Measuring the relationship between intraday returns, volatility spill-overs and market beta during financial distress

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Dissertation submitted in partial fulfillment of the requirements for the degree *Magister Commercii* in Risk Management at the Potchefstroom Campus of the North-West University

Supervisor: Dr A Heymans

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It all starts here ™

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ABSTRACT

The modelling of volatility has long been seminal to finance and risk management in general, as it provides information on the spread of portfolio returns. In order to reduce the overall volatility of a stock portfolio, *modern portfolio theory* (MPT), within an *efficient market hypothesis* (EMH) framework, dictates that a well-diversified portfolio should have a *market beta* of one (thereafter adjusted for risk preference), and thus move in sync with a benchmark market portfolio. Such a stock portfolio is highly correlated with the market, and considered to be entirely hedged against unsystematic risk. However, the risks within and between stocks present in a portfolio still impact on each other. In particular, risk present in a particular stock may spill over and affect the risk profile of another stock included within a portfolio - a phenomenon known as volatility spill-over effects.

In developing economies such as South Africa, portfolio managers are limited in their choices of stocks. This increases the difficulty of fully diversifying a stock portfolio given the volatility spill-over effects that may be present between stocks listed on the same exchange. In addition, stock portfolios are not static, and therefore require constant rebalancing according to the mandate of the managing fund. The process of constant rebalancing of a stock portfolio (for instance, to follow the market) becomes more complex and difficult during times of financial distress. Considering all these conditions, portfolio managers need all the relevant information (more than MPT would provide) available to them in order to select and rebalance a portfolio of stocks that are as mean-variance efficient as possible.

This study provides an additional measure to *market beta* in order to construct a more efficient portfolio. The additional measure analyse the volatility spill-over effects between stocks within the same portfolio. Using intraday stock returns and a residual based test (aggregate shock [AS] model), volatility spill-over effects are estimated between stocks. It is shown that when a particular stock attracts fewer spill-over effects from the other stocks in the portfolio, the overall portfolio volatility would decrease as well. In most cases *market beta* showcased similar results; this change is however not linear in the case of *market beta*. Therefore, in order to construct a

more efficient portfolio, one requires both a portfolio that has a unit correlation with the market, but also includes stocks with the least amount of volatility spill-over effects among each other.

Keywords: Modern portfolio theory, efficient market hypothesis, market beta, volatility spill-over effects, E-GARCH, aggregate shock model.

OPSOMMING

Die modellering van volatiliteit is van seminale belang in die veld van finansies en risikobestuur, omrede dit inligting verskaf oor die verspreiding van portefeulje-opbrengste. Ten einde die algehele volatiliteit van 'n aandeel-portefeulje te verminder, staaf *moderne portefeulje teorie* (MPT) binne die *doeltreffende mark hipotese* (EMH) raamwerk dat 'n goed-gediversifiseerde portefeulje 'n *mark-beta* van een (daarna aangepas vir risiko voorkeur) moet hê, en dus in sinkronisasie met 'n maatstaf mark-portefeulje beweeg. So 'n aandeel-portefeulje wat hoogs gekorreleer is met die mark word beskou as heeltemal verskans teen onsistematiese risiko. Die risiko inherent binne en tussen aandele bied egter steeds 'n opmerklike inpak op 'n portefeulje. In besonder kan die risiko teenwoordig binne 'n bepaalde aandeel oorspoel en 'n invloed uitoefen op die risiko profiel van 'n ander aandeel - 'n verskynsel bekend as volatiliteit-oorspoel effekte.

In 'n ontwikkelende ekonomie soos Suid-Afrika, is portefeuljebestuurders beperk in hul keuse van aandele. Dit verhoog die inspanning om 'n aandeel-portefeulje ten volle te diversifiseer gegewe die volatiliteit-oorspoel effekte wat teenwoordig mag wees tussen aandele. Daarbenewens is aandeel-portefeuljes nie staties nie, en vereis dus konstante herbalansering volgens die mandaat van die besturende fonds. Die proses van herbalansering van 'n aandeel-portefeulje (om byvoorbeeld die mark te volg) raak meer ingewikkeld en moeilikker gedurende tye van finansiële verknorsing. Gegewe al hierdie voorwaardes, is dit noodsaaklik dat portefeuljebestuurders al die relevante inligting (meer as wat MPT kan voorsien) tot hul beskikking het om hul in staat te stel om 'n portefeulje van aandele so doeltreffend as moontlik te kan kies en herbalanseer.

Hierdie studie stel 'n addisionele maatstaaf tot *mark-beta* alleenlik voor ten einde 'n meer doeltreffende portefeulje saam te stel. Die bykomende maatstaaf ontleed die volatiliteit-oorspoel effekte tussen aandele binne dieselfde portefeulje. Met die gebruik van intradag-data, en 'n residueel-gebaseerde toets (die kumulatiewe-skok [AS] model), is volatiliteit-oorspoel effekte bereken tussen die aandele. Daar is bewys dat wanneer 'n bepaalde aandeel minder oorspoel effekte lok vanaf die ander aandele in die portefeulje, die algehele portefeulje-volatiliteit dan opmerklik minder is. In die meeste gevalle het *mark-beta* soortgelyke resultate getoon, hoewel

die verandering in *mark-beta* nie linieêr is nie. Daarom, ten einde 'n meer doeltreffende portefeulje saam te stel, word 'n portefeulje vereis wat beide 'n eenheid-korrelasie met die mark het en aandele insluit wat met die minste hoeveelheid volatiliteit-oorspoel effekte onder mekaar toon.

Sleutelwoorde: Moderne portefeuljeteorie, doeltreffende markhipotese, mark-beta, volatiliteit-oorspoel effekte, E-GARCH, kumulatiewe-skokmodel.

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Wayne Peter Brewer

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"In the prevailing difficult global conditions uncertainty is at an even higher level… and requires that all of us better understand the immediate challenges of the mutating global environment."

~ Gill Marcus, SARB Governor, 2012

CHAPTER 1

INTRODUCTION

Precise modelling of volatility is of vital importance in finance as well as risk management in general. Portfolio managers have long been familiar with *modern portfolio theory* (MPT) and the *efficient market hypothesis* (EMH) where a well-diversified portfolio with a unit correlation with the market is considered entirely hedged against unsystematic risk. However, systematic risk remains even after fully diversifying. In this regard volatility within and between stocks in a portfolio impacts on the profitability of the portfolio, as well as the portfolio's overall risk profile.

From the considerable number of studies done on the EMH, one thing is clear - markets do not exhibit the same level of efficiency (Moix, 2001:61). This is because large markets with a great number of educated traders and high trading volumes exhibit stock returns that are less correlated than that of smaller markets (i.e. a market such as South Africa). Since portfolio managers in smaller economies are limited in their choices of stocks, it becomes increasingly difficult to fully diversify a stock portfolio given volatility spill-over effects between stocks listed on the same exchange.

1.1 Background

Modern portfolio theory (MPT), developed by Markowitz (1952; 1956; 1959) and various authors in the 1960s, most notably Sharpe (1964), has reshaped the way in which portfolio managers approach portfolio risk (Rubinstein, 2002:1044). This theory started by suggesting that portfolio risk is determined by the co-variances of assets included within a portfolio. The product of this was the *capital asset pricing model* (CAPM), which relies on a market related measure of risk, called *market beta*. Furthermore, CAPM is based on a multitude of underlying assumptions, which included the efficiency of the market.[1] This market efficiency was presented by Fama (1965; 1970; 1976) as the efficient market hypothesis (EMH). However, in order to effectively price assets and securities, diversify portfolios and hedge portfolio risk, it is important to gain an in-depth understanding of volatility as well (Harju & Hussain, 2011:82). This understanding should however not only be limited to the co-variance in returns, but should also encompass the volatility transmission between stocks. It is furthermore important to also look at shorter, and more revealing, intraday returns instead of only focusing on the volatility of daily returns. Since the financial market microstructure reveals so much about the patterns in volatility, it is not surprising that a large body of research has been devoted to understanding it (see Tse and Yang (2011)).

Market microstructure analysis is an important tool in discerning the interaction between trading procedures and security price formation, because price formation is related to a security's return volatility (Tian & Guo, 2007:289). For instance, numerous empirical studies found that daily volatility of consecutive opening prices are typically higher than consecutive daily closing prices, and that volatility flattened in between the daily open and close of a security.[2] This is the typical 'U' shape volatility distribution first published by Wood, McInish and Ord (1985).

With the rapid development in information technology and storage capacity, such data can be collected and analysed at extremely high frequencies. In the financial market setting this is especially the case. The specific timing of transaction events in a period of time (such as intraday data as opposed to daily data) is a significant economic variable which needs to be modelled, and for further relevance, forecasted (Cai, Kim, Leduc, Szczegot, Yixiao & Zamfur, 2007:1). Transaction timing of securities and the volatility it implies is therefore an important study in the field of portfolio management. The use of intraday data (or tick data) as opposed to daily squared returns has been seminal in improving volatility forecasts and the management of portfolios (Anderson & Bollerslev, 1998). The use of daily squared returns delivers inferior forecasting potential to the average of intraday squared returns (known as realised volatility) due to

1 See Table 2.1 in section 2.5.3.

2 See for example Bollerslev (1986), Schreiber and Schwartz (1986), Anderson and Bollerslev (1998), Areal and Taylor (2002), Poon (2005) and Tian and Guo (2007).

excessive noise.[3] These financial market microstructure theories are usually tested on an intraday transaction-by-transaction basis in order to improve the modelling of the moments of the return distribution (Cai *et al*., 2007:1).

The analysis of the financial market microstructure has in turn created a need for the development of volatility models to accurately estimate large covariance matrices (McAleer & Veiga, 2008:3). Because of the particular prevalence of distinct intraday volatility patterns, which underlies most of the financial market microstructure literature, higher-frequency returns exemplify highly persistent conditionally heteroskedastic elements together with discrete information arrival effects (Anderson, Bollerslev & Das, 2001:306). For a greater understanding of microstructure elements, such as the presence of heteroskedasticity, volatility must be modelled with an adequate process such as the Generalised Autoregressive Conditional Heteroskedasticity (GARCH). The modelling of heteroskedaticity has its roots in GARCH model of Engle (1982) and Bollerslev (1986), which has spurred the development of various autoregressive conditional volatility models, including Aggregate Shock Models (AS models). The wide-spread use of ARCH-type models is based on their ability to capture several dynamics of financial returns, including time-varying volatility, persistence and clustering of volatility, asymmetric reactions to positive and negative shocks and therefore volatility spill-over effects (McAleer & Veiga, 2008:2). Volatility spill-over effects between different assets refer to causality in return variance, and has seen a great deal of study in the field of financial economics (Kitamura, 2010:158).[4]

According to the *mixture of distribution hypothesis* (MDH), volatility (or the variance in returns) is an increasing function of information arrival.[5] Given the dynamics of this hypothesis, it is reasonable to assume that the volatility spill-over effect between stocks is attributable to information spill-over effects. When there is an interdependent relationship between stocks, these interdependencies will be an increasing function of arrival information relating to the

[3] Realised volatility refers to the volatility estimate calculated using intraday squared returns at short intervals; normally 5 to 15 minutes (Poon, 2005:14).

[4] Causality in return variance is the impact of any previous volatility of a particular asset on the current volatility of another asset.

[5] See Clark (1973) and Tauchen and Pitt (1983).

market (Kitamura, 2010:159). Of particular interest are asymmetric information influences, which is especially prevalent during times of financial turmoil.

1.2 Problem Statement and Research Question

Stock portfolios are dynamic in nature and necessitate constant rebalancing according to the mandate of the managing fund. However, ineffective rebalancing of a stock portfolio can result in higher risk and more volatile returns, especially in times of market turmoil, which may cause the portfolio to underachieve the market portfolio and not attain the investor’s required rate of returns. In order to correctly rebalance a stock portfolio in times of distress it is necessary to uncover the sources of risk within a portfolio, be it the stocks themselves or their effects on other stocks or even the market effects in volatile times.

The problem that comes to the fore is that portfolio managers have mostly relied on the co-variances and *beta* measures when managing a stock portfolio. Although these measures are fairly useful, other measures may be more prominent during times of financial distress as opposed to times of market stability. In order to change their strategies and methods, they need to be informed accordingly about the dynamics of the volatility (risk) that a stock portfolio is exposed to, especially at a microstructure level. The nature of these microstructure level changes mainly manifests as the volatility spill-over effects between stocks present in a portfolio.

In order for strategy adjustments to take place, the volatility spill-over effects of a stock portfolio need to be estimated. Thus, the question that needs to be answered, knowing that volatility transmission on a microstructure level plays an important role in portfolio volatility dynamics, is whether these volatility spill-over effects provide significant information in addition to a more traditional measure, such as *market beta,* for the rebalancing of a stock portfolio?

1.3 Objectives

The objectives to be satisfied are as follows:

i) To measure the portfolio return, volatility and *beta* of the different stocks during the 2008 financial crisis and the subsequent two years,

ii) to measure the volatility spill-over effects between the stocks within a portfolio during this period,

iii) and to analyse whether volatility spill-over effects between the stocks had a significant effect on portfolio volatility.

In this sense intraday volatility spill-over effects need to be estimated between the stocks in order to determine the extent, if any, of these spill-over effects and whether these effects present an alternative to *market beta* when considering portfolio return and volatility.

1.4 Motivation and Research Aim

A limited number of studies have modelled the dynamic intraday interactions between stocks on the Johannesburg stock exchange (JSE) using high-frequency data. This study will partly fill the gap by examining the intraday price volatilities and volatility spill-over effects between 5 stocks listed on the JSE top-40 during, and after, the 2008 financial crisis. Volatility spill-over effects within a market play a vital role in risk management for portfolio managers and assessing the stability of a market for policymakers (Pati & Rajib, 2010:568). These considerations will form part of this study in order to provide portfolio managers with more accurate information regarding the dynamics of volatility in order to effectively rebalance a portfolio.

The aim of this study is thus to investigate the intraday volatility interaction between the top-40 stocks on the JSE using hourly intraday returns between the periods 1 July 2008 to 30 April 2010, which coincides with the 2008 global financial crisis and its fallout. The effects of intraday realised volatility and volatility spill-over effects between the JSE top-40 stocks are analysed during the period under review. In addition to estimating volatility spill-over effects, *market betas* will also be estimated, in order to compare the measures against portfolio return and risk. The comparison is utilised to determine if volatility spill-over effects between stocks exhibit an effect on the characteristics of the portfolio, such as the co-variances (*beta*) exhibit. The study will further aim to test whether these volatility spill-over effects provide the portfolio manager with additional information that will enable him/her to construct a more efficient portfolio.

1.5 Methods

1.5.1 Literature study

The literature study will mainly focus on the following aspects: i) the history of portfolio management, ii) efficient markets, iii) the financial market microstructure, iv) the volatility dynamics of stocks within a portfolio in stable and turbulent market conditions, v) volatility transmission between stocks, and vi) the various models used in previous studies to examine these relationships and their findings.

1.5.2 Empirical study

The software to be used in the empirical study is: i) Microsoft Excel 2010, and ii) EViews 7. The data includes hourly intraday returns of five stocks listed on the JSE top-40 between the 1st of July 2008 and the 30th of April 2010. The JSE all-share index is also utilised during this period as a market portfolio proxy. The data is sourced from the Business Mathematics and Informatics (BMI) department of the North West University – South Africa. The empirical study focuses on the analysis of portfolio return, risk, *beta* and possible spill-over effects among the stocks. Aggregate shock (AS) models are estimated for the purpose of measuring return and volatility spill-over effects, with the error-terms being modelled using a univariate E-GARCH process.

1.6 Provisional Chapter Division

Chapter 2 provides a literature review of portfolio theory from its humble beginnings, up to the present use of *modern portfolio theory* (MPT). The focus is especially placed on Markowitz's (1952; 1956; 1959) and Sharp's (1964) seminal work on *market beta* and the *capital asset pricing model* (CAPM). This is followed by a review of the assumptions underlying MPT, with particular focus on efficient markets.

Chapter 3 includes a review of why capital market anomalies cause discrepancies in efficient markets, and how some of these are captured in intraday data. Secondly, there is a literature review on the importance of using intraday data to model volatility, followed by a description of the characteristics of the price process of stocks (in stable and turbulent market conditions). Thirdly, statistical properties of returns volatility are used to provide insight and model financial microstructure dynamics of the stocks within a portfolio. Fourthly, this chapter provides the methodology for this study, which makes use of ARCH-type models. More specifically the articulation of an aggregate shock (AS) model, used to determine volatility spill-over effects.

Chapter 4 presents the empirical estimation and results. With Eviews 7 and Microsoft Excel 2010, an AS model is constructed which provides estimates of portfolio returns, risk, *market beta* and volatility spill-over effects. The results are obtained from various combinations of a five-stock portfolio over different periods, and compared to one another. The comparison provides further insight into the use of a residual based test for portfolio management in addition to the use of a more traditional measure, such as *market beta*.

Chapter 5 concludes by referring to the aim and objectives of this study. This is followed by a summary of this study. Further conclusions from the results obtained are given with recommendations for portfolio managers about the validity of volatility spill-over effects within the management of a portfolio. Finally, recommendations are provided for further research.

"To achieve satisfactory investment results is easier than most people realise;

to achieve superior results is harder than it looks."

~ Benjamin Graham, the father of value investing

CHAPTER 2

In order to introduce an additional measure for portfolio stock selection during financial distress it is necessary to give an account of the most prevalent measure already in use. This measure is known as *market beta*, and has been of cardinal importance for portfolio management since its inception. It is important to understand the role of *beta* and what information regarding portfolio management it captures. The aim of this chapter (and this study) is not to delve into to intricacies of a mean-variance efficient portfolio (nor the measurement of portfolio efficiency), but rather on an account of the development and the measurement of *beta*. Data constraints prohibit the efficient measurement of a mean-variance portfolio, and therefore would be a suggestion for further study.[6] A clear understanding of *beta* (as portrayed in portfolio theory), however, will help explain why an additional measure – capturing volatility spill-over effects – (as portrayed in chapter 3) is an appropriate compliment to *beta* for portfolio stock selection during times of financial distress.

PORTFOLIO THEORY AND AN EFFICIENT PORTFOLIO

Diversification of a portfolio of assets was a well-known practice long before the seminal paper published by Harry M. Markowitz (1952) on portfolio selection. As an example, since 1941 Arthur Wiesenberger's annual reports on Investment Companies illustrated that firms held a large number of differing securities (Wiesenberger, 1941). These companies were modelled after the investment trusts of Scotland and England.[7] By the middle of the 20^{th} century, diversification of a portfolio was not in its infancy, but an often-practised necessity. However, the drivers that made diversification work were not generally known. The most prominent factor absent, prior to 1952, was adequate theory on investments that explained the effects of diversification when risks

6 See chapter 5, section 5.3, on suggested further research.

7 These Scottish and English investment trusts started in the middle of the 19^{th} century (Markowitz, 1999:5).

are correlated, distinguished between efficient and inefficient portfolios, and analysed risk-return trade-offs on the portfolio as a whole (Markowitz, 1999:5).

2.1 The Foundations of Efficiency (1564-1899)

Dating back to the 16th century, a foremost Italian mathematician named Girolamo Cardano noted, in his book entitled '*Liber de Ludo Aleae*' (The Book of Games of Chance), that gambling simply induced the fundamental principle of equal conditions (Cardano, c. 1564). These equal conditions applied to the opponents, the bystanders, money, the situation, the dice box, and the dice itself. In statistical terminology, it is described as random variables that are independently and identically distributed. This implies that every outcome is independent from the previous outcome, with every outcome having an equal chance of occurrence. By 1602, at least, stock and option markets had come into existence when the Dutch East India Company shares began trading in Amsterdam (de la Vega, 1688). In an eighteenth-century letter, '*Don Quixote'*, Sancho Panza writes, "*It is the part of a wise man to . . . not venture all his eggs in one basket.*" (Perold, 2004:7). However, the proverb "*Do not keep all your eggs in one basket*" dates as far back as Torriano's (1666) '*Common Place of Italian Proverbs'* (Herbison, 2003). Furthermore, in a famous article about the St. Petersburg Paradox published in 1738, a Swiss mathematician named Daniel Bernoulli contends that risk-averse investors should diversify their portfolios: "*...it is advisable to divide goods which are exposed to some small danger into several portions rather than to risk them all together*" (Bernoulli, 1738:26). This principle served investors well over centuries and was based on the premise that markets, and stocks themselves, moved randomly over time. This randomness can best be explained at the hand of Robert Brown's theory of random motion.

In 1828 Robert Brown, a Scottish botanist, reported that grains of pollen demonstrated a rapid oscillatory motion when brought into contact with water (Brown, 1828).[8] This result of particles drifting randomly in fluid was indicative of the fundamental principles of Brownian motion (named after its discoverer). Based on this randomness, a French stockbroker named Jules Regnault noted that as the holding period of a security increased, so did the chance of an investor winning or losing more on its price variation (Regnault, 1863). This price "deviation" was

[8] Oscillation is the repetitive variation about a central value.

directly proportional to the square root of time. Even the first signs of the notion on a random walk appeared as far back as 1880 when a British physicist, Lord Rayleigh, became aware that sound vibrations exhibited a "random walk" (Rayleigh, 1880). In addition, by 1888 the British logician and philosopher, John Venn, clearly comprehended the concept of both a random walk and Brownian motion in the field of logic (Venn, 1888).[9] George Gibson even mentioned efficient markets by 1889 in his book entitled '*The Stock Markets of London, Paris and New York*' (Gibson, 1889). He wrote that when shares were introduced to the public, the prices they acquired could be regarded as the most efficient price concerning available information. The following year Alfred Marshall published '*Principles of Economics*' which established economics as a social science (Marshall, 1890).

2.2 The Era of Unjust Risk and Wasteful Forecasting (1900-1951)

2.2.1 Bachelier (1900): The random walk

In 1900 a French mathematician named Louis Bachelier published his PhD thesis, '*La Théorie de la Spéculation*', which anticipated the *random walk hypothesis* (Bachelier, 1900). Bachelier had developed the mathematics and statistics behind Brownian motion half a decade before Einstein (1905).[10] In addition, he also determined that '*the mathematical expectation of the speculator is zero*' 65 years before efficient markets were described in terms of a martingale by Samuelson (1965).[11] Bachelier published remarkable work that was ahead of its time and was mostly overlooked until its rediscovery in 1954 by Leonard Savage, a statistician (Savage, 1954). Five years after Bachelier's seminal work a Professor and Fellow of the Royal Society, Karl Pearson, introduced the term "*random walk*" (Pearson, 1905). Statistically, the *random walk hypothesis* states that the return process can be expressed as a cumulated series of probabilistic independent shocks. Returns according to the *random walk hypothesis* can be expressed as:

$$R_t = E(R_t) + \varepsilon_t, \qquad \varepsilon_t \sim iid\ N(0, \sigma^2)\ , \tag{2.1}$$

9 John Venn is also renowned for introducing the Venn diagram often used in set theory, probability, logical, statistical, and computer sciences (see Venn, 1880).

10 Bachelier discussed the use of Brownian motion in the evaluation of stock options (Bachelier, 1900).

11 Samuelson (1965) proposed the *martingale hypothesis* that is less restrictive than the random walk hypothesis. This hypothesis does not suffer from first or higher order interdependencies. However, under risk-aversion the martingale property cannot be justified (LeRoy, 1973).

where $E(R_t)$ is the expected return and ε_t is strict white noise. Also in 1905, Albert Einstein, unaware of the research done by Bachelier, formulated the equations that explained the behaviour of Brownian motion (Einstein, 1905). Brownian motion was formally defined a year later by a Polish scientist named Marian von Smoluchowski (von Smoluchowski, 1906). André Barriol made use of Bachelier's arguments in his research on financial transactions (Barriol, 1908). In addition, during that same year, de Montessus utilised Bachelier's work in his research on probability and its applications to finance (de Montessus, 1908). It was also in 1908 that Paul Langevin formulated the stochastic differential equation of Brownian motion (Langevin, 1908). Four years later Bachelier wrote a book entitled '*Le Jeu, la Chance et le Hasard*' (The Game, the Chance and the Hazard) (Bachelier, 1914).

2.2.2 Irving Fisher (1906): variance as a measure of risk

In 1906, variance as a measure of risk was first suggested by Irving Fisher in '*The Nature of Capital and Income'* (Fisher, 1906). Statistically, variance refers to the spread of all likely outcomes around an uncertain variable, usually the mean. Variance, as a measure of risk, is expressed as:

$$\hat{\sigma}^2 = \frac{1}{T-1}\sum_{t=1}^{T}(r_t - E[R_t])^2\ , \tag{2.2}$$

and standard deviation (as a measure of volatility and risk) expressed as:

$$\hat{\sigma} = \sqrt{\frac{1}{T-1}\sum_{t=1}^{T}(r_t - E[R_t])^2}\ . \tag{2.3}$$

where r_t is the return on day t, and $E[R_t]$ is the average (mean) return over the T-day period. It should be noted that variance or standard deviation is not risk, but rather related to risk. Risk is related to an unwanted outcome, where standard deviation measures uncertainty that may be positive or negative. Variance therefore implies uncertainty, and uncertainty (together with abnormal returns) is the reason why forecasting is so appealing.

2.2.3 The great depression in forecasting

The first to note the "riskier" return distributions of assets, which are too "peaked" and "fat-tailed" to comply with Gaussian populations was Wesley Mitchell (Mitchell, 1915).[12] This study noticed for the first time the leptokurtic distribution of asset returns. In 1921, Frank Taussig published a paper, '*Is market price determinate?*', in which he states that the interaction between demand and supply cause short-run "irregularities" (and long-run "normality") in return, and that speculation does not necessarily stabilise an asset's price (Taussig, 1921). This "riskiness" was incorporated in a fundamental notion of efficient markets; in 1923, John Maynard Keynes, the celebrated English economist, distinctly identified that investors on financial markets are rewarded not for predicting future stock returns, but rather for bearing the risk of an investment (Keynes, 1923). Stock returns were evidently too unpredictable. In 1925, this stock price unpredictability (or fluctuations) was described by an economist named Frederick MacCauley as exhibiting a remarkable resemblance to that of a dice toss (MacCauley, 1925). The following year Maurice Olivier provided undisputable proof for the leptokurtosis present in the distributions of asset returns within his doctoral thesis published in 1926 (Olivier, 1926). Further proof of leptokurtic returns was provided by Frederick Mills in '*The Behavior of Prices*' (Mills, 1927). The last event on this timeline-narrative is dated late October 1929, when the *Wall Street Crash* occurred. Taking into account the full scope and duration of its devastating effects, it was more destructive than any other crisis in the history of the U.S. (Schwert, 2011).

2.2.4 Working, Cowles and "*animal spirits*"

In 1930 the Econometric Society with its related journal, '*Econometrica*', was founded and funded by Alfred Cowles, an American economist and businessman. In 1932 he also founded the Cowles Commission for Economic Research. In 1933 Cowles published a paper in which he analysed whether investment professionals could constantly outperform the stock market, and came to the conclusion that forecasters cannot forecast (Cowles, 1933). In corroboration of this

[12] See chapter 3, section 3.4.1, on kurtosis of stock returns.

result were the findings of Holbrook Working in 1934 which concluded that stock returns showcased similar behaviour to numbers from a lottery (Working, 1934). However, in 1936, Keynes published '*The General Theory of Employment, Interest, and Money*' in which he notoriously equalled the stock market to a beauty contest, claiming that most investors' choices are a result of "*animal spirits*" (Keynes, 1936).[13] More logically expressed, it means "*herd behaviour*" where investment choices are driven not by the fundamental factors of stock returns, but rather by what other investors reason and reflect (Keynes, 1936). For this reason, stock returns were seen to be volatile unless you were an expert at predicting behaviour. Furthermore, in 1937 a Ukrainian statistician and political economist, Eugen Slutzky, observed that large sums of independent random variables may be the foundation of cyclical processes (Slutzky, 1937).[14] His research showed that the interaction between chance events could produce periodicity where no such patterns existed initially. In the same year Cowles and Jones discovered substantial evidence of serial correlation in averaged stock price indices (Cowles & Jones, 1937).[15,16] However, in 1944 (in furtherance of his 1933 results), Cowles once again provided research support that investment professionals do not constantly outperform the market (Cowles, 1944). Also in 1944 a rigorous theory on investor risk preferences and decision-making under uncertainty was put forth in the work done by von Neumann and Morgenstern (von Neumann & Morgenstern, 1944). In summary, almost all the research pointed to random future asset and stock returns as was shown by Working, in which he demonstrated that in an efficient futures market it would be unfeasible to accurately predict future price changes (Working, 1949).

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13 Keynes refers to a beauty contest published in a London newspaper of a 100 or so women. Entrants could guess the top-five women based on the consensus, and so would win a prize. Instead of submitting their own choice of women according to their individual perception of beauty, entrants would second-guess the other entrants' perception of beauty. Instead of relying on the fundamental value (profitability based on revenues and costs), investors try to predict "*what the market will do*". This makes investments extremely volatile because returns are not based on fundamentals.

14 Slutzky is well known for the "Slutzky Equation" which is used in the field of microeconomics to separate the *income effect* from the *substitution effect*.

15 See chapter 3, section 3.4.2, on serial correlation of stock returns.

16 This is the only significant research published before 1960 which showcased substantial inefficiencies in stock returns (Sewell, 2011:3).

2.3 The Rise of Uncertainty (1935-1951)

2.3.1 John R. Hicks (1935): theorising uncertainty

John R. Hicks, in his 1935 article named '*A Suggestion for Simplifying the Theory of Money*', argued the need for improving monetary theory by structuring it around the existing theory of value (Hicks, 1935).[17] He contended that monetary theory is intrinsically a function of real events. Furthermore, and more importantly, monetary issues need to be dynamically analysed in sequential context where "time" is imperative. He then developed a specific sequential analysis in which he studies i) what happens within a single period ("*single-period theory*"), and ii) linkages between a series of subsequent periods ("*continuation theory*"). Hicks introduced risk into his analysis, and noted that risk affects investments in two ways, namely: i) by influencing the expected period of investment, and ii) by influencing the expected net yield of the investment. Furthermore, Hicks added that where risk is present, the expected outcome of a riskless situation is substituted by a range of possibilities, all being somewhat probable in occurrence. He stated that these probabilities should be statistically presented by a mean value and a suitable measure of dispersion. However, he also remarked: "*No single measure will be wholly satisfactory, but here this difficulty may be overlooked*" (Hicks, 1935:8). He therefore never proposed variation or standard deviation as a measure of dispersion or when speaking of risk. Hicks was aware of the risk-mitigating effects of diversification rather than holding one particular asset, and he knew that by spreading an investment between "risky" assets, an investor could adjust the risk profile to suit his or her needs, but did not present any supporting empirics. Hicks (1935) was a precursor of Tobin (1985) in trying to explain the demand for money as a result of investor preference for low-risk, high-return investments, but did not present a measure of dispersion, or distinguish between efficient or inefficient portfolios (Markowitz, 1999:12).

2.3.2 Jacob Marschak (1938): articulating uncertainty

Like Hicks, Jacob Marschak also tried to integrate the theory of money with the General Theory of Prices. He writes that in order to improve monetary problems, and more generally, investment

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[17] "Theory of value" is a broad term encompassing all the various theories within economics that try to explain the exchange value (or price of goods and services).

problems, requires a properly generalized Economic Theory (Marschack, 1938). His paper entitled '*Money and the Theory of Assets*' proposed the idea of using the means and the covariance matrix of consumption of commodities as a first order estimate in measuring and maximising consumer utility, subject to a budget constraint.[18] Firstly, Marschak's paper extended the concept of human tastes by considering consumers' aversion to waiting, their desire for safety, and other behavioural characteristics disregarded in the world of perfect certainty as articulated in classical static economics. Secondly, objectively given production conditions were altered into more realistic subjective expectations; because all market transactions are seen as investments. Marschak tried to explain the objective quantities and market prices of goods and claims held, given the subjective preferences and expectations of investors at a certain point in time. He recognised that investors usually prefer high mean and low standard deviation. He also observed that investors prefer "*long odds*", i.e., high positive skewness of yields. Marschak stated that this "yield" is realistically confined by two parameters only, namely: i) the mean expectation ("*lucrativity*") and the coefficient of variation ("*risk*"). From this articulation, the general analysis of portfolio selection is "*the shortest of steps, but one not taken by Marschak*" (Arrow, 1991:14).

Marschack did not advance portfolio theory because no portfolios were considered. The means, standard deviations, and correlations of consumables are directly incorporated within the utility and transformation functions with no analysis on a "portfolio" of goods. However, Marschak did provide a basis for later work on theory of markets where investors act in regard to risk and uncertainty, as developed by Tobin (1958) and related research on the capital asset pricing models (Markowitz, 1999:13).[19]

2.3.3 John B. Williams (1938): fundamentals and intrinsic value

Prior to Williams' argument, economists viewed stock market prices as being largely influenced by expectations and counter-expectations, as had been observed by Keynes in 1936 (Markowitz,

[18] Marschak was Markowitz's supervisor on his influential paper in 1952, but never revealed his earlier work to Markowitz (Markowitz, 1999:12).

[19] Marschak's paper in 1938 is the most advanced research on economics under risk and uncertainty prior to von Neumann and Morgenstern (1944).

1999:13).[20] John B. Williams published a Ph.D. paper in 1938 entitled '*The Theory of Investment Value*', which was pioneering in formulating the theory of Discounted Cash Flow (DCF) based valuation, with special emphasis on dividend based valuation (Williams, 1938). Williams argued that financial markets were only "markets" and that a stock's price should therefore reflect its intrinsic value. Instead of focusing on the expectations based time varying value of a stock, an investor should evaluate the underlying components of a stock. The shift should therefore deviate from forecasting expected stock prices, and focus on future corporate earnings and dividends. Williams proposed that a stock's value should be determined by "*the rule of present worth*". In other words, calculating the present value of future cash flows in the form of dividends and selling price. In its simplest form Williams developed the basis for the dividend discount model (DDM) where the present value of a common stock is expressed as:

$$V_0 = \sum_{t=1}^{\infty} \frac{D_t}{(1+k)^n}, \tag{2.4}$$

where D_t is the expected dividend in period t and k is the required rate of return for the investor.[21] He called this approach (of modelling and forecasting cash flows) "*algebraic budgeting*". Williams also argued that the present worth of all future cash flows was not dependent on a firm's capitalisation; hence anticipating the *Modigliani-Miller theorem*.[22] Considerable emphasis was therefore placed on the "intrinsic value" as the main determinant in current stock value, and as such, Williams was one of the founding developers of fundamental analysis.[23] However, of particular note, Williams did observe that the future dividends of a stock might be uncertain. In such a scenario, he said, probabilities should be estimated for all possible

20 See footnote 13 on the '*Keynesian beauty contest*'.

21 The DDM has been further refined and augmented; most notably the Gordon Growth Model published by Myron J. Gordon in 1959 (Gordon, 1959). The cost of equity capital in this model is the "internal rate of return", which is the discount rate that equates the present value of future cash flows to the current stock price. In this model, the expected dividend stream is $\mathrm{D}, \mathrm{D}(1+\mathrm{g})^1, \mathrm{D}(1+\mathrm{g})^2$, ... The present value of these cash flows, when discounted at rate r, is $\mathrm{D}/(\mathrm{r}-\mathrm{g})$, which, when set equal to the current stock price P, establishes $\mathrm{r} = (\mathrm{D}/\mathrm{P}) + \mathrm{g}$.

22 The *Modigliani–Miller theorem* expresses that, under a market price process (i.e. classically described as a random walk), in the absence of taxes, bankruptcy costs, agency costs, and asymmetric information, and in an efficient market, the value of a firm is uninfluenced by the means of how a firm is financed (Modigliani & Miller, 1958).

23 The DDM developed by Williams remains a popular standard for mean-variance analyses (c.f. Farrell, 1985).

stock values, and the mean of these values used as the value of the stock.[24] In the presence of risk, investing in a portfolio of stocks providing the maximum expected return was proposed, because the law of large numbers will ensure that the actual return almost equals the expected return.[25] This substantiated the notion (as proposed by Williams) that portfolio variance could be completely diversified away by holding a well-diversified portfolio.

2.3.4 Dickson H. Leavens (1945): diversification

Dickson H. Leavens, a former member of the Cowles Commission, published an article on the subject of portfolio diversification in which he examined fifty books and articles on investments (Leavens, 1945). He found that most of these previously published researches referred to the desirability and benefits of diversification. In most of this previously published research, however, the desirability and benefits were generally discussed, and did not clearly state or prove why it was desirable. Leavens, on the other hand, did illustrate the benefits of diversification, although on the assumption that risks are independent between assets. However, Leavens concluded that the assumption of independent risks between assets is an "important" one, albeit an unrealistic restriction in practice. To illustrate the impracticality of independent risks, he mentioned that diversifying between companies in one industry cannot protect a portfolio against factors that might influence the whole industry, nor could diversification between industries protect against unfavourable market conditions. Thus, Leavens intuitively understood that risk between assets are inter-correlated and that some model of covariance is present when analysing an investment, but did not include this notion within his own analysis.

2.4 The Genesis of Modern Portfolio Theory (1952-1959)

2.4.1 Harry M. Markowitz (1952; 1956): mean-variance efficiency

Markowitz writes, in his Nobel Prize autobiography, that he was enlightened with the basic concepts of portfolio theory whilst reading John B. Williams' '*The Theory of Investment Value*' (Markowitz, 1991:292). As talented as Williams was in presenting the first derivation of the *Gordon growth formula*, the *Modigliani-Miller capital structure irrelevancy theorem*, and avidly

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24 Williams did not propose variance as a measure of risk, but rather included a "premium for risk".

25 Williams did not realise that the rule of large numbers could not diversify all the variance within a portfolio of stocks because of stock inter-correlations.

supporting the dividend discount model, he failed to recognise the effects of risk, believing that all risk could be diversified away (Williams, 1938:69).[26] Markowitz (1952) was the first to empirically demonstrate that evaluating securities in isolation, as opposed to evaluating them as a group, provided misleading conclusions on portfolio returns and risk (Rubinstein, 2002:1043). This central idea was evidently missing from Williams (1938) and other authors such as Graham and Dodd (1934). Furthermore, at the time stock prices were structured according to the present value model of Williams (1938). Markowitz revealed that an investor should not analyse each individual security's own risk (measured by security variance), but rather the contribution each security made to the variance of the entire portfolio. He assumed that the beliefs (or projections) about security returns obey the same probability rules that random variables follow. From this assumption, it follows that i) the expected return on the portfolio is a weighted average of the expected returns on individual securities, and ii) the portfolio variance of return is a function of the variances of, and the covariances between, securities and their weights in the portfolio. In general the expected return on a portfolio is given by:

$$E(R_p) = \sum_i w_i E(R_i) \, , \tag{2.5}$$

where R_p is the return on the portfolio, R_i is the return on security i and w_i is the weighting component of asset i (i.e. the share of asset i in the portfolio so that $\sum_i w_i = 1$). The portfolio return variance is given by:

$$\sigma_p^2 = \sum_i w_i^2 \sigma_i^2 + \sum_i \sum_{j \neq i} w_i w_j \sigma_i \sigma_j \rho_{ij} \, , \tag{2.6}$$

where ρ_{ij} is the correlation coefficient between the returns on securities i and j.[27] Therefore $\sigma_i \sigma_j \rho_{ij}$ is the covariance of their returns. In addition, portfolio return volatility (standard deviation) is expressed as:

[26] Numerous authors made the same assumption based on Jacob Bernoulli's (1713) *law of large numbers* (Rubinstein, 2002:1042).

[27] Markowitz's 1952 paper provides the first occurrence of the covariance equation in a published paper on financial economics (Rubinstein, 2002:1043).

$$\sigma_p = \sqrt{\sigma_p^2}\,. \tag{2.7}$$

For a two asset portfolio return and portfolio variance is given by:

$$E(R_p) = w_A E(R_A) + w_B E(R_B)\,, \tag{2.8}$$

$$\sigma_p^2 = w_A^2 \sigma_A^2 + w_B^2 \sigma_B^2 + 2 w_A w_B \sigma_A \sigma_B \rho_{AB}\,, \tag{2.9}$$

and for a three asset portfolio:

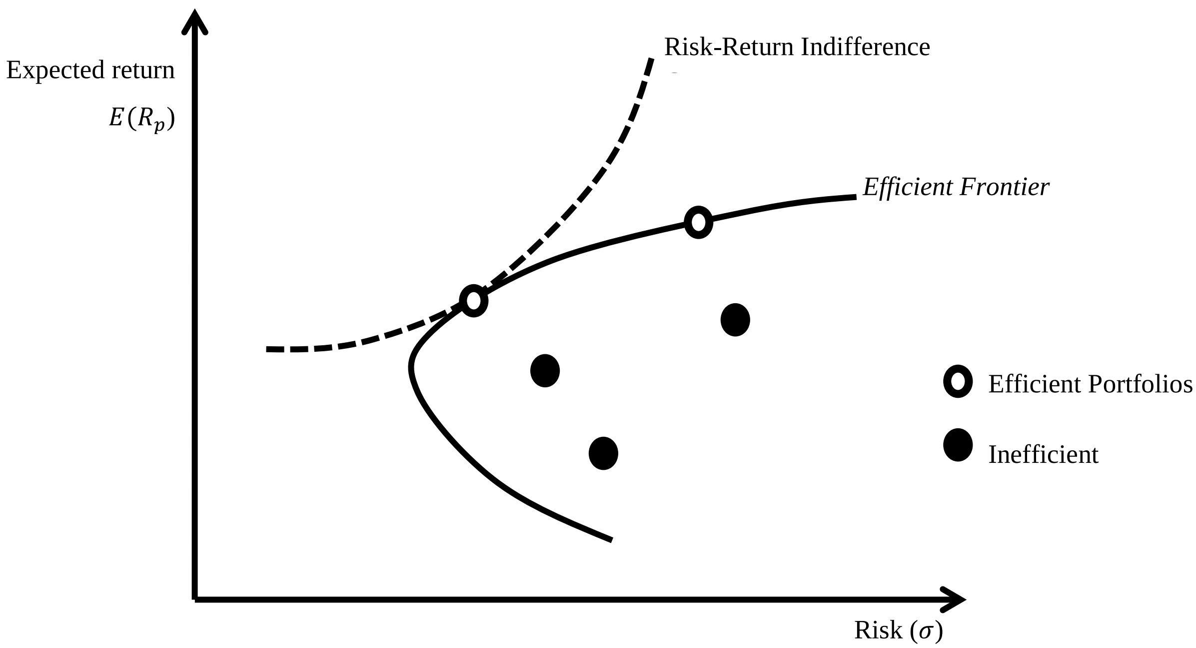
$$E(R_p) = w_A E(R_A) + w_B E(R_B) + w_C E(R_C)\,, \tag{2.10}$$

$$\begin{gathered}\sigma_p^2 = w_A^2 \sigma_A^2 + w_B^2 \sigma_B^2 + w_C^2 \sigma_C^2 + 2 w_A w_B \sigma_A \sigma_B \rho_{AB} \\ +2 w_A w_C \sigma_A \sigma_C \rho_{AC} + 2 w_B w_C \sigma_B \sigma_C \rho_{BC}\,,\end{gathered} \tag{2.11}$$

and so forth. Markowitz did not assume that diversification would eliminate risk, but would rather reduce overall portfolio risk. His paper is therefore the first mathematical formalisation of diversifying a portfolio. In essence, stipulating the financial adaptation of "the whole is greater than the sum of its parts". According to Markowitz, an investor should invest in a portfolio that maximizes expected portfolio return ($E(R_p)$) while minimizing portfolio variance of return (σ_p^2). Investing is therefore a trade-off between risk and expected return. Investors are assumed to be risk averse, and will therefore select the portfolio with the highest expected return given the level of risk, or select the portfolio with the lowest risk given the level of expected return.

Investors could reduce portfolio risk by holding combinations of securities, which are not perfectly correlated (that is where $-1 < \rho_{ij} < 1$). In other words, portfolio risk is reduced by holding a diversified portfolio. A combination of assets (i.e. a portfolio) is seen as "efficient" if it exhibits the best possible level of expected return given the level of risk. In figure 2.1 the combinations of risky assets (without the holding of a risk-free asset) are plotted in the risk-expected return space. The hyperbola is known as the *Markowitz efficient frontier*, and portfolios lying on this frontier are seen as "efficient".[28] The *efficient frontier* lies at the top of the opportunity set (or the feasible set), and is the positively sloped portion of the opportunity set that offers the highest expected return for a given level of risk. The risk-return indifference curve shows all points where an investor obtains the highest possible satisfaction from investing. The point of tangency is where the investor's utility is maximised given all possible risk-return combinations of securities. Differing investors showcase different indifference curves, so the curve may shift, causing the "optimal" portfolio to be located on a separate point of tangency on the *efficient frontier*.

Figure 2.1 The *efficient frontier* for a portfolio of risky assets (source: Compiled by Author).



