CHAPTER 4

PROPOSED POLE-SLIP PROTECTION FUNCTION

“Simplicity is the ultimate sophistication”
Leonardo da Vinci

4.1 INTRODUCTION

Chapter 4 discusses the development of the new pole-slip protection function. The detail design of the pole-slip function is discussed in two parts, namely the steady state calculations and the transient calculations. The design of the pole-slip algorithm is done by means of logics, which can be programmed into a protection relay.

An ABB REM543 relay was selected for this purpose due to the flexibility that the user has with building unique protection algorithms by using the ABB CAP 505 relay programming software. An important part of the design process was to continually test and improve the new pole-slip function as the design progressed. It was convenient to build the new pole-slip algorithm in simulation software called PSCAD, since the algorithm could be tested and de-bugged without having to re-program and test a protection relay multiple times during the design stage.

4.2 DAMAGING EFFECTS OF POLE-SLIPPING

Pole-slipping occurs when the machine internal EMF is 180° out-of-phase with the terminal voltage. During pole-slip operation, the stator current can become nearly as high as the sub-transient terminal three-phase fault current [1].

Although the maximum current during pole-slip or out-of-step operation is smaller than the current for three-phase terminal faults (for which the majority of the machines are designed), pole-slipping is repetitive. Mechanical damage to the stator end-windings can occur due to the repetitive current pulses during pole-slipping.

The normal thermal (electrical) limit on the stator windings should not be exceeded during pole-slipping, since the protection relay thermal overload protection algorithm will typically trip the machine before the windings are thermally damaged. However, if prolonged pole-slipping is allowed, damaging heating can occur at the stator end teeth [23].
The severe pulsating torques produced by pole-slipping can torsionally excite sections of the shaft, exposing them to oscillatory stress [26]. If the shaft material is not sufficiently over-designed, its fatigue life can be used up after relatively few pole-slipping events [2].

Out-of-step operation is typically preceded by a short-circuit and subsequent fault clearance that are both producing impacts on the shaft that can be more severe than the stress caused by pole-slipping itself. The worst conditions occur after slow clearance of a system fault close to the generator.

There is general agreement that generator tripping should be avoided whenever the electrical centre of the system does not pass through a generator or its step-up transformer [1]. This will ensure that supply to local load can be maintained. An unnecessary loss of a generator is likely to worsen the system instability, since the remaining generators will have to pick up the load that was supplied by the tripped generators. With the remaining generators heavier loaded, they are more subjected to instability as will be explained in chapter 5.

### 4.3 Avoiding Damage During Pole-Slipping

As stated earlier, a generator need not be tripped immediately during an out-of-step condition if the electrical centre of the system does not pass through the generator or step-up transformer. The reason for this is that the generator is unlikely to become mechanically (or electrically) damaged if the electrical centre is relatively far from the generator i.e. when the generator is connected via long transmission lines with a large impedance to the rest of the network (as was illustrated in section 2.12).

In the case of long transmission lines where pole-slipping could possibly be allowed to occur more than once, subsynchronous resonance must be taken into account when determining whether the pulsating torque will have a damaging effect on the turbine/generator shaft. When a subsynchronous resonance frequency is triggered, the machine can experience torque pulsations that are amplified to damaging levels.

Apart from the shaft mechanical stress, the mechanical effect on the stator winding overhang must also be taken into account during pole-slipping. As stated earlier, the thermal (electrical) limit on the stator or rotor windings will not be exceeded during pole-slipping since the protection relay thermal overload protection will trip before the windings are thermally damaged. However, the stator end windings can be *mechanically* damaged due to magnetic forces between the windings during pole-slipping.

All of the above concerns can be prevented by tripping a generator before it becomes unstable, which is the aim of the new pole-slip protection function as discussed in the remainder of chapter 4.
4.4 Motor Pole-slippping

Motor pole-slip protection will is not covered in this thesis, but this section briefly introduces the scenarios which can cause large synchronous motors to pole-slip:

- Voltage dips on stator supply
- Loss-of-excitation
- Sudden mechanical failure

4.4.1 Voltage Dips on Stator Supply

When a voltage dip occurs, the excitation system will increase the excitation current in order to deliver more reactive power. The reactive power will increase the terminal voltage on the motor during the voltage dip.

The motor speed will decrease during the voltage dip and the motor can lose synchronism with the network depending on how long the dip is. When the voltage is restored after a dip, the motor will try to resynchronize with the network and can pole-slip if the voltage angle between the motor terminals and the network is too large.

4.4.2 Loss-of-Excitation

During loss-of-excitation, the pull-out torque of the rotor with respect to the stator will be reduced and the motor could possibly pole-slip. The pole-slippering during a complete loss-of-excitation will not cause high torque pulses on the machine shaft, but large stator currents will flow due to the low power factor of the motor. Overheating of the rotor core and damper windings can also occur due to the induction of currents into the rotor material during pole-slippering.

4.4.3 Mechanical Failure

When a sudden mechanical failure occurs, a sudden change in motor speed could cause the synchronously rotating stator magnetic field and the rotor magnetic field to slip with respect to each other.

A gradual mechanical failure (like a bearing seizure) will probably not cause a pole-slip. The large inertia of the motor and load will prevent the motor to change speed instantaneously during a typical bearing mechanical failure.

Although a pole-slip could occur during a sudden mechanical failure, it is most likely that a mechanical failure will occur gradually so that the overload / thermal protection will trip the motor before the
mechanical failure completely locks the rotor. It is, however, possible to have a sudden mechanical failure like a reciprocating motor breaking a connection rod. In such a scenario the motor will pole-slip.

4.5 **GENERATOR POLE-SLIPPING**

The following scenarios can cause synchronous generators to pole-slip:

- Faults near generator terminals
- Power swings
- Loss-of-excitation

4.5.1 **GENERATORS WITHOUT STEP-UP TRANSFORMERS**

Generators can be operated without step-up transformers for power ratings of typically less than 50 MW [39]. These generators should not be paralleled at the switchboard to limit the fault level on the switchboard. Figure 4.1 shows generator arrangements without step-up transformers.

When a fault occurs at the generator terminals, the generator differential protection should trip immediately (Fault zone 1 in Figure 4.1). When a fault occurs in Fault zone 2 of Figure 4.1, the generator is at risk of pole-slip if the fault is not cleared within a certain time. The protection that will clear the fault in Zone 2 is typically the generator thermal overload protection. This protection will not operate instantaneously.

When a fault occurs near the generator terminals, there will be mainly reactive power flowing, since the network fault impedance will be mainly reactive. The electrical active power reduces considerably, while the prime-mover mechanical active power will remain constant for the first few cycles. This will cause the generator frequency to become higher than the network frequency, since the generator speed will increase during the fault.

Once the fault is cleared, the generator active electrical power will start flowing again, which will cause the generator electrical torque to be restored. After the fault is cleared, the generator can pole-slip since the generator is not synchronized with the network anymore.

It must be noted that the generator will not pole-slip during the fault, but will only start pole-slipping once the fault is cleared. The generator can, however, experience large torque pulsations during the fault.
4.5.2 Generators that are paralleled at step-up transformer HV sides

Figure 4.2 shows a typical large power station with more than one generator each connected to its own step-up transformer. The secondary (HV) sides of the step-up transformers are typically paralleled. In such a parallel configuration, all the generators in parallel at the power station will accelerate when there is a fault on one of the transformer’s secondary sides.

4.5.2.1 Scenario 1: Fault at generator terminals

When a fault occurs at Fault zones 1, or 2 (Figure 4.2), the generator and transformer differential protection should operate immediately (D1, D2). It will therefore not be required to build in a pole-slip algorithm for faults in zones 1 and 2.

4.5.2.2 Scenario 2: Fault at step-up transformer HV side

When a fault occurs at the step-up transformer secondary (HV) side (Fault zone 3), all the generators in parallel will contribute equally to the fault. The speed increase of the generators will depend on the pre-fault active power loading of each generator. If one generator pole-slips after the fault is cleared, it does not necessarily mean that the other generators in parallel will also pole-slip. The generator, which had the highest pre-fault active power load, will be most likely to pole-slip after the fault is cleared. The most severe pole-slipping occurs when one generator on a bus loses synchronism with the grid, while the other generators are still in synchronism with the grid. That means the generator that lost synchronism is connected via a very small impedance to the “rest of the network”, which is the generators in parallel to it. There will be no transmission line impedance between the unstable generator and the paralleled generators, which will cause very severe torque pulses on the unstable generator rotor.
It is difficult to determine whether the frequency of the whole network will increase or decrease during a fault on the HV side of a step-up transformer. When the line impedance between the faulted generators and another power station is small, it might happen that the frequency of the whole network increases during the fault. The frequency will increase because the generators of the nearby healthy power station will feed mainly reactive power into the fault at the faulted power station, and will not deliver full active power anymore.

When there are no power stations near the faulted power station, the frequency of the whole network is likely to decrease since more active power will be demanded from the healthy distant power station loads to compensate for the loss in active power at the faulted power station. When the transmission line impedance between the faulted power station and another power station is large, the generators are more likely to fall out-of-step with the rest of the network.

### 4.5.3 Power Swings

Power swings occur when the power transfer angle of two generation units in a network oscillates with respect to each other as was discussed in section 3.3. The most common cause of power swings are faults that occur in the network. Switching of large loads can also cause power swings. The probability of a power swing is higher when the line impedance between the two generation units is high.
4.5.4 LOSS-OF-EXCITATION

During loss-of-excitation, the pull-out torque of the rotor, with respect to the stator, will be reduced. If the excitation is not restored, the generator will fall out-of-step. Induced currents in the rotor damper windings will cause the rotor to overheat if the generator is not tripped soon after excitation is lost.

4.6 ALGORITHM FOR NEW POLE-SLIP PROTECTION FUNCTION

The proposed pole-slip protection algorithm that can trip a synchronous machine before it experiences a damaging pole-slip is presented in the remainder of this chapter. The algorithm consists of Steady-State (pre-fault) and Transient (during-fault and predicted post-fault) calculations. A stability check is continuously performed, which determines if the generator must be tripped.

