit volume in the medium, \( \rho \) is the density and \( c \) is the specific heat of the material. The diffusion equation can also be rewritten in cylindrical coordinates, which is well suited for use in electric machines, as:

\[
\frac{1}{r} \frac{\partial}{\partial r} \left( kr \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial}{\partial \phi} \left( k \frac{\partial T}{\partial \phi} \right) + \frac{\partial}{\partial z} \left( k \frac{\partial T}{\partial z} \right) + \dot{q} = \rho c_p \frac{\partial T}{\partial t}.
\]  (2.6)

The main objective of thermal modelling is determining the temperature distribution in a material due to the boundary conditions acting on it; thus solving the diffusion equation for certain boundary conditions. The other main heat transport mechanism found in electric machines is convection, which is discussed next.

### 2.4 Convection

Convection heat transfer is found between a solid surface and a moving fluid. It is usually categorized according to the source of the fluid motion: forced or natural. In forced convection, the fluid motion is caused by an external force, like a rotating fan. Many studies have been done on forced convection cooling in electric machines. Examples include shaft mounted cooling fans [37], forced air cooling [35] and forced circulation water cooling [34]. In natural convection, the fluid movement is caused by buoyancy forces. A change in the temperature of a fluid will cause a change in density. The less dense (warmer) fluid particles will thus be pushed upwards by more dense (colder) fluid particles [3]. The convection heat (\( q_{conv} \)) transfer can be calculated using:

\[
q_{conv} = hA(T_s - T_\infty),
\]  (2.7)

where \( T_s \) is the surface temperature, \( T_\infty \) is the surrounding fluid temperature, \( A \) is the cross-sectional area and \( h \) is the convection coefficient. It is important to note that conduction heat transfer also occurs between solids and fluids but to a lesser extent. The Nusselt number (Nu)
2.4. CONVECTION

is a dimensionless number equal to the ratio of heat flow due to convection \( q_{\text{conv}} \) and that due to conduction \( q_{\text{cond}} \):

\[
\text{Nu} = \frac{q_{\text{conv}}}{q_{\text{cond}}} \tag{2.8}
\]

A large Nusselt number indicates that the convection heat transfer is much higher than the conduction heat transfer. The convection coefficient can be calculated using:

\[
h = \frac{\text{Nu} k}{L} \tag{2.9}
\]

where \( k \) is the conduction coefficient of the fluid and \( L \) the axial length.

2.4.1 Natural convection

The Rayleigh number (Ra) is another dimensionless number which is the product of the Grashoff (Gr) and Prandtl (Pr) numbers. The Grashoff number is the ratio of buoyancy force and viscous force acting on a fluid and the Prandtl number is the ratio of the kinematic viscosity (\( \nu \)) and thermal diffusivity (\( \alpha \)).

\[
\text{Ra} = \text{GrPr} = \frac{g \beta (T_s - T_\infty) L^3 \text{Pr}}{\nu \alpha} \tag{2.10}
\]

In (2.10), \( g \) is the gravitational acceleration and \( \beta \) is the volume expansivity. It can be seen that Ra is dependent on the temperature difference between the surface and surrounding air temperatures. Various equations ((2.11a), (2.11b), (2.11c)), for determining Nu from Ra have been derived empirically and theoretically. The simplest of the three, (2.11a), is only accurate for \( 10^4 < \text{Ra} < 10^9 \) whereas the other two are applicable for the whole range of Ra [38].

\[
\text{Nu} = 0.59 \text{Ra}^{1/4} \tag{2.11a}
\]

\[
\text{Nu} = \left\{ 0.825 + \frac{0.387 \text{Ra}^{1/6}}{1 + (0.492/\text{Pr})^{9/16}]^{8/27} \right\}^2 \tag{2.11b}
\]

\[
\text{Nu} = 0.68 + \frac{0.670 \text{Ra}^{1/4}}{1 + (0.492/\text{Pr})^{9/16}]^{4/9} \tag{2.11c}
\]