No individual security is expected to lie on the *efficient frontier* due to the benefits of diversification. The *efficient frontier*, in matrix form for a given "risk tolerance" level, $q \in [0, \infty)$, is given by minimising the following equation:

$$w^T \sum w - q * R^T w , \tag{2.12}$$

28 In Markowitz's 1952 paper, the '*efficient frontier*' was addressed as the '*critical line algorithm*'.

where w is a vector of portfolio weights and $\sum_i w_i = 1$, Σ is the covariance matrix for the returns on the assets in the portfolio, $q \geq 0$ is a "risk tolerance" factor, where 0 results in the portfolio with minimal risk and ∞ results in the portfolio infinitely far out on the frontier with both expected return and risk unbounded. It then follows that R is a vector of expected returns, $w^T \sum w$ is the variance of portfolio return and $R^T w$ is the expected return on the portfolio. The complete frontier is parametric on q. A geometrical analysis was therefore used to illustrate the efficient sets, assuming non-negative investments subject to a budget constraint. This model is known as the *HM model* or *Mean-Variance model*.[29]

2.4.2 Arthur D. Roy (July 1952): safety first

Markowitz writes the following about Roy: "*On the basis of Markowitz (1952), I am often called the father of modern portfolio theory (MPT), but Roy (1952) can claim an equal share of this honor*", (Markowitz, 1991:5). Roy (1952) also recommended choosing a portfolio based on its mean and variance as a whole. His approach was coined the *safety-first criterion*. More specifically, he suggested choosing the portfolio that minimizes the probability of a portfolio falling below a certain threshold. Suppose that an investor can choose between portfolio A or B, and has a return threshold of -1%. Then the investor would choose the portfolio that maximises the probability of the portfolio return being at least as high as -1%. The problem an investor meets using the *safety-first criterion* can be expressed as:

$$min \Pr\left(R_p < \underline{R}\right), \tag{2.13}$$

where $\Pr(R_p < \underline{R})$ is the probability of the actual return of the portfolio (R_p) being less than the minimum acceptable return ($\underline{R}$). With the assumption of normally distributed returns, Roy's *safety-first criterion* can be reduced by maximising the *safety-first ratio*:

$$SFRatio_p = \frac{E(R_p - \underline{R})}{\sigma_p}, \tag{2.14}$$

29 *HM model* after the authors name or *Mean-Variance model* due to being based on expected return (mean) and standard deviation (squared variance).

where $E(R_p)$ is the expected return of the portfolio, σ_p is the standard deviation of the portfolio's return, and $\underline{R}$ the minimum acceptable return.[30] Roy's formula for calculating the variance of the portfolio (covariances of stock returns) was similar to the calculation used by Markowitz (1952). The main differences between the Roy and Markowitz's portfolio selection analyses were that i) Markowitz's required non-negative investments whereas Roy's allowed positive or negative investment amounts, and ii) Markowitz allowed the choice of a desired portfolio located on the *efficient frontier* whereas Roy suggested a particular portfolio (Markowitz, 1999:5).[31]

2.4.3 James Tobin (1958): liquidity preference

Tobin hypothesised that the demand for money was distinguishable from other "monetary assets". These monetary assets, including cash, were defined as "*marketable, fixed in money value, free of default risk*." He then presented his seminal theorem, now known as the *Tobin Separation Theorem*. He theorised that the investment process can be separated into two distinct steps, namely: i) the construction of an efficient portfolio, that is invariant to preference, as postulated by Markowitz (1952), and ii) combining the "risky" efficient portfolio with a risk-free investment (cash).[32] A risk-free (R_F) asset has an expected return that is entirely certain, and therefore a standard deviation that is zero ($\sigma_{RF} = 0$). Investor preference determines the optimal allocation between the efficient portfolio and the risk-free asset.[33] Tobin suggested supplementing a portfolio with n risky assets and one risk-free asset, cash.[34] In addition, holdings had to be non-negative. He showed that for a given set of means, variances, and covariances among efficient portfolios containing any cash at all, the mix among risky stocks is always constant. The primary purpose was to improve the theory for holding cash. He concluded that his analysis provides a logically more satisfactory basis for liquidity preference than

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30 Under the assumption of normality, and given an investor's minimum acceptable return is equal to the risk-free rate, the *safety-first ratio* essentially converts to *Sharpe's ratio* (refer to footnote 49).

31 So why did Markowitz, and not Roy, win the Nobel Prize in Economic Sciences in 1990? Maybe it is because Roy basically made this one marvellous contribution and vanished, while Markowitz wrote an assortment of books and articles in the given field (and was therefore more consistently active).

32 A risk-free asset is one that has zero variance, and has no correlation with other assets. Whereas a risky asset has an uncertain return, and this uncertainty is measured by variance, standard deviation or return.

33 This separation between risky and riskless investments was seminal in the conception of the capital market line and in the development of the Capital Asset Pricing Model (Markowitz, 1999:10).

34 Tobin's assets were monetary assets, so the risk was market risk and not default risk.

Keynesian theory, and provides the advantage of explaining diversification between stocks and bonds where Keynesian theory suggests the holding of only one of these risky assets. In practical terms the theorem suggests that an investor can control the risk of a portfolio of risky investments by borrowing at the risk free rate and leveraging the portfolio (and therefore its risk), or lending at the risk free rate and mitigating risk. Since investors are commonly risk-averse, they prefer to supplement a portfolio of risky assets with a risk-free asset, and thus lowering the possible downside risk.[35] Tobin's work, in essence, showed that when investors are able to borrow and lend at the risk-free rate, the *efficient frontier* is simplified.

2.4.4 Markowitz (1959): generalisation and changed views

The primary goal of the book entitled '*Portfolio Selection: Efficient Diversification of Investments*' (published in 1959 by Markowitz) was to simplify the concepts of his seminal paper published in 1952, as well as to reflect how Markowitz's views changed during this period (Markowitz, 1999:7). As with Markowitz (1952), Markowitz (1959) illustrated mean-variance analyses, defined mean-variance efficiency and provided a geometric analysis of efficient sets, but without some errors present in the inaugural paper. The 1959 book also presented a more general derivation of the "*efficient frontier*" which was less restricted, and worked for any covariance matrix. The analyses of such a covariance model, for a large portfolio with many covariances, were too large to analyse the inter-relationships individually, so Markowitz proposed a one-factor (linear) model to ease computation. However, what Markowitz did not realise was that this linear factor model could be used to simplify the computation of the *efficient frontier* as Sharpe (1963) did. Markowitz (1959) also considered what happens to an equal-weight portfolio's variance as diversification increases. He found that when a portfolio with stocks consisting of uncorrelated returns increases its diversification, overall risk approaches zero. However, when returns are correlated, portfolio variance tends to approach "average covariance" as diversification is increased (A term he coined the "*law of the average covariance*").[36] Correlated returns therefore had serious implications for portfolio variance. Markowitz (1959) also made use of semi-variance as a replacement for variance as a measure of

[35] This portfolio of risky and risk-free assets can be termed the stock/bond asset allocation decision.
[36] The average covariance is defined as the sum of all the individual co-varying relationships divided by the total number of co-varying relationships.

risk.[37] In addition, Markowitz insisted that the investor should choose a portfolio to maximise his expected utility according to the Savage (1954) axioms. However, due to the revolutionary ideas of the time, Markowitz (1999), in retrospect, lists the discoveries that he did not anticipate in 1959, which include: i) the *consumption-investment game* he developed was in discrete time rather than in continuous time (see Merton 1969), ii) he did not reveal the discovery of myopic utility functions (see Mossin 1968 & Samuelson 1969), and iii) he did not consider the behaviour of consumers and investors within this *game* (see Sharpe 1964).

2.5 The Capital Asset Pricing Model (1960-1966)

Prior to the development of the Capital Asset Pricing Model (CAPM), estimation of expected returns by an investor was based on the "cost of capital" of an asset; this in turn depended on the asset's manner of finance (see Bierman & Smidt, 1966). The weighted average of the "cost of equity capital" and the "cost of debt capital" denoted the cost of capital of the asset.[38] A popular model for this approach was the Gordon (1959) model.[39] Until the 1960s the empirical measurement of risk and return was in its infancy because of insufficient computing power (Perold, 2004:4). When sufficient computing power became available during this period, academics were capable of collecting, storing and processing market data for the purposes of empirical examination. And so began the narrative of *market beta* as a measure of portfolio risk.

2.5.1 A student-master narrative (1960)

The transition of MPT from the 1950s to the 1960s occurred naturally from one researcher to the next. The thoughts and research of one great academic influenced a subsequent other. Markowitz states that one day in 1960 a Ph.D. hopeful entered his office and asked about research in models of co-variance – for which there was a shortage at the time (Markowitz, 1999:14). This student was William Sharpe, and the encounter set him off on his many lines of research in the field of

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37 Semi-variance is defined like variance except that it only accounts for only deviations below some value.

38 The costs of debt and equity capital were inferred from the long-term yields of those instruments. The cost of debt capital was typically assumed to be the rate of interest owed on the debt, and the cost of equity capital was backed out from the cash flows that investors could expect to receive on their shares in relation to the current price of the shares.

39 Refer to footnote 21.

portfolio theory. From this chance meeting the evolution of the *efficient frontier* into the *capital asset pricing model* (CAPM) was set into motion.

2.5.2 Jack Treynor (1961; 1962): A forgotten bygone

Before Sharpe's seminal work on portfolio theory is discussed, it is worth mentioning that history generally credits Sharpe (1964), Lintner (1965a; 1965b) and Mossin (1966) with the development of the *capital asset pricing model* (CAPM).[40] However, what general history does not often recognise is that Jack Treynor was one of the pioneers in developing CAPM (Black, 1981:14). Treynor also deserves credit for developing CAPM because of his innovative manuscripts ('*Market value, time, and risk*' written in 1961 and '*Toward a theory of market value of risky assets*' written in 1962) that circulated in draft-form, but never got published within a journal during that time.[41] However, both Treynor and Sharpe developed their models independently and almost simultaneously (French, 2003:61).[42]

Treynor (1961, 1962) was arguably the first to derive the linear relationship between expected return and covariance with the market portfolio, and concluded that in equilibrium the market portfolio itself was the only optimal mean-variance efficient portfolio. Treynor approached capital asset pricing from a cost-of-capital decision making perspective (Treynor, 1962).[43] He begins by decomposing expected return into a risk-free component and a risk premium component. Treynor's risk-free component, r, is the perfect "lending rate", which is incorporated within the one-period discount factor b, where:

$$b = 1/(1 + r)\,, \tag{2.15}$$

essentially defining r as the growth factor.[44] Expected performance can therefore be given by:

40 See, for example, Sharpe and Alexander (1978:194), Merton (1990:475), Bodie, Kane & Marcus (1993:242), Reilly (1994:270), and Cochrane (2001:152).

41 Treynor (1962) was eventually published as chapter 2 of Korajczyk (1999). Treynor (1961) remains unpublished.

42 Sharpe started working on his paper in 1960, and submitted his work to the *Journal of Finance* in 1962. However, it was only published in 1964 (French, 2003:61).

43 Treynor took into account the *Modigliani-Miller theorem* (see footnote 22).

44 Lintner's (1965a:16) risk-free component (r^*, the interest rate on risk-less asset or borrowing) is analogous to that of Treynor.

$$EP = rC + (1 + r) \sum x_i a_i \ , \tag{2.16}$$

where rC is the return on capital at the risk-free rate, and $(1 + r) \sum x_i a_i$ the expected return due to the risk premia.[45] The expected portfolio risk premium, μ, is defined as the present value of the portfolio risk premium. He then derives the linear relation between risk and expected return (Treynor, 1962:18-19). Treynor makes use of Markowitz's formula to estimate the portfolio variance and covariance matrix. Treynor also finds a linear relation, which is analogous to the findings of Tobin (1958), providing proof of *Tobin's separation theorem*.[46] Treynor concludes that, in equilibrium i) the same combination of risky assets will be optimal for all investors, ii) the amount invested in risky asset i will be the same as the proportion of asset i in the aggregate market portfolio, and iii) each individual investment must be positive.[47]

2.5.3 William Sharpe (1964), John Lintner (1965a; 1965b) and Jan Mossin (1966)

The development of the *capital asset pricing model* (CAPM) starts off where Markowitz's work in the 1950s ended. Specifically, capital market theory has extended portfolio theory by building a model to price all risky assets (Brown & Reilly, 2009:205). This model is known as CAPM and was developed and further refined by Sharpe (1964), Lintner (1965a; 1965b) and Mossin (1966). CAPM allows an investor to determine the required rate of return for any risky asset. The major contributing factor that caused capital market theory to evolve from portfolio theory is the introduction of a risk-free asset (Hirt & Block, 2009:597). As stated before, a risk-free asset has zero variance, and is therefore uncorrelated with other risky assets. Several authors, including Tobin (1958), experimented with the idea of a risk-free asset after the work done by Markowitz (1952; 1956; 1959). The inclusion of a risk-free asset allowed for the development of a generalised theory of capital asset pricing (such as CAPM) under conditions of uncertainty (flowing on from Markowitz's portfolio theory).

45 Likewise, Lintner's risk premia ($\bar{x}_i$) is analogous to Treynor's a_i.
46 See section 2.4.3 on James Tobin and liquidity preference.
47 Lintner (1965a) came to the same conclusions as Treynor (1962).

A generalised discussion of the development of CAPM by Sharpe (1964), Lintner (1965a; 1965b) and Mossin (1966) follows. As stated earlier, the risk-free asset (R_F) has zero variance ($\sigma_{RF} = 0$), and earns a risk-free rate of return (RFR), which is the expected long-run growth rate of the economy, supplemented for short-run liquidity needs.[48] When a risk-free asset is combined with a portfolio of risky assets (say portfolio Q), then the expected rate of return will be equal to the weighted average of the two returns:

$$E(R_p) = w_{RF}(RFR) + (1 - w_{RF})E(R_Q)\,, \tag{2.17}$$

where $E(R_p)$ is the expected portfolio return, w_{RF} the proportion of the portfolio invested in the risk-free asset, and $E(R_Q)$ the expected return from portfolio Q. Recall from 2.9 that the variation of a two asset portfolio is given by:

$$\sigma_p^2 = w_A^2\sigma_A^2 + w_B^2\sigma_B^2 + 2w_A w_B \sigma_A \sigma_B \rho_{AB}\,. \tag{2.18}$$

By substituting the risk-free asset and risky asset portfolio Q in the equation, it is transformed into:

$$\sigma_p^2 = w_{RF}^2\sigma_{RF}^2 + (1 - w_{RF}^2)^2\sigma_Q^2 + 2w_{RF}(1 - w_{RF}^2)\sigma_{RF}\sigma_Q\rho_{RF,Q}\,. \tag{2.19}$$

However, the variance of a risk-free asset is zero ($\sigma_{RF}^2 = 0$), as well as its correlation with other risky assets ($\rho_{RF,Q} = 0$). After these adjustments, the portfolio variance becomes:

$$\sigma_p^2 = (1 - w_{RF}^2)^2\sigma_Q^2\,, \tag{2.20}$$

and portfolio standard deviation by:

$$\sigma_p = (1 - w_{RF}^2)\sigma_Q\,. \tag{2.21}$$

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48 Government bonds or treasury bills are usually considered risk-free assets (Hirt & Block, 2006:597).

The standard deviation of a portfolio combining a risk-free asset with risky assets is therefore the linear proportion of the standard deviation of the risky asset portfolio. The risk-return relationship between $E(R_p)$ and σ_p can therefore be given by:

$$E(R_p) = RFR + \sigma_p \left[\frac{E(R_p) - RFR}{\sigma_Q}\right]. \tag{2.22}$$

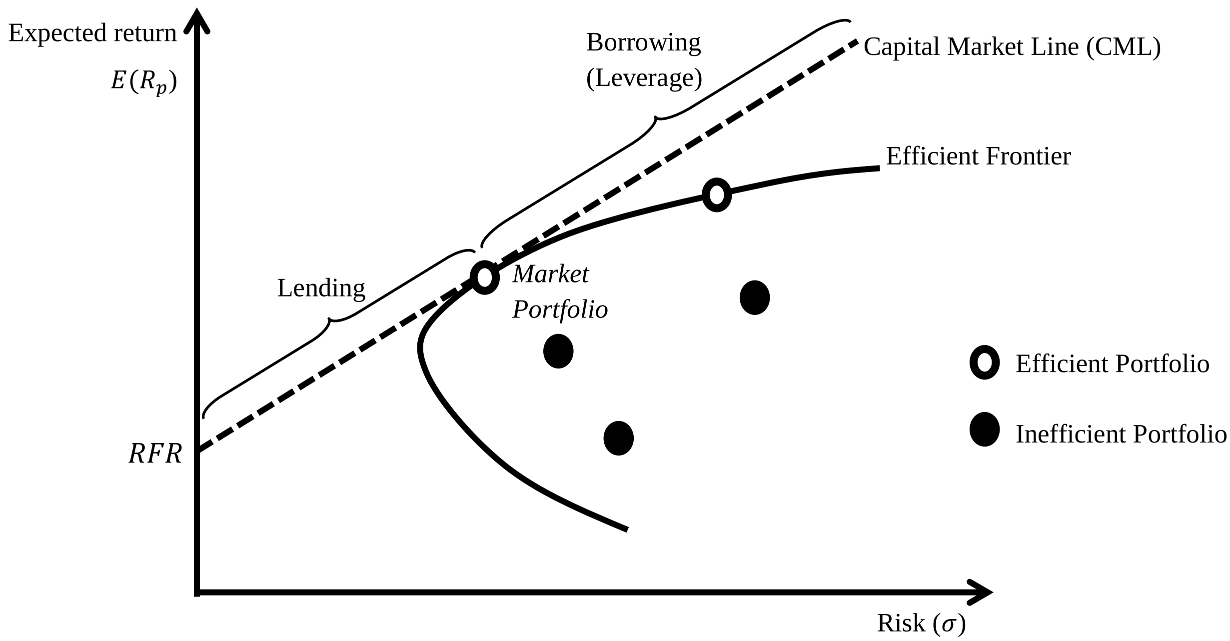
Capital market theory therefore states that an investor can expect a return equal to the risk-free rate plus compensation for the number of risk units (σ_p), times the risk premium ($[E(R_p) - RFR]/\sigma_Q$), they accept. This relationship holds for any combination of a risk-free asset and any collection of risky assets.

Since there is a clear payoff, investors try to maximise their expected return from a portfolio (combining a risk-free asset with a collection of risky assets) given the level of risk they are bearing. Assuming that portfolio Q maximises this risk premium, the portfolio will then be dubbed the "*market portfolio*" (denoted, henceforth, by subscript "M"). By definition, the *market portfolio* contains all risky assets present within the market. It also has the characteristic of receiving the highest expected return per unit of risk for any combination of risky assets. Under such conditions, the risk-return relationship given by:

$$E(R_p) = RFR + \sigma_p \left[\frac{E(R_p) - RFR}{\sigma_M}\right], \tag{2.23}$$

which is called the *capital market line* (CML). Combining the CML with the *Markowitz efficient frontier* shows an investor where he/she will obtain the highest possible expected return per unit of risk, as seen in figure 2.2. The *efficient frontier* is the positively sloped portion of the opportunity set that offers the highest expected return for a given level of risk. The *efficient frontier* lies at the top of the opportunity set (or the feasible set). The risk-return relationship is represented by the CML, which shows all points where an investor obtains the highest possible expected return for a given level of risk. The slope of this line is known as *Sharpe's ratio*.[49] The point of tangency is where the investor obtains a portfolio combining a risk free asset and risky assets with the highest possible expected return per unit of risk (highest *Sharpe ratio*).

Figure 2.2 The *efficient frontier* with a portfolio consisting of a risk-free asset and a combination of risky assets (source: Compiled by Author).



By borrowing and lending, an investor's portfolio would move along the CML line. In accordance with the *separation theorem* proposed by Tobin (1958), an investor can move along the CML with regard to each individual's risk-return preference.[50] This is because all portfolios are perfectly positively correlated with the *market portfolio*. The *market portfolio* is considered to be a completely diversified portfolio because it contains all risky assets. Sharpe (1964), Lintner (1965a; 1965b) and Mossin (1966) suggested that an investor should consider an *external market* measure of risk.

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49 The Sharpe ratio is expressed as $\left[E\left(R_p\right) - RFR\right]/\sigma_p$. This ratio measures excess return (risk premium) in relation to total portfolio risk (Sharpe, 1966).

50 A risk-averse investor will invest only a part into the *market portfolio*, and lend the rest at the RFR by buying risk-free securities (Therefore moving downward on the CML. If an investor prefers more risk, he/she will borrow at the RFR and invest everything into the *market portfolio* (Moving upwards along the CML).

In an efficient market all rational, profit-maximising investors want to hold a completely diversified portfolio with a level of risk and return that is in line with the risk preference of each investor. Under these (and other) assumptions, it was shown that the return of an individual asset moves with the *market portfolio*. This co-integrating movement is known as systematic risk, and is the variance of an asset that is explained by the variance of the *market portfolio*. There is also a non-market related risk present in a portfolio (discussed shortly), which is seen as "unimportant" because it can be diversified away in a large portfolio of stocks. Therefore, the risk premium for an individual asset is a function of an asset's systematic risk, where:[51]

$$Risk\ Premium = f(Systematic\ Market\ Risk).$$

This market measure of risk (systematic risk) has a significant relationship with the fundamental determinants of risk (Thompson, 1976).[52] The risk premium can therefore be expressed as:

$$Risk\ Premium \\ = f(Business-, Financial-, Liquidity-, Exchange\ Rate \\ - \&\ Country\ Risk).$$

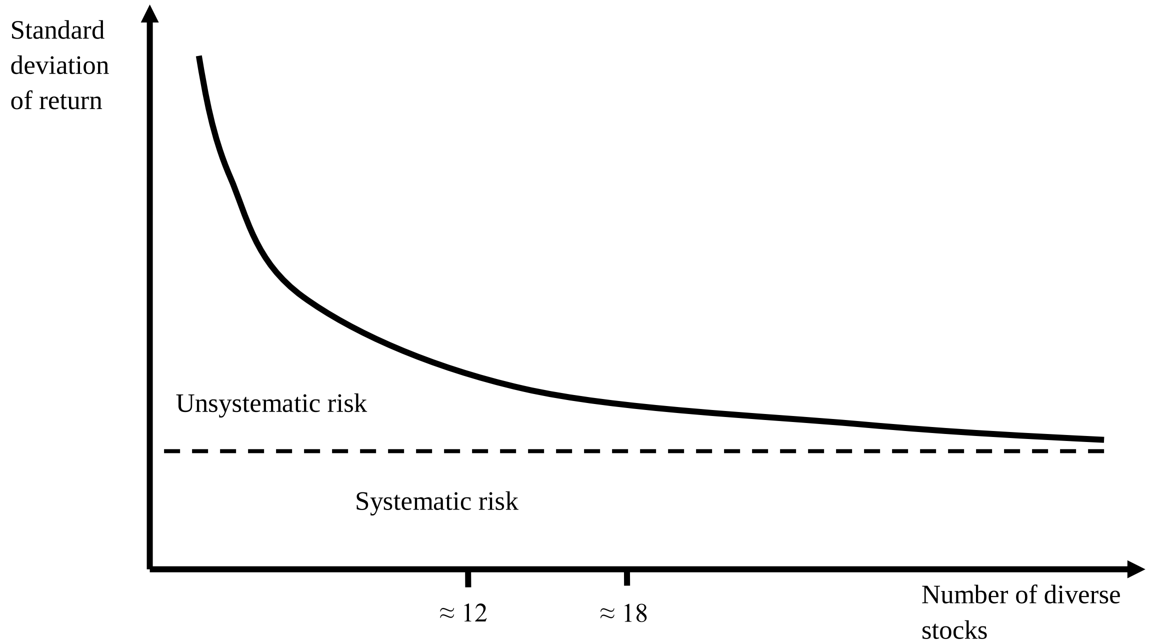
In addition there is a non-market related risk present in an asset's variance called unsystematic risk. This is an asset specific risk and can therefore be diversified away as the number of assets in a portfolio increases.[53] The interaction between systematic and unsystematic risk is shown in figure 2.3. According to efficient *capital market theory*, systematic risk cannot be diversified away, whereas unsystematic risk is mostly diversified away within a portfolio containing 12 or more diverse stocks.

51 This measure is commonly known as *beta*.

52 Fundamental determinants of risk are accounting variables (Brown & Reilly, 2009:20).

53 Research has indicated that an investor needs around 12 to 18 diverse stocks within a diversified portfolio to eliminate a substantial amount of the unsystematic risk (Evans & Archer, 1968; Tole, 1982).

Figure 2.3 Systematic and unsystematic risk (source: Brown & Reilly, 2009:212).



As noted earlier, a portfolio containing a risk-free asset and a set of risky assets are perfectly positively correlated with the *market portfolio*. A completely diversified portfolio will therefore have a unit correlation with the *market portfolio*. This is due to the elimination of unsystematic risk of a completely diversified portfolio. Therefore, the relevant risk measure for a risky asset or portfolio of risky assets is the average covariance it exhibits in relation to the *market portfolio*. Because all individual risky assets are part of the market portfolio, the rate of return of a risky asset can be linearly expressed as:

$$R_{i,t} = a_i + b_i R_{M,t} + \varepsilon_t \,, \tag{2.24}$$

where $R_{i,t}$ is the return for asset i during period t, a_i is the constant term for asset i, b_i is the slope coefficient for asset i, $R_{M,t}$ is the return of the market portfolio during time t, and ε_t is the random error term. The variance of an asset i can therefore be expressed as:

$$\begin{aligned} Var(R_{i,t}) &= Var(a_i) + Var(b_i R_{M,t}) + Var(\varepsilon_t) \\ &= Var(b_i R_{M,t}) + Var(\varepsilon_t) \,. \end{aligned} \tag{2.25}$$

where the $Var(b_i R_{M,t})$ is the variance of asset i that is related to the market portfolio, thus the systematic risk of the portfolio. Also note that the $Var(\varepsilon_t)$ is the unsystematic risk that is attributable to asset i. However, the market portfolio is considered to be completely diversified, exhibiting no unsystematic risk. It is due this characteristic that Sharpe (1964), Lintner (1965a; 1965b) and Mossin (1966) suggested that the variance of a portfolio (σ_p) in the CML expression:

$$E(R_p) = RFR + \sigma_p \left[\frac{E(R_p) - RFR}{\sigma_M}\right], \tag{2.26}$$

can be replaced by a systematically related risk measure when evaluating the expected return of a risky asset or portfolio of risky assets. The expression is then transmuted into:

$$E(R_i) = RFR + \sigma_i \rho_{iM} \left[\frac{E(R_p) - RFR}{\sigma_M}\right], \tag{2.27}$$

where ρ_{iM} is the correlation coefficient between asset i and the *market portfolio*. The expression can be rearranged as follows:

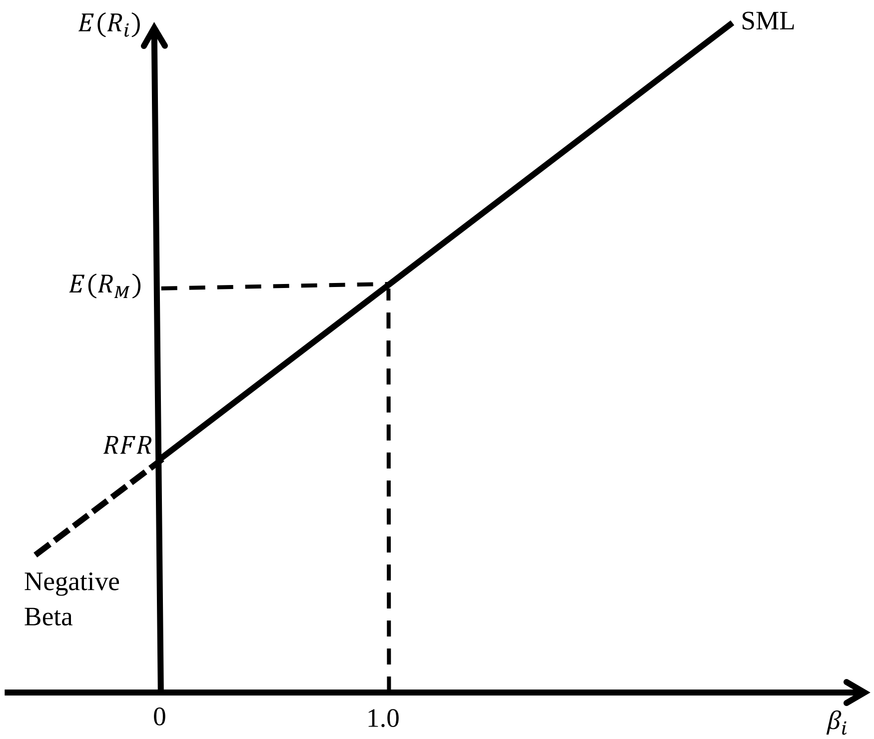
$$E(R_i) = RFR + \beta_i\left[E(R_p) - RFR\right], \tag{2.28}$$

where β_i (*beta*) captures the non-diversifiable portion of an asset's risk which is related to the market as a whole. The market risk premium, $\left[E(R_p) - RFR\right]$, is therefore scaled by an asset's riskiness relative to the market, β_i. *Beta* for asset i can be calculated as:

$$\beta_i = \left(\frac{\sigma_i}{\sigma_M}\right)\left(r_{i,M}\right) = \frac{Cov(R_i, R_M)}{\sigma_M^2}. \tag{2.29}$$

where $Cov(R_i, R_M)$ is the covariance between the return on asset i and the return on the *market portfolio*. In addition, CAPM can be graphically expressed as the *security market line* (SML). This is shown in figure 2.4. The SML is similar to the CML, except that the SML only considers systematic risk, and can be applied to individual assets, instead of just a completely diversified portfolio. In addition, *beta* can be negative.

Figure 2.4 The *security market line* (source: Brown & Reilly, 2009:216).



From figure 2.4 it is expected that all securities should exhibit an estimated rate of return that is consistent with its level of systematic risk. Securities plotting above the SML would be considered undervalued because its estimated rate of return is expected to be above its required rate of return, and *vice versa* for overvalued securities. In a completely efficient market, securities are expected to plot on the SML. CAPM assumes efficient markets (whereby all investors have access to the same information). For example, one of the assumptions is given as: agents have identical knowledge of the market and agree on the same forecasts of asset values (Treynor, 1962:16). Table 2.1 illustrates these assumptions, made by the pioneers of CAPM.

Table 2.1 CAPM assumptions (source: French, 2003:64).

Assumption	Treynor (1962)	Sharpe (1964)	Lintner (1965)	Mossin (1966)
No taxes	Explicit	Implicit	Explicit	Implicit
No frictions (transaction costs)	Explicit	Implicit	Explicit	Implicit
Agents are price takers who all face identical prices	Explicit	Implicit	Explicit	Implicit
Agents maximise expected utility of future wealth	Explicit	Explicit	Explicit	Explicit
Utility represented as a function of return and risk	Explicit	Explicit	Explicit	Explicit
All agents agree that variance (or standard deviation) is the measure of security risk	Explicit	Explicit	Explicit	Explicit
Agents prefer more return to less and display risk aversion	Explicit	Explicit	Explicit	Explicit
A riskless asset (paying an exogenously determined positive rate of interest) exists, and all investors agree that it is riskless	Explicit	Explicit	Explicit	Explicit
All agents share the same subjective probability distribution of expected future prices	Explicit	Explicit	Explicit	Explicit
Fractional stocks may be held	Implicit	Implicit	Explicit	Explicit
Short sales are allowed	Explicitly allowed	Explicitly disallowed	Explicitly allowed	Explicitly allowed
Leverage is allowed	Explicitly allowed	Explicitly disallowed	Explicitly allowed	Implicitly allowed
The number of stocks of each security is constant	Implicit	Implicit	Implicit	Implicit
Agents share the same single period time horizon	Explicit	Explicit	Implicit	Implicit

From the assumptions in table 2.1, it is clear that CAPM is based on efficient markets. Efficient markets and other assumptions have displayed mixed results on the real world application of CAPM (Perold, 2004:22). However, CAPM stimulated a different mind-set within investors. For instance how investors think about the relationship between expected returns and risk, allocation between assets, as well as portfolio performance and capital budgeting. Efficient markets, though, are instrumental for the simplified working of CAPM.

2.6 Efficient Markets (1970-1976)

Macroeconomic factors are an important consideration when trying to assess the functioning of financial markets, especially giving insights into how financial prices move (Moix, 2001:59). These factors, in turn, determine the extent of systematic risk that a completely diversified portfolio is exposed to. The general hypothesis at work is that of informational efficiency within capital markets. The *Efficient Market Hypothesis* (EMH) states that in an efficient capital market, prices of securities adjust instantaneously to the arrival of new information and, therefore, current prices reflect all relevant information about the security (Brown & Reilly, 2009:151).[54] In addition, an efficient market is also one in which investments with higher expected returns have higher levels of risk (Marx *et al*., 2008:31). There are three assumptions that underlie efficient capital markets (Reilly, 1994:195):

i) There are a large number of independent, competing, profit-maximising participants who constantly analyse and value securities.
ii) New information regarding securities comes to the market in a random independent fashion.
iii) Competing investors adjust security prices rapidly to reflect the effect of new information (unbiased adjustment).

These assumptions provide the basis for price changes that are random, and therefore not predictable. In this sense, current prices contain all relevant information, and the best forecast for future prices does not require past price information. Investors also assume that current prices of securities are an accurate reflection of their level of risk. There are, however, various forms of market efficiency that reflect on the informational content of past security prices.

2.6.1 Eugene F. Fama (1965; 1970; 1976): The *efficient market hypothesis*

Early work relating to efficient markets was based on the *random walk hypothesis*, which in turn is based on the notion that changes in stock prices occurred randomly (Brown & Reilly, 2009:153).[55] This hypothesis was extensively tested without any underlying theory.[56] But in three influential articles, Fama (1965; 1970; 1976) attempted to formalise the theory by presenting a *fair game model*. This model stated that investors can be confident about the current market price of a stock because it reflects all relevant information, and therefore stocks provided an expected return consistent with its risk. The *efficient market hypothesis* (EMH) and the

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54 A more accurate definition is that securities are priced in an unbiased fashion at any given time because new information is assumed to arrive in a random, independent fashion, providing no foothold for an upward or downward pricing bias mechanism. Price adjustments are not always perfect, but unbiased, making forecasts unfruitful (Hirt & Block, 2006:256)

55 Samuelson (1965) offered the first proper economic argument for "efficient markets", which focused on the concept of a martingale, rather than a random walk (such as Fama (1970)). Also, see footnote 11.

56 For example, studies including Osborne (1959), Mandelbrot (1963) and Fama (1965).

empirical tests of the hypothesis were separated into three sub-hypotheses on the basis of the information set involved: i) *weak form efficiency*, ii) *semi-strong form efficiency* and iii) *strong form efficiency*.[57]

2.6.1.1 *Weak form efficiency*

Firstly, *weak form efficiency* assumes that current stock prices fully reflect all security market information.[58] This hypothesis therefore implies that past rates of return and other historical market data should have no relationship with future rates of return. If this hypothesis holds then there should be little or no gain from using any trading rule which provides buy or sell signals based on past market data (Bodie *et al.*, 2007).[59] There are two broad groups of tests for *weak form efficiency*, namely, tests of independence and trading rule tests (Brown & Reilly, 2009:154).[60] Tests of independence entail statistical independence tests between rates of return, while trading rule tests focus on a comparison of risk-return results for a trading rule relative to that of a buy-and-hold strategy. These tests seem to uphold *weak form efficiency*, meaning that stock prices seem to be independent over time, or in other words, follow a random walk (Hirt & Block, 2006:257). However, these results are not unanimous (Brown & Reilly, 2009:156).

2.6.1.2 *Semi-strong form efficiency*

Secondly, *semi-strong form efficiency* contends that stock prices fully reflect all public information.[61] This hypothesis incorporates *weak form efficiency* because security market information is considered to be available to the public. The hypothesis therefore implies that investors will not be able to realise above-average returns based on new information that has already become public, because such new public information will already be reflected in the stock's price. No abnormal gain is therefore possible from analysing financial statements or new

57 It was actually Roberts (1967) who distinguished and coined the terms *weak*, *semi-strong* and *strong form* tests, which Fama (1970) used in his later depiction of efficient markets.

58 This includes historical sequences of prices, rates of return, trading volume data, and other market-generated data, such as odd-lot transactions, block trades, and transactions by exchange specialists (Brown & Reilly, 2009:153).

59 Known as chart or technical analyses.

60 As an example of a test of independence see Fama (1965), and a trading rule test see Fama and Blume (1970).

61 This includes all non-market information such as earnings and dividend announcements, price-to-earnings (P/E) ratios, dividend-yield (D/P) ratios, price-book value (P/BV) ratios, stock splits, news about the economy, and political news (Brown & Reilly, 2009:153).

public information.[62] Tests for *semi-strong form efficiency* focus on predicting future rates of return using available public information beyond security market information, or via event studies that examine the speed of adjustment of stock prices to specific economic events (Brown & Reilly, 2009:156).[63] *Semi-strong form efficiency* held unanimously for event studies.[64] However, in contradiction to *semi-strong form efficiency* there are studies on predicting rates of return for a cross-section of stocks that did not support the hypothesis (Hirt & Block, 2006:259).[65] The results of such studies were classified as anomalies, which are discussed further in chapter 4.

2.6.1.3 *Strong form efficiency*

Finally, *strong form efficiency* asserts that stock prices fully reflect all information from public and private sources. No investor or group of investors can constantly earn above-average returns because no one has monopolistic access to information. This hypothesis encompasses *weak form* and *semi-strong form efficiency*. In addition, *strong form efficiency* assumes perfect markets where information is cost-free and immediately available to everyone.[66] Tests for *strong form efficiency* have centred on the analyses of returns, over time, for different investment groups, in order to identify whether any of these groups constantly outperformed the market (Brown & Reilly, 2009:166). Such investors should have access to important private information, or be able to act more quickly on public information.[67] Overall, the research doesn't support *strong form efficiency* (Hirt & Block, 2006:260). For example, specialists on security exchanges were found to achieve above-average rates of return.[68] However, tests on the performance of fund managers indicate that they do not outperform the market, and that market outperformance was negligible when measured against the overall trend (Ippolito, 1993).

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62 Known as fundamental analyses.

63 Studies utilising these tests include Fama *et al*. (1969), Archibald (1972), Kaplan and Roll (1972) and Sunder (1975).

64 These events include stock splits, initial public offerings, world events, economic news, accounting changes and corporate finance events, with mixed results coming from exchange listing research (Brown & Reilly, 2009:166).

65 This includes studies on risk premiums, calendar patterns, and quarterly earnings surprises etc. (Brown & Reilly, 2009:166).

66 No insider trading is possible because no one has access to private information (Brealey *et al*., 1999).

67 Tests analysed the performance of i) corporate insiders, ii) stock exchange specialists, iii) security analysts, and iv) professional money analysts (Brown & Reilly, 2009:167).

68 Further studies debunking *strong form efficiency* include SEC (1963), Peers (1992), Rozeff and Zaman (1988).

The *fair game model* assures investors that they can be confident about the current market price of a stock because it reflects all relevant information, and therefore stocks provided an expected return consistent with its risk; therefore capital market efficiency. CAPM assumes that capital markets are perfect in the sense that i) assets are infinitely divisible, ii) there are no transactions costs, short selling restrictions or taxes, iii) information is costless and available to everyone, and iv) all investors can borrow and lend at the risk-free rate (Perold, 2004:16). Within this framework the capitalisation weighted average of the market *betas* of all stocks is equal to the *beta* of the market portfolio. This implies that the average stock, in an efficient market, has a *market beta* of one. This is thus the benchmark an investor would strive for when holding a portfolio consisting of risky assets.

2.7 Conclusion

Markowitz's 1952 approach, and subsequent evolution of the idea by Sharpe (1964) and others, is the conventional method of approaching portfolio diversification by institutional portfolio managers, both in structuring their portfolios and measuring their performance (Rubinstein, 2002:1044; Sheikh & Noreen, 2012). CAPM was developed on the foundation provided by Markowitz (1952; 1956; 1959) and Tobin (1958). With regards to CAPM, Lintner (1965a) provided the same conclusions that were reached by Treynor (1961, 1962), whereas Mossin (1966) provided a more exact specification within equilibrium conditions than Sharpe (1964) (French, 2003:65). In later studies it was shown that most of CAPM's assumptions could be relaxed. Examples include: i) integrating heterogeneous beliefs (Lintner, 1969; Merton 1987), ii) the effects of taxation (Brennan, 1970), iii) including trading restrictions, transaction costs and information asymmetries (Mayers, 1972), iv) including non-marketable assets (Mayers, 1973) v) allowing for multiple time periods with changing investment opportunities within these periods (Merton, 1973; Breeden, 1979) vi) allowing international investing (Solnik, 1974; Stulz, 1981; Adler & Dumas, 1983), vii) utilising weaker assumptions via arbitrage pricing (Ross, 1976), viii) a zero-*beta* CAPM (Black, 1972), and finally ix) a riskless-asset CAPM (Rubinstein, 1973).

In general, most variants of CAPM have a multi-*beta* expression for expected return, which stems from the same basis of notions (Perold, 2004:22): i) investors hold portfolios that are optimised given their specific needs, constraints and risk preferences, ii) in equilibrium, asset prices reflect these demands, and iii) assets with high expected returns are correlated with a significant, non-diversifiable risk. In essence, capital market theory, as a subset of modern portfolio theory, suggests holding an efficient portfolio with underlying stocks that has a unit correlation with a *market portfolio*. No single portfolio of risky assets is optimal for every investor, but investors rather allocate their wealth differentially among several risky portfolios, which across all investors aggregate to the *market portfolio*.

The application of MPT continued to grow in stature through the late 20th and early 21st centuries, and it is unlikely that its popularity will fade anytime soon (Fabozzi, Gupta & Markowitz, 2002:20). The philosophies, notions and concepts since 1952 have been interwoven into financial economics and portfolio theory to such an extent, that they can no longer be disentangled (Rubinstein, 2002:1044). However, this is subject to the assumption of efficient markets. If markets were not "completely efficient", and stock return patterns did exist to a significant extent (so much so as to realise an abnormal profit), wouldn't it be worthwhile to construct a portfolio with an additional risk measure other than only making use of market *beta*? Chapter 3 discusses the use of just such an additional measure – one that captures volatility dynamics in the form of spill-over effects.

"Shallow men believe in luck or circumstance.

Strong men believe in cause and effect."

~ Ralph Waldo Emerson, essayist, lecturer and poet

CHAPTER 3

The EMH and *beta* have contributed significantly to the advancement of portfolio management. However, due to numerous assumptions and the dynamics of portfolio volatility, *beta* does not encapsulate all the information relevant to a portfolio manager. Volatility dynamics can not be ensnared by only considering its covariance with a market portfolio (*beta*). This chapter looks at the failings of *beta* (especially in the South African context), and why volatility transmission is an important measurement. Volatility may "spill over" from one volatile variable to another (especially during times of financial turmoil), and therefore it is necessary to look deeper into volatility dynamics than just return volatility "covariance". The previous chapter introduced the concept of return covariance, however, this chapter introduces the concept of volatility spill-over, and why such a measure is important when efficient markets are assumed. Using an additional measure that captures volatility dynamics will only be beneficial to a portfolio manager already using a measure that captures return dynamics, especially during times of financial distress.

MODELLING RETURN PATTERNS AND VOLATILITY SPILL-OVER EFFECTS

Some of the most important research over the past few decades has analysed whether capital markets are indeed efficient (Brown & Reilly, 2009:151). If markets were inefficient to a significant extent, the market measure of risk (*beta*) could be ineffective with regard to the construction of a portfolio, prompting the use of another measure for such a task. This research is especially prevalent because of the real-world implication for investors and portfolio managers. In addition, capital market efficiency is one of the most controversial areas in investment research (Marx *et al*., 2008:32). However, a new twist has been added to capital market efficiency by the rapidly expanding field of behavioural finance, which provides interesting insights into many of the financial market anomalies or patterns (Brown & Reilly, 2009:151). In addition, it has been reported that markets can never be perfectly informationally efficient (Grossman & Stiglitz, 1980). Information is costly, resulting in prices that do not perfectly reflect the information that is available, therefore leaving some incentive for information-gathering (security analysis) within a market equilibrium model.

3.1 Stock Return Patterns

As discussed in the previous chapter, Bachelier (1900) had already anticipated the *random walk hypothesis*, which stated that the return process can be expressed as a cumulated series of probabilistic independent shocks. Returns according to the *random walk hypothesis* can be expressed as:

$$R_t = u + \varepsilon_t, \qquad \varepsilon_t \sim iid\ N(0, \sigma^2)\ . \tag{3.1}$$

In equation 3.1, it is assumed that ε_t is identically and normally distributed (Cowles & Jones, 1937; Kendall & Hill, 1953; Roberts, 1959; Osborne, 1959; Working, 1960). This property can be expected if daily returns are constructed from a sum of large numbers of intraday returns with identical distribution and finite variance.[69] However, the sample characteristics of time series data, such as stock returns, are frequently inconsistent with the Gaussian assumption (Moix, 2001:60). Even early studies suggest that a stock market does not follow a random walk (Cootner, 1962; Osborne, 1962). Therefore, several studies have attempted to rematch theory with empirical results by proposing non-Gaussian independent and identically distributed stock returns (Mandelbrot, 1963; Fama, 1965). More recent studies have indicated that the random walk hypothesis does not accurately hold, because stock prices do exhibit patterns during price development (Jegadeesh & Titman, 1993; Lo & MacKinlay, 1999; Malkiel, 2011:13). This proposition is especially of use when considering that stock market anomalies are known to cause serial correlation in returns (Fama, 1965). In more statistical terms, the random walk model does not only rule out dependence in the conditional expectation (i.e. first order interdependence), but also higher order dependencies, of which the most notable is the conditional variance (i.e. second order interdependence) (Moix, 2001:61).

[69] According to the *central limit theorem*.