Since a power system can be modelled in PSCAD, it was the ideal software to validate the effectiveness of the new pole-slip algorithm that runs in parallel to the power system simulation. When the PSCAD power system simulation indicates that a generator is about to fall out of step, the new pole-slip logics (built in PSCAD) must issue a trip before the generator becomes unstable. Chapter 5 discusses the PSCAD validation of the new pole-slip function in detail, while chapter 6 discusses the testing of the new pole-slip function with the new logics programmed into an ABB REM543 relay. An RTDS was used to test the relay with the new logics. Only some parts of the newly developed PSCAD and ABB relay logics are shown in this chapter to assist in clarifying the design methodology.

The new pole-slip protection function logics are available on the attached CD in Appendix D in PSCAD format as well as the ABB REM543 relay format. The input parameters for the new pole-slip protection function are as follows:

**Generator parameters:**

- $S_{\text{base}}$ [MVA] Three phase MVA rating of the generator
- $V_{\text{base}}$ [kV] Line to line voltage rating
- $X_d$ [pu] Direct-axis reactance
- $X'_d$ [pu] Direct-axis Transient reactance
- $X_q$ [pu] Quadrature-axis reactance
- $X_{tx}$ [pu] Transformer reactance
- $H$ [s] Inertia (generator, turbines and coupling / gearbox)
- $f$ [Hz] Nominal Frequency
- $p$ Number of pole pairs
- Rotor Type Salient pole / Round Rotor
**Network parameters:**

- $Z_{\text{line}}$ [pu]: Transmission line impedance to closest other power stations ($R_l + jX_l$)
  - $R_l$: Transmission line resistance
  - $X_l$: Transmission line reactance

From the above inputs, the generator base current is calculated:

$$I_{\text{base}} = \frac{S_{\text{base}}}{\sqrt{3} \cdot V_{\text{base}}}$$  \hspace{1cm} (4.1)

### 4.7 Steady-state (Pre-Fault) Calculations

This section describes the quantities that can be calculated during steady-state conditions before the fault or disturbance occurs. Figure 4.3 shows the steady-state algorithm of the new pole-slip function. The shaded blocks indicate steady-state values that need to be calculated for use in the transient calculations part of the new pole-slip function. The steady state values that need to be exported to the transient calculations section of the algorithm is the pre-fault transfer angle $\delta_q$, the pre-fault EMF $E_q^\prime$ and $X_{q,\text{avg}}$. $X_{q,\text{avg}}$ only needs to be calculated when a round-rotor generator is used as will be explained in section 4.7.4.

![Figure 4.3: Steady State Algorithm for new pole-slip protection function](image)
4.7.1 Calculation of Pre-Fault Transfer Angle

The conventional impedance pole-slip protection relays use the generator subtransient direct-axis reactance $X'_d$ to calculate the impedance locus during power swings. This transfer angle calculation will only be accurate during transient conditions and will not be correct for steady-state operation, since the generator steady-state reactance is larger than $X'_d$. The following equation calculates the actual pre-fault transfer angle.

$$\delta_b = \delta_{gen} + \delta_{trfr} + \delta_{tine}$$  \hspace{1cm} (4.2)

where

- $\delta_b$ is the steady-state transfer angle between the generator EMF and the infinite bus
- $\delta_{gen}$ is the generator steady-state power angle (between the generator EMF and its terminals) as per Table 2.1.
- $\delta_{trfr}$ is the transformer steady-state power angle
- $\delta_{tine}$ is the pre-fault paralleled transmission line steady-state power angle

The steady-state generator power angle $\delta_{gen}$ is calculated for underexcited and overexcited conditions as was shown in Table 2.1. It is important to note that $\delta_{tine}$ in (4.2) is determined by using the paralleled impedance of all the connected transmission lines.

As an example, Figure 4.4 shows the newly developed logics for the ABB REM543 relay that will calculate the steady-state generator power angle $\delta_{gen}$. The rest of the ABB logics will not be shown in this chapter.

The complete set of the new ABB relay pole-slip logics is available in Appendix A.

Figure 4.4: ABB REM543 Logics – Generator Power Angle Calculation
The transformer power angle $\delta_{\text{tx}}$ is:

$$\delta_{\text{tx}} = \sin^{-1} \left( \frac{P \cdot X_{\text{tx}}}{V_{\text{pri}} \cdot V_{\text{sec}}} \right)$$  \hspace{1cm} (4.3)

where $P$ is the transformer active power
$X_{\text{tx}}$ is the transformer reactance
$V_{\text{pri}}$ and $V_{\text{sec}}$ are the transformer primary and secondary voltages respectively

The power angle over the transmission line is calculated as was explained in Figure 3.14 and equation (3.24) as follows:

$$\delta_{\text{Rect}} = \alpha + \phi_{\text{corrected}}$$  \hspace{1cm} (4.4)

The pre-fault transfer angle (equation (4.2)) between the generator EMF and the infinite bus will be used in stability calculations and must continuously be stored in a variable $\delta_0$ until the fault occurs.

### 4.7.2 Calculation of Pre-Fault EMF

This section describes the calculations required in the new pole-slip function to determine the pre-fault generator EMF. With armature resistance neglected, the generator internal steady-state EMF ($\bar{E}_q$) is calculated as (refer to Figure 2.21):

$$\bar{E}_q = \bar{V}_a + jI_a X_d + jI_q X_q$$
$$= jV_d + jV_q + jI_d X_d + jI_q X_q$$  \hspace{1cm} (4.5)

As stated earlier, the machine power angle is defined as the angle between the EMF and the machine terminal voltage $V_a$ and is denoted by the symbol $\delta$. The power factor angle ($\Phi$) is the angle between the terminal voltage $V_a$ and the line current $I_a$.

The voltage and current phasors drawn in their d- and q-axis vector components are as shown in Figure 2.21. Since the EMF is located on the q-axis, the sum of the q-axis components alone can determine the EMF.

Sum of q-axis components (Figure 2.21):

$$V_q + I_q X_q = |\bar{E}_q|$$  \hspace{1cm} (4.6)

The pre-fault EMF $E_q$ can be determined by simply using equation (4.6) during overexcited and underexcited scenarios.
4.7.3 Calculation of Generator Transient EMF (\(E'_q\))

The previous section discussed the calculations required to determine the pre-fault generator EMF \(E_q\). This EMF is not useful in the pole-slip function, since the transient EMF \(E'_q\) that is the voltage behind \(X'_d\) is required to do transient calculations during the fault. Figure 4.5 shows a synchronous machine block diagram with subtransient effects neglected (also refer to section 2.10). The following equations describe the block diagram in Figure 4.5:

\[
E_i = i_d \cdot (X_d - X'_d) + E'_q \\
\therefore E'_q = E_i - i_d \cdot (X_d - X'_d)
\]

where \(i_d\) is the pre-fault (steady-state) direct-axis current

The voltage \(E_i\) is equivalent to the excitation voltage \((E_{fa})\):

\[
E_i = E_{fa} = X_{ad} \cdot i_{fd}
\]

where \(i_{fd}\) is the field winding current

The steady-state field current \(i_{fd}\) can be calculated as follows [15:94]:

\[
i_{fd} = \frac{v_a + R_a i_d + X_d i_d}{X_{ad}}
\]

The voltage \(E_{fa}\) in Figure 4.5 is the excitation voltage applied to the field winding by the excitation system. The following voltages are equal during the steady-state condition:

\[
E_{fa} = E_i = E_q
\]

where \(E_q\) is the steady-state quadrature-axis EMF as calculated by (4.6).
In summary, the transient EMF $E'_q$ can be calculated as follows:

$$E'_q = E_q - i_d \cdot (X_q - X'_d) \quad (4.10)$$

with $E_q$ calculated by equation (4.6).

$i_d$ is the pre-fault (steady-state) direct-axis current.

Substitution of equation (4.6) into (4.10) provides the following:

$$E'_q = V_q + i_d X'_d \quad (4.11)$$

The transient EMF $E'_q$ is the voltage behind the reactance $X'_d$, while $E_q$ is the voltage behind the reactance $X_d$.

### 4.7.4 Determination of $X_{q\text{-avg}}$ for Round Rotor Machines

Round rotor machines and salient pole machines are modelled differently on the q-axis as was explained in section 2.6.8. It was shown in section 2.6.9.2 that salient pole machines are modelled with a reactance $X'_q = X_q$ in transient calculations, whereas round rotor machines are modelled with an $X'_q$ which is
smaller than $X_q$. An extra time constant $T'_{qo}$ is also used in round rotor machines to describe the time characteristics of the transient reactance $X_q$. It was found with by simulations in PSCAD that the quadrature axis reactance value of salient pole machines in the post-fault period is equal to its $X_q$ parameter. This was observed by plotting $V_d/l_q$, where $V_d$ and $l_q$ are the actual values from the PSCAD synchronous machine model. The same graph was plotted for round rotor machines, from which was observed that $V_d/l_q$ ranges from values as small as $X_q$ to values larger than $X_q$.

The aim in this section is to determine an average quadrature axis reactance for round rotor machines during the post-fault period where the generator remains marginally stable, which can be described as follows:

$$X_{q,\text{avg}} = \frac{|V_d|}{|l_q|}$$  \hspace{1cm} (4.12)

Figure 4.6 shows a phasor diagram for a salient pole generator in the transient state, where:

$$\tilde{l}_q = \tilde{l}_q \cos(\delta_{\text{gen}} + \phi)$$
$$\tilde{V}_d = \tilde{V}_d \sin(\delta_{\text{gen}})$$  \hspace{1cm} (4.13)

Figure 4.7 shows the q-axis models of a round rotor and salient pole machine. The phasor diagram in Figure 4.6 is not valid for a round rotor machine during transient conditions, since the vectors indicate that $V_d = l_qX_q$, whereas it can be seen from Figure 4.7 that $V_d \neq l_qX_q$ for round rotor machines in the transient state.

Figure 4.6: Transient Phasor diagram for an overexcited generator (neglecting Ra)
$X_{q,\text{avg}}$ is required to determine the generator transient power angle $\delta_{\text{gen}}$ as will be shown in equation (4.56). Equation (4.12) is therefore not practical to determine $X_{q,\text{avg}}$, since $V_o$ and $I_q$ also depends on the transient generator power angle.

After various PSCAD simulations, it was found that a generator power factor is close to unity after the fault is cleared for a marginally stable fault scenario. This is so because the generator has to supply maximum active power in order to decelerate after the fault is cleared. Minimal reactive power (compared to active power) is supplied in the post-fault period. It can therefore be assumed that the power factor angle $\phi$ in equation (4.12) is close to 0 degrees in the post-fault period.