It is clear that Nu, and thus \( h \), has a non-linear relationship to the temperature difference. By substituting (2.10) into (2.11a) and the result into (2.9) one obtains:

\[
h = \frac{0.59 k}{L} \left( \frac{g \beta L^3 \text{Pr}(T_s - T_\infty)}{\nu \alpha} \right)^{1/4}. \tag{2.12}
\]

The convection resistance is

\[
R_{\text{conv}} = \frac{1}{hA} \tag{2.13}
\]

when derived from (2.7). The non-linearity found in (2.12) makes writing the surface temperature in an explicit form impossible. The surface temperature influences the fluid temperature, which in turn influences the heat flow, again influencing the surface temperature.
The fluid properties ($\beta$, $\nu$, $\alpha$, $k$ and Pr) are all temperature dependent. Table 2.1 shows the effect on $h$ when including the temperature dependence of these variables when the surface temperature is 330 K and the ambient temperature is 290 K. Although the variation in $k$ causes a large variation in $h$, the combined effect is not significant. It can thus be assumed that the fluid properties are constant.

Using (2.12), $h$ is calculated when the surface temperature ranges between 294 K and 333 K, as shown in Figure 2.2. The non-linearity of $h$ can clearly be seen.

### 2.4.2 Forced convection in the air gap

Forced convection heat flow is found in the air gap of high speed electric machines. The fluid movement inside the air gap is caused by the rotor movement. The convection coefficient is dependent on the rotor’s surface speed, air gap width and fluid properties. The fluid flow during forced convection can be one of three types: laminar, vortex or turbulent. The type of flow found between two cylinders can be derived for the Taylor number (Ta), which can be calculated using:

$$Ta = Re \sqrt{\frac{L_g}{r}}$$  \hspace{1cm} (2.14)

where $Re$ is the Reynolds number, $L_g$ is the air gap length and $r$ is the rotor radius [39]. The equation used to calculate $Nu$, is (2.15a) for laminar flow when $Ta < 41$, (2.15b) for vortex flow when $41 < Ta < 100$ and (2.15c) for turbulent flow when $Ta > 100$.

$$Nu = 2$$  \hspace{1cm} (2.15a)

$$Nu = 0.212Ta^{0.63}Pr^{0.27}$$  \hspace{1cm} (2.15b)

$$Nu = 0.386Ta^{0.5}Pr^{0.27}$$  \hspace{1cm} (2.15c)

Table 2.1: Temperature sensitivity of $h$

<table>
<thead>
<tr>
<th>Property</th>
<th>Lower temp [290 K]</th>
<th>Upper temp [310 K]</th>
<th>$%\Delta h_{conv}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>20.74e-6</td>
<td>23.46e-6</td>
<td>3.03</td>
</tr>
<tr>
<td>$\beta$</td>
<td>3.4e-3</td>
<td>3.2e-3</td>
<td>1.65</td>
</tr>
<tr>
<td>$\nu$</td>
<td>15.16e-6</td>
<td>17.02e-6</td>
<td>2.85</td>
</tr>
<tr>
<td>$k$</td>
<td>0.02514</td>
<td>0.0266</td>
<td>5.88</td>
</tr>
<tr>
<td>Pr</td>
<td>0.7309</td>
<td>0.7255</td>
<td>0.18</td>
</tr>
<tr>
<td>Combined</td>
<td></td>
<td></td>
<td>2.08</td>
</tr>
</tbody>
</table>
2.4. CONVECTION

The Reynolds number is the ratio of the inertial force vs. viscous force and is given by:

\[
Re = \frac{\rho u_m l_A}{\mu}
\]  

(2.16)

where \(u_m\) is the average flow velocity, \(\mu\) is the dynamic viscosity and \(l_A\) is the air gap length.