From the considerable number of studies done on the EMH, one thing is clear - markets do not exhibit the same level of efficiency (Moix, 2001:61). This is because large markets with a great number of educated traders and high trading volumes exhibit stock returns that are less correlated than that of smaller markets (i.e. a market such as South Africa). As the professionalism of a stock market increases, so does its efficiency, giving at least some support to *weak* and *semi-strong form efficiency*.

3.2 EMH's Bane: Anomalies

Since research does not support *strong form efficiency,*[70] a closer look will thus be given to *semi-strong form efficiency*. Although *semi-strong form efficiency* is somewhat supported by research, there are exceptions to the rule. These are known as anomalies or deviations from the overall doctrine that the market is informationally efficient (Hirt & Block, 2006:259). This is exhibited in the empirical evidence that testifies that time series returns are not normally distributed (Moix, 2001:62). The reason for this is captured in financial market anomalies (discussed below) and the patterns caused by asymmetry and persistence in volatility (see section 3.4). To understand the studies done on *beta* in the South African market requires a brief description of market anomalies, such as the customary calendar and value effects.

3.2.1 Calendar and value effects

Calendar effects are seasonal patterns that can be found in the returns of stocks (Moix, 2001:62). There is, for example, the *January effect*, where investors engage in tax selling stocks during the end of the year and buying these stocks, or similar stocks, back in the beginning of the year (Branch, 1977; Branch & Ryan, 1980; Branch & Chang, 1985). It puts downward price pressure on stocks during November and December, while putting upward price pressure on stocks during January.[71] Another anomaly is known as the *weekend effect*, where stock returns tend to peak on Fridays and decline on Mondays (Cross, 1973; French, 1980; Harris, 1986). This is due to information released during weekends (a 72-hour non-trading period) that accentuates a stock's volatility on Monday (Moix, 2001:62). This results in stock mean returns that are negative on

[70] See chapter 2, section 2.6.1.3, on *strong form efficiency*.

[71] This "*tax-selling hypothesis*" also held within foreign countries that had different tax laws to those of the United States (Brown & Reilly, 2009: 159).

Mondays, where the average return for the other four days are positive (Brown & Reilly, 2009: 159). This publically available information on seasonal occurrences are inconsistent with *semi-strong form efficiency*.

Secondly, publically available information regarding firms is used by fundamental analysts to identify stocks that may outperform the market (Hirt & Block, 2006:200). One measure that attracts closer examination is the price-earnings (P/E) ratio. Several studies on the examination of the P/E ratio for stocks have found that stocks with a lower P/E ratio consistently outperformed the stocks with higher P/E ratios, on both a risk-adjusted and non-risk-adjusted basis (see Basu, 1977). Similar results were found even after adjusting for firm size, industry effects, and infrequent trading (Peavy & Goodman, 1983). In addition, the *size effect* reflects that smaller firms consistently experience significantly higher risk-adjusted returns than larger firms (see Banz, 1981). It has been contended that the above-average returns, due to a lower P/E ratio, was instead because of the *size effect* (Reinganum, 1981).[72] The reason why the *size effect* is a major efficient market anomaly is because smaller firms' riskiness is measured incorrectly due to less frequent trading, causing rates of return to be higher than the risk estimate (Brown & Reilly, 2009:161). Percentage transaction costs for these traded stocks are also higher, giving credit to the notion that the *size effect* only holds when a long-term buy-and-hold strategy is followed (Stoll & Whaley, 1983). Optimally, the rebalancing of a stock portfolio should only take place once a year when considering the *size effect* (Reinganum, 1983).[73] This is evidence against *semi-strong efficiency* because it implies that investors could use available public information on size and P/E ratios to generate an above-average future return on their portfolios.

Finally, and of significance, is the book value/market value (BM/MV) ratio. It is the ratio between the book value of a firm's equity to the market value of its equity (Brown & Reilly, 2009:162). There is a significant positive relationship between the current BM/MV ratio and future stock returns (Rosenberg, Reid & Lanstein, 1985).[74] This provides evidence against the EMH. In addition, strong support of this finding was provided by testing the joint effects of

72 However, this result was again disputed in favour of P/E ratios (Basu, 1983).

73 Arbel and Strebel (1983) did a study to confirm the *size effect* but also found evidence of a '*neglected-firm effect*'. Beard and Sias (1997) contended the existence of the latter effect after controlling for capitalisation.

74 In many studies the BM/MV ratio is defined as "book-to-market value" (BV/MV) or "price-to-book value" (P/B). The concept stays the same.

market *beta*, size, P/E ratio, leverage and BV/MV ratio (Fama & French, 1992; 1996). The positive relationship between BM/MV persisted even after including all these various other variables. More significantly, leverage and the P/E ratio were significant on their own, but after including both size and the BM/MV ratio, became insignificant. Lending more credibility to the presence of both *size* and *value effects*. In summary, the P/E ratio, *size effect* and the BM/MV ratio provide strong evidence against *semi-strong form efficiency*. However, the more profound influence seems to be captured by including the *size* effect and the BM/MV ratio.[75]

The arbitrage pricing model (APT), formulated by Stephen Ross in 1976, incorporates more than one risk factor, and provides support for a multifactor asset pricing model, as mentioned above (Ross, 1976). However, it should be noted that these individual risk measures are not explicitly about risk, but are at best a proxy for risk (Perold, 2004:22).[76] This causes *size* and *value effects* to be problematic in deriving them, making *beta* a more appropriate risk measure because of its well-defined properties.[77] In this sense, using CAPM above other multi-factor models provides simplicity, and although multi-factor risk models are well-defined, their inference possibilities are limited. However, this study focuses on the South African equity market, and therefore necessitates a review of *beta* within such context.

3.3 Why not only *Beta* in South Africa

The underlying principle of the CAPM is that there is a linear relationship between systematic risk (as measured by *beta*), and expected stock returns (Laubscher, 2002:131). The CAPM attempts to capture this relationship by means of *beta* to explain the differences between the expected returns on various stocks and stock portfolios. The relationship between *beta* and portfolio returns specifies that an investor's expected return is equal to the risk-free rate plus a risk premium, and that the risk premium increases as risk increases. These parameters need to be estimated. The problem is that these parameters are not constant, but variable over time (Ward,

75 It should be noted that a study found these relationships to be significant only during periods of expansive monetary policy (Jensen, Johnson & Mercer, 1997).

76 These risk "proxies" are known as descriptors. Descriptors are not the risk factors but rather candidates for risk factors because of their ability to explain returns (Fabozzi, Gupta & Markowitz, 2002:18)

77 Size cannot be a risk factor that affects expected returns, since this will cause smaller firms to combine into larger firms; furthermore, the *value effect* is based on giving equal weight to small and large firms that is disproportionate to capitalisation-weighted value indexes (Perold, 2004:22).

2000:35); and because of problems with estimating these parameters, it is not possible to apply the CAPM in its purest form (Firer, 1993:25).

Empirical studies done on the Johannesburg Stock Exchange (JSE) regarding the effectiveness of CAPM have mostly drawn the same conclusions as studies done on international markets (Laubscher, 2002:131). According to Bradfield, Barr and Affleck-Graves (1988) CAPM is a reasonable model for explaining the risk/return relationship on the JSE. This was confirmed by Ward (1994) who extended on the work of Bradfield *et al.* (1988) by providing evidence of the validity of the CAPM upon the valuation of JSE stocks. Van Rhijn (1994) also found the CAPM to be explanatory of JSE stock returns and *beta* to be satisfactory in explaining the risk/return relationship. In addition, Bowie and Bradfield (1997) found that the stability of *beta* is similar to those found on the US and UK stock markets after taking the effect of thin trading on the JSE into account.

There are however also various studies that found contrary results. Upsher (1993) for example observed a significantly positive relationship between expected returns and predictable volatility on the JSE, but found that stock returns did not adhere to the CAPM assumptions. Although his findings provided some support for the CAPM, he found overbearing shortcomings with the effectiveness of the CAPM on the JSE. Keogh (1994) also found *beta* to be unstable over time, diminishing the practicality of using the CAPM with regard to the JSE.

The CAPM is of use to portfolio managers as it does describe and explain the risk/return relationship; however, other risk factors, other than *beta*, may also be useful for capturing the risk of stock returns (Laubscher, 2002:131). Studies on whether other risk factors capture the stock return relationship better than *beta* have provided differing conclusions. Ward (1994) provided support for the CAPM on JSE mining and industrial stocks; however, he also provided evidence that a multi-factor *arbitrage pricing theory* (APT) provides a more effective explanation of JSE stock returns than that of CAPM. Van Rensburg (1998) suggested that there are at least two, but no more than three, risk factors that are priced on the JSE. Furthermore, according to Brevis (1998:6), studies done by Knight and Firer (1989), Smith and Chapman

(1994), Gavin (1995) and Meyer (1997) evidence is presented that APT models provide a more explanatory account of returns on JSE stock portfolios than the CAPM.

Regarding value and size effects on the JSE, Graham and Uliana (2001) found that value stocks offer higher returns, and that value stocks outperform growth stocks. In addition, using a Fama and French three-factor model on the JSE, Basiewicz and Auret (2010) show that their model does indeed capture the value effect, but has more difficulty in capturing the size effect. This model significantly improved upon pricing errors observable on the JSE; something that the traditional CAPM could not capture. However, the lack of growth firms (and general illiquidity of the JSE compared to the U.S. markets) causes mispricing of different types of stocks (Basiewicz & Auret, 2010:23). Most interesting was a study done by Van Rensburg and Robertson (2003), who also found persistent size and value effects in the cross-section of stock returns on the JSE, but in addition, found that *beta* had an inverse relationship with returns. This is contrary to the large body of empirical research done on *beta*. Furthermore, Strugnell, Gilbert and Kruger (2011), using data from January 1994 to October 2007, found support for Van Rensburg and Robertson's (2003) earlier findings. However, when *beta's* were estimated by means of the Dimson Aggregated Coefficients method (which takes thin trading into account) with a lead and lag of three months, the negative relationship between *beta* and return loses statistical significance, resulting in *beta* having no predictive power on JSE stock index returns.

Despite the existence of empirical evidence and criticism against the CAPM and *beta* as risk measurement, it remains a beneficial measurement for portfolio managers, especially with regard to the cost of capital and investment performance evaluation (Campbell *et al.* 1997:183; Moyer *et al.* 2001:204). In addition, CAPM does provide insights into the risk/return relationship, and although various empirical studies have refuted the validity of CAPM, the fact that CAPM is stated in terms of *ex ante* parameters, *ex post* analyses cannot provide an absolute rejection of the CAPM and its parameters (Levy 1997:147). However, several of the CAPM assumptions does exhibit severe shortcomings.[78] Of particular interest to this study is the assumption that investors are only interested in the mean and the variance of returns - and therefore does not care about the downside risk or upside risk within certain market conditions (Laubscher, 2002:131). This

78 Also refer to section 3.2 and 3.4.

assumption is insufficient since investors do view risk more intricately as simply considering the mean and variance of returns (Ward 2000:35; Leland 1999:28). Therefore, *beta* is regarded as an incomplete risk measure on its own, and need some additional measures to properly capture the dynamics of portfolio return.

In conclusion, from this review of the South African application of portfolio theory, it can be asserted that the CAPM is not completely miss-specified. However, the practical application of the model does present difficulties (especially in turbulent market conditions and within a less liquid market). Therefore, portfolio managers should be cautious with reliance on the CAPM, and indeed *beta* alone to evaluate investment performance, and should also include other measures of risk to enhance portfolio performance.

3.4 Why Volatility Plays an Important Role in Risk Measurement

The persistent nature of volatility also causes patterns in stock returns. As mentioned in section 3.2, not only does anomalies cause doubt in the efficient application of *beta*, but also asymmetric and persistent volatility. Such patterns can be seen in the second, third and forth moments of the return distribution, as discussed below.

3.4.1 Leptokurtosis and negative skewness

Before continuing the discussion on the third and fourth moments of the return distribution, it is necessary to provide a brief description on asset return variation (volatility).[79] A stylised fact about financial market data is that various asset returns have differing degrees of variation, but most of these exhibit long "tails" compared to a normal distribution (Larson, 1960; Working, 1960; Houthakker, 1961; Alexander, 1961). Characteristically, financial asset return distributions (usually) cross the normal distribution at least three times, with asset returns exhibiting longer left tails and higher peaks. This implies that financial data returns vary within a smaller band

[79] The concept of financial market volatility will firstly be defined on a broader scale and then narrowed down.

than the normal distribution and exhibit significant negative outliers. Financial market data is therefore typecast as being leptokurtic (Maniya, & Magnusson, 2010:10).[80]

In related studies conducted on monthly returns for S&P 500 for each decade between 1926 and 2006, it was found that arithmetical returns were negatively skewed for five out of eight decades, with kurtosis varying between three to nine for these periods (Corrado & Su, 1997; William, 2007).[81] Skewness and kurtosis for a normal distribution is presented as:

$$S = E\left[\frac{(R_t - \mu)^3}{\sigma^3}\right] = 0\,, \tag{3.2}$$

$$K = E\left[\frac{(R_t - \mu)^4}{\sigma^4}\right] = 3\,, \tag{3.3}$$

where μ is the mean and σ^2 the variance of the distribution. A skewness of zero and a kurtosis of three will equal the form and tails of a normal distribution (Gujarati, 2006:66). The distribution in the studies mentioned previously was therefore non-normal. Financial time series data has been found to be significantly leptokurtic with evidence of weak negative skewness. This phenomenon has been recorded in numerous studies for stock returns.[82] The consequence is that normality suppresses the occurrence of large positive and large negative returns (where the third moment is leptokurtic). This postulates a well-known shortcoming when working with financial market data (Maniya, & Magnusson, 2010:10). Of particular interest are the supporting empirics which indicate that the leptokurtosis of a return series increases as the frequency of the data increases (Blattberg & Gonedes, 1974). Evidence for non-normality is therefore weaker for monthly data than intraday data.

80 Leptokurtic, meaning slim or long-tailed, indicates the presence of significant outliers to that of a normal distribution (Gujarati, 2006:67).

81 Kurtosis is known as the third moment of the distribution and skewness as the fourth moment. A normal distribution exhibits a kurtosis of three and a skewness of zero. Negative skewness means a longer left tail (extreme losses) then a right tail (extreme gains).

82 See Fama (1965), Praetz (1972), Blattberg and Gonedes (1974), Kon (1984), Tucker (1992), Kim and Kon (1994), Duffie and Pan (1997), and Hurst and Platen (1997).

3.4.2 Serial correlation in squared returns

Stock returns, r_t, are not serially correlated except for a small possibility at lag one (DeFusco, McLeavey, Pinto & Runkle, 2004).[83] This possible serial correlation at lag one is due to non-synchronous or thin trading (Poon, 2005:7). The lack of serial correlation with stock return patterns is due to the notion that returns are not predictable with regard to speculative assets. This corresponds to *weak form market efficiency*, which states that outperformance of the market portfolio is impossible when based on past values of stock return (Brown & Reilly, 2009:153). In contrast, squared returns exhibit significant serial correlation although it appears roughly uncorrelated (Mandelbrot, 1963; Fama, 1965). This means that a large shock in the returns of a stock's price is likely to be followed by larger variance in the subsequent periods, and vice versa. This phenomenon is more widely described as volatility persistence (Poon, 2005:7). Squared returns are directly linked to the unconditional variance in returns (Moix, 2001:65), which can be expressed as:

$$\begin{aligned} Var[R_t] &= E(R_t - E[R_t])^2 \\ &= E[R_t^2] - (E[R_t])^2 \,. \end{aligned} \tag{3.4}$$

The average term $E[R_t^2]$ is greater than the term $(E[R_t])^2$ by a factor of approximately 700 to one for daily returns (Jorion, 1995). Ignoring this reality will cause bias in the variance estimate of returns (Moix, 2001:65). Serial correlation in the variance of returns is a plausible explanation for the serial correlation in squared returns. Financial asset returns exhibit periods of turbulent trading (high volatility) which are concentrated, followed by periods of tranquil trading (low volatility) which are also concentrated.[84] If a stock experiences a period of highly volatile trading, it tends to experience highly volatile trading in the next period as well, and vice versa (Asteriou &Hall, 2007:249). This phenomenon is simply known as volatility clustering (Poon, 2005:7).

[83] This is of relevance for any type of speculative asset, but for continuity purposes, reference is made to common stock.

[84] Of note is that the normal distribution did not display any volatility clustering due to the property of equal variance.

In summary, all these anomalies (section 3.2 and 3.4) provide insight into why a market is not as efficient as theory postulates, and leaves the door open to exploit such patterns. However, it is less apparent whether violations of *weak form efficiency* exist, proving that unexploited stock price patterns (based on historical data) should not persist in any efficient market (Malkiel, 2011:13). But various other patterns also form when a stock market is entrenched in financial turmoil. For example, from *size* and *value effects* it is noticeable that the price of a stock is linked to a firm's fundamentals in the long-run. However, stock prices do deviate from these fundamentals during "speculative bubbles", where a stock's price becomes influenced by short-term speculation (Sørensen & Whitta-Jacobson, 2010:399). Arguably the most profound argument against the EMH is that security markets have often experienced speculative bubbles (Malkiel, 2011:32). Such a speculative bubble did occur within the US housing market which unravelled during 2008, causing globally induced financial turmoil. There are various implications of such "*financial turmoil induced*" volatility.

3.5 Further Accentuated Volatility: Financial Crises

The financial crisis of 2008 crippled economies in its wake, and pushed the world into a global recession. The crisis shook the very basis of financial market efficiency. Furthermore, there exists a distinguishable pattern during financial distress. These patterns are seen in returns and volatility of differing assets of the same type (e.g. stocks listed on the same exchange) and different markets are biased in moving together (Bauwens, Laurent & Rombouts, 2006:79; Poon, 2005:8). Numerous studies have found that the correlation between asset volatilities is stronger than that of asset returns and that both volatility and return correlations tend to increase during bear markets and financial crises (Longin & Solnik, 1995; Kaminsky & Reinhart, 1998; Maniya & Magnusson, 2010). A stock portfolio's second moment interdependencies therefore tend to become noisier during financial distress, not because of riskier stocks, but rather unstable unsystematic (market) conditions. Volatile market conditions therefore exposes a stock portfolio to harsher volatility circumstances than would otherwise be possible if the stocks only impacted on each other, therefore, positive co-integration.

3.6 Volatility

It is generally accepted that financial volatilities are co-integrated (and therefore tend to move together) over time across assets and markets (Bauwens, Laurent & Rombouts, 2006:79; Poon, 2005:8). Although numerous studies have been done within the field of market volatility spill-over effects, less have been performed on the asset's side.[85] Furthermore, there is a huge volume of literature on modelling daily volatility, and far less on modelling intra-daily volatility (Engle & Sokalska, 2012:56).

Co-integrating volatilities between markets have been tested in numerous studies, of which some are mentioned. So far most studies of first and second order interdependencies (return and volatility spill-over effects) using an aggregate shock (AS) model have focused on inter-market contagion effects. For example, Lin, Engle and Ito (1994) used intra-daily data in an AS model to analyse the international transmission mechanism of volatility and returns between stocks listed on the New York Stock Exchange and the Tokyo Stock Exchange and find limited lagged contemporaneous spill-over effects from New York to Tokyo.[86] Koutmos and Booth (1995) also tested spill-over effects between the New York Stock Exchange and the Tokyo Stock Exchange, further including the London Stock Exchange. They utilised a multivariate E-GARCH to model how quantity and quality of news affects stock prices across markets and found significant volatility spill-over effects between these markets.[87] The significant finding was that bad news created greater sensitivity within markets. A multivariate E-GARCH model is also used by Christofi & Pericli (1999) to test volatility spill-over effects between five Latin American equity markets. Other examples include spill-over effects between European equity markets (Koutmos, 1996), and Asian equity markets during the 1997 Asian Financial Crisis (In, Kim, Yoon & Viney, 2001). Kanas (1998) also made use of an E-GARCH model to test for volatility spill-over effects between the three largest European equity markets before and after the 1987 stock market crash. It was found that volatility spill-over effects essentially doubled during the post-crash

[85] Kearney and Patton (2000) and Karolyi (1995) exemplify such studies. See also Lin, Engle and Ito (1994), NG (2000), Worthington and Higgs (2004), and Piesse and Hearn (2005) to name but a few.

[86] Three models were used to test for contemporaneous spill-overs. These were a GARCH-M model, an AS model, and a single extraction (SE) model. The GARCH-M and AS models performed similarly, and were both superior to the SE model.

[87] A multivariate E-GARCH framework eliminated several problems associated with a univariate framework. For instance, efficiency of repressors and the impact of asymmetry are improved.

period. Finally, Piesse and Hearn (2005) tested the transmission of volatility returns between Sun-Saharan African countries. Using a univariate E-GARCH model they conclude that volatility in the South African market is affected more by bad news than positive news and these effects remain persistent throughout the study.[88] These studies have shed light on the transmission of volatility between markets. However, this particular study aims to shed light on the transmission between stocks listed on the same exchange. The focus will be on a microstructure level rather than on a macrostructure level (as was the case in the studies mentioned above), particularly on intraday volatility spill-over effects between stocks listed on the JSE top-40 in the two years following the 2008 financial crisis. However, in order to understand volatility transmission between stocks listed on the same exchange, knowledge of the modelling of volatility is needed.

The modelling of volatility has become of cardinal importance in finance and effective risk management. Volatility and return dynamics affect how various economic variables react to changing conditions. Portfolio managers, for example, consider the volatility of a given stock before adding it to a portfolio. This results of such an pre-emptive volatility analysis would thus be channelled back to rebalance the portfolio for optimal gains. However, an in-depth understanding of return volatility requires a better understanding of the price process of stocks within the financial microstructure. Therefore, traditional methods of portfolio management must firstly be considered for effective return volatility measurement.

3.6.1 *Beta* and volatility spill-over effects

Portfolio managers have long been familiar with the *efficient market hypothesis* (EMH) where a well-diversified portfolio with a unit correlation with the market is considered entirely hedged against unsystematic risk, in other words, a portfolio with a *beta* equal to one. However, systematic risk still remains even after fully diversifying. In this regard volatility within and between stocks in a portfolio impacts on the profitability of the portfolio, as well as the portfolio's overall risk profile. Since portfolio managers in smaller economies such as South Africa are limited in their choices of stocks, it becomes increasingly difficult to fully diversify a

88 The Sub-Saharan countries include Botswana, Ghana, Kenya, Malawi, Mauritius, Namibia, Nigeria, South Africa, Zambia and Zimbabwe.

stock portfolio given volatility spill-over effects between stocks listed on the same exchange. Such spill-over effects become more prevalent during times of financial distress. Of particular interest are the intraday volatility spill-over effects between stocks, and whether considering this indicator of volatility provides a clearer picture for the effective rebalancing of a stock portfolio (and therefore a risk-return trade-off) than the traditional *market beta* risk measurement on its own.

3.6.2 Measuring volatility

Understanding volatility plays an imperative role in effectively pricing assets and securities, diversifying portfolios and hedging portfolio risk (Harju & Hussain, 2011:82). This is especially true when considering that stock portfolios are prone to downside volatility when financial markets are in turmoil. It is therefore not surprising that a large body of research has been devoted to understanding the financial market microstructure, with special emphasis on intraday stock returns (see Tse & Yang, 2011). Volatility estimates on an intraday level is especially useful to evaluate risk of slow trading or as an additional measure of time-varying liquidity (Engle & Ferstenberg, 2007). However, the most prevalent use of intraday volatility estimates is to optimise strategies for placing limit orders or to schedule trades (Engle & Sokalska, 2012:56).[89]

3.6.2.1 Using intraday data

The past few decades have seen rapid development in information technology and storage capacity, which enabled data to be collected and analysed at extremely high frequencies (Poon, 2005:10). In the financial market setting this is especially the case with the more prevalent availability and use of intraday data. Using intraday (high frequency) data may expose new information about a time series that is not observable in lower data aggregations (Harju & Hussain, 2011:84), and has therefore provided a means for more accurate volatility estimates (Poon, 2005:10).

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89 The literature provides adequate evidence that volatility is an important factor influencing order submission strategies (see Ellul, Holden, Jain & Jennings, 2007; Griffiths, Smith, Turnbull & White, 2000).

Lopez (2001) shows, by using daily squared returns, that the volatility estimator, for forecasting purposes, is exceedingly imprecise.[90] The use of daily return data (as opposed to intraday data) to estimate daily volatility will generate a specifically noisy volatility estimator. This is due to the reality that most financial data is characterised by non-normality and therefore an asymmetric distribution (Poon, 2005:12).[91] A volatility estimator of daily squared returns leads to extremely low R^2 values and weakens the conclusions from such inference (Anderson & Bollerslev, 1998; Christodoulakis & Satchell, 1998). Caution is required in empirical findings that report such a noisy volatility estimator. However, by using an average of intraday squared returns, inference is improved (Anderson & Bollerslev, 1998; Blair, Poon & Taylor, 2001; Fuertes, Izzeldin & Kalotychow, 2009:5).

Blair, Poon and Taylor (2001) report a three- to fourfold increase in R^2 when 5-minute intraday squared returns are used to proxy return volatility instead of daily squared returns.[92] Figure 3.1 illustrates this difference. It shows the volatility estimates over a 7-year period stretching between January 1993 and December 1999. These two graphs look similar but show a clear reduction in noise when intraday data is used as the return volatility estimator.

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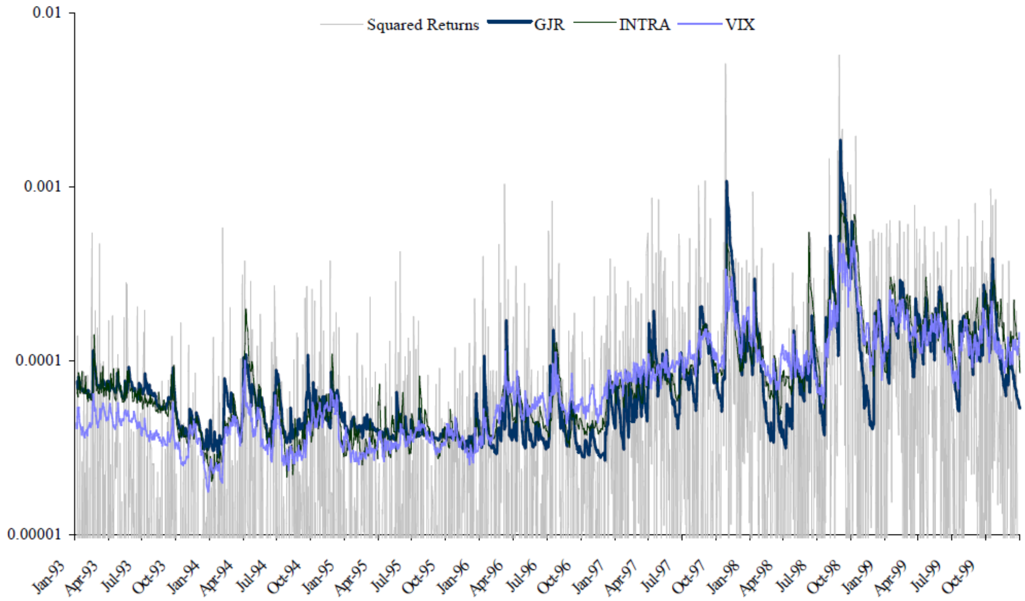
90 Using daily squared returns, Lopez (2001), shows that e_t^2 is an imprecise estimator of σ_t^2 (although unbiased). In the results e_t^2 was found to be 50% greater or smaller than σ_t^2 for about 75% of all estimations.

91 Most financial data is characterised by high kurtosis (leptokurtosis) which leads to fat-tailed distributions (Maniya & Magnusson, 2010:10).

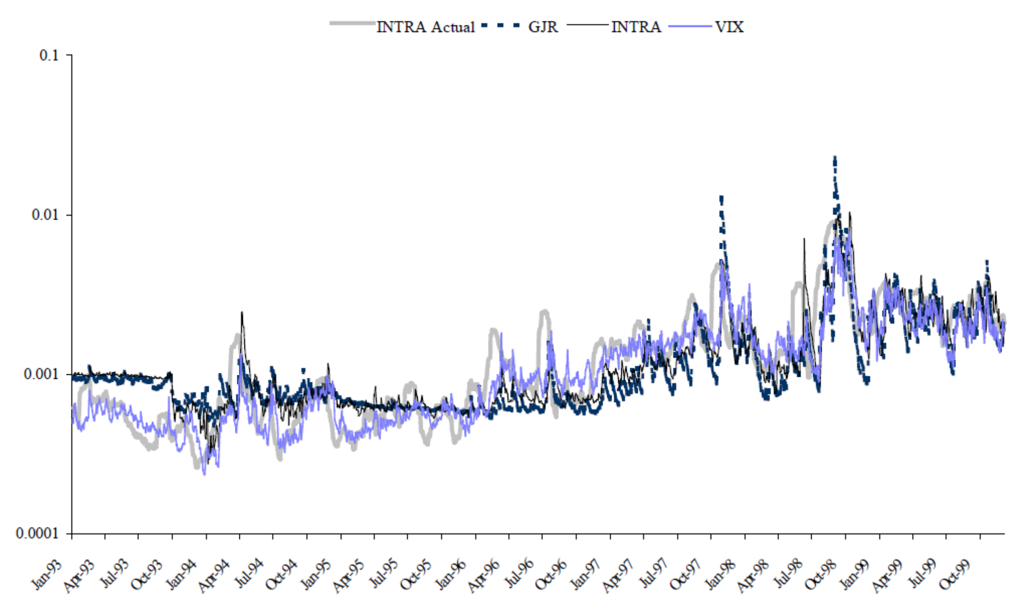
92 The study conducted 1-day-ahead volatility forecasts based on historic return volatilities.

Figure 3.1 S&P100 daily volatility for the period from January 1993 to December 1999 (source: Blair, Poon & Taylor, 2001:32).

(a) Conditional variance proxied by daily squared returns.



(b) Conditional variance derived as the sum of intraday squared returns.



In figure 3.1 the light grey line showcases the conditional variance, with standard deviation greatly reduced when the sum of intraday squared returns is used. However, for forecasting purposes, conventional modelling of intraday volatility was found to be unsatisfactory at the Olsen conference on High Frequency Data Analysis in Zurich in March 1995 (Engle & Sokalska, 2012:55).[93] Estimation of conventional intraday models for different intraday frequencies gives forecasting parameters that are unsatisfactory (Anderson & Bollerslev, 1997). These frequencies are not consistent with theory as proposed by Drost and Nijman (1993). The reason for these inconsistencies is the pronounced intraday pattern in stock returns and trading activity (Engle & Sokalska, 2012:55). These daily patterns indicate that there is great deal of information that is enclosed within intraday data (as opposed to larger data intervals), and the ability to utilise such information will only be beneficial to portfolio managers. The use of intraday data (or tick data) has been seminal in the management of portfolios (Anderson & Bollerslev, 1998). The specific timing of transaction events in a period of time (using intraday data as opposed to daily data) is therefore a significant economic variable which needs to be modelled in order to provide reasonable information regarding return volatility (Cai, Kim, Leduc, Szczegot, Yixiao & Zamfur, 2007:1). Transaction timing of securities and the volatility it implies is therefore an important study in the field of portfolio management.

3.6.2.2 Measuring return volatility

As mentioned previously, the use of daily squared returns delivers inferior statistical inference potential when compared to intraday squared returns (known as realised volatility) due to excessive noise.[94] The use of the latter, high frequency, data is adequate in modelling volatility (Merton, 1980). In more empirical terms, if the log price process is semi-martingale, then the increment of its quadratic variation over a specified time interval can be consistently estimated by its realised volatility (Protter, 2004). In particular, the ARCH type models, together with the

93 Volatility is conventionally modelled using a GARCH framework (Engle, 1982; Bollerslev, 1986).

94 Realised volatility refers to the volatility estimate calculated using intraday squared returns at short intervals; normally 5 to 15 minutes (Poon, 2005:14).

majority of time series volatility models, are squared return models, thus utilising realised volatility (Poon, 2005:11).[95]

There are various other proxies for measuring volatility because, unlike prices and returns, the volatility process is unobserved, even *ex post* (Fuertes, Izzeldin & Kalotychow, 2009:2). Various studies suggest measuring volatility directly from absolute returns because it provides models that are more robust against asymmetry and nonlinearity (Davidian & Caroll, 1987; Ding, Granger & Engle, 1993; Ederington & Guan, 2000). However, using absolute returns exposes the dependence structure to particularly more serial correlation than the squared return. This phenomenon was dubbed the "*Taylor effect*" as it was found that absolute returns of speculative assets have significant serial correlation over long horizons (Taylor, 1986). Furthermore, absolute returns contain a significant number of measurement errors and this impedes reliable inference (see Chan & Fong, 2006). Other studies on intraday volatility extend on the GARCH-framework by including an additional augmented regressor that captures intraday information.[96] These financial market microstructure theories are usually tested on an intraday transaction-by-transaction basis in order to improve the modelling of the moments of the return distribution (Cai *et al*., 2007:1).

The main focus of the above-mentioned models is to bring normality to the return distribution. However, economic time series exhibit times of unusually high volatility followed by periods of tranquil volatility (Asteriou & Hall, 2007:249). Squared returns exhibit significant serial correlation although it appears roughly uncorrelated (Mandelbrot, 1963; Fama, 1965). This is also the case with stock return volatility within a trading day where daily open return volatilities, and to some extent the closing return volatility, is more volatile than the period between these boundaries (Bollerslev, 1986).[97] Analysing the microstructure of stock returns is central to further the understanding of economics and portfolio management, especially during times of financial distress. This creates an environment where ARCH-type models are used to model the

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95 Engle (1982) introduced the first ARCH model. From his seminal work various other ARCH-type models were conceptualised, such as GARCH (Bollerslev, 1986), E-GARCH (Nelson, 1991), TGARTCH (Zakoian, 1994), etc.

96 Examples of the intraday augmented regressor include daily high-low price range (Parkinson, 1980; Taylor, 1987), the number of intraday price changes (Laux & Ng, 1993), daily trading volume (Bessembinder & Seguin, 1993) and the standard deviation of intraday returns (Taylor & Xu, 1997).

97 This phenomenon is known as volatility clustering.

return volatility because such models are meticulously suited to deal with the variance of such time series (Asteriou & Hall, 2007:249). In addition, as stated earlier, ARCH-type models are squared return models. This study therefore focuses not on the unconditional variance of stock returns, but rather the conditional variance using intraday squared returns as a proxy for volatility. It is the price process of stocks within an intraday environment that is of concern in this study, especially the spill-over effects between stocks during certain times in the day. In order to estimate these spill-over effects, intraday stock patterns (price formation) need to be identified.

3.6.3 The price process of stocks

Market microstructure analysis is important in discerning the interaction between trading procedures and security price formation, because price formation is related to a security's return volatility (Tian & Guo, 2007:289). Numerous empirical studies have found that daily open return volatilities are usually higher than close volatilities, with flattened volatility in between the daily open and close of a security.[98] This is the typical 'U' shape volatility distribution first published by Wood, McInish and Ord (1985). Their data consisted of intraday returns of stocks listed on the New York Stock Exchange (NYSE) for the periods of September 1971-February 1972 and the calendar year of 1982, and the results are shown in figure 3.2.

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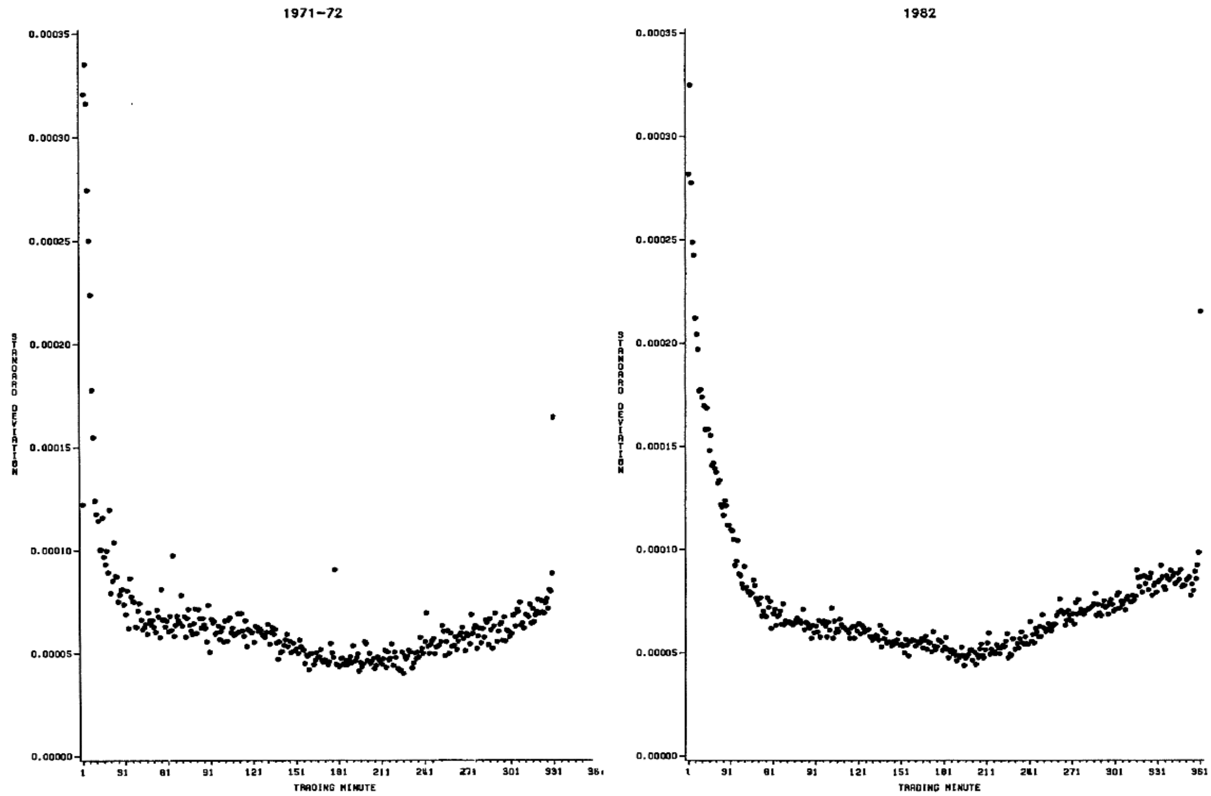
98 See for example Wood, McInish and Ord (1985), Schreiber and Schwartz (1986), Anderson and Bollerslev (1998), Areal and Taylor (2002), Poon (2005) and Tian and Guo (2007).

Figure 3.2 Standard deviation of NYSE returns across days (by minute) (source: Wood, McInish & Ord, 1985:727).



The higher open volatility was first explained via three general views, namely: i) differing trading mechanisms between the opening and closing of a trading day (Amihud & Mendelson, 1987), ii) the monopoly power of the specialist (Stoll & Whaley, 1990), and iii) the long halt (non-trading period) before the market opens (Amihud & Mendelson, 1991). However, the highly volatile opening price of intraday returns is not the result of trading mechanisms (such as call auction) but rather the combined effect of accumulated overnight information and the trading halt effect (Tian & Guo, 2007:290). The end of the trading day is volatile for a different reason – where traders seek to close some of their positions that may be exposed to further overnight information (Ozenbas, Pagano & Schwarts, 2010:45). These intraday patterns therefore originate from differing behaviour of traders during certain periods of the trading day (Wang, Yamasaki, Havlin & Stanley, 2006:2). An intraday volatility pattern does not essentially link higher volatility to a specific driver, but does reveal the price discovery process of stock returns (Tian & Guo, 2007:296). In today’s highly complex markets, data that captures such behaviour is becoming more readily available, painting a clearer picture about how new information and various microstructure factors affect the dynamics of stock returns and volatility within a trading day. Stock prices over longer periods, such as a week or a month, react mostly to the arrival of new information, whereas in short periods, such as daily or intra-daily, stock prices are also influenced by various microstructure factors (Ozenbas *et al.*, 2010:45).[99] More specifically, macro-economic and institutional changes contribute to lower frequency data volatility, whereas trading pressure and turbulence prompt volatility in high frequency data (Daly, 2008:2379).

It is the arrival of new information pertaining to a stock's fundamentals that is the most important factor affecting the market price of a stock, and the sudden arrival of such information results in volatility clustering in high frequency data (Daly, 2008:2379). According to the *mixture of distribution hypothesis* (MDH), volatility (or the variance in returns) is an increasing function of arrival information.[100] Given the dynamics of this hypothesis, it is reasonable to assume that the volatility spill-over effects between stocks are attributable to information spill-over effects. When there is an interdependent relationship between stocks, these interdependencies will be an increasing function of arrival information relating to the market (Kitamura, 2010:159). Of particular interest are asymmetric information influences, which is especially prevalent during times of financial turmoil. The body of literature agrees that negative past information cause larger current volatility than positive past information.[101] This phenomenon is known as the "*leverage effect*" (Christofi & Pericli, 1999:81).[102] Furthermore, volatility asymmetry increases when the fall in a given stock price is larger.[103] In addition, volatility within stock price returns can be traced back to an initial information flow (Karpoff, 1987). This links with the view that overnight information flows, at a microstructural level, cause higher volatility within larger capitalisation stocks, which then leads smaller capitalisation stocks in price discovery (Ozenbas *et al.*, 2010).

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99 Microstructural factors such as transaction costs, blockages, complexities of price discovery and stock volatility spill-overs.

100 See Clark (1973) and Tauchen and Pitt (1983).

101 For example see Mayers (1972), Christofi and Pericli (1999), McAleer and Veiga (2008), Daly (2008), and Kitamura (2010).

102 According to the *leverage effect*, negative stock returns yield a higher debt-to-equity ratio, and therefore higher volatility (Black, 1976; Christy, 1982).

103 The use of realised volatility causes a similar, albeit weaker, relationship in volatility (Poon, 2005:8).

Ozenbas *et al.* (2010) found that when the market opened, price volatility was worse and relative trading was higher for larger capitalisation stocks than smaller capitalisation stocks. This is mainly due to the overnight non-trading period. No such relationship existed at the end of the trading day. Furthermore, it was also reported that larger capitalisation stocks lead smaller capitalisation stocks in price discovery during market opening; providing evidence of volatility spill-over effects from larger capitalisation stocks to smaller capitalisation stocks. Again, no such relationship existed during market closing because traders only accentuate volatility by trying to close out their own positions before the overnight non-trading period. Intraday volatility is therefore a complex variable, but the measurement and understanding thereof leads to greater efficiency of portfolio management. Such efficiency is especially important when considering that a period of financial distress is characterised by significantly higher volatility within stock markets.

3.6.4 The effect of financial crises

The globally devastating financial crisis of 2008 has left economies severely damaged, especially those of the United States and Europe. Emerging economies too, have suffered increased financial turbulence in the wake of increased global financial integration (Boshoff, 2006:61). Due to the emergence of economic shocks emanating from one country, investors in another opt to rebalance their portfolios in the wake of perceived macro-economic risk factors, that become more volatile in the wake of perceived contagion effects. This creates a situation where a non-crisis country faces downward pressures on asset prices because investors are risk-averse, resulting in a crisis being transmitted from one country or market to another (Boshoff, 2006:65).

The crisis has stunned the very basis of modern-day investment theory, which believed the doctrine that financial markets were mostly efficient. Although inefficiency within a crisis is not a given, the general investment public perceives volatility as an indicator of market disruption during a crisis, resulting in unambiguously priced stocks and equity markets not functioning properly (Daly, 2008:2378). Critics have gone so far as to suggest that the *efficient market hypothesis* (EMH) was largely responsible for the 2008 crisis (Malkiel, 2011:1). The EMH does not imply that assets are always “correctly” priced, because prices deviate from their fundamentals, but that all investors accept this “deviated price” as the most efficient (Malkiel, 2011:6; Reilly & Brown, 2012:140). The extent of “deviation” is uncertain, and this is known as a price bubble. A stock portfolio’s second moment interdependencies therefore tend to become noisier during financial distress, not because of riskier stocks, but rather because of unstable unsystematic (market) conditions. Volatile market conditions therefore expose a stock portfolio to harsher volatility circumstances than would otherwise be possible if the stocks only impacted on each other. However, after a price bubble bust, there exists a distinguishable pattern during financial distress. It has been noted that the returns and volatility of differing assets of the same type (e.g. stocks listed on the same exchange) and different markets are inclined to move together (Bauwens, Laurent & Rombouts, 2006:79; Poon, 2005:8). Stock prices do overreact, providing evidence of substantial *weak form* market inefficiencies (De Bondt & Thaler, 1985).[104] Furthermore, stock prices are acknowledged to exhibit irrational price movements (known as fads), especially during an economic crisis (Cho & Yoo, 2011:246). However, stock return volatility across different time frequencies, different industries and different countries exhibited a short-lived, large burst of volatility during late-2008, which lasted for less than a year (Schwert, 2011:18).[105] A positive relationship between high market volatility, economic recessions and financial crises therefore does exist (Daly, 2008:2379). Considering all these influences means that volatility of stock returns encompasses a great deal of information. However, in order for such information to be discerned, stock returns (*beta*) and return volatility (spill-over effects) must somehow be empirically modelled for proper inference.

3.7 From *Beta* to Spill-over Effects

The CAPM helped in decomposing the returns of a portfolio into components that are of concern to portfolio managers (see Merton, 1973). These components are the market return and the risk involved in realising a return on the portfolio (*beta*). Studies done by Campbell (1996) and Chen (2003) take one step further by including additional risk factors for explaining portfolio returns. Firstly, Campbell (1996) reveals that additional relevant risk factors are market excess return and innovations in variables that predict market excess returns. The variables he uses to capture volatility in returns are the aggregate dividend yield (DIV), default premium (DEF), term premium (TERM) and the one-month Treasury bill yield (RF). A multifactor “CAPM”, which

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[104] This study marked the start of behavioural finance (Sewell, 2011:5).