It can be seen from (4.12) that $I_q$ will approach zero as $\delta_{\text{gen}}$ approaches $90^\circ$ (since $\phi$ is assumed to be $0^\circ$ in the post-fault period). The increase in rotor speed will be greater during a fault for a larger generator pre-fault active power ($P_o$). A larger $P_o$ will therefore result in a larger $\delta_{\text{gen}}$ in the post-fault stage. Since $I_q$ will be smaller with a larger $\delta_{\text{gen}}$, it follows from (4.12) that $X_{q,\text{avg}}$ will be larger with a larger $\delta_{\text{gen}}$ (hence $X_{q,\text{avg}}$ will be larger with a larger $P_o$).

It was confirmed with various simulations that $X_{q,\text{avg}}$ can be approximated as follows in the post-fault period after a fault duration that causes the generator to remain marginally stable:

$$X_{q,\text{avg}} = X_q \cdot P_o \quad (4.14)$$

where $P_o$ is the generator pre-fault active power

Equation (4.14) is only valid when $X_q$ and $P_o$ are both given in per-unit.
A generator will be more stable with a larger $X_{q_{\text{avg}}}$. Figure 4.8 defines the post-fault window of importance for the predicted $X_{q_{\text{avg}}}$. The post-fault window of importance covers the period from the instant when the fault is cleared until the time when the speed deviation is back to zero. When the speed deviation is back at zero, it means the generator is stable and no further stability calculations are necessary.

The predicted average value $X_{q_{\text{avg}}}$ (indicated as “$X_{q_{\text{prime algorithm}}}$” in the graphs) is approximately the average of the actual fluctuating $X_{q_{\text{calc}}}$ within the post-fault window of importance. $X_{q_{\text{calc}}}$ is calculated by using equation (4.12). The actual (accurate) PSCAD transient power angle and voltage and current magnitudes were used to calculate the “$X_{q_{\text{calc}}}$” parameter in Figure 4.8.

Figure 4.9 and Figure 4.10 show other generators with different $X_q$ parameters and different pre-fault active powers. These graphs are zoomed into the post-fault window of importance. The pre-fault generator power in Figure 4.9 was $P_{\text{gen}} = 1$ pu. This means $X_{q_{\text{avg}}}$ is predicted to be equal to $X_q$ from equation (4.14).

The simulation in Figure 4.10 had a pre-fault $P_{\text{gen}} = 0.6$ pu, hence $X_{q_{\text{avg}}} = 0.6 \times X_q$. The simulated generator of Figure 4.10 had an $X_q' = 0.49$ pu and $X_q = 2.02$ pu. It is interesting to note that $X_{q_{\text{calc}}}$ can vary between values, as small as $X_q'$, to values larger than $X_q$. This phenomenon was confirmed with MATLAB simulations of the round rotor model presented in Figure 4.7.

Six different generators were simulated on five different power system scenarios (30 simulation in total) to test the accuracy of the new pole-slip function. In all the simulations the $X_{q_{\text{avg}}}$ value was accurately predicted for use in the equal area criteria (refer to chapter 5).

The value of $X_{q_{\text{avg}}}$ is not calculated accurately for fault durations shorter than what is required to put the generator in a marginally stable / unstable scenario. Fault durations shorter than the minimum duration that could cause the generator to remain marginally stable are of no importance to stability calculations, since the pole-slip function will refrain from tripping for these (short/stable) fault scenarios. The methodology developed of predicting $X_{q_{\text{avg}}}$ proved to be working accurately for all fault scenarios where the generator remained marginally stable (which are fault durations approximately 10 ms shorter than unstable faults) and for faults where the generator became unstable.
Figure 4.8: $X_{q,avg}$ prediction – postfault window of importance

Figure 4.9: $X_{q,avg}$ prediction for pre-fault $P_{gen} = 1 \text{ pu}$

Figure 4.10: $X_{q,avg}$ prediction for pre-fault $P_{gen} = 0.6 \text{ pu}$
4.8 Transient (During-Fault and Post-fault) Calculations

4.8.1 Introduction

This section describes the calculations that occur during the transient period. The term “during-fault” is referred to as the period during which the fault occurs, while “post-fault” refers to the period directly after the fault is cleared (i.e. the period directly after the transmission line protection trips the faulted line).

Figure 4.11 shows the block diagram of the transient calculations of the new pole-slip function. The transient pole-slip function must be repeated at a cycle time of maximum 5 ms for pole-slip tripping to be accurate. It must be noted that every block in Figure 4.11 is regarded as a different function in the algorithm. Every function will be executed once in the 5 ms cycle time, except for the iterative function block. The iterative function will undergo five iterations within every 5 ms cycle time to ensure the generator and transformer power angles converge to an accuracy of typically 99.5%.

![Figure 4.11: Transient calculations of new pole-slip algorithm](image)
4.8.2 FAULT DETECTION AND FAULT-CLEARANCE

The first step in the pole-slip algorithm is to detect when a fault occurs, which will cause the generator to accelerate. It is also just as important to detect when the fault is cleared. It is easy to detect a fault by observing the generator current. A protection relay current measurement function block has a typical time delay of 10 ms to 20 ms. In order to detect a fault as quickly as possible, a current increase above 1.2 pu is regarded as a “fault-detected”.

To detect the clearance of a fault is not so easy, since the line current does not drop to the pre-fault value directly after the fault is cleared. In fact, the current can even increase (and terminal voltage decrease) after the fault is cleared due to the rotor inertia. The rotor inertia will tend to keep the rotor above synchronous speed after the fault is cleared, which means the rotor angle still increases after the fault is cleared. A larger rotor angle with respect to the infinite bus means that the current will increase and generator terminal voltage will decrease.

During a fault, Area 1 (refer to Figure 2.7) of the equal area criteria will increase, while Area 2 decreases with time. When the fault is cleared, Area 1 will not increase any further. In order to detect a fault clearance, Area 1 of the equal area criteria must be observed in the pole-slip algorithm. If Area 1 does not increase between one logic cycle and the next, it means that the fault is cleared. The equal area criteria is discussed in section 4.8.12.

Figure 4.12 shows the new PSCAD logics (similar to the new ABB relay logics in Appendix A) for the fault-detected and fault-cleared algorithm by using Area 1. A simpler philosophy was followed when the relay logics were built. The fault-cleared algorithm only observed the generator active power. If the active power increased to the value before the fault occurred, a “fault-cleared” signal is generated. This caused complications since the power measurement function block had a 20 ms to 40 ms time delay. Although the relay logics fault-cleared algorithm worked with reasonable accuracy, the Area 1 method in the PSCAD logics was found to be more accurate. This Area 1 method is the method to be used for the new pole-slip protection function.
4.8.3 **CALCULATION OF ROTOR SPEED INCREASE DURING A FAULT**

After a fault is detected, the generator rotor speed will typically increase. The pole-slip function must accurately determine how much the rotor is accelerating in order to calculate the rotor angle increase with respect to the infinite bus.

The rotor speed can be theoretically determined by measuring the voltage frequency on the generator terminals. The rotor speed deviation $\Delta \omega_{\text{rotor}}$ is calculated as follows:

$$
\Delta \omega_{\text{rotor}} = \frac{2\pi f_p}{2\pi f_y} - \frac{1}{\frac{f}{f_y}} - 1
$$

\hspace{1cm} (4.15)
While testing the REM543 relay on the RTDS, it was found that the voltage frequency could not be properly measured during fault scenarios close to the generator terminals. The reason for the inaccurate measurement was due to distortions in the voltage signal during fault conditions close to the step-up transformer HV terminals.

An alternative way to determining the generator speed is by using the inertia constant $H$. $H$ is expressed in seconds (MW·sec/MVA or MJ/MVA). Section 2.4 explains the use of the $H$-factor in detail.

$H$ must include the inertias of both the generator and its prime-mover (or the inertias of the motor and its load). For a realistic machine, $1 \leq H \leq 9$. It can be computed from the following relation:

$$ H = \frac{5.4831 \times 10^{-9} \cdot J \cdot n^2}{S_{kVA}} \quad (4.16) $$

The inertia ($J$) of the generator rotor is given as follows:

$$ J = m \cdot R^2 \quad (4.17) $$

The acceleration of a machine can be calculated as:

$$ T_m - T_e = \alpha \cdot J \quad (4.18) $$

From (4.18), the speed increase of a generator during a fault can be determined as follows (see section 2.4 for derivation):

$$ \omega = \frac{\omega_{base}}{2H} \int (T_m - T_e) \, dt + \omega_o \quad (4.19) $$

where $\omega_o$ is the speed of the generator before the fault occurred

$\omega_{base}$ is the synchronous speed of the generator

The electrical torque can be approximated by using synchronous speed in the following calculation:

$$ T_e = \frac{P_e}{\omega_o} \quad (4.20) $$

where $P_e$ is the active electrical power during the fault

The approximated electrical torque in (4.20) will be greater than the actual electrical torque when the generator speed increases. This makes the stability calculations conservative towards not giving spurious trips. Synchronous machines will typically not accelerate more than 2% of nominal speed during a fault,
which means the torque calculation in (4.20) will be reasonably accurate during these overspeed scenarios.

Figure 4.13 shows the speed deviation (rad/s) of a generator rotor due to an electrical fault close to the generator. The fault occurs at $t_0 = 10 \text{ s}$ and is cleared at $t_c = 10.24 \text{ s}$. The speed increase of the generator is approximately linear during the fault and it reaches a value of $\Delta \omega_{\text{max}}$ at $t_c$.

By integrating the speed deviation curve during the fault interval, the rotor angle deviation $\Delta \delta_{\text{rotor}}$ is obtained as follows:

$$
\Delta \delta_{\text{rotor}} = \int_{t_0}^{t_c} \Delta \omega \cdot dt
$$

(4.21)

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{speed_deviation.png}
\caption{Generator speed deviation due to an electrical fault}
\end{figure}

The fault cycles in the relay logics are counted for every logic cycle while the fault is detected. With the relay logic cycle time at $5 \text{ ms}$, the fault duration is the number of fault cycles multiplied with $5 \text{ ms}$.

4.8.4 Why Post-Fault Terminal Voltages Need to Be Predicted

Generators in parallel can cause the generator under consideration (say Generator 1) to be more stable or less stable after a fault. It all depends on what the transient EMFs of the generators in parallel are.