2.4.3 Forced convection from the housing

Forced convection cooling is used in electric machines due to the higher heat removal than natural convection. The convection coefficient is: 5 < \(h\) < 25 for natural convection and 10 < \(h\) < 300 for forced convection when air is used as cooling medium [40]. The heat removal can be increased even more by increasing the surface area. This is commonly done by using fins on the outside of the stator housing. The accurate measurement and modelling of the fluid flow in these fins have been researched [41].

In the TWINS machine, forced cooling will be used at the end winding, as discussed in section 1.2. This is an internal flow problem, as opposed to the external flow cooling mentioned in the previous section. The flow can be of two types, laminar or turbulent. The Reynolds number for a circular tube can be calculated using :

\[
Re_D = \frac{\rho u_m D}{\mu}
\]  

(2.17)

where \(u_m\) is the mean fluid velocity and \(D\) is the tube diameter. When fully developed turbulent flow occurs in a non-cylindrical tube, the hydraulic diameter \((D_h)\) must be used to calculate Re and Nu, which can be calculated using:

\[
D_h = \frac{4A_c}{P}
\]  

(2.18)
where $A_c$ is the cross-sectional area and $P$ is the wetted perimeter of the tube. Turbulent flow starts to occur at $Re_D = 2300$ but only fully develops when $Re_D = 10000$. The convection coefficient is still given by (2.9), thus the Nusselt number must be related to $Re_D$. For laminar flow, (2.19a) gives the value of $Nu$. For turbulent flow in a pipe, when $0.5 \leq Pr \leq 2000$ and $3 \times 10^3 \leq Re \leq 5 \times 10^6$, $Nu$ can be calculated using (2.19b).

$$Nu = 4.36$$ (2.19a)

$$Nu = \frac{(f/8)RePr}{1.07 + 12.7(f/8)^{0.5}(Pr^{2/3} - 1)}$$ (2.19b)

where the friction factor $(f)$ can be calculated using (2.20a) for fully developed laminar flow and (2.20b) for fully developed turbulent flow, where $3000 \leq Re \leq 5 \times 10^6$. The latter is based on experimental results.

$$f = \frac{64}{Re}$$ (2.20a)

$$f = 0.79 \times (\ln(Re) - 1.64)^{-2}$$ (2.20b)

This section introduced the convection transfer commonly found in electric machines: natural convection on the outside of the machine, forced convection in the air gap due to the rotor movement and forced convection cooling. The next section introduce another important thermal parameter, interface resistance.

### 2.5 Thermal interface resistance

Thermal interface resistance, also called thermal contact resistance, refers to the thermal resistance between two components. Surface roughness causes the contact to be non-isotropic: parts of the surface are in contact and conduction heat transfer will occur. Other parts are separated by air or some other substance, thus convection or radiation heat transfer dominate. This causes a finite thermal resistance that can have a significant effect on the thermal distribution, especially when the two components are good thermal conductors. Interface resistance is a function of surface smoothness, interface pressure and material hardness to name a few [42]. Manufacturing techniques also has a large influence on the interface resistance. Figure 2.3 illustrates the interface resistance found between laminations and a solid material.
The interface resistance can be reduced by increasing the joint pressure, reducing the surface roughness or ensuring the interface are filled with a good thermal conductor. Soft metals and thermal greases are the two materials commonly used. Metals like lead, silver or tin can be inserted in the interface in a thin foil form. Some thermal greases have a thermal conductivity of up to 50 times that of air and can completely fill the gaps between the two components. If the joints are permanent, lead based solder as well as gold or tin alloys, can be used to ensure a good thermal joint [38].

Interface resistances are common in electric machines since various materials are used for the different parts. The different materials, like laminated steels, multi-core wire and multiple insulation materials, make calculation of interface resistances for electric machines very difficult. Interface resistance is commonly modelled as an air filled cylinder which has a thermal resistance equal to the actual interface resistance. Staton et al. reported some results for the interface resistance between the stator laminations and housing found in industrial motors [43]. Mostly induction motors (4 - 55 kW) were used and the average effective interface gap length is 0.037 mm. The interface gap for a 4 kW totally enclosed forced cooled IM is 0.042 mm.