[105] The countries include the U.S., U.K. and Japan. The frequencies include monthly, daily and intradaily.

includes such innovations (shocks) in variables, significantly increases the explanatory power in the cross-section of average portfolio returns (Petkova, 2006).[106] However, there are some difficulties associated with such multifactor models.[107] Volatility in returns only captures one dimension of portfolio returns. Volatility in return variance also holds invaluable information regarding portfolio returns, especially when there are volatility spill-over effects from the market or other sources. With this in mind, Chen (2003) extends on Campbell's work by introducing fluctuations in market volatility, and its spill-over effects to a portfolio of stocks, as an additional measure. He calls this new risk factor the aggregate volatility shock (AVS), which measures the market volatility spill-over effects to stocks for a given portfolio. Using a GARCH (1,1) model, Chen (2003) found that stock return variance that co-vary positively with this shock earn low average returns, while negative covariance with AVS have high expected returns. Misirli (2011), in turn, extends on Chen’s work by estimating an E-GARCH (1,1) model to measure market volatility spill-over effects, because E-GARCH models capture asymmetric effects between positive and negative market returns. The essence of such asymmetric issues has been discussed by Michayluk *et al*. (2006); however, measuring such effects is a culmination of numerous empirical research projects in various fields.

Measuring volatility spill-over effects between markets has garnered substantial study over the years, even before the widespread use of ARCH-type models. Eun and Shim (1989) used a vector autoregressive (VAR) system to test for spill-over effects between nine developed equity markets, and found substantial evidence of international market integration. However, ARCH-type models, after its introduction by Engle (1982), has gained widespread recognition in its ability to capture volatility dynamics (McAleer & Veiga, 2008:2). For example, Wahab (2012) used an M-GARCH model to study asymmetric volatility spill-over effects from the U.S. to several European markets. Booth, Martikainen, and Tse (1997), and later Krause and Tse (2012),

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106 A multifactor model is also known as an *arbitrage pricing theory* (APT), which was first developed by Ross (1976). Researchers have advocated that the CAPM is flawed with respect to explaining expected returns, and that a multi-factor model such as the APT may provide a better description (Laubscher, 2002:136). The APT advocates that returns are a function of various macroeconomic risk factors - and not just *beta* (Campbell, Lo and MacKinlay 1997:217; Radcliffe 1997:292). See chapter 2, section 2.3.

107 Problems which include: i) risk factors are not identified, ii) it is descriptive by nature (i.e. explaining what is and not what should be), and iii) it does not take into account that risk factors may cause stock returns to change rapidly (Jones 1998:247; Arnott 1993:16).

both use an E-GARCH model to show that volatility transmission among closely-related markets (former) and ETFs (later) is asymmetric in nature, thus exhibiting significant spill-over effects.

The asymmetric volatility phenomenon (AVP) exhibited by markets and financial assets is well documented in the financial literature (see Bekaert & Wu, 2000). Most studies on the subject construe that the AVP is caused by either "leverage effects," or "volatility spill-over effects".[108] It is for this reason that the inclusion of an asymmetric parameter is seminal for capturing volatility spill-over effects in stock indices and financial assets (Bae & Karolyi, 1994). The wide-spread use of ARCH-type models is specifically adapted to capture such asymmetric spill-over effects (McAleer & Veiga, 2008:2). In a study done by Liu, Chiang, and Cheng (2012) on the S&P 500 ETFs for testing spill-over effects, it was found that the E-GARCH model was the most appropriate of four GARCH specifications used. This corroborates Kim and Kon's (1994) findings that demonstrate that Nelson's (1991) E-GARCH model is most descriptive with regard to capturing spill-over effects between financial assets. Moreover, Engle and Ng (1993) and Stevenson (2003) provided support for the E-GARCH model performing remarkably well in capturing asymmetric volatility effects. Furthermore, the aggregate shock (AS) model utilises an E-GARCH model in a two-step process. Tamakoshi and Hamori (2013) made use of a two-step E-GARCH model (similar to the aggregate shock model) to capture volatility spill-over effects between sovereign and banking sector credit default swaps.[109] In addition, Hamao *et al.* (1990), Park (2001), Ng (2000), Christiansen (2003) and Cadarajat and Lubis (2012) used an AS model to analyse volatility and mean spill-over effects between financial assets.

3.8 Modelling Return Volatility and Spill-over Effects

The analysis of the financial market microstructure in turn created a need for the development of volatility models to accurately estimate large covariance matrices (McAleer & Veiga, 2008:3).[110]

108 See Christie (1982), Pindyck (1984), French, Schwert, and Stambaugh (1987), Campbell and Hentschel (1992) and Wu (2001).

109 The only difference being causality tests using weighted cross correlations (Tamakoshi & Hamori, 2013:263).

110 For financial application a portfolio of assets and risks is usually constructed, and subsequently assessed using one of several multivariate models, which specify the risk of one asset as depending dynamically on its own past risk as well as on the past risk of other assets. See Li *et al*. (2002) for a survey of theoretical developments for conditional volatility models, and McAleer (2005) for an examination of a variety of univariate and multivariate, conditional and stochastic, financial volatility models.

Because of the particular prevalence of distinct intraday volatility patterns, which underlie most of the financial market microstructure literature, higher-frequency returns exemplify highly persistent conditionally heteroskedastic elements together with discrete information arrival effects (Anderson, Bollerslev & Das, 2001:306). For a greater understanding of microstructure elements and spill-over influences, such effects must be modelled.

The modelling of heteroskedasticity has its roots in the Autoregressive Conditional Heteroskedasticity (ARCH) model proposed by Engle (1982). The wide-spread use of ARCH-type models is based on their ability to capture several dynamics of financial returns, including time-varying volatility, persistence and clustering of volatility, asymmetric reactions to positive and negative shocks and therefore volatility spill-over effects (McAleer & Veiga, 2008:2). Volatility spill-over effects between different assets refer to causality in return variance, and have seen a great deal of study in the field of financial economics (Kitamura, 2010:158).[111]

3.8.1 The ARCH-family models

Volatility clustering, squared returns, asymmetry and correlation within a stock portfolio on an intraday basis are conjoint in a particular family of models that deal with the prevalent heteroskedastic elements present in such data. As mentioned before, higher-frequency returns exemplify highly persistent conditional heteroskedastic elements together with discrete information arrival effects (Anderson, Bollerslev & Das, 2001:306). To start off, an investor who buys a stock at time t and sells it in time $t+1$would not be satisfied in only knowing the rate of return, nor will unconditional heteroskedasticity be of use. What is of importance is the conditional heteroskedasticity over the holding period as it is an estimate of the riskiness of the stock (Asteriou & Hall, 2007:250).[112]

3.8.2 Engle (1982): ARCH

Traditional econometric models typically assume unconditional heteroskedasticity (Maniya & Magnusson, 2010:9). Therefore, time series data needed to be transformed to exhibit constant

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111 Causality in return variance is the impact of any previous volatility of a particular asset on the current volatility of another asset.

112 Unconditional heteroskedasticity is defined as long-run forecast variance and is therefore treated as constant. Conditional heteroskedasticity is short-run variance and therefore mainly non-constant.

variance in order to be modelled. Engle (1982) present a model that did not assume constant variance, but rather modelled the prevalent conditional heteroskedasticity in time series data. [113] The model proposes that the variance of the residuals at time t depends on past squared error terms. Consider the simple model:

$$Y_t = a + \beta' X_t + u_t \,, \tag{3.5}$$

where X_t is a $k \times 1$ vector of explanatory variables and β is a $k \times 1$ vector of coefficients. Normally u_t is assumed to be independently distributed with a mean of zero and constant variance σ^2. Mathematically shown as:

$$u_t \sim iid\ N(0, \sigma^2) \,, \tag{3.6}$$

Engle's idea starts by allowing the variance of the residuals (σ^2) to depend on its own past history, permitting the modelling of heteroskedaticity as follows:

$$\sigma_t^2 = \alpha_0 + \gamma_1 u_{t-1}^2 \,, \tag{3.7}$$

which is the basic ARCH(1) process with γ_1 representing the effect of the one-lag squared error on present variance. The ARCH(1) model simultaneously models the mean and variance of the time series with the following specification:

$$\begin{gathered} Y_t = a + \beta' X_t + u_t \\ u_t | \Omega_t \sim iid\ N(0, h_t) \end{gathered} \tag{3.8}$$

$$h_t = \alpha_0 + \gamma_1 u_{t-1}^2 \,, \tag{3.9}$$

113 Robert Fry Engle III won the 2003 Nobel Memorial Prize in Economic Sciences, sharing it with Clive Granger, for their seminal research in modelling time-varying volatility in economic time series (ARCH).

where Ω_t is the information set.[114] Equation 3.8 is called the mean equation and equation 3.9 the variance equation. The ARCH(1) model states that when a shock happens in period $t-1$, it is more likely that the value of u_t will increase (in absolute terms due to the squares). That is, when u_{t-1}^2 is large/small, u_t also tends to be large/small. Conditional heteroskedasticity can depend on more than one lag. In general the ARCH(q) process can be presented as:

$$\begin{aligned} h_t &= \alpha_0 + \gamma_1 u_{t-1}^2 + \gamma_2 u_{t-2}^2 + \cdots + \gamma_q u_{t-q}^2 \\ &= \alpha_0 + \sum_{j=1}^{q} \gamma_j u_{t-j}^2 \,, \end{aligned} \tag{3.10}$$

In both these cases the estimated coefficients of γ's have to be positive for positive variance. The ARCH(q) model is covariance stationary when the sum of the autoregressive parameters is less than one (Poon, 2008:38). This model is attractive because it asserts that a large shock in the previous period is expected to cause considerable volatility in the current period (Maniya & Magnusson, 2010:10). ARCH models are therefore versatile on the grounds of capturing important stylised facts of financial data (Daly, 2008:2381).[115] But, ARCH models do exhibit some weaknesses. For example, there is no sure-fire way of determining the number of lags to include, and ARCH models often yield negative estimates of the γ_j's (Asteriou & Hall, 2007:260).[116] For this reason ARCH models are used infrequently. To resolve these shortfalls, Bollerslev (1986) developed the Generalised ARCH (GARCH) model.

3.8.3 Bollerslev (1986): GARCH

One of the shortcomings of the ARCH process was that it looked more like a moving average specification than an auto regression (Engle, 1995). This is where the idea of including the lagged conditional variance terms as autoregressive terms sprouted, starting the family of GARCH models (Asteriou & Hall, 2007:260). The general GARCH(p,q) model of Bollerslev (1986) can be mathematically described as:

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114 Note that the notation of the variance has changed from σ_t^2 to h_t for ease of representation.

115 These stylised facts were described in section 3.2 and 3.4.

116 Sometimes an immense number of lags are required to capture dependence in the conditional variance

$$Y_t = a + \beta' X_t + u_t \tag{3.11}$$
$$u_t | \Omega_t \sim iid\ N(0, h_t)$$

$$h_t = \alpha_0 + \sum_{i=1}^{p} \delta_i\, h_{t-i} + \sum_{j=1}^{q} \gamma_j u_{t-j}^2\ , \tag{3.12}$$

which states that the value of the variance scaling parameter, h_t, now depends both on past values of the shocks (u_{t-j}^2) and on past values of itself (h_{t-i}). Once again the estimated coefficients of γ's and δ have to be positive for positive variance. For $p = 0$ the model is reduced to an ARCH(q) model. The GARCH(p, q) is covariance stationary where:

$$\sum_{i=1}^{p} \delta_i + \sum_{j=1}^{q} \gamma_j < 1\ . \tag{3.13}$$

The simplest GARCH is the GARCH(1,1) model of which the variance equation can be given as:

$$h_t = \alpha_0 + \delta_1 h_{t-1} + \gamma_1 u_{t-1}^2\ , \tag{3.14}$$

This model specification has frequently performed well, and is simple to estimate due to only having three unknown parameters α_0, $\gamma_1$and δ_1 (Asteriou & Hall, 2007:260). There are many alternative specifications that emerged from the GARCH model, one of which, the E-GARCH specification developed by Nelson (1991), is employed in this study.

3.9 Methodology

There are three main differences between the E-GARCH model and the standard GARCH model. Firstly, GARCH models assume that good news and bad news of similar degrees impact volatility in the same way, whereas the E-GARCH model permits good news and bad news to impact volatility differently (Daly, 2008:2385). Thus, the GARCH model only assumes that the magnitude and not the sign of unpredicted excess returns determines h_t (Nelson, 1991:349). A second limitation of GARCH models is the imposition of nonnegative constraints on the estimated coefficients of γ's and δ, which ensure that h_t remains nonnegative for all t with probability one (Nelson, 1991:349). This constraint rules out any random oscillatory behaviour in the h_t process. In more familiar speech, GARCH fails to appropriately capture the volatility that excess negative returns in financial data causes due to leptokurtosis. Finally, the analyses of the persistence of shocks to conditional variance provide a limitation in GARCH modelling. If volatility shocks last indefinitely, it may cause the whole term-structure of risk premia to impact on investment decisions within long-lived capital goods (Nelson, 1991:349). For instance, in a GARCH(1,1) model, shocks may persist in one form and die out in another, which may give rise to exploding conditional moments for a strictly stationary process (Nelson, 1991:350).

The GARCH model's ability to gracefully capture volatility clustering of equity returns makes for an ideal setting for testing market volatility patterns (Nelson, 1991:349). However, despite its popularity in financial application, ARCH and GARCH models are incapable of capturing essential characteristics of financial and economic data. The most captivating of these characteristics being the leverage or asymmetric effect (Daly, 2008:2384).

As stated before, unconditional price or equity returns tend to exhibit fatter tails than a normal distribution, in the form of weak skewness and excess kurtosis (Moix, 2001:62). The E-GARCH model formulated by Nelson (1991) is one of the more successful and effective attempts to model excess conditional kurtosis, which is based on a generalized exponential distribution (Daly, 2008:2394). Nelson's E-GARCH model aims to improve on Bollerslev's GARCH model by eliminating some of the restrictive shortcomings in capturing changes in the volatility of stock market returns (Nelson, 1991:347).

3.9.1 Nelson (1991): E-GARCH

Nelson starts of by stating that if h_t is to be the conditional variance of u_t given information at time t, it must be non-negative with probability one. GARCH models accomplish this by imposing a nonnegative constraint on the estimated coefficients, making h_t a linear combination (with positive weights) of positive random variables (Nelson, 1991:350). Nelson uses an alternative natural device for ensuring that h_t remains non-negative, by making $\ln h_t$ linear in some function of time and lagged h_ts. The E-GARCH model stipulates conditional variance in logarithmic form. Therefore, no nonnegative constraint is required to avoid negative variance. The E-GARCH(p, q) model's variance equation can be given as:

$$\ln h_t = \alpha_0 + \sum_{i=1}^{q} \delta_i \ln h_{t-i} + \sum_{j=1}^{p} \beta_j f\left(\epsilon_{t-j}\right), \tag{3.15}$$

where $f(\epsilon_t) \equiv \left[\theta_j \epsilon_{t-j} + \gamma_j\left(\left|\epsilon_{t-j}\right| - E\left|\epsilon_{t-j}\right|\right)\right]$ and $\beta_1 \equiv 1$. Equation 3.15 can therefore be rewritten as:

$$\ln h_t = \alpha_0 + \sum_{i=1}^{q} \delta_i \ln h_{t-i} + \sum_{j=1}^{p} \left[\theta_j \epsilon_{t-j} + \gamma_j\left(\left|\epsilon_{t-j}\right| - E\left|\epsilon_{t-j}\right|\right)\right], \tag{3.16}$$

with:

$$\epsilon_t = \frac{u_t}{\sqrt{h_t}}. \tag{3.17}$$

In the above formulation h_t depends on both the size and sign of u_t, while α_0 and δ_i are real, non-stochastic, scalar sequences. In addition, this process is covariance stationary only if $\sum_{i=1}^{q} \delta_i < 1$ (Poon 2005:41). The term $\theta_j \epsilon_{t-j}$ resolves the sign effect, and $\gamma_j\left(\left|\epsilon_{t-j}\right| - E\left|\epsilon_{t-j}\right|\right)$ establishes the size effect of innovations (both terms having a mean of zero). If the distribution of ϵ_t is symmetric, these components are statistically significant. Over the range $0 < \epsilon_t < \infty$, $f(\epsilon_t)$ is linear in ϵ_t with slope $\theta + \gamma$, and over the range $-\infty < \epsilon_t \leq 0$, $f(\epsilon_t)$ is linear with slope $\theta - \gamma$. Thus, $f(\epsilon_t)$ permits the conditional variance process h_t to react asymmetrically to rises and falls in stock price. If it is assumed that $\gamma > 0$ and $\theta = 0$, the innovation in $\ln h_{t+1}$ would be positive (negative) when the magnitude of ϵ_t is larger (smaller) than its expected value. Alternatively, if $\gamma = 0$ and $\theta < 0$, the innovation in $\ln h_{t+1}$ is now positive when returns shocks are negative, and vice versa. The E-GARCH specification therefore captures the stylised fact that negative shocks may lead to subsequently higher conditional variance than positive shocks and vice versa.

In relation to the previously discussed GARCH limitations, E-GARCH provides a solution in explaining positive and negative shocks to stock returns. In addition, concerning the second limitation, there are no inequality constraints present in equation 3.15 because the β_j terms can be both negative and positive. Thirdly, because the GARCH model suffers from an inability to evaluate variance shock persistence, the E-GARCH model improves on this by providing a linear process ($\ln h_t$) which is simple to validate for stationarity and ergodicity.[117] The E-GARCH specification further improves on the standard ARCH models since it formulates conditional volatility to be a function of both the magnitude and direction of shocks (Samouilhan, 2006:250).

3.9.2 Aggregate Shock model

The AS model follows a two-step procedure in which fitted values for e_t and h_t in equations 3.18 and 3.20 respectively, are obtained. These fitted values are then respectively substituted into equations 3.19 and 3.21, before revealing the estimated equations. In the AS model, the alternative stock returns in a given portfolio are specified as:

$$B_t = a_1 + \beta_1 AltS_{t-1} + e_t \, , \tag{3.18}$$

where B is the returns of an alternative stock within the same portfolio for period t; e_t captures the factors that affect returns which are unexplained by the autocorrelation of the current period stock returns with the previous period stock returns (persistence or volatility clustering). Thus, e_t represents that part of stock returns which cannot be explained based on available public information when equity trading is initiated at the start of each period. Stock returns at the same period t, can be modelled as:

$$A_t = a_2 + \beta_2 S_{t-1} + \phi e_t + u_t. \tag{3.19}$$

Equation 3.19 includes a coefficient ϕ which is the relationship between the returns on stock B and the returns on stock A, all of which are within the same portfolio. The error term e_t now

117 If shocks to $\ln h$ subside quickly and the deterministic component (α_0) is eliminated, then $\ln h_t$ is strictly stationary and ergodic (Nelson, 1991:351).

represents the unexplained returns on stock B for period t. An E-GARCH(p, q) process is used in order to determine the level of volatility spill-over between stock B and stock A. A univariate E-GARCH approach is followed to eliminate the potential "*curse of dimensionality*".[118] It is assumed that the error term e_t in equation 3.18 is normally distributed with a mean of zero and a variance that follows an E-GARCH(p, q) process:

$$\ln h_{B,t} = \varpi_1 + \delta_1 \ln h_{B,t-1} + \gamma_1 \frac{\varepsilon_{B,t-1}}{\sqrt{h_{B,t-1}}} + \alpha_1 \frac{\left|\varepsilon_{B,t-1}\right|}{\sqrt{h_{B,t-1}}}, \tag{3.20}$$

where the natural log of the conditional variance for e_t in period t is a function of the time invariable mean reversion value, ϖ, the natural logarithm of the past conditional variance, $\ln h_{B,t-1}$, as well as the level of the standardised residuals, $\varepsilon_{B,t-1}/\sqrt{h_{B,t-1}}$, and absolute value of the standardised residuals, $\left|\varepsilon_{B,t-1}\right|/\sqrt{h_{B,t-1}}$. The subscript B denotes an alternative stock (Stock B).[119] Finally, it is assumed that the error term on stock A returns, u_t, is also normally distributed with a mean of zero and a variance that follows an E-GARCH(p, q) process:

$$\ln h_{A,t} = \varpi_2 + \delta_2 \ln h_{A,t-1} + \gamma_2 \frac{\varepsilon_{A,t-1}}{\sqrt{h_{A,t-1}}} + \alpha_2 \frac{\left|\varepsilon_{A,t-1}\right|}{\sqrt{h_{A,t-1}}} + \kappa_i h_{B,t} . \tag{3.21}$$

The model specification of the variance of stock A in 3.21 includes an alternative stock (stock B) measure, $h_{B,t}$, which allows for explicit testing of the relation between stock A volatility and stock B volatility. The $\kappa_i h_{B,t}$ term in equation 3.21 is stock B's conditional variance term, and denotes the relation between stock A's volatility and stock B's volatility. The inclusion of the terms $\varepsilon_{A,t-1}/\sqrt{h_{A,t-1}}$ and $\left|\varepsilon_{A,t-1}\right|/\sqrt{h_{A,t-1}}$ makes it possible to model the asymmetric volatility to past shocks as long as $\gamma_2 \neq 0$. If $\gamma_2 < 0$ then negative shocks (bad news/negative past errors) will have a larger effect on volatility than positive shocks (good news/positive past errors). When $\gamma_2 > 0$, positive shocks cause a greater effect than negative shocks.

118 The relatively "smaller" amount of data results in a parsimonious nature of testing, and avoids the large data sets that may render multivariate models impractical in empirical applications (McAleer & Veiga, 2008:4).

119 That is all the various alternative stocks included within the same portfolio.

3.10 Motivation for E-GARCH in a Univariate Two-Step Process

It should be mentioned that various GARCH-type models have especially been used to model the co-varying movements of volatilities in financial assets (Asai & So, 2012:2). Some of the most prevalent models are the diagonal GARCH model (Bollerslev, Engle & Wooldridge, 1988; Ding & Engle, 2001) and the BEKK model (Engle & Kroner, 1995) which models conditional covariances directly. Other such models also include the M-GARCH model applied by Bollerslev (1990) to model short-run nominal exchange rates, the VARMA-GARCH model of (Ling & McAleer, 2003). The conditional correlation models are also seen in some seminal works; these include the dynamic conditional correlation (DCC) model of Engle (2002), the varying conditional correlation (VCC) model of Tse and Tsui (2002), the generalized DCC model of Bauwens, Laurent & Rombouts (2006), the generalized autoregressive conditional correlation (GARCC) model of McAleer, Chan, Hoti and Lieberman (2008), and the double smooth transition conditional correlation (DSTCC) model of Silvennoinen and Teräsvirta (2009), in which the assumption of constant conditional correlations and dynamic conditional correlations and covariances are relaxed. Finally there are also the generalized orthogonal GARCH model of van der Weide (2002) and the matrix E-GARCH model of Kawakatsu (2006).[120] These multivariate specifications have been developed to incorporate the leverage effects and fat tails (i.e. asymmetric effects) that are typical characteristics of financial time series (Bauwens *et. al.*, 2006).

Multivariate model specifications capturing asymmetric effects, are generally based on either the GJR model (Glosten, Jagannathan & Runkle, 1992) or the E-GARCH model (Nelson, 1991), whereby positive and negative shocks of equivalent magnitude have dissimilar effects on conditional volatility (Asai & So, 2012:3). GJR based models include the VARMA-AGARCH model (McAleer, Hoti & Chan, 2009), as well as an asymmetric BEKK model (Kroner & Ng, 1998). However, Laurent, Rombouts and Violante (2009) tested 16 different multivariate volatility models with the DCC and CCC variations, and found that the GARCH-type models always outperformed the other models (including GJR specifications). Interestingly, the DCC, specifically considers asymmetric effects on volatility and correlation separately (Cappiello, Engle & Sheppard, 2006). Nevertheless, this study utilises *market beta* to capture return

[120] For further details of these models, see the review papers of McAleer (2005) and Bauwens *et. al*. (2006).

correlation, and univariate AS modelling to capture asymmetric volatility co-movement; therefore, the need for multivariate specifications (such as BEKK and DCC) becomes less parsimonious. A serious restriction of DCC-type models is the assumption of common dynamic parameters for correlations across all pairs of stocks; and in the relaxation or absence of such restrictions, the number of parameters to be estimated becomes too large for dimensional models (Chen, 2011:18).[121] Too many variables make the estimation of any multivariate GARCH-type model too complex and, in cases where the amount of data is limited, completely non-feasible (Borovkova & Lopuhaä, 2012). For this reason, individual effects on stock returns cannot be estimated accurately using multivariate specifications, and return correlation and volatility spill-overs need be modelled independently within a univariate framework.

The effect of *beta* needs to be measured separately in order to distinguish the effect from that of volatility spill-over effects. Hecq, Laurent & Franz (2012) suggested that it is more beneficial to firstly look at the individual influences a financial asset has on other financial assets prior to estimating complex multivariate GARCH-type models. This helps in discovering, and separately estimating, the dynamic behaviour certain assets share with one another, which cannot be seen in a whole set of assets.[122] Moreover, univariate E-GARCH modelling in an AS framework is used to eliminate the potential "*curse of dimensionality*".[123] Caporin and McAleer (2009) found that BEKK models especially suffer from this condition. Multivariate GARCH models utilise a one-step maximum likelihood estimation (MLE) procedure which jointly estimates variances and covariances as opposed to a two-step process in which covariances are estimated after variances (Chen, 2011:8).[124] Estimation of variances and covariances in one-step (which guarantees positive semi-definiteness) requires numerous parameters and non-linear estimation, with the number of parameters increasing exponentially with the number of assets. It is for minimising dimensionality and maximising parsimony that this study utilises a two-step process. In addition,

121 The DCC model imposes identical α and β coefficients across all the stocks. However, Chen (2011) shows that the α-coefficients may vary across pairs of stocks.

122 On a side note, Wang and Wu (2012) shows that univariate GARCH-type models allowing for asymmetric effects (such as the E-GARCH) are more accurate than multivariate models when used to capture volatility in the U.S. energy market.

123 The relatively "smaller" amount of data results in a parsimonious nature of testing, and avoids the large data sets that may render multivariate models impractical in empirical applications (McAleer & Veiga, 2008:4).

124 Chen's (2011) study utilised a univariate two-step procedure, and preferred the use of an E-GARCH model to estimate the univariate steps (It was most accurate of several specifications tested).

using only a two-step E-GARCH process provides empirical results that can relate to most studies done on univariate volatility spill-over modelling. Lastly, the E-GARCH model presented by Nelson (1991) still remains "*one of the most popular GARCH-type models for modelling the volatility of financial time series*" (Hafner & Linton, 2013:1).

3.11 Conclusion

Since being presented, the ARCH model has been further developed and refined. Of particular significance is the Generalised Autoregressive Conditional Heteroskedasticity (GARCH) model conceptualised by Bollerslev (1986) which includes a fitted variance from the historical variance. These models have spurred the development of various other autoregressive conditional volatility models, including the E-GARCH model developed by Nelson (1991). The E-GARCH model will also form the basis of measuring spill-over effects within the Aggregate Shock (AS) Model used in this study.[125] The E-GARCH model is of particular interest within this study because it transforms the '*leverage effect*' into being exponential (rather than quadratic), subsequently guaranteeing that the estimates of conditional variance are non-negative (Asteriou & Hall, 2007:268). This causes the E-GARCH specification to be well suited for measuring asymmetries, especially in the sense that good news (positive shocks) generates less volatility than bad news (negative shocks), and consequently enabling researchers to study first and second order interdependencies between return generating assets.

Intraday volatility is not just of cardinal importance for short-term traders, but also long-term investors. This is due to brief-time-interval-volatility being an indicator of the efficiency with which stock prices are set; and inefficient prices lead to overly expensive execution costs (Ozenbas *et al*., 2010:45). Furthermore, microstructure factors that influence stock price volatility dies out in the long-run (Hasbrouck & Schwartz, 1988; Bessembinder & Rath, 2008). This study seeks to shed some additional light on this issue by investigating the asymmetric transmission of intraday volatility between stocks listed on the same exchange during financial distress. This is done by testing for second moment interdependence between stock returns.

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125 As an example of an aggregate shock (AS) model see Lin, Engle and Ito (1994) who used a signal-extraction (SE) model and an AS model to estimate the extent of return volatility correlation of stock indices between Tokyo and New York.

Although most studies testing for first and second order interdependencies (return and volatility spill-over effects) using the AS model have focused on inter-market contagion effects, this study will focus on a microstructure changes rather than looking at a macrostructure level. This is done for the purpose of providing more relevant information when rebalancing a stock portfolio. If significant volatility spill-over effects are found to exist between stocks on an intraday level, a comparison to *market beta* will be made to see if there is any relationship between return co-movement and volatility interdependencies. The performance of the adjusted portfolio will be constructed to be the same as the original portfolio, which was equally weighted, and its market β measured. In conclusion, this study attempts to test whether it is possible to construct a less risky portfolio by additionally limiting volatility spill-over effects between stocks in the wake of turbulent stock exchange conditions, rather than using only *market beta* as a risk measure.

"*Never invest in any ideas you can't illustrate with a crayon.*"

~ Peter Lynch, US fund manager

CHAPTER 4

Chapter 2 gave an account of return dynamics, while chapter 3 provided an account of volatility dynamics. Both these chapters presented a measure for capturing each of these dynamics, and why it is important for a portfolio manager to use such measures in aiding portfolio stock selection. It is only fitting to test whether these measures can be used as compliments to each other, and therefore provide additional information to a portfolio manager who participates in a market where conditions, in itself, is volatile.

EMPIRICAL ESTIMATION AND RESULTS

The data used in this study consists of intraday stock returns from five stocks listed on the JSE top-40. The five stocks encompass those listed by Anglo-America, ABSA, Bidvest, SABMiller and Sasol. The stock price of each of these stocks was refined down to hourly prices for each trading day. This allows for eight data points per trading day, as trading starts at 09:00am on the JSE, and closes at 05:00pm. These hourly prices are converted into hourly returns, which is useful when considering the interpretation of the results that follow. Results are therefore given, for example, as average return per hour.

The data for each of these stocks spans a period from the 1st of July 2008 until the 30th of April 2010. This single period was broken up into 10 lesser periods, which were chosen at random, but still providing thorough coverage of the period as a whole. These 10 periods are each two months in length. A description of these periods is given in table 4.1.

Table 4.1 Period dates and events.

	Start date	End date	Significant Events
Period 1	1 Jul. '08	29 Aug. '08	
Period 2	15 Sep. '08	14 Nov. '08	Lehman Brothers declares insolvency. Thabo Mbeki ousted.
Period 3	1 Des. '08	30 Jan '09	Dow Jones drops 680 basis points.

Period 4	10 Feb. '09	9 Apr. '09	
Period 5	13 Mar. '09	13 May '09	Run-up to Zuma election.
Period 6	22 Jun. '09	21 Aug. '09	
Period 7	3 Sep. '09	2 Nov. '09	
Period 8	9 Nov. '09	8 Jan. '10	Gill Marcus appointed SARB chair.
Period 9	11 Jan. '10	10 Mrt. '10	Haiti Earthquake.
Period 10	1 Mrt. '10	30 Apr. '10	

These periods provide a chance to capture volatility spill-over effects for a period just before the Lehman Brothers bankruptcy on 15 September 2008, and the subsequent periods thereafter (including other global events such as the 2010 Haiti earthquake). In addition, to measure market *beta*, the JSE All Share index (J203) is used. Daily returns on the J203 are estimated using the closing price of each day. The same method is then applied for estimating the returns for each stock in any given portfolio (for comparability).[126] Daily *beta* is utilised because realised (intraday) *beta* is shown to be less persistent and predictable (Anderson, Bollerslev, Diebold & Wu, 2004:14).

4.1 The Basic Idea

This study attempts to expand on the research by Markowitz (1959) that concluded that an equal-weighted portfolio's variance should decrease as diversification increases. As mentioned in chapter 2 above, Markowitz (1959) also found that portfolios consisting of stocks with uncorrelated returns increase its diversification, while the overall risk of the portfolio approaches zero. If returns are correlated, an increase in diversification will cause portfolio risk to approach "average covariance", or as Markowitz coined it, "*the law of the average covariance*".

This study is therefore set up to test for a derivative of uncorrelated stocks by testing whether a reduction in volatility spill-over effects will also lead to lower portfolio risk. In order to test whether volatility spill-over effects between stocks play a noticeable role in overall portfolio risk, it necessitates the creation of proxy stocks of each stock within the portfolio during a given period. These proxy stocks will have similar returns and standard deviations as the actual stock it replaces. Table 4.2 provides the name of the actual stock and its designated proxy.

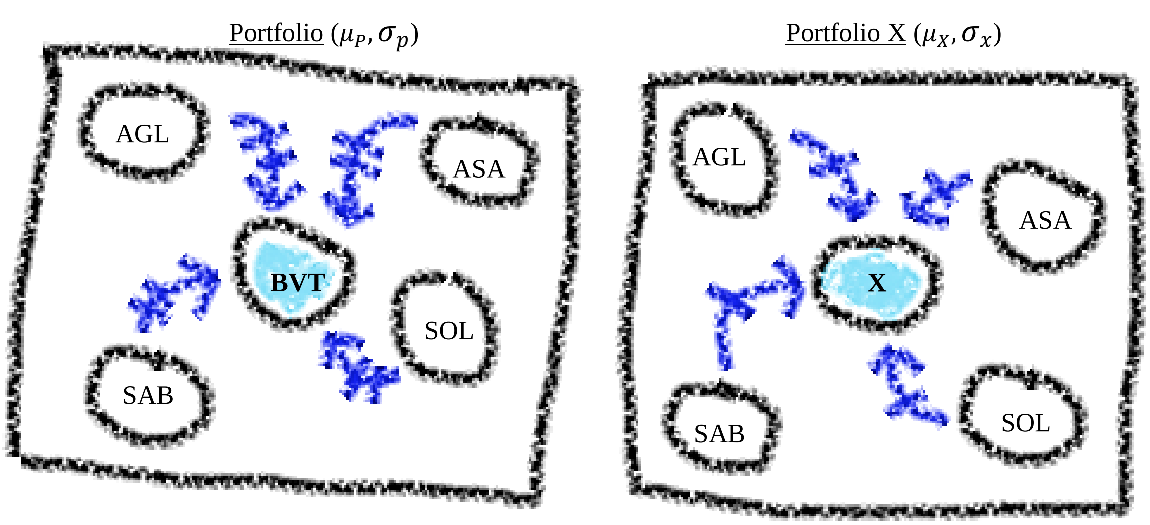
126 The closing price of any given stock is captured in the last (eighth observation) of each trading day.

Table 4.2 Stock JSE codes and proxies.

Company Name	JSE Stock Symbol	Proxy Symbol
Anglo American Plc	AGL	V
ABSA Group Limited	ASA	W
Bidvest Group Limited	BVT	X
SABMiller Plc	SAB	Y
SASOL Limited	SOL	Z

In each period, the mean and standard deviations of the original five-stock portfolio are measured. The proxy stocks are then used to interchangeably replace each of their actual stock counterparts. The portfolio mean and standard deviation will be measured again to gauge the effect of the change in stocks (for each period). In order to distinguish between the portfolios, the original stock portfolio is called "*Portfolio*", while "*portfolio X*", for instance, is the portfolio where stock BVT is replaced by its proxy (stock X), and so forth.[127] The volatility spill-over effects between these stocks will be measured (graphically illustrated in figure 4.1) to determine whether there is noticeable interaction between volatility spill-over effects and overall portfolio risk.

Figure 4.1 Crayon sketch of volatility spill-over effects - refer to Peter Lynch quote (source: Compiled by Author).



[127] Interchangeably substituting stock AGL for stock V (*portfolio V*), stock ASA for stock W (*portfolio W*), stock SAB for stock Y (*portfolio Y*), and stock SOL for Z (*portfolio Z*).

In figure 4.1 each square represents a portfolio containing five stocks. Each portfolio has its own mean (μ) and standard deviation (σ). As was stated earlier, Markowitz (1952) had shown that the co-variances between stocks within the same portfolio play a determinate part in overall portfolio risk. This study, however, focuses on the volatility spill-over effects between the stocks in the same portfolio (denoted by the arrows in the figure). The ultimate aim is to ascertain whether overall portfolio risk declines when overall volatility spill-over effects to the proxy (replacement) stock are less than that of actual stock (in this case "stock X" acting as a substitute for "BVT"), with portfolio return kept constant. It is important to remember that the replacement stocks have that same mean and standard deviation than the actual stock. However, portfolio mean and standard deviation are not bound in the same way, and may change as the volatility spill-over effects differ between the actual stock and its proxy. This study therefore necessitates the use of proxy stocks as to finding an actual replacement stock. A replacement stock does not exhibit the unique return properties of the original stock, since no two stocks are the same. If a replacement stock provides different statistical characteristics within the portfolio than the stock it replaces, capturing the unique volatility spill-over effects becomes an impossible task. It is for this reason that proxy stocks are utilised. The return properties of a proxy stock can be simulated to retain the unique return, volatility and inter-correlated properties of the stock it replaces in the portfolio.

In this sense it is possible to test whether portfolio risk declines when a stock is replaced by another with similar characteristics (but less volatility spill-over effects). In doing this, it should be possible to test whether volatility spill-over effects provide a different dimension to portfolio selection than that of *beta*. It is also important to test whether *beta* change linearly with a reduction or increase in portfolio standard deviation. If there is no linear relationship between *beta* and portfolio standard deviation, this will serve as further proof that volatility spill-over effects should also play a role during portfolio selection.

4.2 Random Normal Stock Returns

A Monte Carlo simulation is utilised in generating each of the proxy stock's returns. This involves a stochastic process that, given the probability, p, simulates random returns which is a normally distributed random variable around a given mean, μ, and standard deviation, σ. The generated random returns provide the given probability. The inverse of the normal (or Gaussian) density function is used to generate these random returns, and is given by:

$$F^{-1}(p;\mu,\sigma^2) = \mu + \sigma\sqrt{2}erf^{-1}(2p-1), \qquad p \in (0,1)\,, \tag{4.1}$$

where the normal density function for general values of μ and σ are expressed as:

$$f(x;\mu,\sigma^2) = \left(\frac{1}{\sigma\sqrt{2\pi}}\right)e^{-\frac{(x-\mu)^2}{2\sigma^2}} \tag{4.2}$$

and the erf (error function) is the integral of the normal density function, expressed as:

$$\Phi(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{\frac{-t^2}{2}}\,. \tag{4.3}$$

4.2.1 More on Monte Carlo

A Monte Carlo simulation, by design, generates a stochastic process that should exhibit no correlation to the original process it simulates. It should then hold that the proxy stocks generated should have no correlation with the stock it replaces. This property is the basis for controlling excessive influences that would have limited the results to pure speculation if real stocks were chosen. It allows the user to test whether a stock exhibiting different volatility characteristics (proxy stock) affects volatility spill-over effects in a different way, when the unique portfolio volatility, portfolio returns, and inter-correlations between the stock in a portfolio is kept constant.

4.2.2 Results

The purpose of the Monte Carlo simulation is to provide a proxy stock with returns that exhibit (approximately) identical returns and risk to the actual stock it replaces within the portfolio (during a particular period). Table 4.3 gives a summary of the actual stock and its simulated version’s mean and standard deviation.

Table 4.3 Mean return and standard deviation of the stocks and their proxies.

		Actual stocks and their simulated counterparts									
		AGL	V	ASA	W	BVT	X	SAB	Y	SOL	Z
Period 1	μ	-0.078%	-0.078%	0.091%	0.091%	0.043%	0.043%	-0.023%	-0.023%	-0.025%	-0.025%
	σ	0.970%	0.970%	1.145%	1.145%	0.940%	0.940%	0.710%	0.710%	0.905%	0.905%
Period 2	μ	-0.128%	-0.128%	-0.030%	-0.030%	-0.034%	-0.034%	-0.041%	-0.041%	-0.068%	-0.068%
	σ	2.355%	2.355%	1.537%	1.537%	1.638%	1.638%	1.316%	1.316%	2.140%	2.140%
Period 3	μ	-0.050%	-0.050%	-0.021%	-0.021%	0.019%	0.019%	0.004%	0.004%	-0.007%	-0.007%
	σ	1.763%	1.763%	1.030%	1.030%	1.057%	1.057%	0.923%	0.923%	1.316%	1.316%
Period 4	μ	-0.023%	-0.023%	0.005%	0.005%	-0.018%	-0.018%	-0.042%	-0.042%	-0.016%	-0.016%
	σ	1.624%	1.624%	1.080%	1.080%	1.315%	1.315%	0.815%	0.815%	1.168%	1.169%
Period 5	μ	0.075%	0.075%	0.016%	0.016%	0.030%	0.030%	0.063%	0.063%	0.042%	0.042%
	σ	1.429%	1.429%	0.995%	0.995%	1.161%	1.161%	0.798%	0.798%	1.090%	1.090%
Period 6	μ	0.027%	0.027%	0.052%	0.052%	0.035%	0.035%	0.017%	0.017%	0.026%	0.026%
	σ	1.017%	1.017%	0.604%	0.604%	0.692%	0.692%	0.514%	0.514%	0.713%	0.713%
Period 7	μ	0.057%	0.057%	0.003%	0.003%	0.014%	0.014%	0.046%	0.046%	0.006%	0.006%
	σ	0.829%	0.829%	0.586%	0.586%	0.505%	0.505%	0.499%	0.499%	0.597%	0.598%
Period 8	μ	0.041%	0.041%	0.021%	0.021%	0.026%	0.026%	0.017%	0.017%	0.016%	0.016%
	σ	0.636%	0.636%	0.506%	0.506%	0.515%	0.515%	0.495%	0.496%	0.456%	0.456%
Period 9	μ	-0.039%	-0.039%	0.023%	0.023%	0.028%	0.029%	-0.016%	-0.016%	-0.020%	-0.021%
	σ	0.714%	0.714%	0.454%	0.454%	0.575%	0.575%	0.427%	0.428%	0.544%	0.544%
Period 10	μ	0.039%	0.039%	0.013%	0.013%	0.018%	0.018%	0.047%	0.046%	0.019%	0.019%
	σ	0.596%	0.596%	0.470%	0.469%	0.535%	0.535%	0.393%	0.393%	0.470%	0.470%

Table 4.3 expresses the similarities between the actual stock and its generated proxy. The difference only becomes apparent at a 1000th of a per cent. In reality it will be difficult to find stocks within a market that replicate another stock's mean and standard deviation; however, these proxy stocks only serve as a simpler means to an end than would have been provided by using actual stocks as replacements. To extend on table 4.3, table 4.4 gives a full representation of the descriptive statistics.

Table 4.4 Descriptive statistics.