If the transient EMF of a generator in parallel is less than the infinite bus voltage, the generator in parallel will tend to consume reactive power from the grid after the fault is cleared. This will cause the terminal voltage of Generator 1 to decrease and reduce stability. On the other hand, generators in parallel can improve the stability of Generator 1 if the paralleled generators are lightly loaded (in terms of active
power) before the fault. After the fault, these parallel generators can supply reactive power, which will increase stability.

During various PSCAD simulations, it was found that if the electrical centre is located in the step-up transformer of a generator in parallel, the parallel generator might be able to supply reactive power to the transformer primary side, but the reactive power will not flow out of the transformer HV terminals. All the reactive power will be consumed in the transformer due to the effect of a short-circuit caused by the electrical centre in the transformer (refer to section 3.4).

This will happen especially if the parallel generators operated at full load before the fault, which will cause a greater rotor acceleration during the fault. This causes a greater transfer angle with respect to the infinite bus. Even after the fault is cleared, the generator will still rotate above synchronous speed (although it is decelerating). The operation above synchronous speed (while decelerating) still causes the transfer angle to increase. The greater the transfer angle becomes, the greater is the reactive power losses and subsequent voltage dip on the generator terminals. Stability reduces with a reducing voltage magnitude on the generator and transformer terminals.

The post-fault terminal voltages of the generator and step-up transformer are also required for calculating Area 2 of the equal area criteria. These post-fault voltages need to be predicted while the fault occurs.

### 4.8.5 Calculation of “Post-fault” Rotor Angle Increase and Correction Factor

This section discusses the function in the pole-slip algorithm that predicts how much the rotor angle will increase during the fault and also how much the rotor angle will increase after the fault is cleared. Figure 4.14 shows a typical rotor angle increase during a fault with the terms “Faulted” and “Post-fault” clearly illustrated. During the “post-fault” period, the rotor is still above synchronous speed due to inertia. This causes the transfer angle to increase during the “post-fault” period. With a larger transfer angle, larger current will flow, which will cause a greater voltage drop on the generator and transformer terminals.

The “post-fault” voltages are important to predict (due to the post-fault rotor inertia) while the fault occurs, since these voltages are used in the equal area criteria to determine generator stability. The rotor kinetic energy increase during the fault must equal the kinetic energy decrease after the fault is cleared. The rotor angle increase during the post-fault period can be determined by using the calculated Area 1 in Figure 4.15. The same area will be present on the Area 2 – side if stability is maintained. This area is indicated as the post-fault area in Figure 4.15. The area on the Area 2 – side will be approximated to be that of a rectangle and a triangle. The rectangle will have a vertical side of length $P_{\text{elec}} \left( \delta_e \right)$, i.e. the value of
active power at the instant that the fault is cleared. The horizontal side of this rectangle will represent the rotor angle increase after the fault is cleared, or $\delta_{\text{max}}$.
Due to the assumption of a rectangular area, $\delta_{\text{max}0}$ will be smaller than the true $\delta_{\text{max}}$. The green triangle in Figure 4.15 is not included in the post-fault power area, and must therefore be added to the rectangle area as shown in Figure 4.15.

The maximum rotor angle $\delta_{\text{max}}$ after the fault is cleared is:

$$\delta_{\text{max}0} = c_1 \cdot \frac{\text{Area}}{P_{\text{elec}}(\delta_{\text{c}}) - P_{\text{mech}}}$$

$$\delta_{\text{max}} = \delta_{\text{max}0} + c_1 \cdot \frac{(P_{\text{elec}}(\delta_{\text{c}}) - P_{\text{elec}}(\delta_{\text{max}0})) \cdot (\delta_{\text{max}0} - \delta_{\text{c}})}{2 \cdot (P_{\text{elec}}(\delta_{\text{max}0}) - P_{\text{mech}})}$$

where $P_{\text{elec}}(\delta_{\text{c}})$ is the power transfer at the instant that the fault is cleared

$P_{\text{mech}}$ is the pre-fault mechanical prime mover power

$c_1$ is a round rotor correction constant

It was discovered that round-rotor synchronous generators are more stable than what the new pole-slip function initially estimated without the correction constant $c_1$. This inaccuracy in the pole-slip function for round-rotor generators is due to the use of equations that neglect saliency.

Section 4.8.7 presents a method to predict the post-fault voltages on the generator and transformer terminals. It will be shown in section 4.8.7 that a Thévenin current is calculated in equation (4.32), which is only accurate if saliency can be neglected. Since saliency cannot be neglected (especially not for round rotor machines in the transient state), the current of round rotor machines in a real power system will be larger than that calculated by equation (4.32). The transient current is larger, since the transient power is larger due to saliency as was shown in Figure 2.20.

The result is that the post-fault voltages that are determined from this Thévenin current are calculated to be smaller for round rotor machines than what they actually are in a real power system. Since stability decreases with a decreased post-fault terminal voltage, the pole-slip function will predict that round rotor generators are less stable than what they actually are in a real power system. For that reason the correction factor is required in equation (4.23) for round rotor machines.

For a correction factor less than 1, equation (4.23) will calculate a smaller increase in post-fault rotor angle, which means a smaller Thévenin current will flow, which will result in less voltage drop on the generator and transformer terminals. The calculated voltage that is larger after the correction factor is included, will cause the new pole-slip algorithm to predict that the generator is more stable (which would be the case in a real power system).
It was illustrated in Figure 2.20 that the transient power angle curve of a synchronous machine is defined by the following equation:

\[
P = \frac{E_d' \cdot V_{\text{gen}}}{X_d} \cdot \sin \delta + \frac{V_{\text{gen}}^2}{2} \left( \frac{1}{X_q} - \frac{1}{X_d} \right) \cdot \sin(2\delta)
\]  

(4.24)

where \( \frac{E_d' \cdot V_{\text{gen}}}{X_d} \cdot \sin \delta \) is the fundamental term

\[
\frac{V_{\text{gen}}^2}{2} \left( \frac{1}{X_q} - \frac{1}{X_d} \right) \cdot \sin(2\delta)
\]

is the saliency term.

Equation (4.24) is used in the equal area criteria to determine generator stability as will be explained in section 4.8.12. For round rotor machines, \( X_{q,\text{avg}} \) needs to be used in equation (4.24) instead of \( X_q \) as was explained in section 4.7.4. When a round rotor generator operates at 1 pu pre-fault power \( P_o \), \( X_{q,\text{avg}} \) will be equal to the round rotor machine \( X_q \) as per equation (4.14). \( X_{q,\text{avg}} = X_q \) for round rotor machines will be considerably larger than the \( X_q \) parameter of a salient pole machine (refer to Table 5.1 for the typical range of salient and round-rotor generator \( X_q \) parameters). Therefore, since the magnitude of \( X_d' \) for round rotor and salient pole machines are typically in a similar range, the saliency term in equation (4.24) will be larger for round rotor machines than what it will be for salient pole machines in the transient state.

When round rotor generators operate at a pre-fault power \( P_o \) of less than 1 pu, the \( X_{q,\text{avg}} \) value will be smaller (as per equation (4.14)) than the value it would have when the generator operates at \( P_o = 1 \) pu. With a smaller \( X_{q,\text{avg}} \), the round rotor generator will behave closer to a salient pole generator in terms of the transient power curve. Figure 4.16 shows round rotor machine transient saliency power curves for \( P_o = 1 \) pu and \( P_o = 0.25 \) pu. A salient pole machine will have a transient saliency curve closer to the \( P_o = 0.25 \) pu curve, no matter what the pre-fault loading of the salient pole machine was. The correction factor can be adjusted in terms of the round rotor generator pre-fault loading \( P_o \) as follows:

\[
c_1 = 0.5 \cdot [2 - P_o]
\]  

(4.25)

The equation above is designed to allow for correction factors as per the table below:

<table>
<thead>
<tr>
<th>Pre-fault power ( P_o )</th>
<th>Correction factor ( c_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.5</td>
</tr>
<tr>
<td>0.75</td>
<td>0.625</td>
</tr>
<tr>
<td>0.5</td>
<td>0.75</td>
</tr>
<tr>
<td>0.25</td>
<td>0.875</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

Salient pole machines were tested successfully with a correction factor of \( c_1 = 1 \), which means no compensation for saliency is required for salient pole machines in the transient state. It is important to
note that round rotor machines have less saliency than salient pole machines during steady state, but the effect of saliency is greater on round rotor machines during transient conditions than what the effect of saliency is on salient pole machines during transient conditions.

Figure 4.16: Round Rotor Generator Power Curves in Transient State

4.8.6 Thévenin Circuits for Voltages and Power Angle Calculations

Thévenin equivalent circuits must be developed in order to solve complex circuits as part of the new pole-slip protection function. The Thévenin theory is used extensively in this section, and is therefore briefly reviewed below. Any combination of sources (generators) and impedances can be replaced by a single voltage source $V_{th}$ and a single series impedance $Z_n$, as shown in Figure 4.17. The value of $V_{th}$ is the open circuit voltage at the terminals. The value of $Z_n$ is the effective impedance over A and B with the voltage source short-circuited.

Figure 4.17: Review of the Thévenin-theory
The circuits in Figure 4.18 represent a typical power system of a generator under consideration (Generator 1) with a generator in parallel (Generator 2). This power system circuit must be simplified to a Thévenin circuit in order to calculate the voltage magnitudes and angles of Generator 1 and Transformer 1. These voltage magnitudes and angles will be used in the equal area criteria to predict stability of the generator while the fault occurs.

Figure 4.18: Typical power system Thévenin circuit simplification for pole-slip function

Figure 4.19 shows the different steps required to solve the Thévenin circuits. The steps shown in Figure 4.19 are explained as follows:

**Step 1:**

Consider only the generator in parallel (Generator 2) with the shunt load connected, but with the transmission lines disconnected. Calculate the voltage $\bar{V}_{tm} \angle \delta_{tm}$ and Thévenin impedance $\bar{Z}_{tm}$ for use in the next step. It is important to note that the transmission line impedance $\bar{Z}_{line}$ in Figure 4.19 is the paralleled impedance of all the transmission lines connected to the power station, except for the faulted transmission line. It is assumed that the faulted transmission line will be tripped by the line protection
relay. After the fault is cleared, the generator current (and consequent voltages on the different buses) is calculated by excluding the faulted transmission line impedance.

**Step 2:**

Use $\bar{V}_m \angle \delta_m$ and $\bar{Z}_m$ as calculated in Step 1 with the transmission line connected. Generator 1 remains disconnected during this step. Calculate $\bar{V}_m \angle \delta_m$ and $\bar{Z}_m$ for use in the next step.