2.6 Solution techniques

The theory of heat transfer has been discussed in the previous sections. This section explores the techniques used to calculate thermal distributions.

2.6.1 Distributed analytical methods

Distributed solution of the diffusion equation is usually in a closed form and only dependent on the directional variables \((x, y, z, r, \phi)\), the material constants and time \((t)\). Distributed models are useful when internal generation is present and more than one dimension must be modelled. When modelling 1-D heat flow, with no internal generation, the lumped parameter (LP) and distributed methods are the same, thus LP is a special case of the distributed method.

Solving the diffusion equation in four dimensions (3 directional and time) can be very complex. In most cases, assumptions can be made to reduce the complexity. Usually the partial differential equation (PDE) need only be solved in 2-D for steady state conditions. The assumption that heat flow in one of the three dimensions is not significant can usually be made in practical situations.

Distributed models are widely used in magnetic field modelling of electric machines [44, 45]. Radial flux machines are the most commonly investigated and are modelled in the \(r\phi\) - plane. Most textbooks thus focus on this plane [46]. Linear and axial flux machines should be modelled in the \(rz\) - plane, but this is less common [47]. Magnetic fields usually have complex flux sources but simple boundary conditions and material properties. Boundary dependent heat transfer phenomena like radiation and convection can complicate boundary conditions in thermal models.
Solutions of the diffusion equation using analytical methods have been around for many years. Textbooks exist that list solutions for common geometries and boundary conditions [5, 48]. Solution methods include separation of variables, Laplace transforms and Green’s function [49]. Research in these areas are mostly done by mechanical engineers and mathematicians [50].

Some of the challenges receiving attention are the time required to converge [51] and the computation of eigenvalues and residue. A solution for the latter has been proposed by Lu et al. by combining Laplace transform, separation of variables and variable transformation. The technique then avoids residue calculation and was applied to composite cylindrical slabs [52] as well as composite cylinders [53]. Both showed good correlation with numerical results. These works are focused on theoretical proofs rather than practical implementation.

Exact analytical solutions can be used to verify approximate solutions determined using numerical methods. It is computationally much less expensive than numerical techniques and is thus best suited for optimization problems where the solution must be calculated many times [54]. Exact analytical solutions are commonly used when determining material properties from experiments, sometimes called the inverse problem. Since the introduction of powerful computers, numerical solution techniques have been used much more than exact analytical solution for thermal modelling. Distributed models take more time to derive than the implementation of numerical methods. Complex geometries complicate the derivation of a distributed model to an extent where the effort does not justify the rewards. In extreme cases, it is impossible to derive a distributed model.

### 2.6.2 Lumped parameters

An analogy can be drawn between heat distribution and electrical charge distribution. Thermal resistances and capacitances can be used to model conduction in one-dimension when no internal generation is present. The analogy is drawn between temperature difference and voltage difference, heat transfer rate and current. The thermal conduction resistance in a plane wall is

\[
R_{t,x} = \frac{\Delta T}{q_x} = \frac{L}{kA}
\]

where \(L\) is the thickness of the plane wall, \(A\) is the cross-sectional area and \(k\) is the conduction coefficient. The radial thermal resistance of a cylinder is

\[
R_{t,r} = \frac{\ln(r_2/r_1)}{2\pi Lk}
\]

where \(r_1\) is the inside radius, \(r_2\) is the outside radius and \(L\) is the axial length. The axial thermal resistance of a cylinder is

\[
R_{t,a} = \frac{L}{\pi k(r_2^2 - r_1^2)}.
\]

All of these can be derived by solving the one directional diffusion equation. The lumped parameter (LP) method has been used widely in electric machines. Most regard the work by Mellor et al. for electric machine modelling as the roots of this trend [4]. A general cylindrical component was derived and used to model all the main parts of a totally enclosed, forced
cooled (TEFC) induction machine. The LP method is used in MotorCAD®, a thermal design tool for electric machines. This package has been used to model various machine types, including induction motors [41, 43, 55, 56], permanent magnet synchronous machines [57–59], as well as shared thermal problems [60–64]. It has grown from the PhD study of Dave Staton to a user friendly and well verified product. A screenshot of a LP model compiled by MotorCAD® is shown in Figure 2.4. LPs are the only thermal modelling method found in electric machine design textbooks [65].