	Descriptive Statistics									
Period 1	AGL	ASA	BVT	SAB	SOL	V_AGL	W_ASA	X_BVT	Y_SAB	Z_SOL
Obs.	343	343	343	343	343	343	343	343	343	343
Mean	-0.078%	0.091%	0.043%	-0.023%	-0.025%	-0.078%	0.091%	0.043%	-0.023%	-0.025%
Max.	3.348%	6.863%	3.770%	3.483%	2.977%	2.794%	2.827%	2.798%	2.206%	2.334%
Min.	-4.614%	-4.639%	-2.768%	-2.431%	-3.472%	-2.891%	-3.248%	-2.403%	-2.292%	-2.179%
Std. Dev.	0.970%	1.145%	0.940%	0.710%	0.905%	0.970%	1.145%	0.940%	0.710%	0.905%
Skewness	-0.1720	1.0120	0.4506	0.3865	0.0580	-0.1154	-0.0727	0.0034	0.0653	0.0430
Kurtosis	5.1929	9.2248	4.6692	5.8688	4.4157	2.8590	2.7424	2.7399	3.3386	2.6395
Period 2	AGL	ASA	BVT	SAB	SOL	V_AGL	W_ASA	X_BVT	Y_SAB	Z_SOL
Obs.	351	351	351	351	351	351	351	351	351	351
Mean	-0.128%	-0.030%	-0.034%	-0.041%	-0.068%	-0.128%	-0.030%	-0.034%	-0.041%	-0.068%
Max.	9.704%	6.417%	7.043%	6.381%	8.950%	6.653%	4.634%	7.941%	4.550%	5.950%
Min.	-8.994%	-5.752%	-6.404%	-4.531%	-8.691%	-7.398%	-4.395%	-3.814%	-3.876%	-6.010%
Std. Dev.	2.355%	1.537%	1.638%	1.316%	2.140%	2.355%	1.537%	1.638%	1.316%	2.140%
Skewness	0.4636	0.2131	0.2952	0.4901	-0.1814	0.0840	0.1073	0.3985	-0.0533	0.1332

Kurtosis	5.9978	5.7760	5.9923	6.1010	5.6436	3.2177	2.9439	3.8047	3.2027	3.1298
Period 3	AGL	ASA	BVT	SAB	SOL	V_AGL	W_ASA	X_BVT	Y_SAB	Z_SOL
Obs.	311	311	311	311	311	311	311	311	311	311
Mean	-0.050%	-0.021%	0.019%	0.004%	-0.007%	-0.050%	-0.021%	0.019%	0.004%	-0.007%
Max.	6.884%	3.987%	3.609%	3.438%	4.085%	4.759%	2.416%	2.636%	2.250%	3.952%
Min.	-8.534%	-3.846%	-4.019%	-3.264%	-6.880%	-5.071%	-2.930%	-2.858%	-2.474%	-3.652%
Std. Dev.	1.763%	1.030%	1.057%	0.923%	1.316%	1.763%	1.030%	1.057%	0.923%	1.316%
Skewness	0.0624	-0.1008	0.1977	0.1528	-0.3011	0.0210	-0.1072	0.0171	-0.1201	-0.1027
Kurtosis	5.4990	5.0734	4.5259	4.2940	5.6326	2.6578	2.7810	2.7709	2.7258	3.1363
Period 4	AGL	ASA	BVT	SAB	SOL	V_AGL	W_ASA	X_BVT	Y_SAB	Z_SOL
Obs.	343	343	343	343	343	343	343	343	343	343
Mean	-0.023%	0.005%	-0.018%	-0.042%	-0.016%	-0.023%	0.005%	-0.018%	-0.042%	-0.016%
Max.	9.613%	3.666%	6.353%	4.742%	5.748%	4.497%	3.674%	4.080%	2.004%	3.092%
Min.	-5.935%	-3.188%	-5.402%	-2.736%	-3.551%	-4.792%	-3.168%	-3.617%	-2.470%	-3.335%
Std. Dev.	1.624%	1.080%	1.315%	0.815%	1.168%	1.624%	1.080%	1.315%	0.815%	1.169%
Skewness	0.5733	0.2900	0.1262	0.5018	0.4222	0.0488	-0.0010	0.0961	-0.0212	-0.1373
Kurtosis	7.2389	3.9093	6.1135	6.5205	6.5540	3.0087	3.4770	2.9963	2.7019	2.8015
Period 5	AGL	ASA	BVT	SAB	SOL	V_AGL	W_ASA	X_BVT	Y_SAB	Z_SOL
Obs.	311	311	311	311	311	311	311	311	311	311
Mean	0.075%	0.016%	0.030%	0.063%	0.042%	0.075%	0.016%	0.030%	0.063%	0.042%
Max.	9.613%	3.685%	5.181%	4.742%	5.748%	4.095%	3.463%	3.149%	2.348%	2.874%
Min.	-4.424%	-3.408%	-3.950%	-2.912%	-4.331%	-3.931%	-2.900%	-3.326%	-2.283%	-3.520%
Std. Dev.	1.429%	0.995%	1.161%	0.798%	1.090%	1.429%	0.995%	1.161%	0.798%	1.090%
Skewness	0.9571	0.1615	0.1421	0.6145	0.2089	-0.0821	0.1233	0.0166	-0.0788	-0.0454
Kurtosis	9.7642	4.4232	5.1619	7.0610	6.4121	2.6844	3.3252	2.9132	3.0677	2.9929
Period 6	AGL	ASA	BVT	SAB	SOL	V_AGL	W_ASA	X_BVT	Y_SAB	Z_SOL
Obs.	351	351	351	351	351	351	351	351	351	351
Mean	0.027%	0.052%	0.035%	0.017%	0.026%	0.027%	0.052%	0.035%	0.017%	0.026%
Max.	4.198%	2.050%	2.580%	1.784%	3.207%	2.774%	1.699%	2.032%	1.434%	1.878%
Min.	-4.158%	-2.130%	-2.346%	-1.529%	-2.749%	-2.898%	-2.126%	-1.858%	-1.652%	-2.004%
Std. Dev.	1.017%	0.604%	0.692%	0.514%	0.713%	1.017%	0.604%	0.692%	0.514%	0.713%
Skewness	-0.1199	0.1287	0.2163	0.2332	0.3052	-0.1428	-0.0819	0.1155	-0.0741	-0.0934
Kurtosis	5.1758	4.0992	5.1023	4.1034	5.6767	2.9436	3.3609	2.9855	3.1026	3.0091
Period 7	AGL	ASA	BVT	SAB	SOL	V_AGL	W_ASA	X_BVT	Y_SAB	Z_SOL
Obs.	335	335	335	335	335	335	335	335	335	335
Mean	0.057%	0.003%	0.014%	0.046%	0.006%	0.057%	0.003%	0.014%	0.046%	0.006%
Max.	2.458%	1.595%	2.220%	1.939%	2.338%	2.346%	1.798%	1.232%	1.474%	1.629%
Min.	-3.992%	-2.447%	-1.796%	-1.511%	-2.549%	-1.924%	-2.130%	-1.467%	-1.296%	-1.483%
Std. Dev.	0.829%	0.586%	0.505%	0.499%	0.597%	0.829%	0.586%	0.505%	0.499%	0.598%
Skewness	-0.3003	-0.3774	0.1947	0.2226	-0.2408	0.0806	-0.1482	-0.1939	0.0613	-0.0856
Period 8	AGL	ASA	BVT	SAB	SOL	V_AGL	W_ASA	X_BVT	Y_SAB	Z_SOL
Obs.	319	319	319	319	319	319	319	319	319	319
Mean	0.041%	0.021%	0.026%	0.017%	0.016%	0.041%	0.021%	0.026%	0.017%	0.016%
Max.	3.738%	1.764%	1.783%	2.762%	1.473%	1.858%	1.675%	1.420%	1.379%	1.344%
Min.	-3.430%	-2.251%	-2.385%	-1.617%	-2.077%	-2.205%	-1.412%	-1.241%	-1.505%	-1.476%
Std. Dev.	0.636%	0.506%	0.515%	0.495%	0.456%	0.636%	0.506%	0.515%	0.496%	0.456%
Skewness	0.3553	-0.1240	-0.2282	0.9280	-0.6740	-0.0039	0.1113	0.0133	0.0292	0.0561
Kurtosis	10.2255	4.9326	5.2928	7.9140	6.1919	3.0773	3.1811	2.6365	3.0112	3.2283
Period 9	AGL	ASA	BVT	SAB	SOL	V_AGL	W_ASA	X_BVT	Y_SAB	Z_SOL
Obs.	335	335	335	335	335	335	335	335	335	335
Mean	-0.039%	0.023%	0.028%	-0.016%	-0.020%	-0.039%	0.023%	0.029%	-0.016%	-0.021%
Max.	2.096%	2.067%	2.399%	1.617%	2.354%	2.259%	1.371%	1.876%	1.413%	1.873%
Min.	-3.296%	-3.254%	-3.623%	-2.440%	-3.333%	-2.586%	-1.218%	-1.384%	-1.080%	-1.820%
Std. Dev.	0.714%	0.454%	0.575%	0.427%	0.544%	0.714%	0.454%	0.575%	0.428%	0.544%
Skewness	-0.5318	-0.5868	-0.4961	-0.7010	-0.3918	-0.0719	-0.0908	0.2215	0.3459	-0.0788
Kurtosis	5.3468	12.8554	8.9849	7.1277	7.8850	3.5090	2.7302	2.8513	3.0433	3.1566
Period 10	AGL	ASA	BVT	SAB	SOL	V_AGL	W_ASA	X_BVT	Y_SAB	Z_SOL
Obs.	319	319	319	319	319	319	319	319	319	319
Mean	0.039%	0.013%	0.018%	0.047%	0.019%	0.039%	0.013%	0.018%	0.046%	0.019%

Max.	1.708%	1.604%	2.600%	1.856%	2.376%	1.589%	1.262%	1.452%	1.086%	1.443%
Min.	-3.148%	-2.415%	-2.138%	-1.388%	-2.176%	-1.710%	-1.346%	-1.924%	-1.083%	-1.313%
Std. Dev.	0.596%	0.470%	0.535%	0.393%	0.470%	0.596%	0.469%	0.535%	0.393%	0.470%
Skewness	-0.5618	-0.2782	0.4585	0.7446	0.2019	-0.1221	-0.1641	-0.0100	-0.1344	0.0274
Kurtosis	5.7903	6.7356	7.1891	6.0339	8.0333	3.0519	2.6462	3.1644	2.8828	2.9093
Kurtosis	5.6865	4.5372	4.8577	4.2514	5.6325	2.9471	3.3342	2.7968	3.0016	2.7663

In table 4.4 it can be seen that the number of observations for each two-month period ranges from 351 at most, to 311 at least. This provides an adequate number of data points to commence with statistical inference. The main descriptive statistics that this study will focus on have already been described in the discussion of table 4.3. In addition, the maximum and minimum values display a measure volatility of each stock. These values move in smaller bands for the proxy stocks, making them less prone to outliers than the actual stock they emulate. This observation can be (more formally) seen in the measure of kurtosis for each of the stocks. The kurtosis for the original stocks exhibit long "tails" (leptokurtic) compared to a normal distribution, which complies with studies done by Larson (1960), Working (1960), Houthakker (1961) and Alexander (1961). The random normal proxy stock returns exhibit random distributions, as expected. Although the actual stocks and their proxies have the same returns and standard deviation, their distribution characteristics start to differ from the 4^{th} moment onwards. However, the aim of this study is not on distribution characteristics, but to investigate whether these proxy stocks can provide proof of the correlation between lower portfolio risk and less volatility spill-over effects, or vice versa. However, the presentation of the results in table 4.4 was somewhat informal.

4.3 Formal Testing

Formal testing of the link between portfolio risk and volatility transmission will consist of four parts. The first part provides a motive for the use of stock returns for reasons of stationarity. The second part will focus on the level of integration between the five original stocks and their proxies, by implementing Granger causality. However, these aforementioned tests are only for completeness, and did not form an integral part of the study. The third part presents a comparison of portfolio risk, portfolio return, and *beta* according to the methodology of Markowitz (1952) and Sharpe (1964). In the final part of the empirical study, the focus shifts towards establishing the level of volatility transmission between the respective stocks and their proxies by applying an AS Model to each.[128]

4.3.1 Stationarity and heteroskedasticity

Stationarity is an important concept underlying a time series process. When a variable is non-stationary (or trended), incorrect inferences could easily be made since the assumptions of the classical linear regression model are violated, leading to unreliable t-tests, F-tests or R-squared values (Asteriou & Hall, 2007:295). The order of integration (testing for a unit root) of a variable discloses significant information about its stationarity.

An Augmented Dickey Fuller test (ADF) is utilised to test the order of integration of each stock. A series y_t is integrated of order one and contains a unit root, if y_t is non-stationary but Δy_t is stationary (Asteriou & Hall 2007:290).[129] The three forms of the ADF tests are given as:

$$\Delta y_t = \gamma y_{t-1} + \sum_{i=1}^{p} \beta_i \Delta y_{t-i} + u_t \, , \tag{4.4}$$

$$\Delta y_t = \alpha_0 + \gamma y_{t-1} + \sum_{i=1}^{p} \beta_i \Delta y_{t-i} + u_t \, , \tag{4.5}$$

$$\Delta y_t = \alpha_0 + \gamma y_{t-1} + \alpha_2 t + \sum_{i=1}^{p} \beta_i \Delta y_{t-i} + u_t \, , \tag{4.6}$$

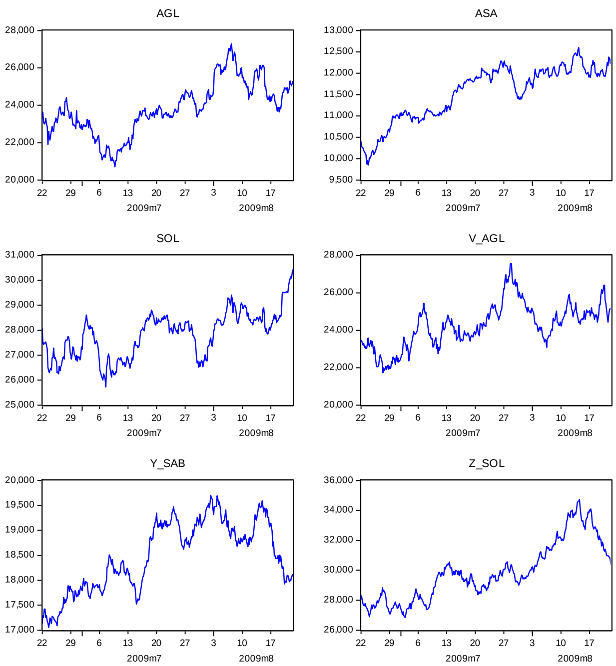
where α_0 and $\alpha_2 t$ are deterministic elements. The results obtained from performing the ADF test indicated that the null hypothesis of a unit root cannot be rejected when using stock prices. However, when using stock returns (which enables comparability between stocks), the null hypothesis of a unit root can be rejected as the probability of the t-statistic is smaller than 0.05

[128] This study employs a similar methodology to the one used by Lin, Engle and Ito (1994) and Samouilhan (2006) with regards to the use of an AS Model.

[129] A non-stationary time series y_t might need to be differenced more than once to make it stationary, resulting in a higher order of integration.

($p < 0.05$). This confirms that all the stock returns are integrated of order zero ($y_t \sim I(0)$), therefore containing no unit root, and exhibiting stationarity. Accordingly, the stock returns need not be differenced in order to induce stationarity.[130] For illustrative purposes figure 4.2 and 4.3 provides a graphical view of period six's stock price and stock returns.[131]

Figure 4.2 Period six stock price line graph.



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130 In contrast, stock prices will need to be differenced to induce stationarity. However, stock returns are utilised in this study.

131 The line graphs of the other stocks are presented in the appendix, sections A1 and A2.

Figure 4.3 Period six stock returns line graph.

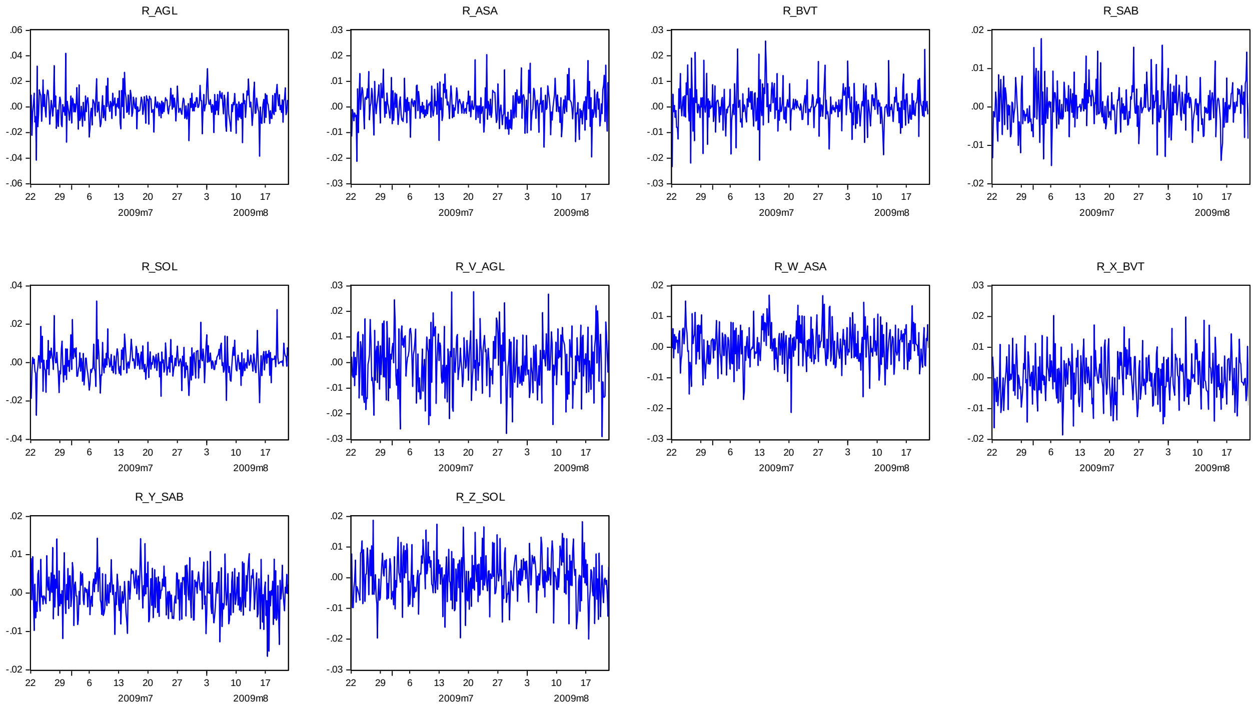

Figure 4.2 shows the trend present in stock prices, which contains a unit root, and therefore does not exhibit stationarity. However, in figure 4.3, the trend is eliminated by taking the hourly returns of every stock, resulting in stationarity.

In addition, another central statistical inference measure is to test for the presence of heteroskedasticity, which is the serial correlation in error terms; the presence of which affects the distribution of the parameter estimators, and makes the estimators of the OLS inefficient (Asteriou & Hall, 2007:116). Heteroskedasticity, like non-stationarity, also impacts hypothesis testing, by estimating t-statistics and F-statistics that are not reliable (Asteriou & Hall, 2007:116). For completion, White's test is used (after obtaining OLS results) to test for heteroskedasticity. The null hypothesis of heteroskedasticity was rejected as all of the p-values were significantly lower than the five per cent level of statistical significance. However, these results are of no use in this study as the focus is on the presence of ARCH effects (serial correlation in the variance of error terms).

4.3.2 Granger causality

Contagion effects can be tested via the Granger causality methodology, however, this approach only allows one to capture the unidirectional spill-over (mean returns). In contrast, developments in the autoregressive conditional heteroskedastic (ARCH) family of models have made it possible to study the conditional volatility, and the transmission of such volatility (Worthington & Higgs, 2004:2). Nevertheless, Granger causality test will allow for a greater understanding of the direction and speed in which the stocks affect other and the level of integration.

A time series X Granger causes time series Y, if it is possible to reveal that the X values impart statistically significant information on future values of Y. In common terminology, the test for Granger causality involves testing the null hypothesis that x_t does not cause y_t, by means of the following two regressions (Asteriou & Hall, 2007:285):

$$\Delta y = \sum_{i=1}^{m} a_i y_{t-i} + \sum_{j=i}^{n} b_j x_{t-j} + e_t \,, \tag{4.7}$$

$$y = \sum_{i=1}^{m} a_i y_{t-1} + e_t \,, \tag{4.8}$$

where testing $b_j = 0$ for every j, and the assumption of uncorrelated disturbances. The unidirectional causality from all the various stocks (including the proxy stocks) will be tested for all periods. For stock A to Granger-cause stock B, the estimated coefficients on lagged stock A values must be statistically different from zero, and the estimated coefficients on lagged stock A values should not be statistically different from zero (Gujarati, 2003:697). For illustrative purposes table 4.5 only presents the Granger causality tests of the first period of the ten periods estimated.[132]

132 The remainder of the Granger causality test are presented in the appendix, section A3.

Table 4.5 Granger causality tests.

Null Hypothesis	Obs.	Prob.	Null Hypothesis	Obs.	Prob.
Period 1					
R_ASA does not Granger Cause R_AGL R_AGL does not Granger Cause R_ASA	341	0.5077 0.0077*	R_SOL does not Granger Cause R_BVT R_BVT does not Granger Cause R_SOL	341	0.7592 0.4802
R_BVT does not Granger Cause R_AGL R_AGL does not Granger Cause R_BVT	341	0.0159* 0.9632	R_V_AGL does not Granger Cause R_BVT R_BVT does not Granger Cause R_V_AGL	341	0.7348 0.7430
R_SAB does not Granger Cause R_AGL R_AGL does not Granger Cause R_SAB	341	0.8623 0.2047	R_W_ASA does not Granger Cause R_BVT R_BVT does not Granger Cause R_W_ASA	341	0.9105 0.9342
R_SOL does not Granger Cause R_AGL R_AGL does not Granger Cause R_SOL	341	0.0029* 0.7583	R_X_BVT does not Granger Cause R_BVT R_BVT does not Granger Cause R_X_BVT	341	0.7191 0.5163
R_V_AGL does not Granger Cause R_AGL R_AGL does not Granger Cause R_V_AGL	341	0.2927 0.1714	R_Y_SAB does not Granger Cause R_BVT R_BVT does not Granger Cause R_Y_SAB	341	0.7764 0.6288
R_W_ASA does not Granger Cause R_AGL R_AGL does not Granger Cause R_W_ASA	341	0.4815 0.1968	R_Z_SOL does not Granger Cause R_BVT R_BVT does not Granger Cause R_Z_SOL	341	0.5076 0.9207
R_X_BVT does not Granger Cause R_AGL R_AGL does not Granger Cause R_X_BVT	341	0.9336 0.5893	R_SOL does not Granger Cause R_SAB R_SAB does not Granger Cause R_SOL	341	0.0765^ 0.8805
R_Y_SAB does not Granger Cause R_AGL R_AGL does not Granger Cause R_Y_SAB	341	0.0777^ 0.5340	R_V_AGL does not Granger Cause R_SAB R_SAB does not Granger Cause R_V_AGL	341	0.4815 0.5161
R_Z_SOL does not Granger Cause R_AGL R_AGL does not Granger Cause R_Z_SOL	341	0.3715 0.9907	R_W_ASA does not Granger Cause R_SAB R_SAB does not Granger Cause R_W_ASA	341	0.8189 0.4701
R_BVT does not Granger Cause R_ASA R_ASA does not Granger Cause R_BVT	341	0.2106 0.1529	R_X_BVT does not Granger Cause R_SAB R_SAB does not Granger Cause R_X_BVT	341	0.7603 0.9656
R_SAB does not Granger Cause R_ASA R_ASA does not Granger Cause R_SAB	341	0.5593 0.0044*	R_Y_SAB does not Granger Cause R_SAB R_SAB does not Granger Cause R_Y_SAB	341	0.2812 0.9141
R_SOL does not Granger Cause R_ASA R_ASA does not Granger Cause R_SOL	341	0.0745^ 0.0027*	R_Z_SOL does not Granger Cause R_SAB R_SAB does not Granger Cause R_Z_SOL	341	0.0099* 0.7658
R_V_AGL does not Granger Cause R_ASA R_ASA does not Granger Cause R_V_AGL	341	0.8460 0.2257	R_V_AGL does not Granger Cause R_SOL R_SOL does not Granger Cause R_V_AGL	341	0.3054 0.5359
R_W_ASA does not Granger Cause R_ASA R_ASA does not Granger Cause R_W_ASA	341	0.7454 0.0572^	R_W_ASA does not Granger Cause R_SOL R_SOL does not Granger Cause R_W_ASA	341	0.5810 0.2153
R_X_BVT does not Granger Cause R_ASA R_ASA does not Granger Cause R_X_BVT	341	0.0117* 0.4464	R_X_BVT does not Granger Cause R_SOL R_SOL does not Granger Cause R_X_BVT	341	0.9067 0.0881^
R_Y_SAB does not Granger Cause R_ASA R_ASA does not Granger Cause R_Y_SAB	341	0.5321 0.3054	R_Y_SAB does not Granger Cause R_SOL R_SOL does not Granger Cause R_Y_SAB	341	0.0969^ 0.3467
R_Z_SOL does not Granger Cause R_ASA R_ASA does not Granger Cause R_Z_SOL	341	0.2191 0.7403	R_Z_SOL does not Granger Cause R_SOL R_SOL does not Granger Cause R_Z_SOL	341	0.1901 0.2959
R_SAB does not Granger Cause R_BVT R_BVT does not Granger Cause R_SAB	341	0.4926 0.0519^			

* Indicates statistical significance at the 95% level.
^ Indicates statistical significance at the 90% level.

Table 4.5 presents the Granger-causality test results. For illustrative purposes only one of the ten periods is shown. Where the causality is statistically significant, the null hypothesis is rejected, resulting in return causality from a given stock to another. In all the periods, most of the statistically significant causality in returns is present among the actual stocks. The proxy stocks exhibit far less influence in being Granger-caused or Granger-causing any return transmission at a significant level. Stated differently, the causality between the original stocks in the five-stock portfolio outweighs the causality present when a proxy stock is introduced. However, Granger causality is a scarce occurrence, and therefore does not provide any significant explanatory capacity. For this reason the study moves on to the return and volatility spill-over effects when the stocks interact with each other within a five-stock portfolio.

4.3.3 Portfolio risk, return and *beta*

Modern portfolio theory (MPT), developed in the 1950s and gaining momentum in the 1960s, saw a shift in the management of portfolios. The introduction of the concept of co-variance between stocks provided a different facet to how stocks affect each other when co-existing in the same portfolio. In addition, portfolio managers have long been familiar with the *efficient market hypotheses* (EMH) where a well-diversified portfolio with a unit correlation (*beta* equal to one) with the market is considered entirely hedged against market (unsystematic) risk.[133] Table 4.6 provides the portfolio risk, portfolio returns and *beta* measures of all the different five-stock portfolios in all the different periods under study.

Table 4.6 Portfolio risk, return and *beta*.

	Volatility Measures					
Period 1	Portfolio	Portfolio V	Portfolio W	Portfolio X	Portfolio Y	Portfolio Z
Standard deviation	0.538%	0.480%	0.506%	0.500%	0.497%	0.496%
Return	0.001%	0.001%	0.001%	0.001%	0.001%	0.001%
Beta	0.7005	0.4505	0.6992	0.5633	0.5739	0.6864
Period 2	Portfolio	Portfolio V	Portfolio W	Portfolio X	Portfolio Y	Portfolio Z
Standard deviation	1.333%	1.069%	1.180%	1.181%	1.246%	1.134%
Return	-0.060%	-0.060%	-0.060%	-0.060%	-0.060%	-0.060%
Beta	0.8805	0.6214	0.6884	0.7245	0.8085	0.7061
Period 3	Portfolio	Portfolio V	Portfolio W	Portfolio X	Portfolio Y	Portfolio Z
Standard deviation	0.828%	0.682%	0.767%	0.754%	0.770%	0.672%
Return	-0.011%	-0.011%	-0.011%	-0.011%	-0.011%	-0.011%
Beta	0.8363	0.5091	0.7798	0.7436	0.7634	0.5959
Period 4	Portfolio	Portfolio V	Portfolio W	Portfolio X	Portfolio Y	Portfolio Z
Standard deviation	0.830%	0.690%	0.753%	0.728%	0.771%	0.725%
Return	-0.019%	-0.019%	-0.019%	-0.019%	-0.019%	-0.019%
Beta	1.0935	0.6878	1.0439	0.8834	1.0610	0.9135
Period 5	Portfolio	Portfolio V	Portfolio W	Portfolio X	Portfolio Y	Portfolio Z
Standard deviation	0.754%	0.613%	0.674%	0.670%	0.692%	0.668%
Return	0.045%	0.045%	0.045%	0.045%	0.045%	0.045%
Beta	0.9054	0.5425	0.8174	0.7589	0.8789	0.7316
Period 6	Portfolio	Portfolio V	Portfolio W	Portfolio X	Portfolio Y	Portfolio Z
Standard deviation	0.482%	0.389%	0.419%	0.430%	0.454%	0.416%
Return	0.031%	0.031%	0.031%	0.031%	0.031%	0.031%
Beta	0.8024	0.3874	0.7035	0.6747	0.7218	0.5997

133 Refer to chapter 2.

Period 7	Portfolio	Portfolio V	Portfolio W	Portfolio X	Portfolio Y	Portfolio Z
Standard deviation	0.408%	0.327%	0.357%	0.373%	0.372%	0.366%
Return	0.025%	0.025%	0.025%	0.025%	0.025%	0.025%
Beta	0.9092	0.6116	0.6950	0.7244	0.8562	0.7918
Period 8	Portfolio	Portfolio V	Portfolio W	Portfolio X	Portfolio Y	Portfolio Z
Standard deviation	0.320%	0.277%	0.293%	0.295%	0.289%	0.283%
Return	0.024%	0.024%	0.024%	0.024%	0.024%	0.024%
Beta	0.7583	0.4826	0.5904	0.7085	0.7768	0.6603
Period 9	Portfolio	Portfolio V	Portfolio W	Portfolio X	Portfolio Y	Portfolio Z
Standard deviation	0.360%	0.314%	0.337%	0.321%	0.339%	0.303%
Return	-0.005%	-0.005%	-0.005%	-0.005%	-0.005%	-0.005%
Beta	0.8345	0.4772	0.6773	0.7212	0.8134	0.6280
Period 10	Portfolio	Portfolio V	Portfolio W	Portfolio X	Portfolio Y	Portfolio Z
Standard deviation	0.330%	0.274%	0.292%	0.298%	0.302%	0.286%
Return	0.027%	0.027%	0.027%	0.027%	0.027%	0.027%
Beta	0.8582	0.6813	0.7195	0.6875	0.7212	0.7028

From table 4.6 it is evident that the portfolio returns in each period match each other equally. This is possible because of the proxy stock that replaces each stock in the five proxy portfolios effectively have the same returns. The differences of each portfolio are captured by portfolio standard deviation and market *beta*. Period one to four portfolios all have negative returns, as these were the periods hardest hit by the financial crisis (except period one, which is more of a lead up to the crisis). Period two captures the defining moment of the 2008 financial crisis, as it encompasses the two months after Lehman Brothers declared insolvency. It is for this reason that the portfolios in period two exhibit the highest portfolio risk. All the periods, except period four, exhibit portfolios that have a *beta* less than one. Period four signals the beginning of stabilisation of the JSE. However, the international exposure of some of the large capitalisation stocks may explain the excess riskiness of the stocks within period four portfolios (higher *beta* means a greater exposure to market risk than that of the market portfolio). From period five onwards portfolio returns are positive, giving impetus to recovery from the initial shock of the financial crisis. Period nine, in contrast, had a small volatility hike, with negative returns. This could possibly be attributed to the irrational investor behaviour due to the Haiti earthquake. Stabilisation of the JSE and stocks subsequently commenced thereafter (period 10).

Interestingly, the original five-stock portfolio always exhibits a larger portfolio standard deviation than the five proxy portfolios in the same period. *Market beta* captures this effect to a significant extent. As the portfolio standard deviation decreases, the *beta* decreases (except for one case), and therefore the market exposure of the particular portfolio. In other words, when the volatility of the market increases (such as in a financial crisis), the volatility of the portfolio(s), with a *beta* less than one, increases to a lesser extent. However, *beta* does not decrease linearly as portfolio standard deviation decreases. In one particular case, the *beta* of a proxy portfolio was higher than the original portfolio (period 8).

Additionally, portfolio performance measures were estimated. These measures did not present any new information or further this study, but serve as an addendum for completeness.[134] However, the nature of a portfolio's risk-return relationship provides telling insights into whether a portfolio achieved a desirable return in relation to the risk incurred for this return. Therefore, several portfolio performance measures were developed to capture this element, most notably the measures used henceforth. Firstly, Sharp's performance index (SPI) is expressed as the portfolio's return per unit of risk, and is expressed as (Sharpe, 1966):

$$SPI = \frac{(r_p - r_f)}{\sigma_p}, \tag{4.9}$$

where $r_p$is the return on the portfolio, σ_p the standard devition of the portfolio, and r_f the risk free rate. The SPI does not distinguish between systematic or unsystematic risk, but uses standard deviation as a measure of total risk.[135] However, unsystematic risk can be effectively hedged by diversifying, leaving only the systematic (market) risk to impact on portfolio performance. Treynor's performance index (TPI) indicates the portfolio's return per unit of market risk, and is expressed as (Treynor, 1965):

$$TPI = \frac{(r_p - r_f)}{\beta_p}, \tag{4.10}$$

134 Because portfolio returns were kept constant, and the risk free rate remaining relatively constant, the risk measure was the only measure exhibiting variability. Therefore, if *beta* or standard deviation were lower in each period, the measure would automatically be more favourable.

135 This property makes the SPI attractive as it can measure how well diversified a portfolio is when used in conjunction with other measures such as the TPI (which assumes a well-diversified portfolio).

where a portfolio achieved superior performance if the TPI exeeds the market risk premium $(r_p - r_f)$.[136] In addition, Jenson's alpha (α) also uses portfolio *beta* (more specifically *CAPM*) as the basis for portfolio performance measurement, and can be expressed as (Jenson, 1968):

$$\alpha = r_p - [r_f + \beta_p(r_m - r_f)]\,, \qquad (4.11)$$

where r_m is the return of the market portfolio. α indicates the actual excess return over the return required as per the *CAPM*. Table 4.7 presents the results of these commonly used portfolio performance measures.

Table 4.7 Portfolio performance measures.

	Performance Measures					
Period 1	Portfolio	Portfolio V	Portfolio W	Portfolio X	Portfolio Y	Portfolio Z
SPI	-0.00153	-0.00172	-0.00163	-0.00165	-0.00166	-0.00166
TPI	-0.00001	-0.00002	-0.00001	-0.00001	-0.00001	-0.00001
α	0.00018	0.00011	0.00018	0.00014	0.00015	0.00018
Period 2	Portfolio	Portfolio V	Portfolio W	Portfolio X	Portfolio Y	Portfolio Z
SPI	-0.04638	-0.05783	-0.05239	-0.05235	-0.04962	-0.05452
TPI	-0.00070	-0.00099	-0.00090	-0.00085	-0.00076	-0.00088
α	0.00008	-0.00013	-0.00008	-0.00005	0.00002	-0.00006
Period 3	Portfolio	Portfolio V	Portfolio W	Portfolio X	Portfolio Y	Portfolio Z
SPI	-0.01549	-0.01881	-0.01672	-0.01701	-0.01666	-0.01909
TPI	-0.00015	-0.00025	-0.00016	-0.00017	-0.00017	-0.00022
α	-0.00008	-0.00010	-0.00008	-0.00008	-0.00008	-0.00009
Period 4	Portfolio	Portfolio V	Portfolio W	Portfolio X	Portfolio Y	Portfolio Z
SPI	-0.02509	-0.03018	-0.02766	-0.02861	-0.02701	-0.02872
TPI	-0.00019	-0.00030	-0.00020	-0.00024	-0.00020	-0.00023
α	0.00045	0.00020	0.00042	0.00032	0.00043	0.00034
Period 5	Portfolio	Portfolio V	Portfolio W	Portfolio X	Portfolio Y	Portfolio Z
SPI	0.05726	0.07043	0.06406	0.06444	0.06239	0.06463
TPI	0.00048	0.00080	0.00053	0.00057	0.00049	0.00059
α	0.00012	0.00024	0.00015	0.00017	0.00013	0.00018
Period 6	Portfolio	Portfolio V	Portfolio W	Portfolio X	Portfolio Y	Portfolio Z
SPI	0.06053	0.07500	0.06963	0.06785	0.06426	0.07013
TPI	0.00036	0.00075	0.00041	0.00043	0.00040	0.00049
α	0.00006	0.00018	0.00009	0.00009	0.00008	0.00012
Period 7	Portfolio	Portfolio V	Portfolio W	Portfolio X	Portfolio Y	Portfolio Z
SPI	0.05680	0.07087	0.06492	0.06213	0.06230	0.06332
TPI	0.00025	0.00038	0.00033	0.00032	0.00027	0.00029
α	0.00007	0.00013	0.00011	0.00011	0.00008	0.00009
Period 8	Portfolio	Portfolio V	Portfolio W	Portfolio X	Portfolio Y	Portfolio Z
SPI	0.06930	0.08005	0.07568	0.07517	0.07673	0.07836
TPI	0.00029	0.00046	0.00038	0.00031	0.00029	0.00034

136 The market index, by default, has a *beta* of one and therefore exhibits a TPI equal to $r_p - r_f$.

α	0.00004	0.00011	0.00008	0.00005	0.00004	0.00007
Period 9	Portfolio	Portfolio V	Portfolio W	Portfolio X	Portfolio Y	Portfolio Z
SPI	-0.01896	-0.02174	-0.02025	-0.02126	-0.02013	-0.02253
TPI	-0.00008	-0.00014	-0.00010	-0.00009	-0.00008	-0.00011
α	-0.00009	-0.00008	-0.00009	-0.00009	-0.00009	-0.00009
Period 10	Portfolio	Portfolio V	Portfolio W	Portfolio X	Portfolio Y	Portfolio Z
SPI	0.07629	0.09188	0.08621	0.08448	0.08336	0.08802
TPI	0.00029	0.00037	0.00035	0.00037	0.00035	0.00036
α	0.00011	0.00014	0.00014	0.00014	0.00014	0.00014

The results in table 4.7 further accentuate those in table 4.6. The original five-stock portfolio always exhibits a lower portfolio performance measure than the five proxy portfolios in the same period. This holds when the portfolio returns are positive and higher than the prevailing risk free rate (rfr) for that period. Due to portfolio returns being kept constant, no economic value can be inference from periods were the portfolio returns were negative or less than the risk free rate (periods 1, 2, 3, 4 and 9).[137] Because of turbulent economic conditions, investing in a money market account during these periods of negative access returns would have been a more sound option, which lends no credibility to portfolio performance measures. In addition, the use of daily *beta* could have limited the stability and accuracy of the *beta* measure. However, *beta* does explain portfolio risk to a significant extent, with the exception of some deviations and non-linearity. It is for the later reasons that a closer look will be given to volatility spill-over effects as a supportive measure.

4.3.4 Aggregate shock models

4.3.4.1 Lag specification

Various lag specifications are estimated for each stock within each portfolio, with the appropriate lag specification for the E-GARCH(p,q) term chosen where the Akaike information criterion (AIC) and Schwarz criterion (SC) values are minimised. The AIC is computed as (Quantitative Micro Software LLC, 2009):

$$AIC = -2\frac{l}{T} + 2\frac{k}{T}. \tag{4.12}$$

137 A constant negative access return for all portfolios will lead to higher negative return per unit of risk when the portfolio risk measure becomes smaller (less risky). This leads to wrongful interpretation.

The SC is an alternative to the AIC and imposes a larger penalty for additional coefficients (Quantitative Micro Software LLC, 2009):

$$SC = -2\frac{l}{T} + 2\frac{(k \log T)}{T}. \tag{4.13}$$

Where l is the value of the log likelihood function, T is the number of observations, and k, the number of parameters. Both these information criterion are based on –2 times the average log likelihood function, adjusted by a penalty function. These criterions allow for choosing the appropriate aggregate shock model for each one estimated.

4.3.4.2 Results

The aggregate shock (AS) model allows one to formally test the relationship of both returns and volatility on the stocks within each five-stock portfolio. Recall that the AS model, as presented in chapter three, culminated in equation 3.21. The end product of the E-GARCH process is given by:

$$\ln h_{A,t} = \varpi_2 + \delta_2 \ln h_{A,t-1} + \gamma_2 \frac{\varepsilon_{A,t-1}}{\sqrt{h_{A,t-1}}} + \alpha_2 \frac{|\varepsilon_{A,t-1}|}{\sqrt{h_{A,t-1}}} + \kappa_i h_{B,t}\,. \tag{4.14}$$

Furthermore, the following summary of the coefficients with regards to the output estimation (given by equation 4.14) will be useful to interpret the results obtained from the AS model applied on each stock within each portfolio of each period. Therefore, from equation 4.14, ϖ_2 is the invariable mean reversion value, α_2 indicates the past conditional variance, γ_2 captures the asymmetric function of volatility, δ_2 indicates the volatility persistence, and κ_i captures the level of volatility spill-over. Table 4.8 provides the AS model output. Both the mean and variance equation coefficients are given, as well as the SC and AIC for each model.

Table 4.8 AS model results (mean and variance equations).

	Period 1									
	Mean Equation			Variance Equation						
	Variable	Coeff.	Dep.Lag	ϖ	α	γ	δ	κ	AIC	SC
AGL	ASA(e)	0.1383*	-0.0283	-5.7445*	-0.0862*	0.1927*	-0.7667*	1.1893*	-6.1879	-6.0870
	BVT(e)	0.2294*	-0.0467	-5.5449*	0.2056*	0.0391	-0.7619*	0.5482*	-6.2601	-6.1479
	SAB(e)	0.3956*	-0.0997*	-11.4796*	0.0130	0.0655*	-0.9408*	0.8162*	-6.2104	-6.1095
	SOL(e)	0.4188*	0.0673	-2.7551	0.3690*	-0.0695	-0.5139*	0.5959*	-6.6585	-6.5351
V	ASA(e)	0.0535	-0.0222	1.3553*	-0.1076	0.0387	0.6346^	0.3016*	-6.3988	-6.2755
	BVT(e)	-0.0796	-0.0207	-10.4198*	0.0495	0.0128	-0.9954*	-0.1258	-6.4099	-6.2978
	SAB(e)	-0.0750	-0.0116	-29.8176*	0.0173	-0.0342	-0.8437*	0.4079*	-6.4153	-6.3032
	SOL(e)	0.0901^	-0.0200	-2.7380*	-0.1898*	0.0102	0.1508	-0.2317*	-6.4161	-6.2928
ASA	AGL(e)	0.1383*	-0.0283	-5.7445*	-0.0862*	0.1927*	-0.7667*	1.1893*	-6.1879	-6.0870
	BVT(e)	0.2963*	-0.0594	-12.2864*	0.0436	0.0128	-1.0068*	0.9149*	-6.2491	-6.1370
	SAB(e)	0.4071*	-0.1114*	-8.8475*	0.0565^	-0.0506*	-0.9364*	1.0437*	-6.2106	-6.0985
	SOL(e)	0.1048^	-0.0356	-26.7160*	0.0891*	-0.0231	-0.9815*	-0.1912*	-6.1835	-6.0826
W	AGL(e)	0.1030	0.0268	-10.5173*	0.1052	0.0106	-0.7743*	0.7272^	-6.0815	-5.9694
	BVT(e)	0.0332	0.0336	-0.3476	0.0801	0.0194	0.9302*	0.0391	-6.0711	-5.9589
	SAB(e)	0.0421	0.0364	-4.1492*	0.0705*	-0.0201*	0.8326*	-0.1053*	-6.0701	-5.9579
	SOL(e)	0.0295	0.0373	-5.3074	0.1593^	-0.0087	0.6774^	0.0331	-6.0877	-5.9756
BVT	AGL(e)	0.1266*	-0.1558*	-11.9072*	0.1802*	0.0795	-0.9119*	0.7199*	-6.6071	-6.4838
	ASA(e)	0.1627*	-0.1471*	1.3588*	-0.1679	0.0763*	0.9636*	0.1881*	-6.6316	-6.5307
	SAB(e)	0.4001*	-0.1630*	0.5655*	-0.1025	0.1153*	0.1441*	0.0797*	-6.6557	-6.5436
	SOL(e)	0.1430*	-0.1561*	-16.8218*	0.1974*	0.0361^	-0.9439*	0.2305*	-6.6096	-6.5087
X	AGL(e)	0.0584	-0.0500	-2.9325*	-0.0357*	0.0253	0.8428*	-0.2328*	-6.4594	-6.3361
	ASA(e)	-0.0053	-0.0363	-1.6954	0.0703	-0.0789	0.9144*	-0.0648	-6.4749	-6.3628
	SAB(e)	-0.0178	-0.0609	-1.9731*	-0.1058*	-0.0966	0.8024*	-0.1461*	-6.4878	-6.3757
	SOL(e)	0.0453	-0.0583	-19.2730*	-0.0405	-0.1222	-0.5829*	0.4399*	-6.4853	-6.3731
SAB	AGL(e)	0.1481*	0.0297	-5.3463*	0.0187	-0.0232*	-0.9995*	0.0811	-7.2312	-7.1303
	ASA(e)	0.1347*	0.0954^	0.2486*	-0.2282*	0.0461*	0.9181*	0.1017*	-7.1296	-7.0287
	BVT(e)	0.1758*	0.0454	-3.4920*	0.0383	-0.0050	-1.0072*	0.2753*	-7.2726	-7.1605
	SOL(e)	0.0211	0.0412	-4.1625*	0.0497*	0.1150	-0.9731*	0.2521*	-7.1789	-7.0555
Y	AGL(e)	0.0033	-0.0496	-14.5232*	-0.1338	-0.1564*	-0.7126*	-0.4281	-7.0351	-6.9230
	ASA(e)	-0.0161	-0.0492	-16.3014*	-0.1060	-0.1630*	-0.5450*	-1.0296	-7.0441	-6.9320
	BVT(e)	-0.0677^	-0.0188	-29.2527*	-0.1884^	0.1236	-0.7107*	-0.7298	-7.0583	-6.9461
	SOL(e)	-0.0024	-0.0303	-11.7294*	-0.2294	-0.2289*	-0.2241	0.1792	-7.0423	-6.9189
SOL	AGL(e)	0.3943*	0.0510	-1.2960	0.2874*	-0.2453*	-0.3771*	0.9306*	-6.8112	-6.6991
	ASA(e)	0.0394*	0.1294*	-0.2255*	-0.5846*	-0.1758*	0.9118*	0.0455*	-6.6920	-6.5911
	BVT(e)	0.1383*	0.0992^	-1.6402*	-0.5906*	-0.1784*	0.8366*	-0.0299*	-6.6885	-6.5876
	SAB(e)	0.0157	0.1332*	-16.1268*	0.4084*	-0.2983*	-0.5334*	-0.6849*	-6.6337	-6.5215
Z	AGL(e)	0.0463	0.0410	-17.8822*	0.0190	-0.1062	-0.7296*	-0.6523	-6.5404	-6.4282
	ASA(e)	-0.0765	0.0361	-13.5048*	0.1470	0.0420	-0.6752*	0.8555*	-6.5623	-6.4502
	BVT(e)	0.0389	0.0339	-18.3297*	0.1348	-0.0005	-0.9589*	0.4851	-6.5500	-6.4378
	SAB(e)	-0.1273^	0.0574	-22.5883*	-0.0353	0.0926*	-0.9346*	0.6801^	-6.5776	-6.4655

* Indicates statistical significance at the 95% level.
^ Indicates statistical significance at the 90% level.