**Step 3:**

Use $\bar{V}_m \angle \delta_m$ and $\bar{Z}_m$ as calculated in Step 2 with Generator 1 connected. It is now possible to calculate the voltage magnitudes and voltage angles on the terminals of Generator 1 and Transformer 1 ($\bar{V}_{gen} \angle \delta_{gen}$ and $\bar{V}_{tx} \angle \delta_{txHV}$). The angle $\delta_{gen}$ is the internal power angle (with saliency neglected) of Generator 1. $\delta_{txHV}$ is the voltage angle difference between the EMF $\bar{E}_q \angle \delta_{q2}$ of Generator 1 and the voltage angle on the HV terminals of Transformer 1. It was proven with various PSCAD simulations that $\delta_{txHV}$ is accurate enough for stability calculations. $\delta_{gen}$ was calculated without taking the effect of saliency into account. Therefore $\delta_{gen}$ and $\delta_{txHV}$ will be used in an iterative calculation algorithm to calculate the accurate transient power angle of the generator (with the effect of saliency included).

![Diagram of Thévenin circuit simplification process](image)

**Figure 4.19:** Steps in the Thévenin circuit simplification process
The abovementioned steps must be followed for “during-fault” and “post-fault” voltage angles and magnitudes. The generator EMF angles with respect to the infinite bus $\delta_{\text{fault}}^1$, $\delta_{\text{fault}}^2$, $\delta_{\text{postfault}}^1$ and $\delta_{\text{postfault}}^2$ are used as is shown in pole-slip algorithm transient block diagram Figure 4.11 (and as is described below):

*Use $\delta_{\text{postfault}}^1$ and $\delta_{\text{postfault}}^2$ to achieve the following:*

- Predict post-fault voltages magnitudes on Generator 1 ($V_{\text{gen1,postfault}}$) and Transformer 1 HV terminals ($V_{\text{tx1,postfault}}$)

- Use these post-fault voltage magnitudes (during the fault) to predict what the size of Area 2 (decelerating area) would be after the fault is cleared in the equal area criteria

*Use $\delta_{\text{fault}}^1$ and $\delta_{\text{fault}}^2$ to achieve the following:*

- Calculate the real-time voltage magnitudes and angles of Generator 1 and Transformer 1.

- Use the real time voltage magnitudes and angles in an iterative calculation to determine the real time generator power angle with the effect of saliency included.

- These real-time voltage angles (with the effect of saliency included) is used in the equal area criteria (together with the post-fault voltage magnitudes) to determine stability

### 4.8.7 Calculation of Expected “Post-fault” Currents and Voltages

This section describes the part of the pole-slip algorithm that can predict the post-fault voltage on Generator 1 and the HV terminals of Transformer 1 by using Thévenin calculations. Area 2 of the equal area criteria is the “decelerating area” after the fault is cleared. The post-fault voltages are only used to predict Area 2 of the equal area criteria.

These voltages are required to be used in the equal area criteria for Generator 1 and Transformer 1 respectively to determine stability after the fault is cleared. For the calculations to follow, it is assumed that the infinite bus voltage is $\overline{V}_{\text{inf}} = 1\angle 0^\circ$.

Figure 4.20 shows the complete network circuit that must be simplified with Thévenin circuits. It is advised to refer back to Figure 4.19 regularly while reading this section, since Figure 4.19 provides a handy overview of the methodology that is followed in this section. Figure 4.21 to Figure 4.23 refer to the same “step numbers” as is shown on Figure 4.19.
The aim is to determine the effective Thévenin equivalent network to which Generator 1 is connected. This Thévenin network consists of the generator in parallel (Generator 2) and the transmission line and shunt loads. The first step is to determine the current of Generator 2 ($I_{th} \angle \theta$) as if only the shunt loads were connected to Generator 2 (refer to Figure 4.21). This current is then used to determine the Thévenin voltage $V_{th} \angle \delta_{th}$.

$$I_{th} \angle \theta = \frac{E_{th} \angle \delta_{postfault}}{jX_{d2} + jX_{t2} + R_{sh} + jX_{sh}}$$ (4.26)

where $\delta_{postfault} = \delta_{max}$ as calculated in (4.23)

It is important to note that $P_{elec}(\delta)$ and $P_{mech}$ in (4.23) is applicable to Generator 2 in this case.

$\delta_{1\_postfault}$ for Generator 1 is also determined from equation (4.23) for calculations to follow later.
From Figure 4.21:

\[
\bar{V}_{\text{th}} \angle \delta_{\text{th}} = E_{\text{q2}} \angle \delta_{\text{q2}_\text{postfault}} - \bar{i}_{\text{th}} \angle \theta \cdot (jX_{d2} + jX_{\alpha2})
\]  

\(\bar{i}_{\text{th}} \angle \theta\) does not include the current of the generator under consideration (i.e. Generator 1). \(\bar{i}_{\text{th}} \angle \theta\) represents only the currents of the generators that are paralleled with Generator 1. In this example, there are only two generators in parallel. Therefore \(\bar{i}_{\text{th}} \angle \theta\) is the current of Generator 2 with the shunt loads included.

![Diagram](image)

**Figure 4.22: “Post-fault” Thévenin Circuit – Step 2**

The voltage \(\bar{V}_{\text{th}} \angle \delta_{\text{th}}\) as calculated in equation (4.27) is used as shown in Figure 4.22 to determine the current \(\bar{i}_{\text{th}} \angle \theta\). This current is equivalent to the current that would flow in the transmission line when Generator 1 is disconnected, but with the shunt loads and Generator 2 connected. In order to calculate \(\bar{i}_{\text{th}} \angle \theta\), the Thévenin impedance \(Z_{\text{th}}\) must be determined as follows:

\[
\bar{Z}_{\text{th}} = \frac{(jX_{d2} + jX_{\alpha2}) \cdot (R_{\text{sh}} + jX_{\text{sh}})}{jX_{d2} + jX_{\alpha2} + R_{\text{sh}} + jX_{\text{sh}}}
\]  

\(\bar{i}_{\text{th}} \angle \theta\) = \(\bar{V}_{\text{th}} \angle \delta_{\text{th}} - \bar{V}_{\text{inf}} \angle 0^\circ\)  

\(\bar{V}_{\text{th}} \angle \delta_{\text{th}} = \bar{V}_{\text{th}} \angle \delta_{\text{th}} - \bar{i}_{\text{th}} \angle \theta \cdot (R_{\text{th}} + jX_{\text{th}})\)

![Diagram](image)

**Figure 4.23: “Post-fault” Thévenin Circuit – Step 3**
The voltage \( \vec{V}_m \angle \delta_m \) as calculated in (4.30) is used as shown in Figure 4.23 to determine the current of Generator 1, namely \( \vec{I}_1 \angle \theta_1 \). In order to calculate \( \vec{I}_1 \angle \theta_1 \), the Thévenin impedance \( \vec{Z}_m \) must be determined as follows:

\[
\vec{Z}_m = \frac{\vec{Z}_m \cdot \vec{Z}_l}{\vec{Z}_m + \vec{Z}_l}
\]

(4.31)

\[
\vec{I}_1 \angle \theta_1 = \frac{\vec{E}_q \angle \delta_{\text{postfault}} - \vec{V}_m \angle \delta_m}{jX_q + jX_m + jX_{\text{th}} + R_{\text{th}}}
\]

(4.32)

The post-fault generator terminal voltage magnitude \( V_{\text{gen1, postfault}} \) and transformer secondary voltage magnitude \( V_{\text{tx1, postfault}} \) required for the equal area criteria is calculated as follows:

\[
V_{\text{gen1, postfault}} = \left| \vec{E}_{q1} \angle \delta_{\text{postfault}} - \vec{I}_1 \angle \theta_1 \cdot jX_{d1} \right|
\]

(4.33)

\[
V_{\text{tx1, postfault}} = \left| \vec{E}_{q1} \angle \delta_{\text{postfault}} - \vec{I}_1 \angle \theta_1 \cdot (jX_{d1} + jX_{\text{tx1}}) \right|
\]

(4.34)

Area 2 of the equal area criteria is the “decelerating area” after the fault is cleared. The post-fault voltages as calculated above are used solely to predict Area 2 of the equal area criteria. Note that although the post-fault voltages are predicted, they are used during the fault to predict what Area 2 will be after the fault is cleared. The post-fault voltages need to be used together with the “during fault” voltage angles to do the equal area calculations.

It is important to note that the calculation of the generator current in (4.32) is only an approximation with the generator saliency effect neglected. Correction factors were introduced in section 4.8.5 to compensate for inaccuracies incurred in neglecting saliency in the Thévenin equivalent circuits.

### 4.8.8 Calculation of “During-Fault” Currents and Voltages

This section describes the part of the pole-slip algorithm that determines the “during-fault” voltage angles on the generator and transformer terminals.

The “during-fault” voltage angles that are calculated in this section are not accurate enough to be used in the equal area criteria, since the Thévenin circuits do not include the effect of generator saliency. The aim of this section is to calculate the voltage angle difference between the EMF of Generator 1 and the HV terminals of Transformer 1 \( \delta_{\text{tx1, HV}} \). The angle \( \delta_{\text{tx1, HV}} \) is effectively the sum of the internal power angles of Generator 1 and Transformer 1. An iterative calculation will be required to determine what portion of
\( \delta_{\text{1,HV}} \) is made up by the generator power angle, and what portion is made up the transformer power angle.

For the calculations to follow, it is assumed that the infinite bus voltage \( V_{\text{inf}} = 1 \angle 0^\circ \). Figure 4.24 shows the complete network circuit that must be simplified with Thévenin circuits. It is advised to refer back to Figure 4.19 regularly while reading this section, since Figure 4.19 provides a handy overview of the methodology that is followed in this section. Figure 4.25 to Figure 4.27 refer to the same “step numbers” as is shown on Figure 4.19.