LPs are regularly used on induction machines since these are the most commonly used industrial machines [66–69]. Simplification of the LP model to include only the prominent behaviour of temperature sensitive parts has also been done [35, 70–72]. The LP method has also been used in modelling other types of electrical devices including: inductors [73], supercapacitors [74], transformers [75] as well as modelling coolant flow [76]. Other machine types like switched reluctance [77, 78] and synchronous [79] have also been analyzed using LPs. PM machine thermal models based on LPs have been created in [34, 80–86] and LPs were also used in axial flux PM machine models [22, 87].

By combining three sets of one-dimensional thermal networks, Wrobel and Mellor have created a three-dimensional model for a cube [88]. This better approximates the anisotropic parts of an electric machine, like the windings and laminations. This technique has been used to model laminations and copper windings as found in a high energy density inductor [89] and a segment of a stator winding [90].

LP methods have some advantages. LP thermal methods require solving simple electrical cir-
circuits, a concept well known to electric engineers. Since electric machines are designed by electric engineers, LP methods are easily mastered. LP methods are also used in equivalent magnetic circuits which are also commonly used by electric engineers. The effect one thermal resistance has on the total heat flow can easily be identified and fitting the model to the measured results is a simple task. When heat flow in a certain direction is dominating, neglecting the other dimensions will not result in large errors and will reduce the model complexity significantly.

When truly 2-D heat flow occurs, LP results will not be correct. The errors are more pronounced in machines with large geometries and complex machine structures where it cannot be assumed that the radial and axial heat flow components are independent. It is assumed that heat generation is constant throughout a section by Mellor et al. This is true for current carrying winding but not for eddy current induced losses like that found in lamination materials and the rotor. The magnetic flux distribution is not constant in these areas and thus the losses will also not be constant. Rotor thermal analysis of induction machines have not been done extensively since these machines would most likely fail due to stator winding overheating, not rotor bar overheating. PM machines can experience failure due to demagnetization, thus the rotor temperature is important [91]. These are some of the disadvantages of LP methods.

2.6.3 Numerical methods

Various techniques can be used to solve the diffusion equation using numerical methods. Finite difference, finite element, boundary element and the finite volume method are the most well known. Whereas the temperature can be determined at any point using analytical methods, numerical methods only calculate the temperature at discrete points. The number of points strongly influence the accuracy of the result since the temperature at a point represents the average temperature of the surfaces around it. By applying the conservation of energy, the PDE is reduced to a system of linear, algebraic equations. Numerical results can be verified using exact solutions [3]. Numerical methods can approximate complex geometries and boundary conditions that cannot be solved analytically [48].

Numerical methods are widely researched and published in various fields, including mechanical stress, acoustics and aerodynamics, to name a few. Specialist numerical packages used for thermal modelling include FLUX2D [92], COMSOL® [84] and ANSYS®. Coupled models take into account the coupling between, for example, the thermal and electromagnetic domains. This has been done using FEM [93–95]. Figure 2.5 shows the temperature in a brake disk as simulated by COMSOL®. FEM has also been implemented in mathematical packages like MATLAB® [96]. Computational fluid dynamics (CFD) is another growing application of numerical methods used in electrical machine modelling [43, 97, 98]. CFD is used to predict fluid movement, used to predict the cooling in electric machine. Recent advances in computer technology have made CFD more accessible for use in electric machines.