	Period 2									
	Mean Equation			**Variance Equation**						
	Variable	Coeff.	Dep.Lag	ϖ	α	γ	δ	κ	AIC	SC
AGL	ASA(e)	0.5428*	-0.1245*	-1.3447*	-0.1500*	0.0008	0.9215*	-0.1244*	-4.9085	-4.7983
	BVT(e)	0.6521*	-0.0998*	-0.3195*	-0.1841*	0.1443*	0.5021*	0.0208*	-4.8787	-4.7795
	SAB(e)	0.8852*	-0.0610	0.9172	-0.0914^	-0.2113*	0.5534*	0.1877*	-4.9216	-4.8114
	SOL(e)	0.6338*	-0.1078*	-12.5434*	0.1289*	0.0248	-0.9459*	0.5805*	-5.0977	-4.9765
V	ASA(e)	0.0014	0.0168	-2.9748*	0.0613	-0.0738^	0.8915*	-0.1749*	-4.6444	-4.5011
	BVT(e)	-0.0299	0.0127	-1.4484	0.2410*	0.0978*	0.9037*	0.0859	-4.6881	-4.5448
	SAB(e)	-0.0491	0.0397	-0.5776	0.1764*	0.0178	0.9478*	0.1109	-4.6554	-4.5341
	SOL(e)	0.0234	-0.0204	-0.5598*	-0.2727*	0.1044*	0.6155*	-0.0217*	-4.6394	-4.5181
ASA	AGL(e)	0.2917*	-0.0793	0.8324	0.2823*	-0.1471*	-0.3963*	1.3919*	-5.7975	-5.6983
	BVT(e)	0.4205*	-0.1517*	-3.0858*	-0.0518	-0.0074	-0.9961*	0.2427*	-5.8919	-5.7817
	SAB(e)	0.2780*	-0.1379*	-13.8546*	0.0760*	0.0044	-0.9854*	0.4719*	-5.6231	-5.5239
	SOL(e)	0.3559*	-0.1196*	-15.2229*	0.0886	-0.2749*	0.0628	-0.0350	-5.8660	-5.7558
W	AGL(e)	-0.0752*	0.0212	-10.2612*	0.3910*	0.0736^	-0.3689^	-0.4520*	-5.5070	-5.3747
	BVT(e)	0.0203	0.0328	-3.7790	0.1520*	0.0552	0.8199*	-0.1151	-5.4968	-5.3755
	SAB(e)	0.0602	0.0156	-6.1584*	0.1269^	-0.0481	0.7308*	-0.3890*	-5.5165	-5.3953
	SOL(e)	0.0198	0.0080	-6.2748*	0.1009	0.0516	0.7561*	-0.4915^	-5.4933	-5.3720
BVT	AGL(e)	0.3246*	-0.0915*	-5.1269*	0.1110*	0.0420*	0.3409*	-0.2367*	-5.7192	-5.6090
	ASA(e)	0.5241*	-0.1742*	-2.7320*	0.0160	-0.0699	-0.2818*	-0.3087*	-5.6830	-5.5618
	SAB(e)	0.2415*	-0.1141*	-6.7093*	0.0971*	0.0539	-0.9632*	0.1788*	-5.4239	-5.3136
	SOL(e)	0.2780*	-0.1217*	-2.2194*	0.0701*	-0.0283^	-0.9697*	0.3822*	-5.6407	-5.5415
X	AGL(e)	-0.0139	0.0292	-8.3891*	0.6065	-0.1183	0.4228^	-0.1876	-5.3970	-5.2757
	ASA(e)	-0.0304	0.0337	-6.0605	0.5748*	-0.1343	0.4303^	0.1030	-5.3971	-5.2759
	SAB(e)	0.0483	0.0180	-5.5141	0.5984*	-0.1200	0.4083^	0.2199	-5.4002	-5.2790
	SOL(e)	-0.0014	0.0154	-11.5625*	0.7112*	-0.0651	0.0122	-0.5347	-5.4017	-5.2915
SAB	AGL(e)	0.2670*	-0.0398	6.6108	0.2327*	-0.0650	0.2940^	1.7496*	-6.1169	-6.0288
	ASA(e)	0.1605*	-0.0653	-2.0471*	0.1704*	0.0738^	0.8654*	-0.1511*	-5.9092	-5.7990
	BVT(e)	0.1201*	-0.0302	-1.4245^	-0.2824*	-0.0544	-0.7194*	0.4708*	-5.9296	-5.8193
	SOL(e)	0.1763*	-0.0083*	0.5104	0.2819*	0.0261	0.9270*	0.1768*	-6.0318	-5.9216
Y	AGL(e)	0.0412	0.0486	-1.3485	0.2343*	-0.0538	-0.6194*	-0.0357	-5.8360	-5.7037
	ASA(e)	0.0084	0.0587	-1.7104^	0.0231	0.0825*	0.4227	-0.1672^	-5.8321	-5.6888
	BVT(e)	-0.0047	0.0728	-1.1832	0.1531^	0.0562	0.7863*	0.0064	-5.8064	-5.6851
	SOL(e)	0.0147	0.0543	2.2315	0.1664*	0.0526	0.7721*	0.5110	-5.8137	-5.6925
SOL	AGL(e)	0.4848*	-0.0093	-3.3265*	-0.0929*	0.1154*	0.7681*	-0.4175*	-5.2907	-5.1695
	ASA(e)	0.5051*	-0.0483	-0.3180*	-0.1815*	0.0486	0.9385*	-0.0100*	-5.2192	-5.0979
	BVT(e)	0.4101*	-0.0144	-3.3508^	0.1600*	-0.0923	0.1277*	1.2244*	-5.1278	-5.0175
	SAB(e)	0.5680*	-0.0646	1.0504*	-0.4523*	-0.1360*	0.9262*	0.1665*	-5.0489	-4.9497
Z	AGL(e)	0.0700	-0.0558	-3.4083*	-0.1812*	-0.0421	0.7034*	-0.2950*	-4.8267	-4.7055
	ASA(e)	0.1299*	-0.0574	-3.4037*	-0.2144*	-0.0744	0.7780*	-0.2577*	-4.8345	-4.7133
	BVT(e)	-0.0983^	-0.0371	-1.4779*	-0.2011*	-0.0178	0.1029	-0.0190*	-4.8099	-4.6776
	SAB(e)	0.1415	-0.0458	-7.9782*	0.1070*	0.0088	-0.4826*	0.3920*	-4.8407	-4.7084

* Indicates statistical significance at the 95% level.

^ Indicates statistical significance at the 90% level.

Period 3										
	Mean Equation			Variance Equation						
	Variable	Coeff.	Dep.Lag	ϖ	α	γ	δ	κ	AIC	SC
AGL	ASA(e)	0.3947*	0.0994*	-16.1586*	0.0880*	-0.0897*	-1.0035*	0.0084	-5.3960	-5.2755
	BVT(e)	0.3367*	0.0934*	-30.9449*	-0.1851^	0.0878^	-0.6580*	-0.9849*	-5.2709	-5.1504
	SAB(e)	0.7305*	0.0957^	0.7785	-0.1053	-0.0783	0.8231*	0.3467	-5.4447	-5.3242
	SOL(e)	0.7250*	0.1048	-14.9991*	-0.1960	0.0446*	-0.9333*	0.3716*	-5.6099	-5.4773
V	ASA(e)	0.0866	-0.1025^	-4.3271*	0.1322*	-0.0430^	-0.9631*	0.0843^	-5.2490	-5.1285
	BVT(e)	0.0137	-0.0593	-1.4715*	-0.1622	-0.0366	-0.1885	-0.0387	-5.1967	-5.0641
	SAB(e)	-0.1273	-0.0506	-8.6641	-0.0369	-0.1115*	-0.3334	0.5135^	-5.2201	-5.0634
	SOL(e)	-0.0578	-0.0435	-8.8800^	-0.1909	-0.0665	-0.1163	-0.7390	-5.2031	-5.0705
ASA	AGL(e)	0.1353*	0.0051	-11.9491*	0.1743^	-0.1866*	-0.7938*	-0.0878	-6.3912	-6.2707
	BVT(e)	0.2934*	-0.0412	-5.1700^	0.5188*	-0.2338*	-0.3615*	0.7222*	-6.4487	-6.3282
	SAB(e)	0.1485*	0.0115	-6.8079*	0.0616*	-0.0137	-0.9978*	-0.1141*	-6.4434	-6.3229
	SOL(e)	0.2409*	0.0110	-0.0118	0.0178	0.0835*	-0.9496*	0.6460*	-6.5229	-6.4024
W	AGL(e)	0.0555^	-0.0094	-2.8354	0.1329	0.0983	0.7451*	-0.0072	-6.2858	-6.1533
	BVT(e)	-0.0267	0.0061	-1.6866*	0.0375	0.0430	0.0373*	0.0198	-6.2745	-6.1298
	SAB(e)	0.0073	-0.0035	-3.9970*	0.0121	0.0135	0.7140*	0.1317*	-6.3090	-6.1764
	SOL(e)	0.0837*	-0.0038	-0.2196	0.1370	0.1175*	0.7658*	0.2450	-6.2906	-6.1580
BVT	AGL(e)	0.1414*	-0.1379*	-3.2223^	0.2766*	0.1701*	0.7137*	0.0513	-6.3777	-6.2571
	ASA(e)	0.2924*	-0.1791*	-9.8685*	0.4661*	0.1109*	-0.0827*	0.6560*	-6.4297	-6.3092
	SAB(e)	0.2204*	-0.1398*	-1.6729	0.2275*	-0.1597*	0.2135	0.0014	-6.3588	-6.2382
	SOL(e)	0.1919*	-0.1589*	-21.5316*	0.5295*	0.0065	-0.6104*	0.5282*	-6.4223	-6.3017
X	AGL(e)	-0.0707	0.0414	-3.7963^	-0.3394*	-0.2685*	0.1878	-0.0759	-6.2602	-6.1276
	ASA(e)	-0.0419	0.0585	-3.2993*	-0.5567*	-0.2170*	0.3855*	-0.1921*	-6.2777	-6.1451
	SAB(e)	-0.0490	0.0357	-1.5526	-0.3297*	-0.2860*	0.0324	0.2407	-6.2521	-6.1195
	SOL(e)	-0.0178	0.0329	-5.7669	-0.3117^	-0.2719*	0.1243	-0.2141	-6.2464	-6.1138
SAB	AGL(e)	0.2063*	-0.0047	-15.2187*	0.3905*	-0.1088*	-0.7163*	0.6015*	-6.7304	-6.6098
	ASA(e)	0.1163*	-0.0054	-3.7334*	0.0732	-0.0498^	-0.8767*	0.2551*	-6.6138	-6.4932
	BVT(e)	0.2557*	-0.0304	-36.6389*	0.2351*	0.0552*	-0.8893*	-0.4966^	-6.5675	-6.4470
	SOL(e)	0.1169*	0.0155	0.6367	0.0709	-0.0620*	-0.8941*	0.7160*	-6.6381	-6.5176
Y	AGL(e)	-0.0280	0.0320	-1.6320*	-0.2166*	-0.2087	-0.3933	-0.0585*	-6.4856	-6.3531
	ASA(e)	0.0187	-0.0214	-11.4052	0.1051	0.0249	0.7308*	-0.1977^	-6.5146	-6.3579
	BVT(e)	0.1156*	-0.0091	-31.0539*	-0.0041	0.0444	-0.9535*	0.2023	-6.5134	-6.3928
	SOL(e)	-0.0109	0.0544	1.7676^	-0.2340*	0.1038^	-0.1039*	0.2686*	-6.4862	-6.3295
SOL	AGL(e)	0.3943*	0.0790	-17.6043*	0.1761*	0.1418*	-0.7404*	1.0834*	-6.1612	-6.0407
	ASA(e)	0.4268*	0.0087	-28.3331*	0.1539*	0.1186*	-0.8893*	0.0536	-5.9632	-5.8427
	BVT(e)	0.2877*	-0.0028	-17.8021*	0.1559	-0.1518*	0.5252*	-1.0437*	-5.9429	-5.8224
	SAB(e)	0.2994*	0.0318	-18.9544*	0.0388	0.0585*	-0.9907*	0.2783*	-5.8620	-5.7294
Z	AGL(e)	-0.1254*	0.0346	-14.5515^	-0.2539	0.0725^	-0.1728	0.5866*	-5.8289	-5.7083
	ASA(e)	-0.0125	-0.0808*	-1.8788*	0.3458*	-0.0286*	-0.8816*	-0.0874*	-5.8583	-5.7377
	BVT(e)	0.0315	-0.0610	-13.2777*	0.0881	-0.0923^	-0.9964*	-0.0134	-5.8127	-5.6801
	SAB(e)	-0.1238	0.0008	-7.8394*	-0.0359	-0.0790*	-0.9003*	0.4518*	-5.8398	-5.7072

* Indicates statistical significance at the 95% level.

^ Indicates statistical significance at the 90% level.

Period 4

	Mean Equation			Variance Equation						
	Variable	Coeff.	Dep. Lag	ϖ	α	γ	δ	κ	AIC	SC
AGL	ASA(e)	0.3292*	0.0488	-18.2040*	-0.1034*	-0.0545*	-0.5917*	0.2349*	-5.5096	-5.3974
	BVT(e)	0.3381*	0.0562	-0.2752*	-0.2250*	-0.0673*	0.6079^	0.0704*	-5.5204	-5.4082
	SAB(e)	0.6721*	-0.0210	-0.1601*	-0.2919*	0.1136*	0.1432*	0.0482*	-5.6218	-5.5096
	SOL(e)	0.7422*	0.0754	-16.4602*	0.1864*	-0.0545*	-0.9685*	0.0929*	-5.7627	-5.6618
V	ASA(e)	0.2103	-0.0242	-10.5186*	0.2323*	-0.0521*	-0.9324*	-0.2635*	-5.4257	-5.3136
	BVT(e)	0.0287	-0.0209	-2.5442*	0.1863*	0.0126	-0.9421*	0.1482^	-5.4104	-5.2983
	SAB(e)	-0.1199	0.0004	0.0289	-0.1562	-0.0370	0.9223*	0.0565*	-5.3842	-5.2833
	SOL(e)	-0.0113	-0.0297	-7.2650*	-0.1197	-0.0914	0.8160*	-0.5759*	-5.3809	-5.2576
ASA	AGL(e)	0.1495*	0.0533	-3.2148*	-0.1377	0.0577	0.7219*	0.2158*	-6.2977	-6.1519
	BVT(e)	0.2608*	0.0154	-5.5078*	0.0936^	0.2088*	-0.9613*	0.2060*	-6.3374	-6.2141
	SAB(e)	0.3772*	0.0138	-3.5216*	0.1500*	0.1439^	-0.9560*	0.1763*	-6.3709	-6.2364
	SOL(e)	0.2974*	0.0210	-21.2197*	0.0467	0.1910^	-0.9815*	0.0171	-6.3195	-6.1961
W	AGL(e)	0.0120	-0.0267	-1.5814*	0.0046	0.0047	0.9165*	-0.1011*	-6.2440	-6.1431
	BVT(e)	0.0225	-0.0362	-3.1585	0.2690^	0.0210*	-0.4910*	0.4495^	-6.2028	-6.0907
	SAB(e)	0.0205	-0.0101	-3.8978*	0.1520^	1.3394*	-0.8029*	0.0446	-6.2002	-6.0993
	SOL(e)	-0.0304	-0.0254	3.9920^	0.0321	1.5744*	-0.6065*	0.4819	-6.2163	-6.1154
BVT	AGL(e)	0.2516*	-0.1513*	-2.1409*	0.1568*	0.0806^	0.8936*	-0.1257*	-5.9893	-5.8772
	ASA(e)	0.4009*	-0.1957*	-1.2514*	-0.2271*	-0.0491^	0.9744*	-0.1213*	-6.0694	-5.9685
	SAB(e)	0.2672*	-0.1651*	-20.8295*	-0.1478	0.1283^	-0.5319	-0.7477*	-5.8791	-5.7670
	SOL(e)	0.3384*	-0.1482*	-23.6445*	0.2292	0.0851	-0.3857*	-0.5467*	-5.9719	-5.8598
X	AGL(e)	-0.0192	0.0146	-15.9240*	-0.1906^	0.0781	-0.8607*	0.1461	-5.8229	-5.7108
	ASA(e)	-0.0310	-0.0209	-7.7611^	-0.4666*	0.0067	0.1307	-0.2419	-5.8128	-5.7007
	SAB(e)	-0.0827	0.0103	-13.2877*	0.0356	0.0786^	-0.8736*	0.4012	-5.8277	-5.7155
	SOL(e)	-0.0734	0.0103	-16.7581*	-0.1772^	0.0792	-0.8725*	0.0634	-5.8262	-5.7140
SAB	AGL(e)	0.2035*	0.0164	-18.6046*	0.0720	0.0043	-0.9157*	0.6558*	-6.9434	-6.8312
	ASA(e)	0.1622*	0.0050	-0.7843	-0.1762	-0.0107	-0.6324*	0.2118*	-6.8434	-6.7313
	BVT(e)	0.0903*	-0.0337	-4.7725	-0.2942*	0.0267	0.6678*	-0.2057	-6.8144	-6.7135
	SOL(e)	0.2228*	-0.0233	-2.8138*	-0.2548*	0.1089	0.4208*	0.2968*	-6.9059	-6.8050
Y	AGL(e)	0.0377	-0.1212*	-0.3751*	-0.3249^	-0.0285	0.9386*	0.0164*	-6.7891	-6.6881
	ASA(e)	-0.0276	-0.1145*	-3.0905	0.3325*	0.0072	0.4384^	0.4668^	-6.7629	-6.6171
	BVT(e)	0.0090	-0.1280*	-0.2875*	-0.2973*	0.0011	0.9684*	-0.0066*	-6.7940	-6.6931
	SOL(e)	0.0251	-0.0872^	-0.0815	0.1779*	-0.0587*	0.9594*	0.1252	-6.7815	-6.6582
SOL	AGL(e)	0.3598*	0.0353	-0.7059*	-0.3070*	0.0650*	0.9257*	-0.0206*	-6.4326	-6.3205
	ASA(e)	0.3472*	0.0397	-3.0246	0.2315*	0.1513*	0.5821*	0.2055	-6.1618	-6.0496
	BVT(e)	0.2561*	0.0011	-3.4127*	-0.0617^	0.0206	-0.9930*	0.2318*	-6.2571	-6.1449
	SAB(e)	0.4565*	0.0311	-6.7252*	0.4181*	-0.1641*	-0.6053*	0.8778*	-6.2040	-6.1031
Z	AGL(e)	0.0501	-0.0084	-14.8625*	-0.0040	-0.0172	-0.7959*	0.8350*	-6.0356	-5.9235
	ASA(e)	-0.0470	-0.0176	-0.5936*	-0.1018*	-0.0448	0.1505*	-0.0477*	-6.0403	-5.9281
	BVT(e)	0.0837^	-0.0011	-34.6718*	0.0013	-0.0536	-0.4884	-0.2599^	-6.0429	-5.9195
	SAB(e)	0.0796	-0.0069	0.1781	0.0653	-0.0232	0.9676*	0.0891	-6.0378	-5.9145

* Indicates statistical significance at the 95% level.
^ Indicates statistical significance at the 90% level.

	Period 5									
	Mean Equation			**Variance Equation**						
	Variable	Coeff.	Dep. Lag	ϖ	α	γ	δ	κ	AIC	SBC
AGL	ASA(e)	0.5951*	-0.0643	-10.1934*	-0.0239	0.0305	0.8058*	-0.6460*	-5.8362	-5.6915
	BVT(e)	0.3931*	0.0244	1.2163	0.1350*	0.0188	-0.9707*	0.7341*	-5.9585	-5.8380
	SAB(e)	0.7225*	0.0127	-8.7034*	0.0517	0.0507	-0.7109*	0.6448*	-5.8285	-5.7200
	SOL(e)	0.6793*	-0.0183	1.8197*	-0.1032	0.1618*	0.8456*	0.3381*	-6.0337	-5.9252
V	ASA(e)	-0.0396	-0.0873*	-3.1573*	0.4058*	0.0558^	-0.9631*	0.1585	-5.6840	-5.5514
	BVT(e)	-0.0231	-0.0441	-11.8713^	0.1773^	0.1144	0.3123	-0.6733	-5.6256	-5.4930
	SAB(e)	0.0366	-0.0574	-13.6554*	0.2635^	0.1423	-0.5630^	0.1933	-5.6218	-5.4892
	SOL(e)	-0.0368	-0.0759	1.2438*	-0.4226*	0.0456	0.0237	0.2053*	-5.6205	-5.4879
ASA	AGL(e)	0.2313*	-0.1357*	-1.8478	0.1983^	-0.1312^	-0.3924*	0.9368*	-6.5256	-6.4171
	BVT(e)	0.3136*	-0.0200	-3.0595*	-0.5474*	-0.0837	0.7224*	-0.0564*	-6.5553	-6.4227
	SAB(e)	0.3281*	0.0085	-10.5693*	0.3630*	0.0065	-0.6768*	0.5438*	-6.4724	-6.3760
	SOL(e)	0.2080*	-0.1077*	-12.3048*	0.3929*	-0.0907*	-0.8812*	0.6184*	-6.5767	-6.4803
W	AGL(e)	0.0254	-0.0372*	-7.4678*	0.1600*	-0.0114	-0.9725*	0.0984	-6.4094	-6.2769
	BVT(e)	0.0100	-0.0160	-1.7021	-0.2455	0.1894*	0.6825*	0.1312*	-6.3358	-6.2153
	SAB(e)	-0.0731^	0.0468*	-22.4150*	0.2471*	0.0201	-0.9405*	-0.3033^	-6.3948	-6.2743
	SOL(e)	-0.1303*	-0.0168	-10.1868*	0.2584*	-0.0403	-0.7562*	0.2827	-6.3509	-6.2303
BVT	AGL(e)	0.2046*	-0.3097*	-7.9965*	-0.2629*	0.0327	-0.6409*	0.7033*	-6.2598	-6.1393
	ASA(e)	0.3764*	-0.3434*	-19.5935*	0.1442*	-0.0062	-0.9718*	0.1505	-6.3276	-6.2192
	SAB(e)	0.2478*	-0.2717*	-1.3711*	-0.4795*	0.1224*	0.6257*	-0.0451*	-6.2583	-6.1257
	SOL(e)	0.3663*	-0.3047*	-2.6836*	-0.3395*	0.1160*	0.7301*	-0.1805*	-6.3678	-6.2352
X	AGL(e)	-0.0189	0.0478	-6.5399*	-0.2637*	0.1104	-0.2047	-0.3058^	-6.0520	-5.9435
	ASA(e)	0.0459	0.0843	-5.5829*	-0.1416*	0.0712*	0.8945*	-0.5545*	-6.0722	-5.9517
	SAB(e)	-0.1798*	0.0233	-9.4422*	-0.2540*	0.1781*	-0.5802*	-0.0828	-6.0627	-5.9542
	SOL(e)	0.0012	0.0540	-15.4375*	-0.1963	0.0944	-0.4550^	-0.5909^	-6.0539	-5.9334
SAB	AGL(e)	0.1679*	-0.0196	-5.3965*	-0.4532*	0.1015*	0.6038*	0.1351*	-7.0532	-6.9326
	ASA(e)	0.1245*	-0.0957*	-5.1294*	-0.4372*	0.1664*	0.7022*	-0.0721^	-6.9761	-6.8555
	BVT(e)	0.1411*	-0.0853*	-1.0731*	-0.2806*	0.0956*	0.9084*	-0.0427*	-6.9331	-6.8367
	SOL(e)	0.1442*	-0.1136*	-0.4952*	-0.4369*	-0.0044	0.4351*	0.0921*	-6.9385	-6.8180
Y	AGL(e)	-0.0368	0.0085	-0.3255	-0.0748	0.1388^	0.6864*	0.3046	-6.8101	-6.7016
	ASA(e)	-0.0038	0.0039	5.7725*	-0.1853*	0.0264^	0.9284*	0.6802*	-6.8256	-6.7171
	BVT(e)	0.0480	0.0054	-6.6940*	-0.0482	-0.0550	-0.9866*	0.2511	-6.7932	-6.6847
	SOL(e)	0.0284	0.0140	0.9013	-0.1250	0.0719	0.6975*	0.7257*	-6.8198	-6.6872
SOL	AGL(e)	0.3524*	0.0243	-4.7334*	0.1301^	-0.0466*	-0.9829*	0.1410*	-6.6113	-6.4908
	ASA(e)	0.2244*	0.0003	0.3471	0.2313*	0.0456^	-0.9496*	0.6891*	-6.4298	-6.2972
	BVT(e)	0.3304*	0.0543	2.2485^	0.2303*	-0.0367*	-0.9559*	0.9209*	-6.5254	-6.4048
	SAB(e)	0.3188*	0.0175	-3.8600*	0.1838*	-0.0003	-0.9646*	0.1656*	-6.4289	-6.3084
Z	AGL(e)	0.0849*	-0.1640*	-14.5942*	0.4494*	0.0665	-0.7995*	0.2672	-6.2321	-6.1236
	ASA(e)	0.1018^	-0.1663*	-12.3548*	-0.2611	0.0750	-0.5793*	0.2239	-6.2178	-6.0972
	BVT(e)	0.0630	-0.1453*	-0.3638	-0.0316	0.1514^	0.6785*	0.2790	-6.1899	-6.0934
	SAB(e)	0.1666*	-0.1283*	-1.2977	0.0273	0.1571*	0.7909*	0.0630	-6.2042	-6.1077

* Indicates statistical significance at the 95% level.
^ Indicates statistical significance at the 90% level.

Period 6

	Mean Equation			Variance Equation						
	Variable	Coeff.	Dep. Lag	ϖ	α	γ	δ	κ	AIC	SBC
AGL	ASA(e)	0.5122*	-0.0303	-11.0043*	-0.0018	-0.0522	0.8557*	-0.9410*	-6.4627	-6.3745
	BVT(e)	0.5740*	-0.1737*	-14.0011*	0.4572*	0.0436	-0.8720*	0.1763*	-6.5428	-6.4215
	SAB(e)	0.0797	0.0071	0.2054*	-0.0223	-0.0253	0.9759*	0.0391*	-6.3563	-6.2681
	SOL(e)	0.7224*	-0.0625^	-0.9587*	-0.1153*	-0.0213	0.8385*	-0.0833*	-6.6558	-6.5566
V	ASA(e)	-0.1954*	-0.0616	-5.1198*	0.2140*	0.0622*	0.8921*	-0.2694	-6.3335	-6.2122
	BVT(e)	-0.1363*	0.0265	-2.9866	0.2219*	-0.0245	0.8198*	0.0572	-6.3444	-6.2231
	SAB(e)	-0.1824	-0.0287	-17.4458*	-0.1744^	0.0418	-0.7947*	0.0874	-6.3084	-6.2092
	SOL(e)	-0.1533*	-0.0349*	-9.8678*	0.2359*	0.0267^	-0.9393*	0.0033	-6.3521	-6.2419
ASA	AGL(e)	0.1777*	-0.0194	-12.0904*	-0.0817	0.1231*	-0.8523*	0.2712	-7.4870	-7.3878
	BVT(e)	0.2087*	-0.0962*	-1.5737*	0.2160*	-0.0323	0.7224*	-0.1386*	-7.4826	-7.3723
	SAB(e)	-0.0171	-0.0229	-37.8032*	-0.1096*	0.0101	-0.8746*	-0.1418*	-7.4343	-7.3351
	SOL(e)	0.3355*	-0.0738*	-15.7422*	0.2388*	0.0268	-0.9366*	0.0126	-7.5864	-7.4652
W	AGL(e)	-0.0643^	-0.0057	-19.0049	0.0622	-0.0741	0.1516	-0.2935	-7.3526	-7.2534
	BVT(e)	-0.0402	0.0020	-33.7269*	-0.0309	-0.0145	-0.9196*	-0.1225^	-7.3526	-7.2534
	SAB(e)	-0.0558	-0.0089	-20.0929*	0.0586	-0.0728	-0.8732*	-0.0813	-7.3537	-7.2655
	SOL(e)	-0.1060*	0.0273	-21.4567*	0.1510*	-0.0465*	-0.9539*	-0.1193*	-7.3907	-7.3025
BVT	AGL(e)	0.2204*	-0.1529*	4.7065	0.3326*	0.1286^	0.0020	1.6474*	-7.3278	-7.2396
	ASA(e)	0.1541*	-0.2429*	-17.5795*	0.1631*	0.1401*	-0.9527*	-1.0927*	-7.4259	-7.3157
	SAB(e)	0.0840	-0.1310*	-9.9028*	0.5536*	-0.0845	-0.3140^	0.0857	-7.2043	-7.0941
	SOL(e)	0.2778*	-0.1757*	-0.0895	-0.4208*	0.1205^	0.7321*	0.2689*	-7.3178	-7.2186
X	AGL(e)	0.0052	0.0152	-4.7449*	0.1113	0.1300	0.0889	-0.2615*	-7.1092	-6.9880
	ASA(e)	0.0476	0.0219	-9.7326*	-0.3388*	-0.2813*	0.3836*	-0.3734	-7.1258	-7.0377
	SAB(e)	-0.0075	0.0265	-6.9235^	-0.3261*	-0.2896*	0.4534*	-0.1611	-7.1265	-7.0383
	SOL(e)	0.0380	0.0237	-5.7040*	-0.3477*	-0.2705*	0.4391*	-0.0366	-7.1228	-7.0347
SAB	AGL(e)	0.1409*	0.0308	12.6836*	0.2864*	0.2032*	-0.1581	2.2838*	-7.8324	-7.7332
	ASA(e)	0.1419*	-0.0084	-29.8013*	0.3974*	0.0606	-0.2080	0.4184*	-7.7652	-7.6439
	BVT(e)	0.1406*	-0.0661^	-34.0891*	0.3331*	0.0237	-0.8216*	-0.2387*	-7.7746	-7.6643
	SOL(e)	0.1624*	0.0937^	-3.5329	0.2672*	0.0620*	0.7971*	0.1574*	-7.8111	-7.6898
Y	AGL(e)	0.0278	-0.0727^	-9.2210	0.1958^	0.0047	0.8314*	-0.2712*	-7.7224	-7.6011
	ASA(e)	0.0019	-0.0293	-19.1923*	-0.0217	0.0156	-0.9517*	0.1445*	-7.7058	-7.6176
	BVT(e)	0.0328	-0.0348	-17.5690*	-0.1611	-0.0113	-0.6771*	0.0033	-7.6782	-7.5790
	SOL(e)	0.0846*	-0.0501	-4.8538^	0.2099	0.0160	0.5484*	-0.2664^	-7.6885	-7.5672
SOL	AGL(e)	0.3290*	0.0934*	-11.8064*	0.1969*	0.0418*	-0.9460*	0.4573*	-7.4068	-7.2856
	ASA(e)	0.3711*	0.0316	-3.9206*	0.1829^	0.2932*	0.9669*	-0.3497*	-7.2403	-7.1301
	BVT(e)	0.3087*	0.0743	-3.4609*	0.1114^	0.2608*	0.8661*	-0.2011*	-7.1956	-7.0964
	SAB(e)	0.1256*	0.0903*	-28.7509*	0.0168	-0.3490*	-0.4318*	-0.0365*	-7.1909	-7.0697
Z	AGL(e)	0.0093	0.0290	0.8771	-0.2907*	0.0042	0.4832*	0.6279	-7.0321	-6.9439
	ASA(e)	-0.0806	0.0434	-8.1754*	-0.0767	-0.0066	-0.5759*	0.0860	-7.0269	-6.9056
	BVT(e)	0.0707	0.0633	-7.4581*	0.0868	0.0055	-0.7384*	0.1314	-7.0289	-6.9076
	SAB(e)	-0.0411	0.0479	-11.3586*	-0.0609	0.0080	-0.6021*	-0.1549	-7.0238	-6.9026

* Indicates statistical significance at the 95% level.

^ Indicates statistical significance at the 90% level.

Period 7										
	Mean Equation			Variance Equation						
	Variable	Coeff.	Dep. Lag	ϖ	α	γ	δ	κ	AIC	SC
AGL	ASA(e)	0.4711*	-0.0146	-5.4828*	-0.3962*	-0.0396	-0.6949*	-0.2342*	-7.0610	-6.9583
	BVT(e)	0.1793*	-0.0463^	-0.6587*	-0.8476*	0.0401	0.6555*	0.1928*	-6.9235	-6.8208
	SAB(e)	0.3900*	0.0022	-4.4282*	-0.2646^	-0.0311	-0.7147*	-0.0611*	-6.9534	-6.8392
	SOL(e)	0.5643*	0.0186	-7.2593*	0.0654*	-0.0147*	-1.0094*	-0.1361*	-7.1979	-7.0838
V	ASA(e)	-0.0389	0.1214*	-2.7010*	-0.1848*	0.0882^	0.7556*	-0.0480*	-6.7450	-6.6537
	BVT(e)	-0.1877*	0.1275*	-1.1266	-0.1873^	0.0629	0.7754*	0.0833	-6.7596	-6.6683
	SAB(e)	0.0191	0.0764^	-9.3938*	-0.2929*	0.0876^	0.7543*	-0.6823*	-6.7758	-6.6845
	SOL(e)	-0.0776	0.1348*	-5.5590*	-0.1825*	0.1149*	0.7191*	-0.2909*	-6.7477	-6.6564
ASA	AGL(e)	0.2832*	-0.0832	-8.5153*	0.1335	-0.0426	-0.7240*	0.3756^	-7.6268	-7.5127
	BVT(e)	0.2822*	-0.1156*	-31.0191*	0.3454*	0.0328	-0.8653*	-1.0436*	-7.5075	-7.4048
	SAB(e)	0.2420*	-0.0628	-13.8137*	0.0444	-0.1214*	-0.9720*	-0.0773	-7.5144	-7.4003
	SOL(e)	0.3696*	-0.0401	-2.0005	0.2974	0.0103	-0.0654	0.9236*	-7.5655	-7.4628
W	AGL(e)	-0.0448	0.0360	-20.9649*	0.0714	-0.0875	-0.6685*	-0.3832^	-7.4391	-7.3478
	BVT(e)	0.0079	-0.0233	-24.3070*	0.1641*	-0.0596^	-0.9653*	-0.5014*	-7.4431	-7.3290
	SAB(e)	-0.0070	0.0395	-12.3666*	0.0145	0.0266	-0.9889*	0.0790	-7.4123	-7.3096
	SOL(e)	-0.0334	0.0284	-10.9788*	-0.0156	-0.0022	-0.9638*	0.2977*	-7.4147	-7.3120
BVT	AGL(e)	0.1787*	-0.1258*	2.1031^	0.0046	-0.0259	-0.3974*	0.3029*	-7.8485	-7.7344
	ASA(e)	0.2098*	-0.1599*	-20.8273*	0.3770*	-0.0229*	-0.9820*	0.1170^	-7.8698	-7.7671
	SAB(e)	0.1240*	-0.0648	-8.2453^	0.3026*	-0.1783*	-0.4568*	0.7093*	-7.7909	-7.6882
	SOL(e)	0.2696*	-0.0835*	-16.4920*	0.2175*	0.0313^	0.5931*	-0.0124	-7.9005	-7.7749
X	AGL(e)	-0.0188	-0.1439*	-1.9056*	-0.3223*	0.1059*	0.8016*	-0.1240*	-7.7533	-7.6392
	ASA(e)	-0.0447	-0.0866	-20.3243*	0.1293	0.2012*	0.0690	-1.0029^	-7.7304	-7.6391
	SAB(e)	0.0467	-0.0618	-17.3833*	0.1915	0.1935*	-0.1991	-0.4244	-7.7213	-7.6301
	SOL(e)	-0.0244	-0.0815	-1.5397*	-0.3022*	-0.0976	0.3655*	-0.1051*	-7.7281	-7.6026
SAB	AGL(e)	0.1526*	-0.0701	-3.1489*	0.0759	0.0199	-0.8915*	0.6185*	-7.9013	-7.7986
	ASA(e)	0.1842*	-0.0272	-15.5058*	0.2379*	-0.0799	-0.7333*	0.6889*	-7.8089	-7.6834
	BVT(e)	0.1322*	-0.0888	-1.5455*	-0.3523*	-0.0111	0.9222*	-0.0766*	-7.7960	-7.6819
	SOL(e)	0.3080*	-0.1142*	-4.1389	0.3264*	0.0381	-0.9182*	0.5167*	-7.8801	-7.7660
Y	AGL(e)	-0.0460	0.0137	-8.1262*	0.0052	0.0251	-0.9892*	-0.1122^	-7.7525	-7.6498
	ASA(e)	-0.0352	0.0010	-36.3216*	-0.1332	0.0039	-0.7076*	-1.0954*	-7.7523	-7.6496
	BVT(e)	0.0274	0.0229	-0.4262	-0.0013	0.0474	0.7316*	0.2274	-7.7270	-7.6357
	SOL(e)	-0.0304	0.0305	-24.8247*	-0.0223	0.0014	-1.0154*	0.0478	-7.7412	-7.6385
SOL	AGL(e)	0.3307*	-0.0217	-5.1192*	-0.0596	-0.1202*	-0.8389*	0.6190*	-7.6633	-7.5378
	ASA(e)	0.3095*	-0.0581	-9.1502*	-0.1494*	-0.0250*	-0.9637*	0.3571*	-7.6624	-7.5369
	BVT(e)	0.2811*	0.1014*	-9.3476*	-0.1535*	0.0002	-0.9796*	0.3110*	-7.6346	-7.5090
	SAB(e)	0.2315*	0.0067	-24.1498*	0.1753*	0.0843*	-0.8738*	0.4454*	-7.4795	-7.3768
Z	AGL(e)	0.0724^	0.0385	-10.2425*	-0.0032	0.0268	-0.9882*	0.1532^	-7.4061	-7.3034
	ASA(e)	0.0573	0.0189	-27.9389*	0.0752	-0.0413	-0.9345*	0.1549	-7.3693	-7.2552
	BVT(e)	0.0648	0.0487	-12.0488*	-0.0626	0.0023	-0.9953*	-0.1779^	-7.3884	-7.2857
	SAB(e)	0.1468*	0.0283	-25.3462*	-0.1367	-0.0473	-0.5688*	-0.8770*	-7.3953	-7.3040

* Indicates statistical significance at the 95% level.
^ Indicates statistical significance at the 90% level.

	Period 8									
	Mean Equation			**Variance Equation**						
	Variable	Coeff.	Dep. Lag	ϖ	α	γ	δ	κ	AIC	SC
AGL	ASA(e)	0.2800*	0.0576	-36.1535*	-0.0748^	0.1088*	-0.9795*	-0.9530*	-7.4642	-7.3459
	BVT(e)	0.1480*	0.0763*	-11.8349*	0.0371	0.1627*	-0.7712*	1.3266*	-7.3919	-7.2736
	SAB(e)	0.2295*	0.1300*	-1.6858*	0.1336*	-0.3287*	0.9685*	-0.1168*	-7.5081	-7.4016
	SOL(e)	0.3468*	-0.0121	-1.6486	0.2244*	-0.0101	0.6967*	0.0422	-7.4332	-7.3149
V	ASA(e)	-0.0452	-0.0930	-1.3089^	-0.2057	-0.0337	0.8044*	-0.0927	-7.2704	-7.1521
	BVT(e)	0.0214	-0.0723	0.5072*	-0.2335*	-0.0141	0.9663*	0.0708*	-7.2762	-7.1698
	SAB(e)	0.2025*	-0.1169*	-5.8430*	0.0309	-0.0153	-0.9375*	0.0250	-7.2645	-7.1343
	SOL(e)	-0.0120	-0.0587	-30.5497*	0.0842	-0.0004	-0.4674^	-0.1415	-7.2493	-7.1429
ASA	AGL(e)	0.2507*	-0.0725^	-11.2517*	-0.2720*	-0.0305	0.8314*	-0.9511*	-7.8537	-7.7590
	BVT(e)	0.2308*	-0.0495	-1.7423*	-0.1304*	-0.2068*	-0.6029*	-0.0448*	-7.7818	-7.6635
	SAB(e)	0.0514	-0.0432	-2.1460*	-0.0531^	-0.0361	-0.5508*	-0.0819*	-7.7283	-7.6100
	SOL(e)	0.2669*	-0.0486	-0.7268*	-0.0040	0.0047	0.8067*	0.1049*	-7.8034	-7.6851
W	AGL(e)	0.0220	-0.0153	-19.2045*	0.1682	-0.1155	-0.7592*	-0.0335	-7.6942	-7.5759
	BVT(e)	-0.0271	0.0170	0.6955*	-0.0626	0.0135	0.9580*	0.1033*	-7.7343	-7.6396
	SAB(e)	0.0495	0.0493	-4.4520*	0.1380	-0.0117	-0.7661*	0.0586	-7.7080	-7.6015
	SOL(e)	0.0523	0.0123	-6.4144*	0.1399^	-0.0225	-0.9153*	0.1157	-7.7130	-7.6065
BVT	AGL(e)	0.1405*	-0.1194*	-13.8430*	-0.2174*	0.1613*	-0.6678*	-0.8583*	-7.7334	-7.6151
	ASA(e)	0.2031*	-0.1142*	-18.7487*	-0.1440^	0.0435	-0.8820*	-0.8224*	-7.7566	-7.6383
	SAB(e)	0.1055*	-0.1030*	-6.0085*	0.1693*	0.0866*	-0.9701*	-0.0512*	-7.8525	-7.7224
	SOL(e)	0.3641*	-0.2740*	-28.6042*	0.2517*	-0.0138*	-0.9384*	0.2851*	-7.9681	-7.8498
X	AGL(e)	-0.0048	-0.0158	0.3046*	0.0963*	0.0090	0.9734*	0.0523*	-7.6613	-7.5549
	ASA(e)	-0.0628	-0.0281	-0.0547	-0.1337*	-0.0183	0.6038*	0.0333	-7.6635	-7.5571
	SAB(e)	0.0956	-0.0307	-0.0662	0.2357*	0.0407	0.9800*	0.0107*	-7.6625	-7.5442
	SOL(e)	0.0116	0.0103	-11.6207*	-0.0815	0.0169	-0.9781*	-0.1002	-7.6692	-7.5627
SAB	AGL(e)	0.1562*	-0.1090^	8.2337*	-0.5008*	-0.0337	-0.5705*	1.4140*	-8.0223	-7.9040
	ASA(e)	0.1163*	-0.1219^	-2.3116	0.6237*	0.1482*	0.0001	0.8937^	-7.8986	-7.7803
	BVT(e)	0.0523*	-0.0795	-14.4497*	0.5568*	-0.1549*	-0.7044*	-0.7146*	-7.9122	-7.7939
	SOL(e)	0.1603*	-0.2526*	-5.4892*	0.9477*	0.0263	-0.4211*	0.4164*	-7.9371	-7.8307
Y	AGL(e)	0.0283	0.0424	-16.4203*	-0.0670	-0.0560*	-0.9539*	0.4789	-7.7505	-7.6441
	ASA(e)	-0.1198*	0.0471	-5.3039*	-0.1855	0.0461	0.4475*	-0.3102*	-7.7594	-7.6411
	BVT(e)	-0.1289*	0.0787^	0.3321*	-0.1859*	-0.0652*	0.9373*	0.0807*	-7.7797	-7.6851
	SOL(e)	-0.0106	0.1182*	-13.9304*	0.1822*	0.0409	-0.9589*	0.1868*	-7.7840	-7.6538
SOL	AGL(e)	0.2146*	-0.0647	-28.1608*	0.1286*	0.0704*	-0.8730*	-1.8571*	-8.0867	-7.9803
	ASA(e)	0.2395*	-0.0846	-13.7935*	0.4841*	0.0871	-0.8223*	0.6729	-8.0805	-7.9622
	BVT(e)	0.3201*	-0.0695	-1.8438*	-0.7147*	-0.0398	-0.0939*	-0.1438*	-8.1517	-8.0334
	SAB(e)	0.1976*	-0.0639	-11.8029*	-0.1457^	0.0062	-0.8317*	0.2099*	-8.0497	-7.9196
Z	AGL(e)	0.0095	-0.0307	-15.5738*	-0.0449	0.0979*	-0.9189*	0.3821	-7.9165	-7.8100
	ASA(e)	0.0118	0.0089	-13.6070*	-0.1429*	0.0283	-0.9530*	-0.1083	-7.9220	-7.8156
	BVT(e)	-0.0474	-0.0114	-13.0162*	0.0337*	0.0330	-0.9162	0.7298^	-7.9018	-7.7953
	SAB(e)	0.0255	-0.0479	-8.4101*	0.1786*	-0.0134	-0.9611*	0.0862*	-7.9471	-7.8170

* Indicates statistical significance at the 95% level.

^ Indicates statistical significance at the 90% level.