Figure 4.24: Complete Power System Circuit (Fault Calculations)

The effective Thévenin equivalent network to which Generator 1 is connected must be determined. This Thévenin network consists of the generator in parallel (Generator 2) and the transmission line and shunt loads. The first step is to determine the current of Generator 2 (\( I_{\text{th}} \angle \theta \)) as if only the shunt loads were connected to Generator 2 (refer to Figure 4.25). This current is then used to determine the Thévenin voltage \( V_{\text{th}} \angle \delta_{\text{th}} \) as:

\[
I_{\text{th}} \angle \theta = \frac{E_{\text{2}} \angle \delta_{\text{2, fault}}}{jX_{d2} + jX_{tr2} + R_{\text{sh}} + jX_{\text{sh}}} \quad (4.35)
\]

The angle \( \delta_{\text{2, fault}} \) is the instantaneous angle between the EMF of Generator 2 and the infinite bus during the fault, and is calculated as:

\[
\delta_{\text{2, fault}} = \delta_{0, \text{gen2}} + \Delta \delta_{\text{rotor2}} \quad (4.36)
\]

where \( \delta_{0, \text{gen2}} \) is the pre-fault transfer angle of Generator 2 as calculated by (4.2)

\( \Delta \delta_{\text{rotor2}} \) is the increase in Generator 2 rotor angle due to the fault.
From Figure 4.25:

\[ \vec{V}_n \angle \delta_n = \vec{E}_{q1} \angle \delta_{q1 \text{ fault}} - \vec{I}_n \angle \theta \cdot (jX'_{q1} + jX_{ov2}) \]  \hspace{1cm} (4.37)

\( \vec{I}_n \angle \theta \) does not include the current of the generator under consideration (i.e. Generator 1). \( \vec{I}_n \angle \theta \) represents only the currents of the generators that are paralleled with Generator 1.

\[ \vec{I}_n \angle \theta \] does not include the current of the generator under consideration (i.e. Generator 1). \( \vec{I}_n \angle \theta \) represents only the currents of the generators that are paralleled with Generator 1.

The voltage \( \vec{V}_n \angle \delta_n \) as calculated in (4.37) is used as shown in Figure 4.26, to determine the current \( \vec{I}_n \angle \theta \). This current is equivalent to the current that would flow in the transmission line when Generator 1 is disconnected. The Thévenin impedance \( \vec{Z}_n \) remains the same as is calculated in (4.28):

\[ \vec{I}_n \angle \theta = \frac{\vec{V}_n \angle \delta_n - \vec{V}_{int} \angle 0^\circ}{\vec{Z}_n + R_l + jX_l} \]  \hspace{1cm} (4.38)

\[ \vec{V}_n \angle \delta_n = \vec{V}_n \angle \delta_n - \vec{I}_n \angle \theta \cdot (R_{sh} + jX_{sh}) \]  \hspace{1cm} (4.39)

The voltage \( \vec{V}_n \angle \delta_n \) as calculated in (4.39) is used as shown in Figure 4.27 to determine the current of Generator 1, namely \( \vec{I}_n \angle \theta \). The Thévenin impedance \( \vec{Z}_n \) remains the same as is calculated in (4.31).
The angle \( \delta_{\text{fault}} \) is the instantaneous angle between the EMF of Generator 1 and the infinite bus during the fault, and is calculated as:

\[
\delta_{\text{fault}} = \delta_{0, \text{gen}\, 1} + \Delta \delta_{\text{rotor} 1}
\]  

(4.41)

where \( \delta_{0, \text{gen}\, 1} \) is the pre-fault transfer angle of Generator 1 as calculated by (4.2)

\( \Delta \delta_{\text{rotor} 1} \) is the increase in Generator 1 rotor angle due to the fault

The generator terminal voltage \( V_{\text{gen}\, 1, \text{fault}} \angle \delta_{\text{gen}\, 1} \) and transformer secondary voltage \( V_{\text{tx}\, 1, \text{fault}} \angle \delta_{\text{tx}\, 1, \text{HV}} \) are:

\[
V_{\text{gen}\, 1, \text{fault}} \angle \delta_{\text{gen}\, 1} = \bar{E}_{q 1} \angle \delta_{\text{fault}} - \bar{I}_1 \angle \theta_1 \cdot jX_{q 1}
\]

(4.42)

\[
V_{\text{tx}\, 1, \text{fault}} \angle \delta_{\text{tx}\, 1, \text{HV}} = \bar{E}_{q 1} \angle \delta_{\text{fault}} - \bar{I}_1 \angle \theta_1 \cdot (jX_{q 1} + jX_{x 1})
\]

(4.43)

It must be noted that the angle \( \delta_{\text{gen}\, 1} \) as calculated in (4.42) is not accurate due to the saliency of the generator. Only \( \delta_{\text{tx}\, 1, \text{HV}} \) is considered accurate enough for further use in iterative calculations to determine the actual transient power angle (with the effect of saliency included) of the generator and transformer during the fault. Section 4.8.11 discusses the iterative calculation in detail.

**4.8.9 Thévenin Calculations for Multiple Generators in Parallel**

Before proceeding to the iterative calculations, it is worth noting that the network that is simplified with Thévenin circuits in the previous sections will typically be more complex in real power systems. The preceding sections dealt only with the scenario where one generator is operating in parallel to the generator under consideration (Generator 1). When more than one generator is operating in parallel with Generator 1, the Thévenin equivalent circuit of all the generators in parallel to Generator 1 must be determined as the first step.
Figure 4.28: Thévenin circuit of multiple generators in parallel with Kirchhoff current loop indicated

Figure 4.28 shows a Thévenin circuit with five generators in parallel with Generator 1. Similar calculations can be done for any other number of generators in parallel. The aim is to determine the Thévenin circuit without Generator 1 and without any transmission lines or shunt loads connected to the HV side of the transformers as a first step. The dotted lines in Figure 4.28 show the equipment that does not form part of the Thévenin circuit of step 1.

Two scenarios can be considered, namely with the HV tie-breaker closed or with the HV tie-breaker open.

**Scenario 1: HV Tie-Breaker Closed:**

The Thévenin impedance is calculated with the number of generators \(i\) in parallel as follows. Note the number \(i\) generators includes Generator 1, which means \(i = 6\) in the example of Figure 4.28:

\[
jX_{\text{th, par, gen}} = \left( \frac{1}{j(X_{d2} + X_{x2})} + \frac{1}{j(X_{g3} + X_{x3})} + \ldots + \frac{1}{j(X_{d(i-1)} + X_{x(i-1)})} + \frac{1}{j(X_{d(i)} + X_{x(i)})} \right)^{-1} \tag{4.44} \]
It can be shown with multiple Kirchhoff current loops that \( I_i \) as shown in Figure 4.28 can be expressed as follows with a number \( i \) generators operating in parallel:

\[
I_i = \frac{\bar{E}_{q2} \angle \delta_2 - \bar{E}_{q3} \angle \delta_3 + j(X_{q2} + X_{v2}) \left( \bar{E}_{q2} \angle \delta_2 - \bar{E}_{q3} \angle \delta_3 \right) \ldots + j(X_{q2} + X_{v2}) \left( \bar{E}_{q2} \angle \delta_2 - \bar{E}_{q3} \angle \delta_3 \right) \ldots}{1 + \left( \frac{X_{q2} + X_{v2}}{X_{q3} + X_{v3}} \right) + \left( \frac{X_{q2} + X_{v2}}{X_{q3} + X_{v3}} \right) + \ldots + \left( \frac{X_{q2} + X_{v2}}{X_{q3} + X_{v3}} \right) + \left( \frac{X_{q2} + X_{v2}}{X_{q3} + X_{v3}} \right)}
\]

(4.45)

If all the paralleled generators have identical impedances and if all the transformers have identical impedances, equation (4.45) can be simplified as follows:

\[
I_i = \frac{\bar{E}_{q2} \angle \delta_2 - \bar{E}_{q3} \angle \delta_3 + (i-3) \left( \bar{E}_{q2} \angle \delta_2 \right) - \bar{E}_{q3} \angle \delta_3 - \ldots - \bar{E}_{q(i-1)} \angle \delta_{i-1} - \bar{E}_{q(i)} \angle \delta_i}{2 \cdot j \left( X_{q2} + X_{v2} \right)}
\]

(4.46)

From Figure 4.28, \( \bar{V}_{m_{HV}} \) can be determined as follows:

\[
\bar{V}_{m_{HV}} = \bar{E}_{q2} \angle \delta_2 - I_i \cdot j \left( X_{q2} + X_{v2} \right)
\]

(4.47)

The number \( i \) will be equal to the number of generators in operation. If one or more generators are out of service, the number \( i \) will reduce with the number of generators out of service.

**Scenario 2: HV Tie-Breaker Open:**

When the HV Tie-Breaker is open, only Generators 2 and 3 are considered as parallel generators to Generator 1. Although generators 4 to 6 are located physically next to generators 1 to 3, they are electrically separated by a long transmission line from generators 1 to 3. Generators 4, 5 and 6 therefore form another group of generators that forms part of the infinite bus. The open/closed status of the HV tie-breaker(s) is therefore important to feed back to the pole-slip protection relay.

The Thévenin impedance is calculated as follows:

\[
jX_{m_{par\_gen}} = \left( \frac{1}{j \left( X_{q2} + X_{v2} \right)} + \frac{1}{j \left( X_{q3} + X_{v3} \right)} \right)^{-1}
\]

(4.48)

The Thévenin voltage at the HV bus is calculated as follows:

\[
\bar{V}_{m_{HV}} = \bar{E}_{q2} \angle \delta_2 - \left( \bar{E}_{q2} \angle \delta_2 - \bar{E}_{q3} \angle \delta_3 \right) \left( \frac{X_{q2} + X_{v2}}{X_{q2} + X_{v2} + X_{q3} + X_{v3}} \right)
\]

(4.49)

When more than 2 generators are operating in parallel to Generator 1 on that bus, an equation similar to equation (4.45) can be written, where \( i \) will be the number of generators on that bus.


4.8.10 **Thévenin Circuit for Shunt Loads at Generator Terminals**

The preceding section dealt with the scenario that more than two generators can be connected in parallel at a power station. It was shown how this scenario must be simplified with Thévenin circuits. This section deals with the case when shunt loads are connected at the generator terminals (transformer LV side). Another additional step must be included in the Thévenin circuit calculations to deal with shunt loads at the generator terminals. Shunt loads at the generator terminals are typically “house loads” for power stations.

