A practical implementation of the higher-order transverse-integrated nodal diffusion method

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This work is dedicated to the act of dedication, motivated by the need to keep moving and justified by the dream of becoming. It is not for us to measure how we will be measured, to define how we will be judged or to value what we have created. We may humbly give it life and watch it grow. As such and without expectation, I allow myself a modicum of vanity to hope that this work somehow contributes to the people I work with, the company I work for, the family I care for and the body of knowledge which remains long after all of these reach their respective destinations.
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It is with the deepest gratitude that I gladly acknowledge the role players which facilitated, supported and made this work possible. It is no small accomplishment to guide and develop others toward independence. It requires a special balance of knowledge, wisdom, humor, discipline and the most difficult of all, humility. For these characteristics and many others, I am honoured to thank, and acknowledge the contributions of my two promoters and mentors during this work, namely Dr. Djordje I Tomašević and Prof. Harm Moraal. These special individuals have contributed greatly to both the work in this thesis as well as to the person behind it and I have no words to fully express my gratitude. I wish to acknowledge the contributions, through discussion and interaction, of my colleagues at Necsa and elsewhere, with specific mention of Dr. Pavel Bokov, Dr. Wessel Joubert and Dr. Erwin Müller. A special word of gratitude is extended to Hantie Labuschagne for many hours of tireless editing and corrections. To Necsa, the company I work for and who sponsored me, I express my appreciation for the financial support provided. I would like to thank my line-management, in particular Dr. Gawie Nothnagel, for actively supporting an environment in which this work was possible.

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Samevatting

Transversaal geïntegreerde nodale diffusie metodes verteenvoordig steeds die standaard in reaktor berekeninge. Die primêre tekortkoming in hierdie benadering is die gebruik van die sogenaamde kwadratiese transversale lekkasie aanname. Hierdie aanname word algemeen gebruik in die berekening van ligte water reaktore, maar is sonder teoretiiese grondslag. Dit is nie direk afleibaar van die diffusie oplossing nie en kan akkuraatheid- en konvergensi-probleme tot gevolg hê. In hierdie werk word ’n verbeterde, konsekente hoër-orde lekkasie aanname geformuleer. Die kritiese suksesfaktore in so ’n metode is gekoppel aan beide akkuraatheid en effektiwiteit (berekeningskoste), en gevolglik word ’n reeks iterasiemetodes verder ontwikkel om die voorgestelde oplossing van praktiese waarde te maak. Die mees belowe van hierdie skemas gebruik die hoër-orde lekkasie aanname om korreksiefaktore vir die standaard kwadratiese transversale lekkasie aanname te bereken. Numeriese resultate word producer aan die hand van ’n reeks standaard toetsprobleme. Verder word die toepassing van die metode ook demontreer op ’n stel realistiese SAFARI-1 reaktor berekeninge. Die uiteindelike voorgestelde oplossing is geïmplimenteer in a losstaande FORTRAN-90 module wat naatloos aan bestaande nodale kodes gekoppel kan word. Ter illustrasie word die module ook aan die OSCAR-4 kodesisteem gekoppel, wat oor dertig jaar by Necsa ontwikkel is en wat as primêre berekeningskode vir ’n aantal internationale navorsingsreaktore gebruik word.
Abstract

Transverse-integrated nodal diffusion methods currently represent the standard in full core neutronic simulation. The primary shortcoming of this approach is the utilization of the quadratic transverse leakage approximation. This approach, although proven to work well for typical LWR problems, is not consistent with the formulation of nodal methods and can cause accuracy and convergence problems. In this work, an improved, consistent quadratic leakage approximation is formulated, which derives from the class of higher-order nodal methods developed some years ago. In this thesis a number of iteration schemes are developed around this consistent quadratic leakage approximation which yields accurate node average results in much improved calculational times. The most promising of these iteration schemes results from utilizing the consistent leakage approximation as a correction method to the standard quadratic leakage approximation. Numerical results are demonstrated on a set of benchmark problems and further applied to realistic reactor problems for particularly the SAFARI-1 reactor operating at Necsa, South Africa. The final optimal solution strategy is packaged into a standalone module which may be simply coupled to existing nodal diffusion codes, illustrated via coupling of the module to the OSCAR-4 code system developed at Necsa and utilized for the calculational support of a number of operating research reactors around the world.

keywords: transverse leakage, nodal diffusion, higher order methods
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Nomenclature

ANM Analytic Nodal Method
ANOV Analysis of Variance
BOC Beginning of Cycle
CCSI Chebyshev Cyclic Semi-Iterative
CLA Consistent (Transverse) Leakage Approximation
CQLA Consistent Quadratic Leakage Approximation
EOC End of Cycle
FHO Full Higher-Order
HOTR Higher-Order Transverse Leakage and Reconstruction (code module)
MANM Multi-group Analytic Nodal Method
MGRAC Multi-group Reactor Analysis Code
MR (Higher-order) Model Reduction
NEM Nodal Expansion Method
OSCAR Overall System for the Calculation of Reactors
PLC Partial Leakage Convergence
QLAC (Standard) Quadratic Leakage Approximation Correction
RLCS Reduced Leakage Correction Scheme
SANS Standard Analytic Nodal Solver
SFSIM Standard Fission Source Iterative Method
SQLA Standard Quadratic Leakage Approximation

TLSIM Transverse Leakage Source Iterative Method