Period 9										
	Mean Equation			Variance Equation						
	Variable	Coeff.	Dep. Lag	ϖ	α	γ	δ	κ	AIC	SC
AGL	ASA(e)	0.4137*	0.0976^	-0.3346*	-0.2385*	-0.0921*	-0.2780*	0.0442*	-7.1533	-7.0392
	BVT(e)	0.3376*	0.0961*	-1.7803*	-0.0893^	-0.1247*	0.8660*	-0.1518*	-7.1606	-7.0465
	SAB(e)	0.2921*	0.1139*	-14.4283*	0.1200^	-0.1674*	-0.6563*	0.7810*	-7.0812	-6.9785
	SOL(e)	0.6610*	-0.0388	-5.5290*	0.1646*	0.0314^	-0.9709*	0.0486*	-7.4066	-7.2925
V	ASA(e)	0.1403*	-0.0428	-0.7646*	-0.0479*	-0.0544*	0.9277*	-0.0175*	-7.0333	-6.9306
	BVT(e)	0.0135	-0.0009	-15.2124*	0.0481	0.0511	-0.7496*	0.2077	-7.0092	-6.9065
	SAB(e)	0.3086*	-0.0188	-26.4224*	0.2250*	-0.0867*	-0.9306*	-0.6370*	-7.0802	-6.9889
	SOL(e)	0.0872	-0.0139	-5.5782^	-0.0561	-0.0436	0.0469	-0.2878^	-7.0272	-6.9245
ASA	AGL(e)	0.0965*	-0.0326	-4.8045*	-0.4355*	-0.1665*	0.7349*	-0.0697	-8.1075	-7.9934
	BVT(e)	0.1936*	-0.0070	-11.5906*	0.0978*	0.0681*	-0.8975*	1.5953*	-8.0872	-7.9731
	SAB(e)	0.0620*	-0.0353^	-3.0669*	-0.1346*	-0.1244*	0.4997*	0.0218*	-8.0997	-7.9856
	SOL(e)	0.2306*	0.0416	-8.1179*	0.0583	0.0431	-0.7563*	0.6185*	-8.1260	-8.0233
W	AGL(e)	0.0724*	-0.0388	-2.9566	0.1052	-0.0209	0.9101*	0.0205	-7.9533	-7.8277
	BVT(e)	0.0198	0.0079	-2.2517	0.1376^	-0.0132	0.5913*	0.5401	-7.9473	-7.8218
	SAB(e)	-0.0214	-0.0357	-3.5800	0.1290	-0.0278	0.8805*	0.0376	-7.9322	-7.8067
	SOL(e)	0.1159*	0.0161	-21.9755*	0.0400	0.0083	-0.9402*	-0.0874	-7.9424	-7.8511
BVT	AGL(e)	0.2593*	-0.1853*	-23.6123*	0.0235	0.0751*	-0.9951*	0.4892	-7.6393	-7.5366
	ASA(e)	0.3697*	-0.1510*	-0.1477*	-0.3631*	0.0611*	0.9660*	0.0120*	-7.6449	-7.5422
	SAB(e)	0.2158*	-0.1964*	-1.0246*	0.1037*	-0.0094	-0.9862*	0.0235^	-7.6417	-7.5162
	SOL(e)	0.2536*	-0.0857*	-10.3132*	0.1446*	0.0310	-0.8113*	0.1174*	-7.7663	-7.6407
X	AGL(e)	-0.0294	0.0511	-2.3523*	0.2464*	-0.0349	-0.9266*	0.0119	-7.5063	-7.3808
	ASA(e)	0.0219	0.0571	-2.3640*	0.2485*	-0.0382	-0.9197*	0.0059	-7.5005	-7.3749
	SAB(e)	-0.0836	0.0456	-5.4178^	0.1511	0.1356^	0.5900*	-0.1043	-7.4540	-7.3513
	SOL(e)	-0.0213	0.0682*	-4.6687	0.2305*	0.0336	0.7957*	0.0385	-7.4773	-7.3517
SAB	AGL(e)	0.1289*	-0.0511	-3.8521*	0.1116*	0.0214	-0.9850*	0.2531*	-8.3559	-8.2418
	ASA(e)	0.0866*	-0.0736*	-5.8582*	0.0705*	-0.0066	-0.9970*	0.0549*	-8.2798	-8.1543
	BVT(e)	0.1517*	0.1103*	-0.2397*	-0.5295*	-0.1263*	0.9627*	0.0077*	-8.1934	-8.0907
	SOL(e)	0.1945*	-0.0157	-6.1187*	0.1546*	-0.0759^	-0.8885*	0.1218*	-8.1906	-8.0765
Y	AGL(e)	0.0348	0.0003	-8.7981*	-0.0974^	0.0354	-1.0024*	0.0848	-8.0476	-7.9449
	ASA(e)	0.0270	0.0136	-1.7036*	-0.0120	-0.0068	-0.9822*	0.0227^	-8.0505	-7.9478
	BVT(e)	-0.0354	0.0095	-16.3869	-0.0150	0.0524	0.5109^	0.0382	-8.0396	-7.9255
	SOL(e)	0.0843*	0.0108	-9.7971*	-0.0988*	0.0382	-0.9950*	-0.0200	-8.0542	-7.9515
SOL	AGL(e)	0.2758*	0.0815^	-13.4358*	0.0231	-0.1689*	-0.7903*	0.2027	-7.9056	-7.7800
	ASA(e)	0.3777*	-0.0140	-16.4408*	0.2609	0.1279^	-0.3550*	0.5427*	-7.7897	-7.6641
	BVT(e)	0.2251*	0.0006	-23.7168*	0.4014*	0.1987*	-0.5209*	0.2224	-7.7420	-7.6165
	SAB(e)	0.1935*	-0.0687	-22.6544*	0.6767*	0.0120	-0.9978*	0.0043	-7.7579	-7.6553
Z	AGL(e)	-0.0186	-0.0901^	-1.6640*	-0.1226^	0.0197	0.8627*	-0.0330*	-7.5687	-7.4775
	ASA(e)	-0.0852	-0.0880	-0.9893*	-0.1173*	0.0081	0.9128*	-0.0155*	-7.5704	-7.4791
	BVT(e)	0.0239	-0.0820	0.4282	-0.0980*	0.0194	0.9051*	0.1295*	-7.5647	-7.4734
	SAB(e)	0.0216	-0.0612	-1.3029*	-0.2674*	0.0524*	0.8789*	-0.0229*	-7.5892	-7.4979

* Indicates statistical significance at the 95% level.
^ Indicates statistical significance at the 90% level.

Period 10

	Mean Equation			Variance Equation						
	Variable	Coeff.	Dep. Lag	ϖ	α	γ	δ	κ	AIC	SC
AGL	ASA(e)	0.5074*	0.0585	-15.5350*	0.3314*	0.0413	-0.6987*	0.2314*	-7.5714	-7.4767
	BVT(e)	0.2522*	0.1015*	-2.2456*	-0.3818*	-0.0328	0.7142*	-0.2105*	-7.5485	-7.4302
	SAB(e)	0.6249*	-0.0175	-35.6720*	0.0600	-0.0932*	-0.9255*	0.1006*	-7.5887	-7.4704
	SOL(e)	0.4454*	-0.0141	-26.1076*	0.1921*	0.0034	-0.4519*	0.2183*	-7.5521	-7.4456
V	ASA(e)	-0.0715	-0.0278	7.1676	-0.1765	0.0276	0.8406*	0.7989^	-7.4007	-7.2824
	BVT(e)	-0.0246	-0.0041	-11.5922*	0.0790	0.0724*	0.4872*	0.1869*	-7.4047	-7.2863
	SAB(e)	-0.0658	-0.0228	2.1924	-0.1355	0.0383	-0.3519	0.3490^	-7.3897	-7.2714
	SOL(e)	0.0773	-0.0435	-0.4421	-0.1699*	0.0348	-0.3910^	0.1330^	-7.3861	-7.2560
ASA	AGL(e)	0.3223*	-0.1034^	-11.6932*	0.2429*	0.1273*	-0.6730*	1.1143*	-8.1017	-7.9952
	BVT(e)	0.2386*	-0.2121*	-15.1903*	0.2189*	-0.0455	-0.9606*	0.6512*	-8.1557	-8.0374
	SAB(e)	0.3539*	-0.1597*	5.9589*	-0.1333*	-0.2563*	0.6777*	0.7377*	-8.0186	-7.9002
	SOL(e)	0.2558*	-0.2071*	-14.3728*	0.1650*	0.0547*	0.2707*	0.1387*	-8.0212	-7.9148
W	AGL(e)	0.0025	0.0636	5.7033	-0.0612	0.0262	0.8057*	0.8870*	-7.8742	-7.7441
	BVT(e)	-0.0985	0.0714	-7.6043	-0.1814	0.0902	0.3317	-0.5261	-7.8607	-7.7306
	SAB(e)	-0.0504	0.0618	2.4776	0.0982	-0.1067	0.5999^	0.6066^	-7.8671	-7.7607
	SOL(e)	0.0247	0.0648	-20.9176*	0.0527	0.0121	-0.9507*	-0.0940	-7.8740	-7.7557
BVT	AGL(e)	0.2843*	-0.2516*	-36.3820*	0.2092*	-0.0151	-0.9538*	0.1114*	-7.8339	-7.7156
	ASA(e)	0.2848*	-0.1470*	3.9718^	-0.3269*	0.1088*	0.6370*	0.3912*	-7.8268	-7.7085
	SAB(e)	0.3960*	-0.1745*	2.7521*	-0.1581	0.1189*	0.9743*	0.2744*	-7.7668	-7.6604
	SOL(e)	0.2829*	-0.2194*	-21.7580*	0.2688*	-0.0588^	-0.9581*	0.2472*	-7.8438	-7.7137
X	AGL(e)	0.0743	-0.0029	0.4896	0.0716	-0.0304	0.8158*	0.3946*	-7.6076	-7.4775
	ASA(e)	0.0216	0.0327	-7.1264*	-0.0951	-0.1946*	0.5172*	-0.5502*	-7.5933	-7.4750
	SAB(e)	-0.0439	0.0437	-4.7089*	-0.1352	-0.1662*	0.4918^	-0.3000	-7.5957	-7.4774
	SOL(e)	0.1120^	0.0304	-2.8826	0.0549	0.1230*	0.4826	0.2324	-7.5970	-7.4787
SAB	AGL(e)	0.2040*	-0.0043	-7.9483*	0.1013*	0.0432*	-0.9931*	-0.1453	-8.5709	-8.4526
	ASA(e)	0.2393*	-0.0095	-16.7888*	0.1435	0.2102*	-0.6074*	0.6261*	-8.3789	-8.2606
	BVT(e)	0.1663*	0.0620	2.8875	-0.4458*	0.1233*	-0.3404*	1.0402*	-8.3450	-8.2267
	SOL(e)	0.1315*	0.0294	-21.3422*	0.1025^	0.0226	-0.9968*	0.1662*	-8.4402	-8.3219
Y	AGL(e)	0.0519	-0.0730	-34.6410*	-0.0900	-0.0176	-0.9075*	0.1782	-8.2148	-8.0965
	ASA(e)	-0.0264	-0.0386	-36.3414*	-0.0021	0.0149	-0.9228*	0.2031*	-8.2249	-8.1066
	BVT(e)	0.0138	-0.0769	-35.1121*	-0.0605	-0.0105	-0.9336*	0.1622^	-8.2088	-8.0905
	SOL(e)	-0.0108	-0.0816	-3.5695*	-0.0307	0.0982	0.8538*	-0.1824^	-8.2132	-8.1067
SOL	AGL(e)	0.2558*	-0.0571	-21.9988*	0.4549*	0.2062*	-0.4824*	0.5517*	-8.1161	-7.9978
	ASA(e)	0.2334*	-0.1441*	-10.3657*	0.3930*	0.2886*	-0.2845*	0.8839*	-8.0531	-7.9348
	BVT(e)	0.2207*	-0.0485	-15.0857*	0.4123*	0.1481*	-0.6670*	0.4955*	-8.0480	-7.9416
	SAB(e)	0.3855*	-0.1782*	-8.7174^	0.6077*	0.1789*	-0.3383	0.6357*	-8.0699	-7.9634
Z	AGL(e)	-0.0781*	0.0451	-26.2365*	0.1025^	-0.0113	-0.9864*	0.3074^	-7.8903	-7.7838
	ASA(e)	-0.0747	0.0312	-21.4630*	0.1029	0.0009	-0.9559*	0.3799^	-7.8658	-7.7475
	BVT(e)	-0.0650	0.0317	-25.4160*	0.2962*	0.0554	-0.8201*	0.1829	-7.8679	-7.7496
	SAB(e)	-0.0117	0.0562	-25.0512*	0.1888^	-0.0740	-0.8220*	-0.2314	-7.8669	-7.7604

* Indicates statistical significance at the 95% level.
^ Indicates statistical significance at the 90% level.

In table 4.8 the stocks and their proxies in the first column represent the stock that is analysed for volatility spill-over effects from the stocks in the second column. The second column represents the coefficient of the error term of the stock causing a spill-over (or the ϕ-coefficient in equation 3.19). The actual stock and its replacement proxy are sequenced directly below each other to ease comparability. This allows for each eight rows, respectively, representing the volatility spill-over effects of all the other stocks present in the portfolio to one of the five stocks (first four rows) and similarly for its proxy (second four rows). This allows for better comparison in testing whether replacing a stock that exhibits less volatility spill-over effects from all the other stocks present, reduces overall portfolio risk.

The first peculiarity is present in the error term (ϕ) coefficients in the mean equation (first highlighted column), which represents the relationship between the stocks (or the part of a given stock's return that is captured in the error term of another). In all of the estimated AS model regressions the ϕ-term is statistically significant for at least three of the four stocks for all the original stocks in each period. In contrast, the proxy stock replacements, occasionally and at most, exhibit only one statistically significant influence from the four ϕ-terms. This means that the ϕ-terms of each stock affected the returns of the proxy stocks minimally (if at all), whereas the ϕ-terms captured a great deal of the returns present in the original stocks. The returns of the proxy stocks are therefore negligibly integrated with the returns of the other stocks present in the portfolio. In this regard, each proxy stock replaces a stock that exhibits statistically significant integration with the other stocks. This result is consistent with the Granger causality estimates. It is important to note that this reduction of returns spill-over does not affect the overall portfolio returns.

Secondly, and most significant for this study, are the estimates for the variance equation. The γ parameter measures the asymmetry or the leverage effect of volatility. A negative sign indicates a negative asymmetric impact, which means that negative shocks cause a larger reaction in volatility than positive shocks, and vice versa. In all the periods and portfolios, the γ-coefficient is extensively more significant for the actual stocks than that of the proxy stocks. Also of importance is the δ-coefficient, which indicates the degree of volatility persistence. The majority of volatility persistence coefficients are significant for both the original and proxy stocks. In addition, all of the coefficients were found to be smaller than one (except for the odd two out of 400 AS models), which, according to Xu and Fung (2005), is a requirement for the stability of volatility persistence terms. A high level of volatility persistence implies that fluctuations will remain for an extended period. The focus of this study is on the κ-coefficients (in the second highlighted column). The κ-terms capture the actual volatility spill-over effects of a given stock from all the other stocks present in the given portfolio. Once again, the more profound statistically significant volatility transmission effects occurred between the original stocks included in the five-stock portfolio. As soon as a proxy stock is introduced, the overall volatility spill-over effects decrease. However, it is important to note that some stocks may transmit more volatility to a proxy stock than to the original stock, but that overall, the combined volatility spill-over effects (from all the other stocks) are greater for the original stocks than for the proxy stocks. In addition, volatility spill-over effects increased during the periods surrounding the height of the 2008 financial crisis (periods two to four), with volatility spill-over effects subsequently decreasing as the market started to calm. This is in accordance with studies that testify to the heightened integration between stocks (with regard to returns and volatility) during periods of financial turmoil.[138]

Within all the portfolios, in all the periods, one thing stands out; the proxy stocks attract significantly less volatility spill-over effects from the other stocks in the portfolios than the original stocks attracts from the same stocks. This is evident from the considerably fewer statistically significant coefficients from both the mean equations (ϕ-terms) and the variance equations (γ-, δ- and κ-terms). Therefore, based on all the AS model estimations, the proxy stocks are far less prone to volatility transmission than the actual counterparts that they replace.

4.4 Conclusion

These results exhibit strong support for the inclusion of a volatility spill-over measure when constructing a portfolio based on past price information. Not only did this measure provide insight into the information captured within the residuals, but it also provides consistently stable results (even during times of financial distress). Both these advantages offer portfolio managers a greater range of stock selection and allocation, which are not captured by simply using the market *beta*. These results by no means rule out the use of *beta*, for *beta* captures co-variances in the mean equation. However, using *beta* estimates in conjunction with a residual based test (such as the AS model) will provide a more stable and reasonable choice in stock selection when trying to construct a mean-variance efficient portfolio. It should be noted that the construction of such a portfolio is based solely on past price information, and that future price movements remain unpredictable.

[138] See Longin and Solnik (1995), Kaminsky and Reinhart (1998), and Maniya and Magnusson (2010).

Considering that the proxy stocks, when replacing the actual stocks, are less prone to volatility spill-over effects (in all cases), and also reduce portfolio standard deviation (in all cases), it is reasonable to infer that when contemplating choices between stocks that have similar returns and risk, the obvious choice would be the one that exhibits the least volatility spill-over effects from the stocks included in a portfolio. The Granger causality tests also testify to this notion. However, *beta* also provides a tried and tested (and simple) method to help portfolio managers in portfolio allocation. *Beta* was shown to mostly co-move with portfolio standard deviation, although not linearly. In some instances, *beta* moved contrary to both portfolio standard deviation and volatility spill-over effects. This result is significant in that it proves that *market beta* will not always render an accurate depiction of the portfolio's risk profile. It is important to remember why this is so. Since *beta* is an indication of the co-movement of portfolio returns and the returns of the market portfolio, it does not capture the ultimate risk position of the portfolio.

These results attest to the fact that portfolio managers ought to look deeper into the second moment relationships among stocks in the same proposed portfolio. It is very possible for a portfolio to achieve the same returns from a group of stocks with high portfolio risk, which could easily have performed just as well with another combination of stocks that possessed less causal relationships and therefore lower portfolio risk. Thus, when a stock within a portfolio has the same past return record as another stock, selection between these stocks should be made on the grounds of its co-variance with the other stocks and the market (*beta*), and its volatility spill-over effects received from other stocks within the portfolio. For optimal portfolio selection, the *beta* measure should therefore be equal to one (adjusted for risk preference), and the stocks selected for such a portfolio should exhibit the least volatility spill-over effects among each other, provided they deliver the same portfolio return. It should also be noted that during financial distress, volatility gets accentuated, which may cause results to be biased. However, the time periods under question did include times of financial instability, and the results did hold, to a lesser extent. Such behaviour was expected.

"For having lived long, I have experienced many instances of being obliged, by better information or fuller consideration, to change opinions, even on important subjects, which I once thought right but found to be otherwise."

~ Benjamin Franklin, author, politician, scientist, musician and inventor.

CHAPTER 5

CONCLUDING REMARKS AND RECOMMENDATIONS

Modern portfolio theory has long been placing emphasis on using *market beta* as a measure in constructing an efficient portfolio; and because of its simplicity, *beta* has seen substantial integration into modern-day portfolio management. Portfolio managers have long been familiar with a unit-*beta*, under the *efficient market hypothesis* (EMH), translating to a well-diversified portfolio with a unit correlation with the market, and therefore considered entirely hedged against unsystematic risk. However, under these assumptions the *beta* measure does not capture all the relevant information pertaining to stock selection and allocation. Firstly, *beta* has been known to be an unstable measure in the wake of stock market anomalies, and secondly, *beta* only tells a portfolio manager more about the mean returns between stocks. In addition, systematic risk still remains even after fully diversifying (in accordance with *beta*). In this regard volatility within and between stocks in a portfolio impacts on the profitability of the portfolio, as well as the portfolio's overall risk profile. There is a wealth of information that is captured within the price fluctuations of stocks in a portfolio. These price fluctuations also exhibit "co-varying" properties. Understanding this concept is of utmost importance when a particular economy only offers a limited number of stocks.

Since portfolio managers in smaller economies such as South Africa are limited in their choices of stocks, it becomes increasingly difficult to fully diversify a stock portfolio given volatility spill-over effects between stocks listed on the same exchange. In such a setting, using only *beta* to construct a portfolio that tracks the market or market index, may therefore lead to substandard portfolio returns, which could have been negated if one considered the impact of volatility spill-over effects during portfolio construction. If a limited number of stocks in an equity market is not enough to persuade the use of additional measures, then times of financial distress should necessitate the use of them. In this sense, the study focused on volatility spill-over effects as an additional measure to enhance portfolio selection and allocation.

5.1 Research Aim, Question and Objectives

The problem with conventional portfolio management has been the reliance on co-variances and *beta* measures when tracking the market and fully diversifying. However, other measures may be equally telling. Volatility spill-over effects between stocks may also provide useful information regarding portfolio rebalancing, especially when used in conjunction with return co-variances. Considering both these measures will enhance portfolio management by providing an incentive for strategy adjustment. In order for strategy adjustments to take place, volatility spill-over effects had to be determined and measured. Volatility transmission was measured on a microstructure level to estimate whether this transmission measure provides significant information as opposed to *market beta* for the rebalancing of a stock portfolio.

To answer this proposed question the portfolio return, volatility and *beta* of the different stocks had to be estimated (during and following the 2008 financial crisis) in order to compare and estimate volatility spill-over effects between the stocks within a given portfolio. Volatility spill-over effects were then analysed to determine if it had a significant effect on portfolio volatility.

5.2 Findings and Recommendations

The main findings were that volatility spill-over effects did indeed provide an additional measure for portfolio stock selection and allocation. Portfolio volatility decreased when a stock was replaced by one that attracts less volatility spill-over effects from the other stocks in the portfolio. This was true for all the observed portfolios in each period. In addition, when portfolio volatility decreased, *portfolio beta*, followed suit in most instances. Both these measures were accurate in capturing the reduced portfolio volatility. However, volatility spill-over effects and *beta* should not be seen as substitutes, but rather as complementing measures to enhance stock selection and allocation. This is because *beta* captures information related to the mean equation, and volatility spill-over effects captures to the variance equation, both of which are important when considering that the more information is available, the more efficient the portfolio could be constructed. It is therefore recommended that both these measures be utilised in constructing an efficient portfolio. However, these findings are based on randomly generated stocks (for which parameters could be fixed), and is therefore only valid in theory. Further research and implementation will be needed to vindicate the performance of a portfolio which utilises volatility spill-over effects as an input in portfolio construction.

5.3 Suggested Further Research

These findings, however influential they may be, have only used the aggregate shock (AS) model as the preferred method of residual based testing. The most relevant further research, however, would be the estimation of mean-variance efficient portfolios if data constraints do not prohibit it. These mean-variance efficient portfolios' *betas* and volatility spill-over effects can then be tested to substantiate if the results from this study does, in fact, cause a more efficient portfolio. An outcome of this particular study was to ascertain if there was indeed scope to include a volatility spill-over measure to aid *beta* measures in portfolio selection and construction. The natural route to follow would be to test whether this combination of measures actually increases mean-variance efficiency of portfolios in South Africa during financial distress, or even financial stability.

To further authenticate these results, other residual based tests may be used to test the validity of volatility spill-over effects. Other testable frameworks may include (but are not limited to) i) a multivariate E-GARCH (Karolyi, 1995), ii) GARCH-BEKK (Maniya & Magnusson, 2010), or the dynamic conditional correlation (DCC) model (Engle, 2002). Expanding on such volatility spill-over results may require the use of a "volatility spill-over index" as done by Diebold and Yilmaz (2008). Such an index could be expanded beyond the stocks within a portfolio, and may even include all the relevant stocks listed on an exchange. The index should include an up-to-date method of measuring the volatility spill-over effects to a specific stock from a selection of stocks (to be included within a portfolio). Computationally it will be a daunting task; however, it will significantly facilitate the process of stock selection for any portfolio manager. Finally, these results can be tested on other emerging markets, or by using larger data intervals (such as daily stock returns), or when market conditions have significantly improved. However, the overall period of testing in this study did include various two-month-periods in which volatility calmed.

On a more microstructure and trading-related topic the intraday predictability and persistence in the cross-section of stock return and volatility spill-over effects can be estimated (Heston, Korajczyk & Sadka, 2010). In essence, such a study will capture the hourly (or shorter interval) cross-section spill-over effects between stocks on a day-to-day basis, and assess how returns and volatility spills over between stocks during certain times of the day, and how long such effects persist. Results from this study will especially be helpful for actively trading investors that seek adequate information before executing a trade.

5.4 Conclusion

Taking into consideration that the mandate of a managing fund requires the constant rebalancing of a stock portfolio, and that this process of rebalancing becomes more complex and difficult during times of financial distress, the need for adequate information becomes more important. This study provided an additional measure to *market beta* in order to construct a more efficient portfolio, namely, volatility spill-over effects between stocks within the same portfolio. Using intraday stock returns and a residual based test (aggregate shock (AS) model), volatility spill-over effects were estimated between stocks. It was shown that when a particular stock attracts fewer spill-over effects from the other stocks in the portfolio, then the overall portfolio volatility decreased as well. In most cases *market beta* showcased similar results. Therefore, in order to construct a more efficient portfolio (which is effectively hedged against unsystematic risk), requires both a portfolio that has a unit correlation with the market, but also include stocks with the least amount of volatility spill-over effects among each other. Stock selection and allocation is a fine art in itself, therefore be vigilant of the various factors that may influence the return capability of a portfolio.

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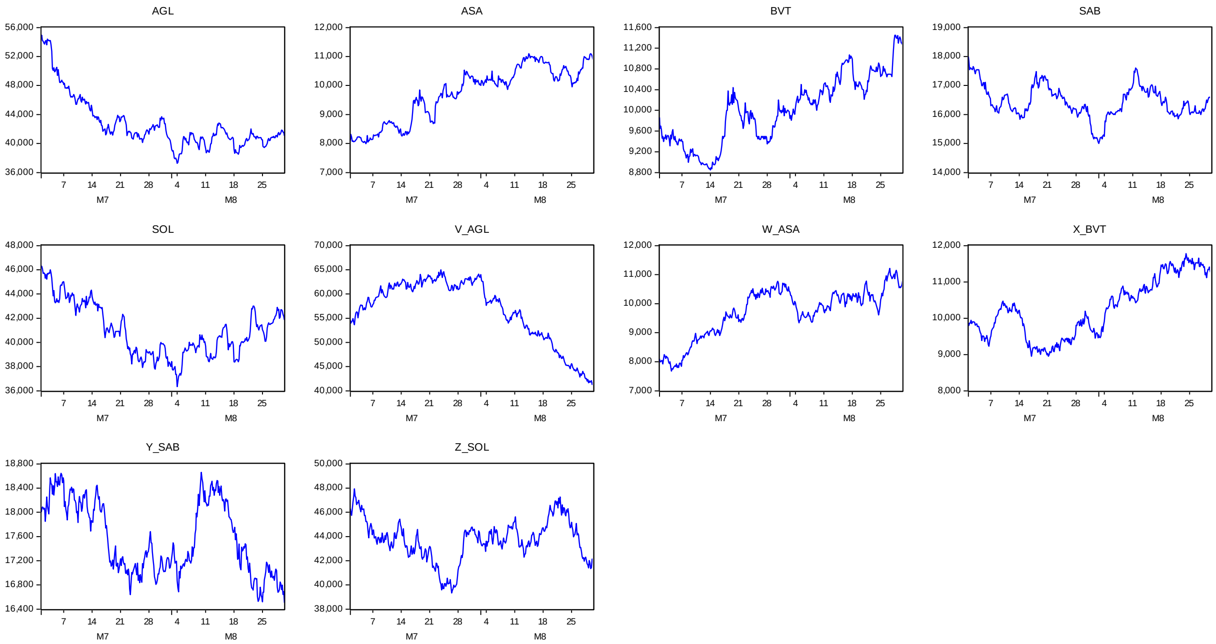
APPENDIX

A1 Stock price line graphs.

Period 1



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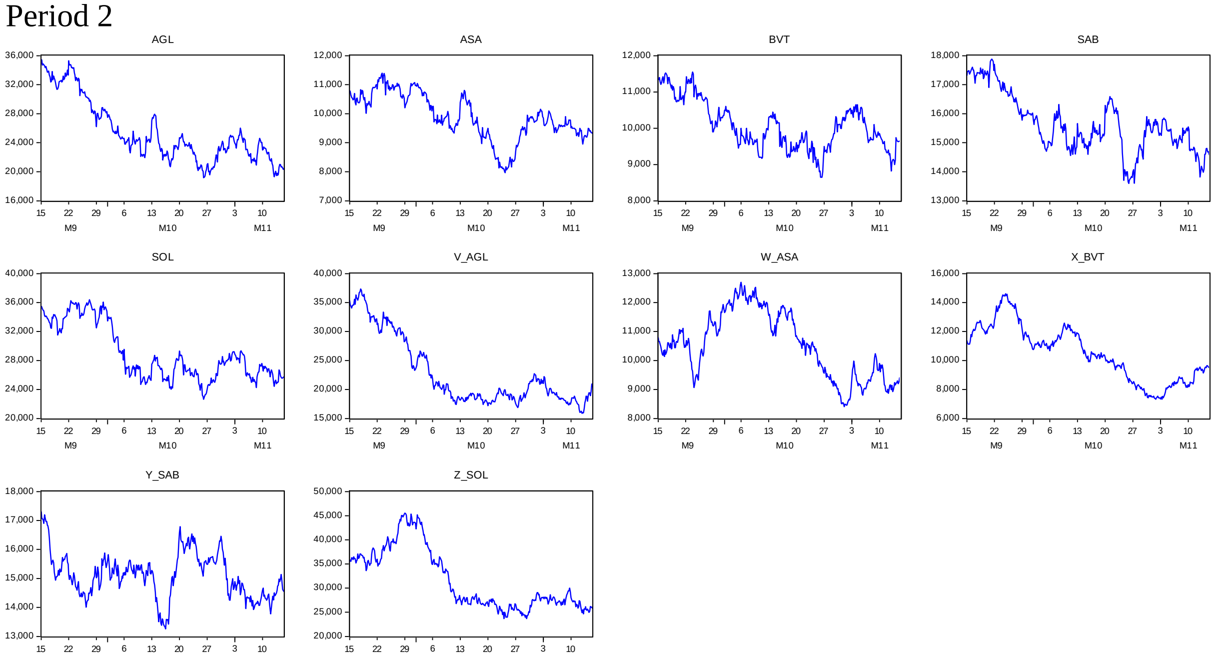

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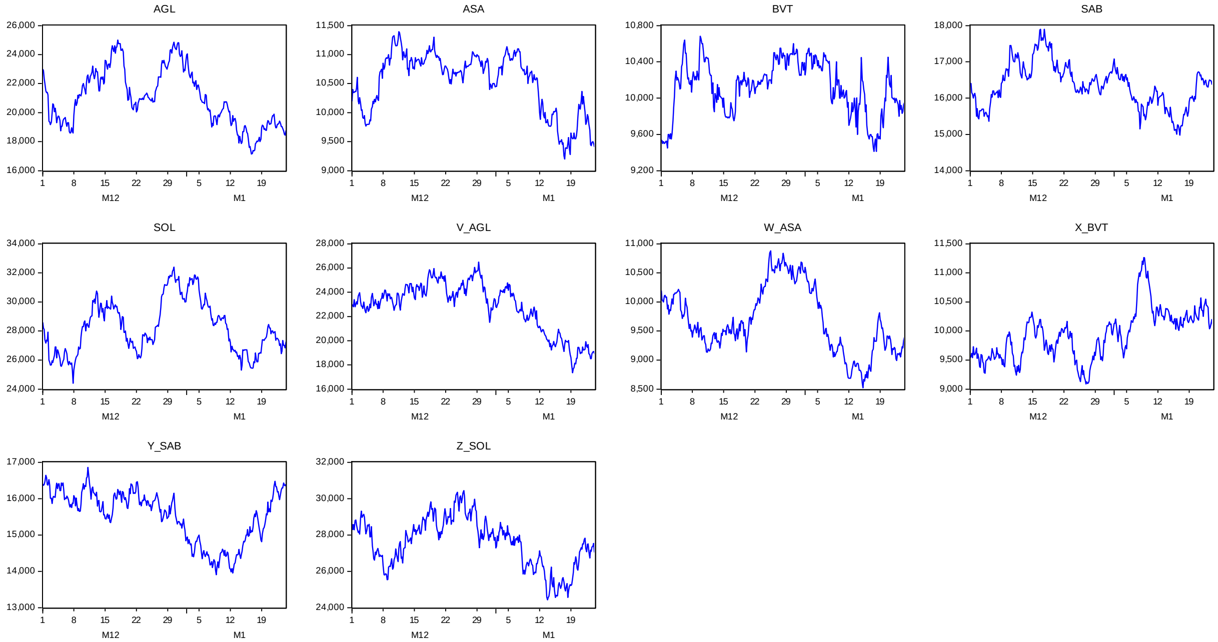

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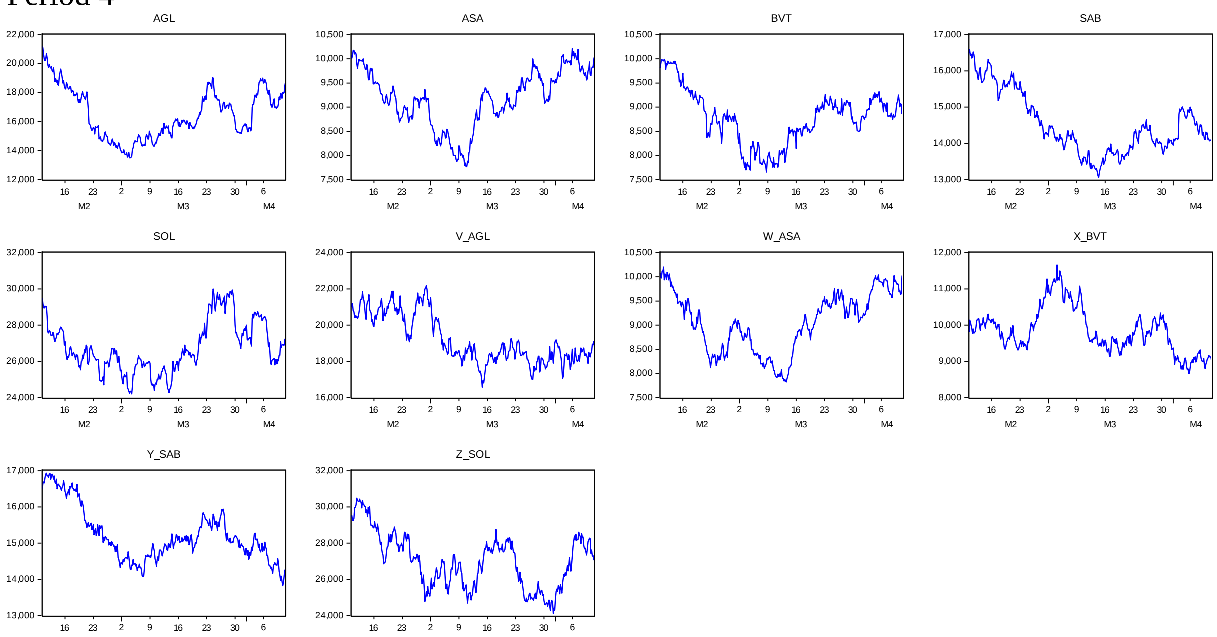

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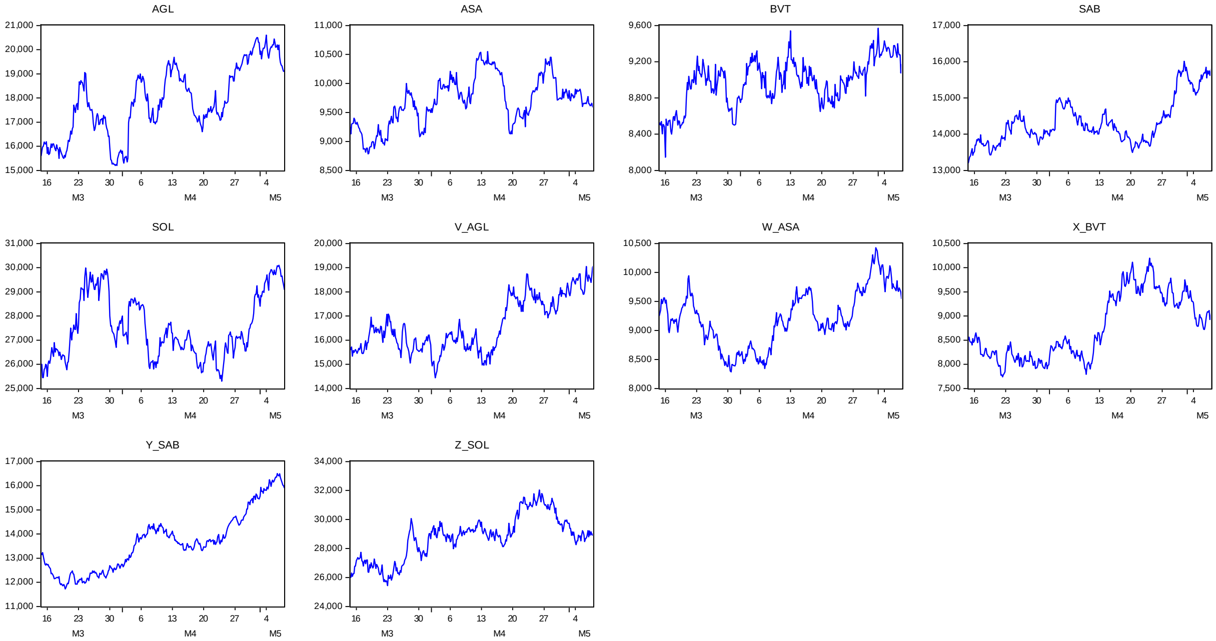
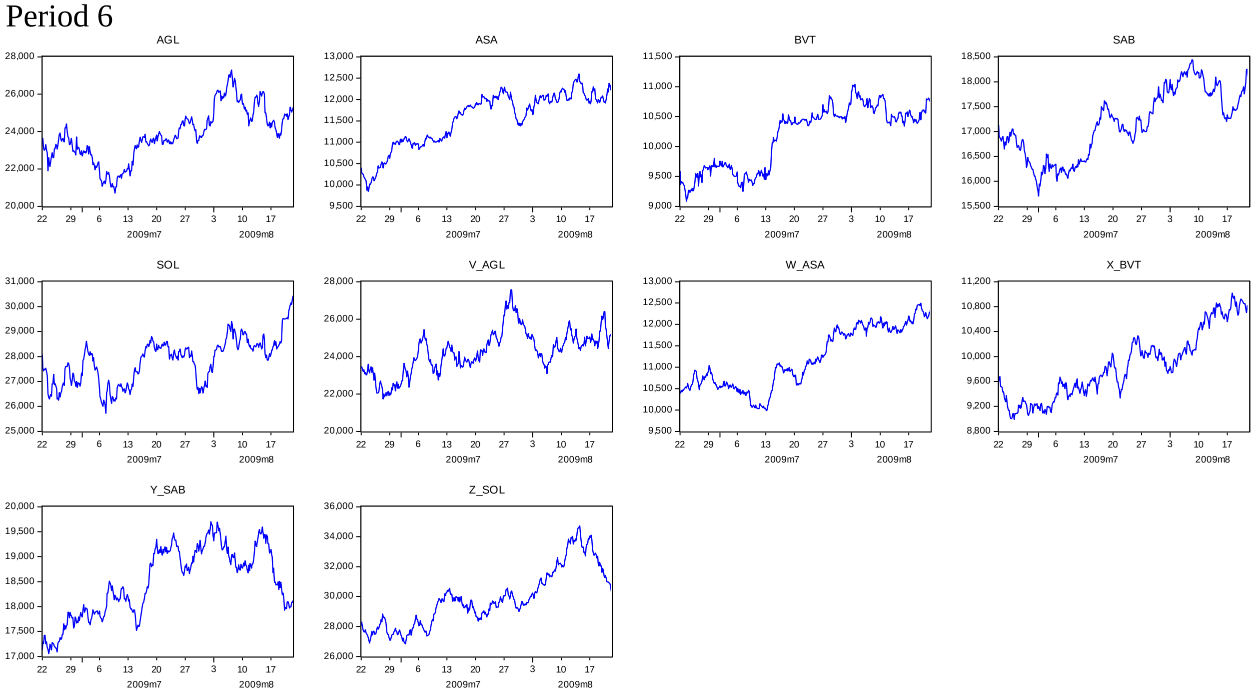
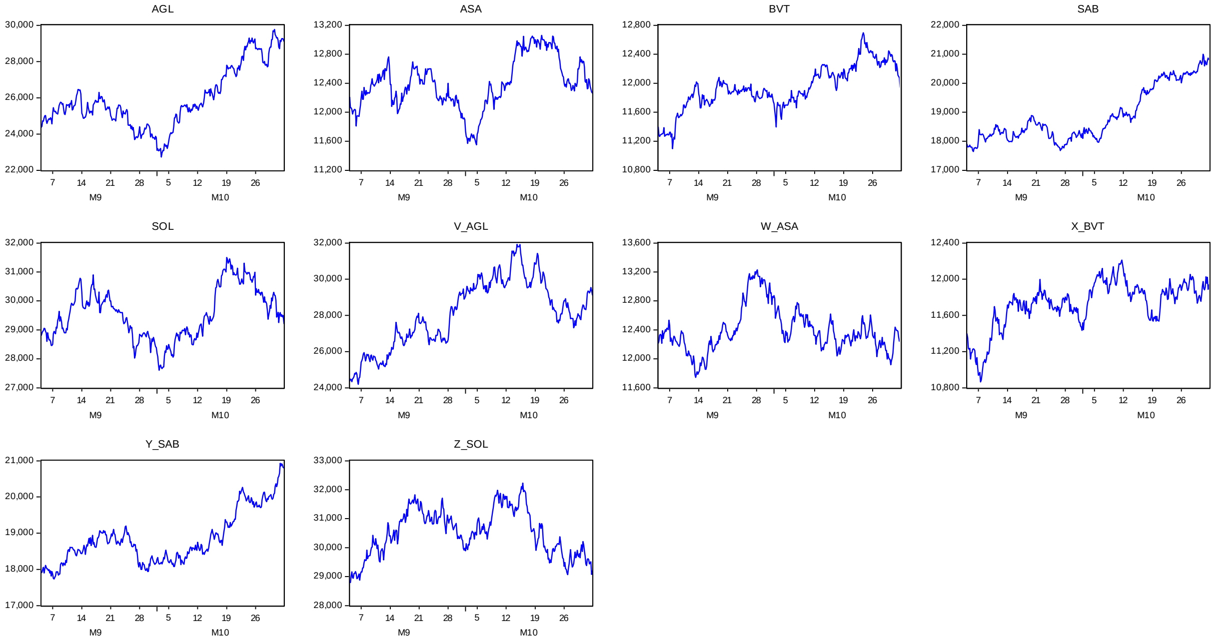