When there are shunt loads connected at the generator terminals, a Thévenin circuit must be developed for each generator with its shunt load as a first step. The Thévenin impedance will be determined as:

$$ Z_{th\_gen\_shunt} = \left(1 + \frac{1}{Z_{gen\_shunt}} - jX_d \right)^{-1} $$

(4.50)

The Thévenin voltage is:

$$ E_{th\_shunt\_gen} = \frac{E_q}{jX_d + Z_{gen\_shunt}} $$

(4.51)

$$ V_{th\_shunt\_gen} = E_q - \frac{E_q}{jX_d} $$

(4.52)

$V_{th\_shunt\_gen}$ is used instead of $E_q$ in the post-fault equations (4.26), (4.27), (4.32), (4.33) and (4.34) when shunt loads are present at the generator terminals. The use of $V_{th\_shunt\_gen}$ instead of $E_q$ is similarly done for the “during fault” calculations when shunt loads are present at the generator terminals.

4.8.11 **Iterative Calculation of “During-Fault” Voltage Angles**

The preceding sections dealt with simplifying a typical power system with Thévenin circuits to determine the voltage angle between the generator EMF and the transformer HV side terminals. The generator power angle $\delta_{gen}$ and the voltage angle over the transformer $\delta_{tx}$ are required for use in the equal area criteria. These angles could not be accurately calculated by using Thévenin circuits, since the effect of saliency in the generator needs to be taken into account to determine the actual transient generator power angle.

When the transformer has a large reactance, “instability” can occur in the transformer, while the generator internal power angle remains normal. The generator transient reactance is typically larger than the reactance of the step-up transformer, which means it is unlikely that the transformer internal power angle will reach $180^\circ$ while the generator power angle remains normal. Although this is an unlikely
scenario, it was decided to perform two equal area criteria calculations, namely one for Generator 1 and one for the Transformer 1. Since two equal area criteria calculations will be done, both the generator power angle and the transformer internal power angle needs to be calculated. The voltage angle between the generator EMF and the transformer HV side $\delta_{tx_{\text{HV}}}$ (as was determined in the Thévenin circuits) was verified with PSCAD simulations as accurate to use in the calculations to follow.

The initial estimate for $\delta_{\text{gen}}$ is obtained from equation (4.42):

$$\delta_{\text{gen}}(0) = \text{Angle}\left(\overline{V}_{\text{gen1}_{\text{fault}}} \angle \delta_{\text{gen1}}\right)$$

$\delta_{tx_{\text{HV}}}$ is calculated by equation (4.43) as follows:

$$\delta_{tx_{\text{HV}}} = \text{Angle}\left(\overline{V}_{\text{tx1}_{\text{fault}}} \angle \delta_{tx_{\text{HV}}}\right)$$

Note that $\delta_{\text{gen}}(0)$ is the initial estimate of the Generator 1 internal power angle, while $\delta_{tx_{\text{HV}}}$ is the voltage angle difference between the EMF $E'_q$ of Generator 1 and the voltage on the HV terminals of transformer 1.

The initial estimate of the internal power angle of transformer 1 is:

$$\delta_{tx}(0) = \delta_{tx_{\text{HV}}} - \delta_{\text{gen}}(0)$$

Since there exists no explicit solution to the following equations, an iterative solution must be followed in order to determine $\delta_{\text{gen}}$ and $\delta_{tx}$. The generator power ($P_{\text{gen}}$) and transformer power ($P_{tx}$) are determined in terms of the iteration index (i) from Figure 4.27:

$$P_{\text{gen}}(i) = \frac{|E'_q|}{X_d} |V_{\text{gen1}_{\text{fault}}}| \sin(\delta_{\text{gen}}(i)) + \frac{|V_{\text{gen1}_{\text{fault}}}|}{2} \left( \frac{1}{X_q} - \frac{1}{X_d} \right) \sin(2\delta_{\text{gen}}(i))$$

$$P_{tx}(i) = \frac{|V_{\text{tx1}_{\text{fault}}}|}{|jX_{\alpha}|} \sin(\delta_{\alpha}(i))$$

The voltage magnitudes in the above equations are those calculated in equations (4.42) and (4.43), namely $|\overline{V}_{\text{gen1}_{\text{fault}}}|$ and $|\overline{V}_{\text{tx1}_{\text{fault}}}|$. The value $V_{th}$ is as calculated from equation (4.39). The quadrature axis reactance $X_q$ must be replaced with $X_{q\text{avg}}$ as calculated in equation (4.14) for round rotor machines.

$P_{\text{gen}} = P_{tx}$ when resistance losses are neglected. The aim is to find a solution for $\delta_{\text{gen}}$ with $P_{\text{gen}} = P_{tx}$. By substituting (4.55) into (4.57), $\delta_{\text{gen}}$ can be solved. As stated above, there is no explicit solution for $\delta_{\text{gen}}$.
when (4.56) is set equal to (4.57). For that reason an iterative solution must be found for \( \delta_{\text{gen}} \). For an iterative index chosen as \((i)\):

\[
\delta_{\text{gen}}(i) = \left[ \delta_{\text{gen}}(i-1) - (P_{\text{gen}}(i-1) - P_{\text{tx}}(i-1)) \cdot K_1 \right] 
\]

\(4.58\)

\[
\delta_{\text{tx}}(i) = \delta_{\text{txHV}} - \delta_{\text{gen}}(i) 
\]

\(4.59\)

The angle \( \delta_{\text{txHV}} \) used in the above equation is calculated from equation (4.43). The constant \( K_1 \) was tested for various scenarios. It was found that the solution converges the fastest when \( K_1 \) is chosen to be \( K_1 = 7/V_{\text{gen}} \). The choice for \( K_1 \) is valid when the angles are in degrees and the active power \( P \) in per unit.

Figure 4.29 presents a typical convergence curve for the accuracy in \( \delta_{\text{gen}} \) and \( \delta_{\text{tx}} \). The iterative algorithm reaches a solution that is typically 99.5% of the true value within 5 iterations.

It must be kept in mind that the abovementioned iterative process takes place while the fault occurs. After a solution for \( \delta_{\text{gen}}(i) \) and \( \delta_{\text{tx}}(i) \) was found, the instantaneous value of \( \delta_{\text{txHV}} \) as calculated by (4.43), is used to determine \( \delta_{\text{gen}}(i) \) and \( \delta_{\text{tx}}(i) \) again in the iterative process. In parallel with this process, the equal area criteria of the generator and transformer are calculated to determine if instability will occur when the fault is cleared.

The aim is to calculate the value of the generator power angle continuously during the fault to see what the power angle would be at the instant when the fault is cleared. This calculation of the power angle during the fault is used in the equal area criteria to determine stability.
It is important to note that only the value of the power angles during the fault and at the instant when the fault is cleared is of importance. The power angles after the fault is cleared are of no importance, since it is not used in the equal area criteria.

If a scenario is ever detected that causes the iterative solution not to converge, the constant $K_2$ must be adjusted to cater for that scenario as well. In the unlikely scenario that the iterative algorithm does not converge to at least 99% accuracy, the pole-slip function will refrain from tripping, and the conventional pole-slip protection will serve as a backup to trip the generator after it has fallen out of step.

4.8.12 Equal Area Criteria

This section describes the equal area criteria function in the new pole-slip algorithm. The equal area criteria states that the accelerating area under the active power curve $Area\ 1$ must be less than the decelerating area $Area\ 2$ for stability to be maintained as is explained in section 2.5. The mechanical prime-mover power is denoted as $P_0$ in Figure 2.7.

Two separate equal area calculations are performed, namely one for the generator and one for the step-up transformer. In the (unlikely) event that the step-up transformer has an impedance that is almost as high as the generator $X'_d$, it can happen that the electrical centre is in the transformer during a power swing, which means stability can be lost “inside” the transformer. This can be predicted by performing an equal area criteria calculation on the transformer. It must be noted that this is an unlikely event. The generator will typically have a larger $X'_d$ than the transformer reactance and therefore stability is more likely to be lost in the generator, i.e. the generator will pole-slip.

The equal area criteria calculations are done for the generator (with saliency effects included) and for the step-up transformer. If instability is predicted to occur in the generator / step-up transformer, the generator must be tripped before the fault is cleared.

During a fault, the increase in rotor angle $\Delta \delta_{rotor}$ must be added to the pre-fault transfer angle $\delta_0$ to obtain the post-fault transfer angle between the generator EMF and the network infinite bus (refer to section 4.7). The post-fault transfer angle is calculated as:

$$\delta_f = \delta_0 + \Delta \delta_{rotor}$$

where $\Delta \delta_{rotor}$ is the increase in rotor angle due to the fault.
Area 1 is calculated by integrating from the pre-fault transfer angle (δ₀) to the post-fault transfer angle δ_c as determined by equation (4.60). Area 1 for the generator and step-up transformer can be calculated as follows:

\[
Area_1 = \int_{\delta_0}^{\delta_c} (P_0 - P_{\text{fault}}) d\delta
\]
\[
= P_0 [\delta_c - \delta_0] - \int_{\delta_0}^{\delta_c} P_{\text{fault}} d\delta
\]
\[
= \sum_{t_0}^{t_c} [P_0 \cdot \Delta \delta_{\text{rotor}}(t)] - \sum_{t_0}^{t_c} [P_{\text{fault}}(t) \cdot \{\delta(t) - \delta_0\}]
\]
\[
= \sum_{t_0}^{t_c} [P_0 \cdot \Delta \delta_{\text{rotor}}(t)] - \sum_{t_0}^{t_c} [P_{\text{fault}}(t) \cdot \Delta \delta_{\text{rotor}}(t)]
\]
\[
= \sum_{t_0}^{t_c} [(P_0 - P_{\text{fault}}(t)) \cdot \Delta \delta_{\text{rotor}}(t)]
\]

(4.61)

The minimum settable cycle time (or integration step) of an ABB REM543 relay logics is 5 ms. The minimum integration time step as programmed into the relay was therefore 5 ms. The rotor angle difference in one logic cycle during the fault is:

\[
\Delta \delta_{\text{rotor}}(t) = \Delta \omega_{\text{rotor}} \cdot 0.005 \quad (4.62)
\]

where \( \Delta \omega_{\text{rotor}} \) is the rotor speed increase in one logic cycle.