The biggest advantage of numerical methods lies in modelling complex geometries. The simulation packages that use numerical methods are easy to use and geared to give the user a solution quickly. Even though solution times are longer compared to analytical methods, the
2.7. LOSSES IN ELECTRIC MACHINES

The heat generated in an electric machine is due to mechanical and electromagnetic losses. Surface frictional loss of the high speed rotor can be significant as shown by Saari, but is also strongly geometry dependent [35]. Electromagnetic losses can be divided into two main areas: losses due to current flow and losses due to magnetic domain changes. The former will be discussed first and makes up a large portion of the total machine losses.
2.7.1 Losses due to current flow

The loss in a material due to current flow can be characterized in terms of the current source. In the stator windings, current flow is caused by the voltage difference applied to the terminals by the drive or utility supply. The loss in the winding is commonly called copper loss \( P_{\text{copper}} \), and can be calculated using (2.24) when a direct current (DC) \( I \) is flowing through an element with resistance \( R \).

\[
P_{\text{copper}} = I^2 R
\]  

When alternating current (AC) flows through a conductor, it is surrounded by a magnetic field, according to Biot-Savart’s law. This causes the current density in the conductor to become higher close to the outside of the conductor and smaller in the middle of the conductor. The thickness of the current carrying upper layer is called the penetration or skin depth. The skin depth is an approximation of the continuously varying current density. The skin depth is influenced by the frequency of the voltage source and the conductor’s properties. By the same mechanism, the magnetic fields of conductors in close proximity influence one another, resulting in the proximity effect. Both effects on the current density can be seen in Figure 2.6. In all the conductors the current density is smaller in the center than closer to the edge. When the current flows in the same direction (left), the higher current density is where the conductors are closest. The inverse is observed when current flows in opposite directions (right). The skin and proximity effects increase the winding effective resistance resulting in an increase in the copper loss. The use of pulse width modulation (PWM) also has a significant influence on the skin and proximity effects since PWM currents contain high frequency components [99–101]. Skin and proximity effects can be minimized by using thin, specially arranged, insulated wire, called Litz wire [102]. Advanced control methods can be used to minimise the copper loss during machine operation [103].

![Figure 2.6: Current density in copper wires with current flowing in the same direction (left) and in opposite directions (right)](image)

\(1049 \times 10^6\)

\(4036 \times 10^7\)
The second current flow found in an electric machine is caused by induced voltages. When a changing magnetic flux is applied to an electrical conductive material, a voltage will be induced in this material, according to Faraday’s law [104]. An electric current will flow inside the material. According to Lenz’s law, this current will be directed in such a way that the resultant magnetic flux opposes the initial magnetic flux variation. This resultant current is called an eddy-current since its flow pattern is similar to the circular flow patterns found in small whirlpools [105]. Figure 2.7 shows the eddy current in a solid cylinder due to alternating current flow in a wire. The current flow causes energy to be converted to heat through resistive loss, similar to copper loss. Eddy current loss can be modelled using analytical [85, 106, 107] or numerical methods [108–111]. The average eddy current loss \( P_e \) in a laminated magnetic core can be calculated using

\[
P_e = k_e f^2 \delta^2 B_m^2 V
\]

where \( k_e \) is a material dependent constant, \( f \) is the frequency, \( \delta \) is the lamination thickness, \( B_m \) is the maximum flux density and \( V \) is the volume. To minimise the eddy current loss, the electrical resistance in the direction of current flow must be as high as possible. This is commonly done by using a laminated magnetic core. In PMSMs, the eddy-current loss on the rotor is significant in the case of surface mounted PMs. This can be reduced by using segmented magnets [112] or a shielding cylinder [13]. A laminated shielding cylinder will have lower eddy-current loss but the mechanical strength is still under investigation [113].

![Figure 2.7: Eddy currents in a solid cylinder due to alternating current flowing in a wire](image)

### 2.7.2 Losses due to magnetic domain changes

Each time the magnetization direction in a ferromagnetic material changes, the direction of the magnetic domains inside the material must also change. The power needed to change the magnetic domain directions is called hysteresis loss. When a maximum flux density \( B_m \) exists throughout a material with volume \( V \), the hysteresis loss \( P_h \) can be calculated using:

\[
P_h = k_h f B_m^2 V
\]
where \( k_h \) is a material dependent constant and \( n \) is the Steinmetz exponent. This equation was determined from experimental observations. Since the hysteresis losses are dependent on the magnetic flux density, it varies throughout a material with a non-uniform magnetic flux density. When the magnetic flux density distribution is known, the hysteresis losses can be calculated more accurately [79]. Numerical [68, 114] and analytical [10] methods have been employed to calculate the magnetic field. This is a topic which is receiving continued attention, especially when PWM supplies are used [115, 116]. This type of excitation causes mini-hysteresis loops to coexist with the main hysteresis loops [117, 118]. Hysteresis loss is mostly found in the stator laminations of PMSMs. The hysteresis loss is directly proportional to the frequency of the flux changing and maximum flux density. Consequently, slotless stators are commonly used in high speed PMSMs [119]. Even though this results in a large air gap which is degrading to the torque generation, the reduction in hysteresis loss is significant.

### 2.7.3 Mechanical losses

The two main mechanical losses found in electric machines are the bearing and windage losses.

Bearing loss is caused by friction between the moving parts of the bearing, i.e. between the balls and sleeves in the case of ball bearings. Lubrication is used to lower the frictional loss, but the viscosity of the lubrication also influences the total bearing loss. Other factors that influence bearing loss are shaft speed, bearing type and bearing load. The latter is composed of static and dynamic loading, the former is associated with gravitational forces and the latter with dynamic effects like unbalance forces and rotor dynamics. The bearing loss \( P_{\text{bearing}} \) can be calculated using:

\[
P_{\text{bearing}} = 0.5 \omega k_f r F d_{\text{Bearing}}
\]

(2.27)

where \( k_f \) is the frictional coefficient of the bearing, \( F \) is the dynamic bearing load and \( d_{\text{Bearing}} \) is the inner diameter of the bearing [65]. The bearing load is greatly increased when a bearing needs to carry loads in a direction it was not designed for, e.g. axial loading in a bearing designed for radial operation.

The second important mechanical loss mechanism is windage loss. The mechanical air gap in an electric machine is filled with a fluid, usually air. The fluid is moving at the same speed as the rotor on the rotor surface and on the stator surface, it is stationary. This results in a drag torque and frictional loss. The windage loss \( P_{\text{wind}} \) can be calculated using [65]:

\[
P_{\text{wind}} = \frac{1}{32} k_s C_M \pi \rho_a \omega^3 d^4 L
\]

(2.28)

where \( k_s \) is the surface roughness coefficient (usually between 1-1.4), \( C_M \) is the torque coefficient, \( \rho_a \) is the air density, \( d \) is the rotor outside diameter and \( L \) is the active length of the machine. The torque coefficient is dependent on the Reynolds number and can be calculated
using:

\[
C_M = 10 \frac{(2L_A/d)^{0.3}}{Re}, \quad \text{Re} < 64, \quad (2.29a)
\]

\[
C_M = 2 \frac{(2L_A/d)^{0.3}}{Re^{0.8}}, \quad 64 < \text{Re} < 5 \times 10^2, \quad (2.29b)
\]

\[
C_M = 1.03 \frac{(2L_A/d)^{0.3}}{Re^{0.5}}, \quad 5 \times 10^2 < \text{Re} < 10^4, \quad (2.29c)
\]

\[
C_M = 0.065 \frac{(2L_A/d)^{0.3}}{Re^{0.2}}, \quad 10^4 < \text{Re}. \quad (2.29d)
\]

were \( L_A \) is the air gap length.