By substituting equation (4.62) into (4.61), Area 1 is determined as follows:

\[
Area_1 = \sum_{t-t_0}^{t-c} [(P_0 - P_{\text{fault}}(t)) \cdot (\Delta \omega_{\text{rotor}} \cdot 0.005)]
\]

(4.63)

The decelerating area (Area_{2_gen}) for the generator is:

\[
Area_{2_{\text{gen}}} = \int_{\delta_{\text{rotor}}}^{\delta_{\text{rotor}} \cdot \Delta \omega_{\text{rotor}} \cdot 0.005} \left[ \frac{E_{\text{avg}} \cdot V_{\text{gen,postfault}}}{X_q} \sin \delta + \frac{V_{\text{gen}}^2}{2 X_q - \frac{1}{X_d}} \sin(2 \delta_{\text{gen}}) \right] \cdot d\delta_{\text{gen}}
\]
\[
= \frac{1}{4} V_{\text{gen,postfault}}^2 \left( 1 - \frac{1}{X_d} \right) \left( \cos(2 \delta_{\text{rotor}}) - \cos(2 \delta_{\text{rotor}}) \right)
\]
\[+ \frac{E_{\text{avg}} \cdot V_{\text{gen,postfault}}}{X_d} \left( \cos(\delta_{\text{rotor}}) - \cos(\delta_{\text{rotor}}) \right) - P_0 (\delta_{\text{rotor}} - \delta_{\text{rotor}})
\]

(4.64)

The voltage \( V_{\text{gen,postfault}} \) is calculated from equation (4.33). Note that the predicted quadrature axis reactance \( X_{q,\text{avg}} \) must be used in (4.64) for round rotor machines instead of \( X_q \). The normal quadrature axis reactance \( X_q \) must be used for salient pole machines (refer to section 4.7.4).
The decelerating area \((\text{Area}_{2_{\text{tx}}})\) for the transformer is calculated as follows:

\[
\text{Area}_{2_{\text{tx}}} = \int_{\delta_{\text{c,tx}}}^{\delta_{\text{c,tx}} - \pi} \left[ \frac{V_{\text{gen}} \cdot V_{\text{tr}_{\text{sec}}}}{X_{\text{tr}}} \sin \delta_{\alpha} - P_0 \right] \cdot \sin \delta_{\alpha} d\delta_{\alpha}
\]

\[
= \frac{V_{\text{gen}} \cdot V_{\text{tr}_{\text{sec}}}}{X_{\text{tr}}} \left( \cos \delta_{\text{i,tx}} - \cos \delta_{\text{i,tx}} - P_0 \left( \delta_{\text{i,tx}} - \delta_{\text{i,tx}} \right) \right)
\]

(4.65)

As an example of how to test the equal area criteria in PSCAD, Figure 4.30 shows the PSCAD logics (similar to relay logics) for the calculation of \(\text{Area} \, 2\) for the generator and the step-up transformer. The variable \(\delta_{\text{c,gen}}\) is represented by the iterative calculated variable “delta_gen_i5_rad” and \(\delta_{\text{c,tx}}\) by “delta_tx_i5_rad”.

It is advised to refer back to Figure 4.11 since this block diagram shows a handy overview of the input and output variables of the equal area criteria function block in the new pole-slip algorithm.
The aim in the remainder of this section is to calculate the maximum rotor angle $\delta_L$ that can be reached (after the fault is cleared) while the generator remains stable. This maximum rotor angle will be higher for machines with saliency than for “theoretical machines” without saliency. The maximum rotor angle must therefore be calculated by including the effects of generator saliency. The angle $\delta_L$ indicated on
Figure 4.32 is the maximum allowable transfer angle (with saliency neglected) for stability to be maintained and can be described as follows:

\[ P_0 = \frac{E_g \cdot V_{gen}}{X_{total}} \cdot \sin \delta \]  

(4.66)

\[ \therefore \delta = 180^\circ - \sin^{-1} \left( \frac{P_0 \cdot X_{total}}{E_g \cdot V_{gen}} \right) \]  

(4.67)

Where \( X_{total} = X_d + X_{network} \)

- \( X_{network} \) is the network impedance from the generator terminals up to the infinite bus
- \( X_d \) is the generator transient direct-axis reactance

There is no explicit solution for \( \delta \) for the saliency power curve in equation (4.56). The generator power curves with saliency included and with saliency neglected are shown in Figure 4.31. A trigonometric solution for \( \delta \) is obtained in Figure 4.32 by using the prime-mover power as 1 pu in this illustration.
A vertical line can be drawn from point B where \( P_{\text{gen,approx}} \) (without saliency) intersects with the prime-mover power, to point A where this vertical line intersects with \( P_{\text{gen,salient}} \). A line can be drawn from 180° (point D) to point A. The intersection with the prime-mover power (point E) can be regarded as \( \delta' \) (the maximum calculated generator angle for stability to be maintained with saliency effects taken into account).

It can be seen from Figure 4.32 that the portion of \( P_{\text{gen,salient}} \) from \( \delta' \) to 180° is close to a straight line. This method of calculating \( \delta' \) was tested for various operating conditions. The error in \( \delta' \) was consistently less than 0.5°. It is assumed that the accuracy in \( \delta' \) is sufficiently accurate for equal area calculations.

The gradient \( m \) of line AD in Figure 4.32 is determined as follows:

\[
m = \frac{AB}{BE} = \frac{AC}{CD} = \frac{180' - \delta}{\delta_{\text{salient}}}\]

\[m = \frac{180' - \delta}{\delta_{\text{salient}}}\]  \hspace{1cm} (4.68)

where \( \delta' \) is determined by equation (4.67)

Following from (4.68):

\[
BE = \frac{AB}{m} = \frac{P_{\text{salient}}(\delta') - P_o}{m}
\]

where \( P_o \) is the prime-mover power \((P_o = 1 \text{ pu in this illustration})\)

\( \delta' \) is calculated as follows:
\[ \therefore \delta_L^* = \delta_L + BE \] (4.70)

By substituting (4.67), (4.68) and (4.69) into (4.70) gives:

\[ \delta_L^* = 180' - \sin^{-1}\left(\frac{P_o \cdot X_{total}}{E_{gen} \cdot V_{ref}^2}\right) + (P_{salienc}(\delta_L) - P_2) \cdot \frac{P_{salienc}(\delta)}{180' - \delta_L} \] (4.71)

Figure 4.33 shows the PSCAD logics required in the pole-slip function to determine the maximum angle \( \delta_L^* \) with generator saliency taken into account. The logics in Figure 4.33 describes equation (4.71).

4.8.13 **Trip Condition**

Figure 4.34 shows the logics ABB relay logics that determines when the generator must be tripped. Note that the ABB REM543 relay was only programmed to check stability for the generator. No equal area criteria logics for the transformer were developed for the relay, although the transformer equal area criteria calculations were fully tested with PSCAD. The equal area states that the machine shall be tripped if the following condition is met for either the generator or the transformer:

Trip if: Area_1 > Area_2
4.8.14 Communication Setup Required for New Pole-Slip Function

The previous sections described the new pole-slip function in detail. This section focuses on the communication required between different pole-slip relays and other electrical equipment to ensure the new pole-slip protection works properly. The typical communication setup that is required for the new pole-slip function is shown in Figure 4.35. The VT and CT connections are hardwired to each relay. A software or hardware link can be used to transfer the tie-breaker status to each relay. A software link is required for transmitting the magnitudes of $E_{q}'$, $\delta_{\text{post-fault}}$ and $\delta_{\text{fault}}$ from each generator protection relay to all the other generator protection relays.

Each generator protection relay needs to know the magnitudes of the $E_{q}'$, $\delta_{\text{post-fault}}$ and $\delta_{\text{fault}}$ variables of all the other generators. However, only $\delta_{\text{post-fault}}$ and $\delta_{\text{fault}}$ is required during transient conditions. $E_{q}'$ is regarded as constant during transient conditions. The pre-fault $E_{q}'$ value for all the generators will be stored in each protection relay for use in transient conditions. The communication link that needs to transmit $\delta_{\text{post-fault}}$ and $\delta_{\text{fault}}$ to and from all the generators needs to update the variables at least every 5 ms to ensure accurate tripping in pole-slip scenarios.

The step-up transformers tap changer setting is important to know, since the voltage base $V_{\text{base}}$ of the transmission lines will change every time the step-up transformer tap changer changes position. Only the tap-changer position of the step-up transformer at the generator is important to know. The tap changer settings of transformers in the network (far away from the power station under consideration) will have a less important effect on stability calculations. The nominal tap position for transformers far away from the power station can be assumed.
4.9 SUMMARY

Chapter 4 presented the design of the new pole-slip protection function. The damaging effects that pole-slip has on synchronous machine rotors and the importance of tripping a machine before it pole-slips were once again emphasized. Some scenarios were discussed, which could cause synchronous motors and generators to pole-slip.

The new pole-slip protection function was designed and explained in detail. Both the steady-state and transient calculations were discussed. The new pole-slip protection function uses the equal area criteria as a basis to predict stability. The equal area criteria is only useful if the post-fault voltages on the generator and transformer terminals as well as the transient power angles of the generator and transformer is known. Since the post-fault voltages are not known during the fault, these voltage magnitudes have to be predicted while the fault occurs.
The Thévenin theory was used to simplify the network with paralleled generators and shunt loads. Protection relay limitations were kept in mind while designing the pole-slip function. Logics for an ABB REM543 relay as well as PSCAD logics were presented to indicate how the new pole-slip algorithm equations could be implemented in typical protection relay logic format.

The transient state vectors presented in textbooks for synchronous machines are typically only valid for salient pole machines. Round rotor generators have a more complex transient power curve characteristic than salient pole machines because of a transient quadrature axis reactance $X_q'$, which is not present in salient pole machine models. An algebraic expression had to be developed that can describe the post-fault power curve characteristics of round rotor machines, which could be used to determine Area 2 of the equal area criteria.

A new parameter ($X_{q\text{avg}}$) was introduced, which is the average value of the transient quadrature axis reactance of round rotor generators during the post-fault period. $X_{q\text{avg}}$ had to be predicted for use in the equal area criteria. It was found that the value of $X_{q\text{avg}}$ is not calculated accurately for faults shorter than what is required to put the generator in a *marginally* stable / unstable scenario. Fault shorter than faults that could cause the generator to remain *marginally* stable is of no importance to stability calculations, since the pole-slip function will refrain from tripping for these (short/stable) fault scenarios. The methodology developed of predicting $X_{q\text{avg}}$ proved to be working accurately for all fault scenarios where the generator remained marginally stable and for faults where the generator became unstable.

Correction factors were introduced to compensate for errors where equations were used during Thévenin circuit simplifications that neglect saliency. It was found that the correction factors only needed to be introduced for round-rotor machines and not for salient pole machines. The reasons for this finding were discussed in detail. The correction factors were designed to be “self-tuning” for different pre-fault generator loading conditions.