### 2.7.4 Loss mechanisms in high speed slotless PMSMs

In this subsection, the focus is on the main loss mechanisms found in a slotless high speed PMSM. Figure 2.8 gives a graphical overview.

**Stator losses**

The stator consists of the winding and laminations. Eddy currents will be induced in both. The magnetic nature of the laminations leads to hysteresis loss. Current flow in the winding results in some conduction loss \((I^2R)\).

The magnetic field distribution in the stator is caused by the PMs’ magnetic flux as well as the magnetic fields due to the winding currents.

[Figure 2.8: Loss types and their location in a high speed slotless PMSM like the TWINS]
CHAPTER 2. BACKGROUND

Rotor losses

The shielding cylinder, PM and laminations are the loss locations found on the rotor. All these parts are made from conductive materials. If an alternating magnetic field is present in any of these parts, eddy currents will be induced. Hysteresis loss will exist in the rotor laminations since it is a magnetic material.

An alternating magnetic field in the rotor is caused by the stator magnetic field, produced by the stator winding currents.

Thermal modelling can only be done if the heat sources, i.e., losses, are accurately modelled.

2.8 Conclusion

High speed and PM machines were discussed in the beginning of this chapter. The PMSM is a very good candidate for high speed operation but the thermal problems associated with the PM must be analyzed. This chapter also presented the most important heat flow mechanisms found in electric machines: conduction and convection. Both can be modelled using various techniques: distributed analytical solutions, lumped parameter analytical solutions as well as numerical techniques (FEM, CFD etc.). Of these, LPs and numerical methods are the most common. Distributed analytical solutions have not been used as much for thermal modelling, but are widely used in the electromagnetic design of electric machines.

2.8.1 Research direction

It is clear from the preceding paragraphs that losses and thermal modelling of electric machines are still an important research topic. This paragraph discusses some of the areas with unsolved issues. End winding space has complex fluid flow and the winding has a complex shape. The airflow can be modelled using CFD but experimental verification is difficult since measuring the airflow usually changes the airflow. Bearings are modelled by an equivalent air gap. This equivalent air gap is determined by fitting experimental results to the model. The same approach is followed for interface gaps between lamination stacks and housings/shafts.

Much work is being done in machine thermal protection without measuring the temperature directly. By modifying the control currents [120] or current injection [121], the change in stator resistance can be determined. From this change in resistance, the change in temperature in the stator winding and even the PM on the rotor can be calculated [122]. Indirect measuring methods are still widely researched and have not reached commercial maturity. Using resin with a high thermal conductivity can help improve heat removal in external rotor machines [123]. Understanding the limiting factors in very high speed machines are also receiving attention [30]. Cooling is very important in high speed machines due to high power density and small heat extraction surface.

All the modelling domains are coupled, thus the ultimate objective is simultaneously solving
all the domains. Material properties like electric conductivity is influenced by mechanical as well as thermal conditions. Electrical conductance is temperature dependent and hysteresis loss depends on mechanical stress, for example. In PM machines, the temperature dependence of magnetic remanence and coercivity are very important. In extreme cases demagnetization can occur, rendering the machine useless. Even when the magnets are not heated to their Curie temperature, machine operation can still be hampered since the machine will not be able to deliver design torques (motor) or generated voltage (generator). It is also possible that only sections of the PM can be demagnetized.

2.8.2 Towards a contribution

From the literature it is clear that the following are contributions:

**Distributed thermal model:** A 2-D distributed thermal model of the rotor PM of a high speed PMSM should be investigated. The rotation results in convection heat transfer on the three sides in contact with air and conduction heat flow towards the rotor occurs in the fourth.

**Hybrid thermal model:** Combining the distributed and LP models to achieve a detailed temperature distribution of the PM. Hybrid models combining LP and numerical methods have been used but not the combination proposed here.

**Experimental investigation:** A large part of thermal modelling is verification through experiments. No literature was found on the empirical investigation of a INCONEL® shielding cylinder. Convection and interface resistances are also usually determined through experiments.