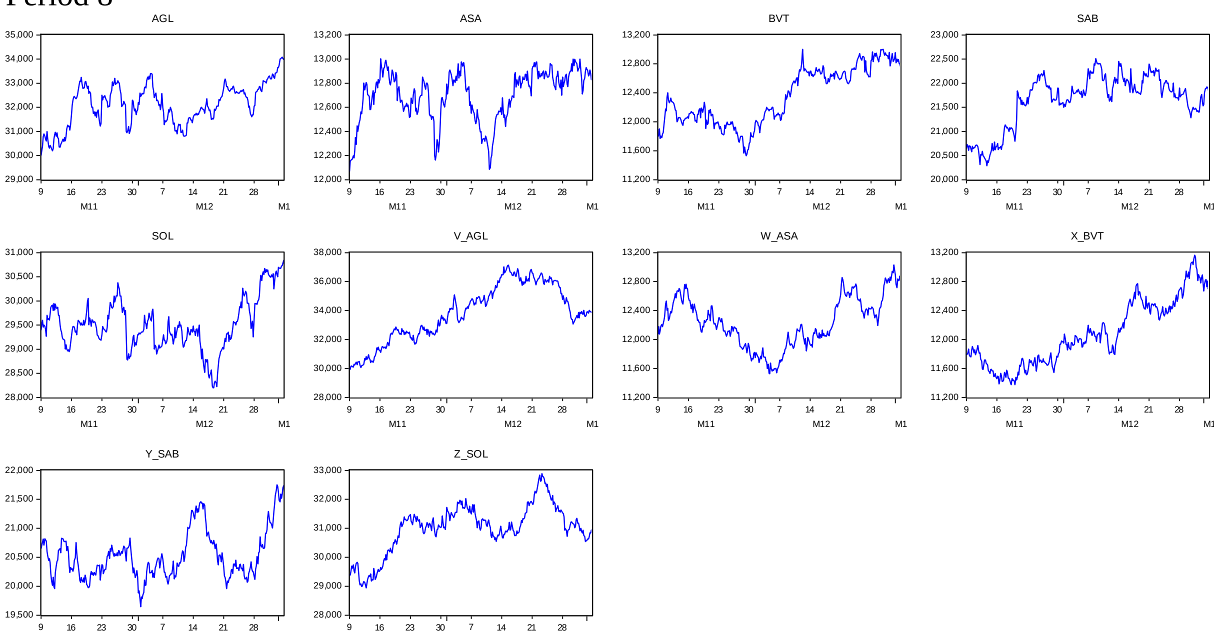
Period 6



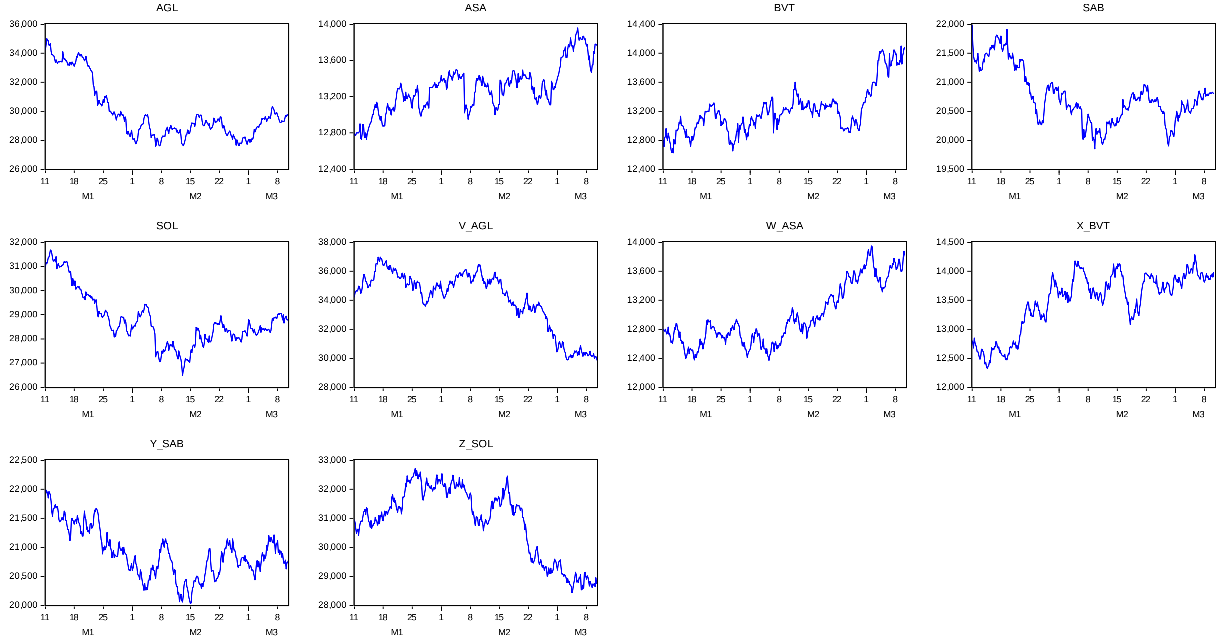
Period 7



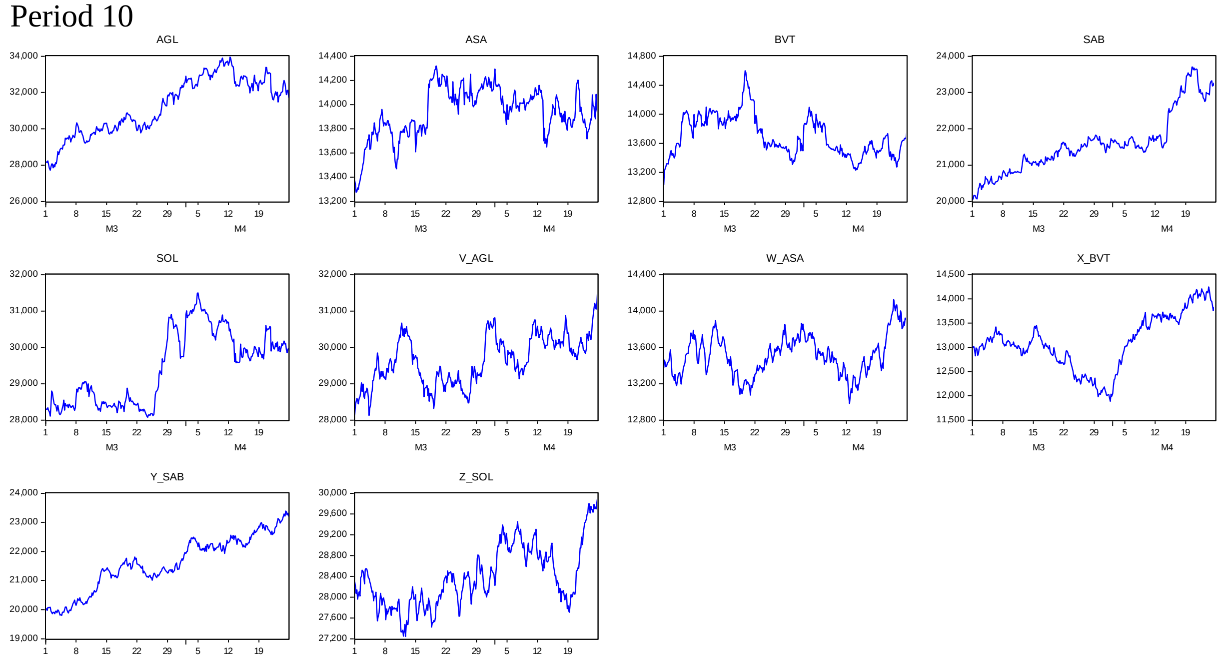
Period 8



Period 9

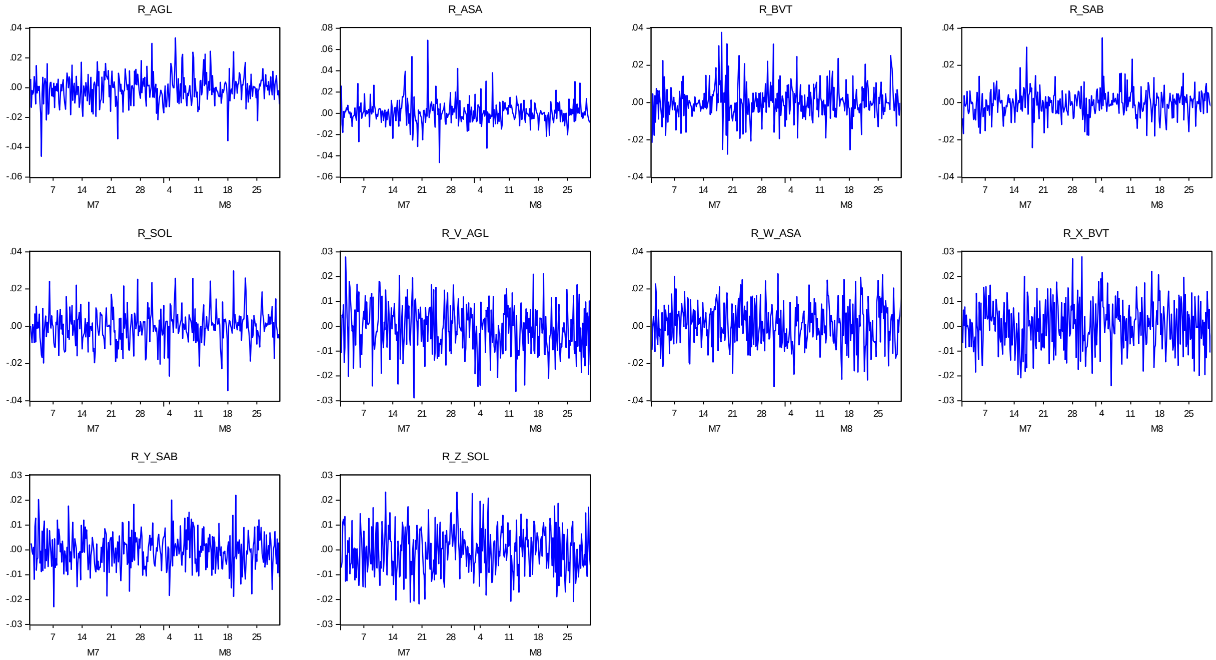


Period 10

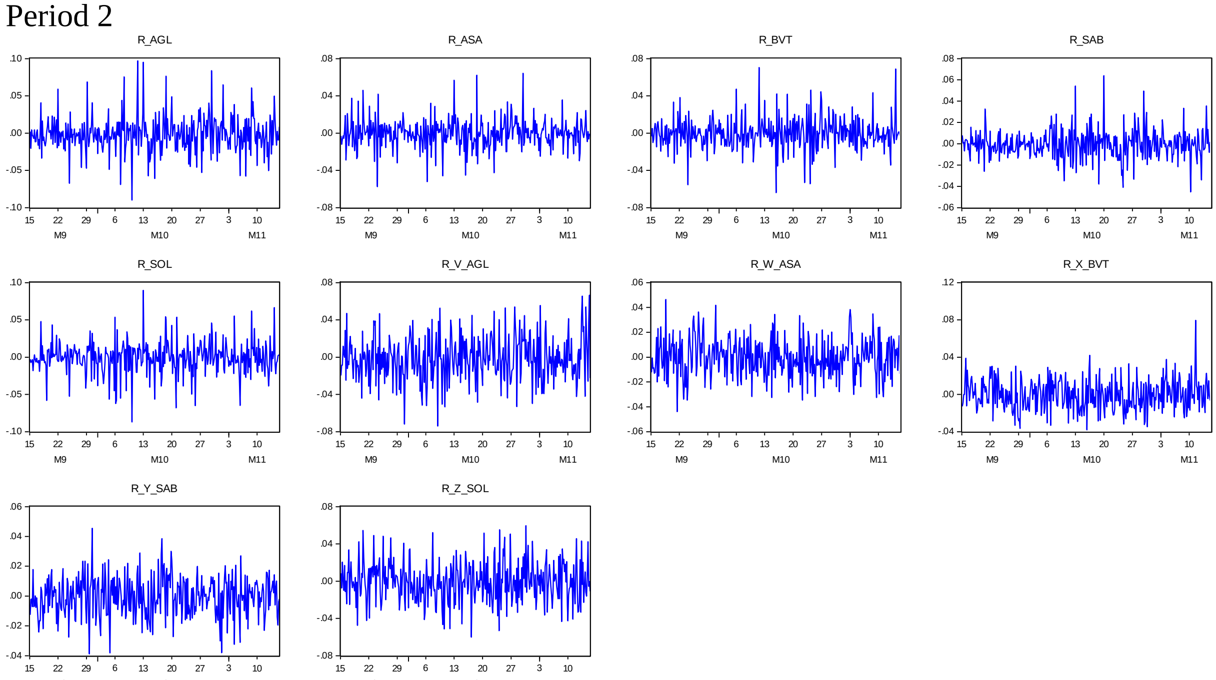


A2 Stock returns line graphs.

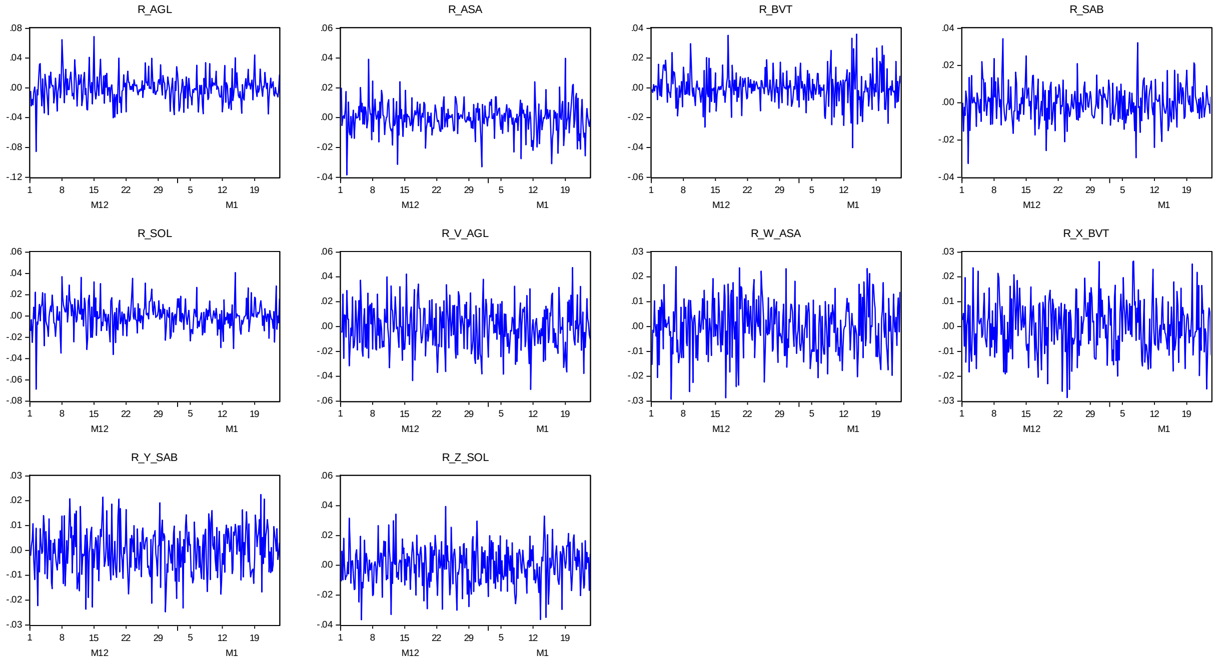
Period 1



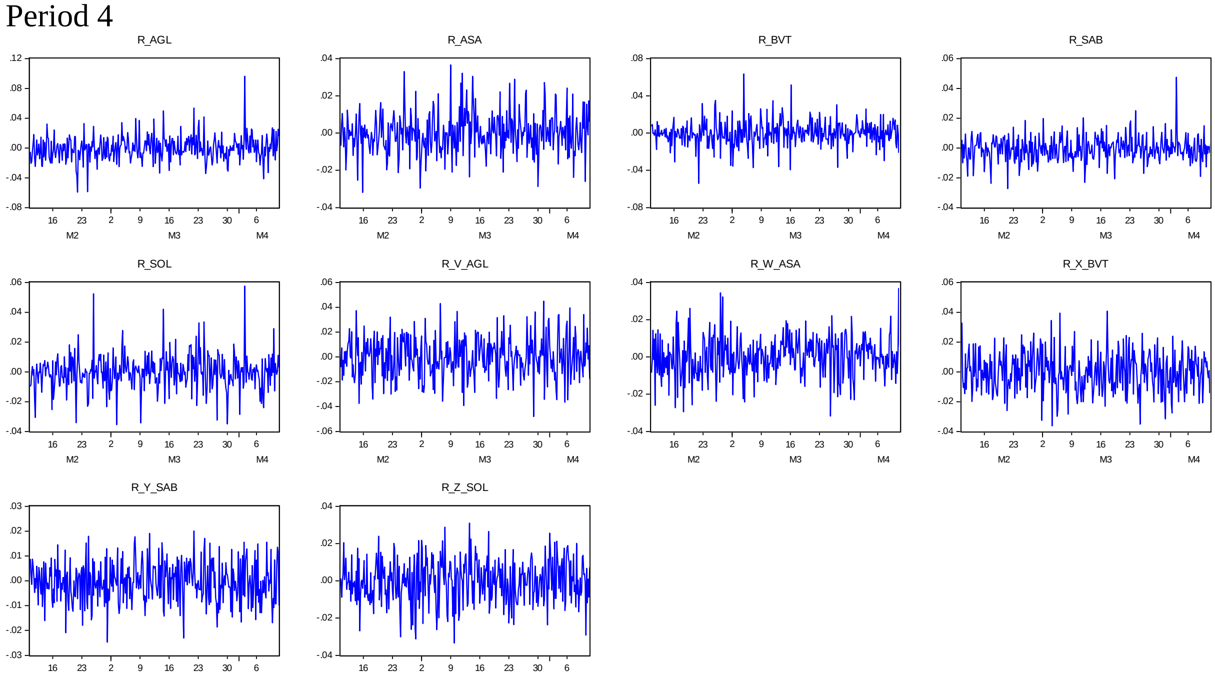
Period 2



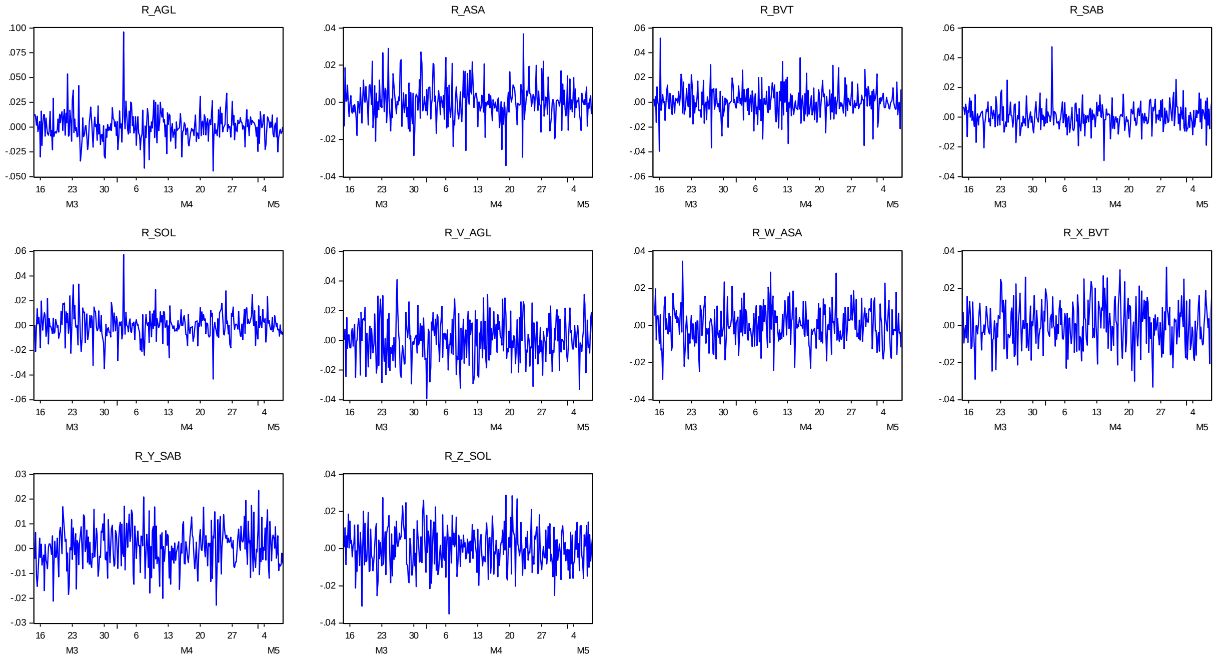
Period 3



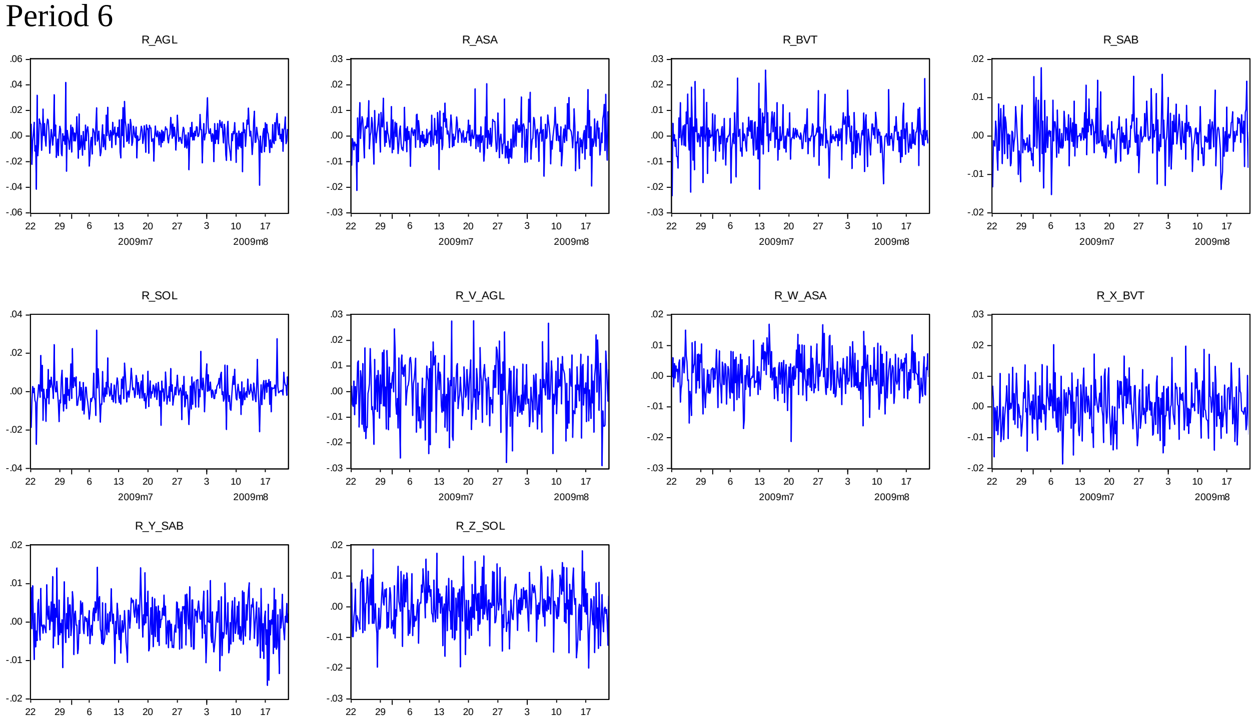
Period 4



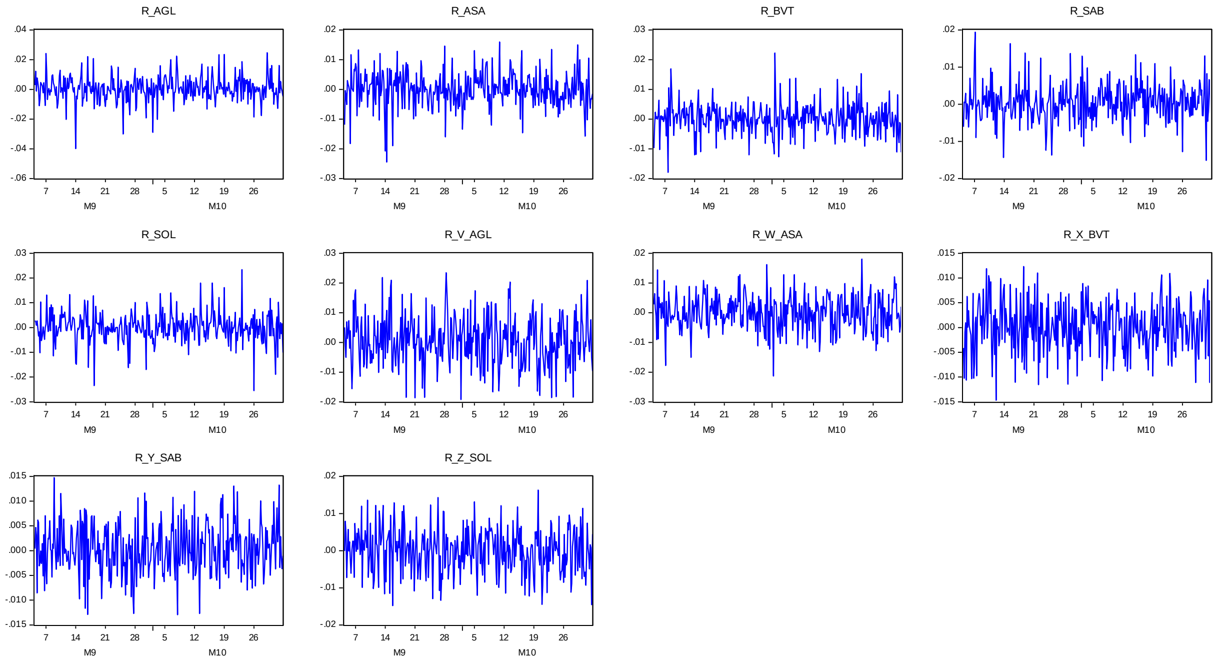
Period 5



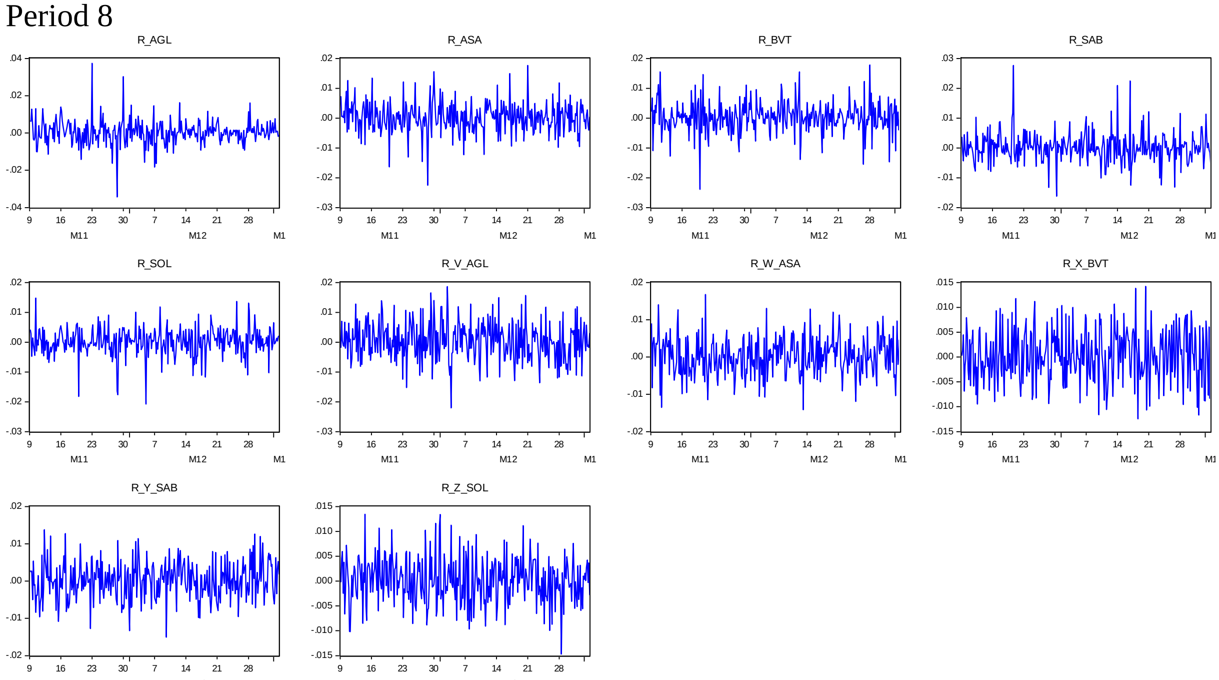
Period 6



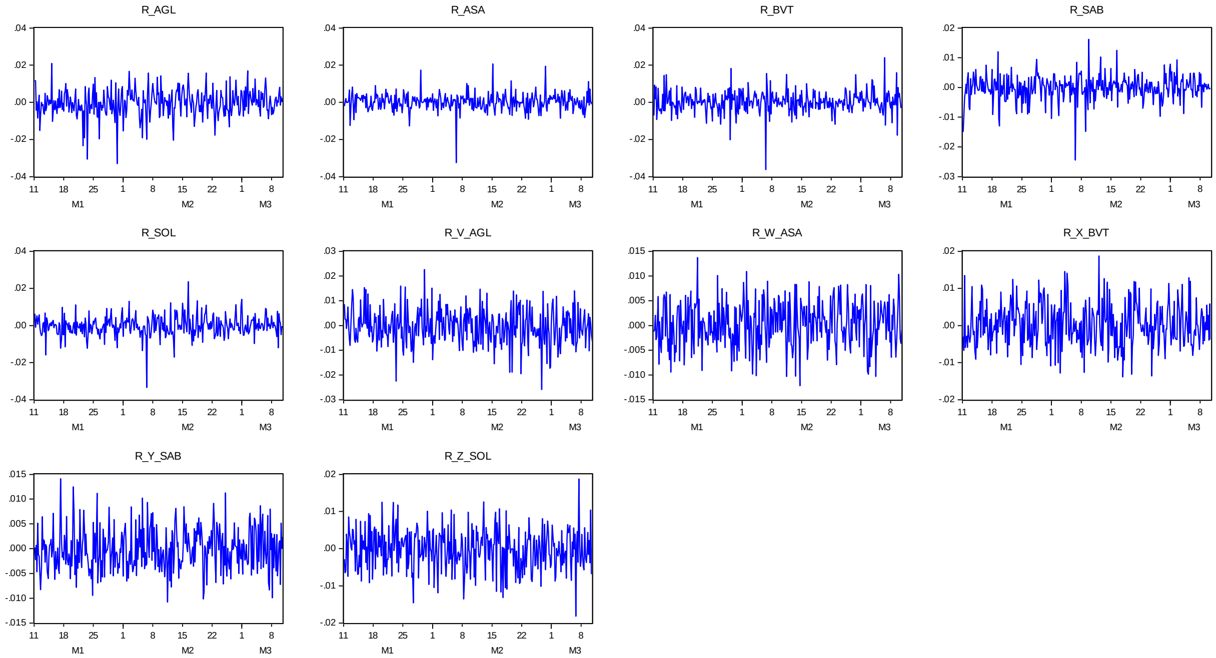
Period 7



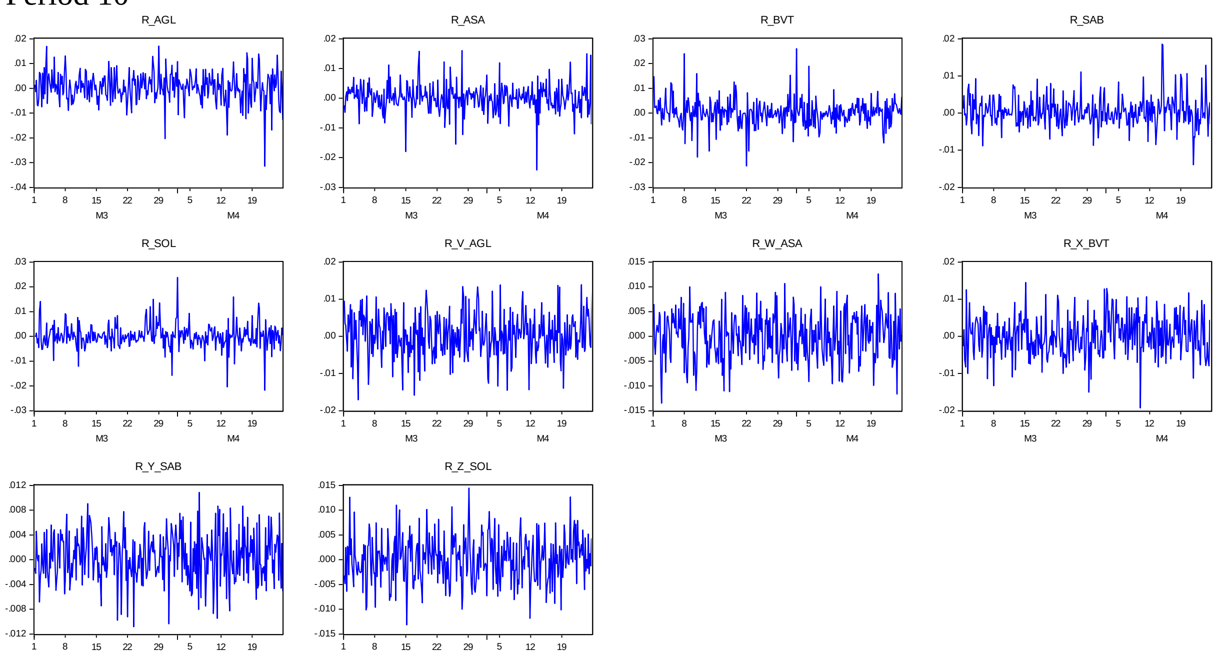
Period 8



Period 9



Period 10



A3 Granger causality tests.

Period 2					
Null Hypothesis	Obs.	Prob.	Null Hypothesis	Obs.	Prob.
R_ASA does not Granger Cause R_AGL	349	0.1749	R_SOL does not Granger Cause R_BVT	349	0.3106
R_AGL does not Granger Cause R_ASA		0.0010*	R_BVT does not Granger Cause R_SOL		0.5255
R_BVT does not Granger Cause R_AGL	349	0.7678	R_V_AGL does not Granger Cause R_BVT	349	0.8105
R_AGL does not Granger Cause R_BVT		0.0355*	R_BVT does not Granger Cause R_V_AGL		0.4110
R_SAB does not Granger Cause R_AGL	349	0.5512	R_W_ASA does not Granger Cause R_BVT	349	0.3576
R_AGL does not Granger Cause R_SAB		0.9508	R_BVT does not Granger Cause R_W_ASA		0.3385
R_SOL does not Granger Cause R_AGL	349	0.6701	R_X_BVT does not Granger Cause R_BVT	349	0.4796
R_AGL does not Granger Cause R_SOL		0.0114*	R_BVT does not Granger Cause R_X_BVT		0.1540
R_V_AGL does not Granger Cause R_AGL	349	0.3853	R_Y_SAB does not Granger Cause R_BVT	349	0.5174
R_AGL does not Granger Cause R_V_AGL		0.6009	R_BVT does not Granger Cause R_Y_SAB		0.4030
R_W_ASA does not Granger Cause R_AGL	349	0.3458	R_Z_SOL does not Granger Cause R_BVT	349	0.9387
R_AGL does not Granger Cause R_W_ASA		0.1860	R_BVT does not Granger Cause R_Z_SOL		0.9376
R_X_BVT does not Granger Cause R_AGL	349	0.4483	R_SOL does not Granger Cause R_SAB	349	0.7976
R_AGL does not Granger Cause R_X_BVT		0.5832	R_SAB does not Granger Cause R_SOL		0.9466
R_Y_SAB does not Granger Cause R_AGL	349	0.5334	R_V_AGL does not Granger Cause R_SAB	349	0.9191
R_AGL does not Granger Cause R_Y_SAB		0.0036*	R_SAB does not Granger Cause R_V_AGL		0.1112
R_Z_SOL does not Granger Cause R_AGL	349	0.1672	R_W_ASA does not Granger Cause R_SAB	349	0.0043*
R_AGL does not Granger Cause R_Z_SOL		0.8466	R_SAB does not Granger Cause R_W_ASA		0.3715
R_BVT does not Granger Cause R_ASA	349	0.2059	R_X_BVT does not Granger Cause R_SAB	349	0.9186
R_ASA does not Granger Cause R_BVT		0.2126	R_SAB does not Granger Cause R_X_BVT		0.2841
R_SAB does not Granger Cause R_ASA	349	0.1625	R_Y_SAB does not Granger Cause R_SAB	349	0.7039
R_ASA does not Granger Cause R_SAB		0.7160	R_SAB does not Granger Cause R_Y_SAB		0.1206
R_SOL does not Granger Cause R_ASA	349	0.0185*	R_Z_SOL does not Granger Cause R_SAB	349	0.1583
R_ASA does not Granger Cause R_SOL		0.8710	R_SAB does not Granger Cause R_Z_SOL		0.3527
R_V_AGL does not Granger Cause R_ASA	349	0.9264	R_V_AGL does not Granger Cause R_SOL	349	0.7795
R_ASA does not Granger Cause R_V_AGL		0.0275*	R_SOL does not Granger Cause R_V_AGL		0.2415
R_W_ASA does not Granger Cause R_ASA	349	0.5079	R_W_ASA does not Granger Cause R_SOL	349	0.3707
R_ASA does not Granger Cause R_W_ASA		0.5383	R_SOL does not Granger Cause R_W_ASA		0.7736
R_X_BVT does not Granger Cause R_ASA	349	0.0273*	R_X_BVT does not Granger Cause R_SOL	349	0.0720^
R_ASA does not Granger Cause R_X_BVT		0.6311	R_SOL does not Granger Cause R_X_BVT		0.6316
R_Y_SAB does not Granger Cause R_ASA	349	0.9425	R_Y_SAB does not Granger Cause R_SOL	349	0.6262
R_ASA does not Granger Cause R_Y_SAB		0.4727	R_SOL does not Granger Cause R_Y_SAB		0.2251
R_Z_SOL does not Granger Cause R_ASA	349	0.2649	R_Z_SOL does not Granger Cause R_SOL	349	0.6151
R_ASA does not Granger Cause R_Z_SOL		0.9794	R_SOL does not Granger Cause R_Z_SOL		0.3512
R_SAB does not Granger Cause R_BVT	349	0.0584^			
R_BVT does not Granger Cause R_SAB		0.7484			

* Indicates statistical significance at the 95% level.
^ Indicates statistical significance at the 90% level.

Period 3					
Null Hypothesis	Obs.	Prob.	Null Hypothesis	Obs.	Prob.
R_ASA does not Granger Cause R_AGL	309	0.9678	R_SOL does not Granger Cause R_BVT	309	0.4646
R_AGL does not Granger Cause R_ASA		0.5159	R_BVT does not Granger Cause R_SOL		0.6190
R_BVT does not Granger Cause R_AGL	309	0.5855	R_V_AGL does not Granger Cause R_BVT	309	0.5547
R_AGL does not Granger Cause R_BVT		0.7181	R_BVT does not Granger Cause R_V_AGL		0.7960
R_SAB does not Granger Cause R_AGL	309	0.5593	R_X_BVT does not Granger Cause R_BVT	309	0.5613
R_AGL does not Granger Cause R_SAB		0.3080	R_BVT does not Granger Cause R_X_BVT		0.5192
R_SOL does not Granger Cause R_AGL	309	0.6661	R_W_ASA does not Granger Cause R_BVT	309	0.3873
R_AGL does not Granger Cause R_SOL		0.0024*	R_BVT does not Granger Cause R_W_ASA		0.8998
R_V_AGL does not Granger Cause R_AGL	309	0.3431	R_Y_SAB does not Granger Cause R_BVT	309	0.7598
R_AGL does not Granger Cause R_V_AGL		0.9965	R_BVT does not Granger Cause R_Y_SAB		0.8328

R_X_BVT does not Granger Cause R_AGL R_AGL does not Granger Cause R_X_BVT	309	0.4067 0.2652	R_Z_SOL does not Granger Cause R_BVT R_BVT does not Granger Cause R_Z_SOL	309	0.3515 0.4165
R_W_ASA does not Granger Cause R_AGL R_AGL does not Granger Cause R_W_ASA	309	0.3534 0.0119*	R_SOL does not Granger Cause R_SAB R_SAB does not Granger Cause R_SOL	309	0.5538 0.0762^
R_Y_SAB does not Granger Cause R_AGL R_AGL does not Granger Cause R_Y_SAB	309	0.3402 0.0783^	R_V_AGL does not Granger Cause R_SAB R_SAB does not Granger Cause R_V_AGL	309	0.6312 0.2752
R_Z_SOL does not Granger Cause R_AGL R_AGL does not Granger Cause R_Z_SOL	309	0.2196 0.7735	R_X_BVT does not Granger Cause R_SAB R_SAB does not Granger Cause R_X_BVT	309	0.4965 0.6146
R_BVT does not Granger Cause R_ASA R_ASA does not Granger Cause R_BVT	309	0.8549 0.7085	R_W_ASA does not Granger Cause R_SAB R_SAB does not Granger Cause R_W_ASA	309	0.5519 0.6031
R_SAB does not Granger Cause R_ASA R_ASA does not Granger Cause R_SAB	309	0.0004* 0.9697	R_Y_SAB does not Granger Cause R_SAB R_SAB does not Granger Cause R_Y_SAB	309	0.9738 0.1881
R_SOL does not Granger Cause R_ASA R_ASA does not Granger Cause R_SOL	309	0.1080 0.3617	R_Z_SOL does not Granger Cause R_SAB R_SAB does not Granger Cause R_Z_SOL	309	0.6145 0.3162
R_V_AGL does not Granger Cause R_ASA R_ASA does not Granger Cause R_V_AGL	309	0.1948 0.6886	R_V_AGL does not Granger Cause R_SOL R_SOL does not Granger Cause R_V_AGL	309	0.2607 0.3554
R_X_BVT does not Granger Cause R_ASA R_ASA does not Granger Cause R_X_BVT	309	0.5612 0.0989^	R_X_BVT does not Granger Cause R_SOL R_SOL does not Granger Cause R_X_BVT	309	0.9934 0.3648
R_W_ASA does not Granger Cause R_ASA R_ASA does not Granger Cause R_W_ASA	309	0.4660 0.4465	R_W_ASA does not Granger Cause R_SOL R_SOL does not Granger Cause R_W_ASA	309	0.6892 0.1380
R_Y_SAB does not Granger Cause R_ASA R_ASA does not Granger Cause R_Y_SAB	309	0.2058 0.4169	R_Y_SAB does not Granger Cause R_SOL R_SOL does not Granger Cause R_Y_SAB	309	0.6861 0.0472*
R_Z_SOL does not Granger Cause R_ASA R_ASA does not Granger Cause R_Z_SOL	309	0.9716 0.7399	R_Z_SOL does not Granger Cause R_SOL R_SOL does not Granger Cause R_Z_SOL	309	0.2142 0.1479
R_SAB does not Granger Cause R_BVT R_BVT does not Granger Cause R_SAB	309	0.3867 0.5409			

* Indicates statistical significance at the 95% level.
^ Indicates statistical significance at the 90% level.

Period 4					
Null Hypothesis	Obs.	Prob.	Null Hypothesis	Obs.	Prob.
R_ASA does not Granger Cause R_AGL R_AGL does not Granger Cause R_ASA	341	0.0175* 0.0040*	R_SOL does not Granger Cause R_BVT R_BVT does not Granger Cause R_SOL	341	0.4685 0.6719
R_BVT does not Granger Cause R_AGL R_AGL does not Granger Cause R_BVT	341	0.3413 0.0636^	R_V_AGL does not Granger Cause R_BVT R_BVT does not Granger Cause R_V_AGL	341	0.2849 0.4064
R_SAB does not Granger Cause R_AGL R_AGL does not Granger Cause R_SAB	341	0.8001 0.0309*	R_X_BVT does not Granger Cause R_BVT R_BVT does not Granger Cause R_X_BVT	341	0.0700^ 0.6849
R_SOL does not Granger Cause R_AGL R_AGL does not Granger Cause R_SOL	341	0.7180 0.4715	R_Z_SOL does not Granger Cause R_BVT R_BVT does not Granger Cause R_Z_SOL	341	0.0286* 0.4050
R_V_AGL does not Granger Cause R_AGL R_AGL does not Granger Cause R_V_AGL	341	0.5224 0.9983	R_W_ASA does not Granger Cause R_BVT R_BVT does not Granger Cause R_W_ASA	341	0.0200* 0.7074
R_X_BVT does not Granger Cause R_AGL R_AGL does not Granger Cause R_X_BVT	341	0.6643 0.7733	R_Y_SAB does not Granger Cause R_BVT R_BVT does not Granger Cause R_Y_SAB	341	0.5214 0.2664
R_Z_SOL does not Granger Cause R_AGL R_AGL does not Granger Cause R_Z_SOL	341	0.5488 0.7039	R_SOL does not Granger Cause R_SAB R_SAB does not Granger Cause R_SOL	341	0.8442 0.4015
R_W_ASA does not Granger Cause R_AGL R_AGL does not Granger Cause R_W_ASA	341	0.0906^ 0.2886	R_V_AGL does not Granger Cause R_SAB R_SAB does not Granger Cause R_V_AGL	341	0.9510 0.5247
R_Y_SAB does not Granger Cause R_AGL R_AGL does not Granger Cause R_Y_SAB	341	0.0610^ 0.5781	R_X_BVT does not Granger Cause R_SAB R_SAB does not Granger Cause R_X_BVT	341	0.8319 0.1262
R_BVT does not Granger Cause R_ASA R_ASA does not Granger Cause R_BVT	341	0.5979 0.0094*	R_Z_SOL does not Granger Cause R_SAB R_SAB does not Granger Cause R_Z_SOL	341	0.4018 0.8207
R_SAB does not Granger Cause R_ASA R_ASA does not Granger Cause R_SAB	341	0.4022 0.2971	R_W_ASA does not Granger Cause R_SAB R_SAB does not Granger Cause R_W_ASA	341	0.7386 0.1429
R_SOL does not Granger Cause R_ASA R_ASA does not Granger Cause R_SOL	341	0.1871 0.2876	R_Y_SAB does not Granger Cause R_SAB R_SAB does not Granger Cause R_Y_SAB	341	0.5510 0.4449
R_V_AGL does not Granger Cause R_ASA	341	0.7313	R_V_AGL does not Granger Cause R_SOL	341	0.6429

R_ASA does not Granger Cause R_V_AGL		0.4652	R_SOL does not Granger Cause R_V_AGL		0.3782
R_X_BVT does not Granger Cause R_ASA R_ASA does not Granger Cause R_X_BVT	341	0.4886 0.6632	R_X_BVT does not Granger Cause R_SOL R_SOL does not Granger Cause R_X_BVT	341	0.9560 0.7664
R_Z_SOL does not Granger Cause R_ASA R_ASA does not Granger Cause R_Z_SOL	341	0.9496 0.1042	R_Z_SOL does not Granger Cause R_SOL R_SOL does not Granger Cause R_Z_SOL	341	0.7966 0.4064
R_W_ASA does not Granger Cause R_ASA R_ASA does not Granger Cause R_W_ASA	341	0.0240* 0.1765	R_W_ASA does not Granger Cause R_SOL R_SOL does not Granger Cause R_W_ASA	341	0.0381* 0.9719
R_Y_SAB does not Granger Cause R_ASA R_ASA does not Granger Cause R_Y_SAB	341	0.4193 0.8549	R_Y_SAB does not Granger Cause R_SOL R_SOL does not Granger Cause R_Y_SAB	341	0.0778^ 0.7962
R_SAB does not Granger Cause R_BVT R_BVT does not Granger Cause R_SAB	341	0.8927 0.8472			

* Indicates statistical significance at the 95% level.
^ Indicates statistical significance at the 90% level.

Period 5					
Null Hypothesis	Obs.	Prob.	Null Hypothesis	Obs.	Prob.
R_ASA does not Granger Cause R_AGL R_AGL does not Granger Cause R_ASA	309	0.3325 0.0349*	R_SOL does not Granger Cause R_BVT R_BVT does not Granger Cause R_SOL	309	0.6072 0.2794
R_BVT does not Granger Cause R_AGL R_AGL does not Granger Cause R_BVT	309	0.7348 0.3673	R_V_AGL does not Granger Cause R_BVT R_BVT does not Granger Cause R_V_AGL	309	0.8996 0.3212
R_SAB does not Granger Cause R_AGL R_AGL does not Granger Cause R_SAB	309	0.9401 0.0489*	R_W_ASA does not Granger Cause R_BVT R_BVT does not Granger Cause R_W_ASA	309	0.6689 0.3172
R_SOL does not Granger Cause R_AGL R_AGL does not Granger Cause R_SOL	309	0.1025 0.0746^	R_X_BVT does not Granger Cause R_BVT R_BVT does not Granger Cause R_X_BVT	309	0.0584^ 0.0269*
R_V_AGL does not Granger Cause R_AGL R_AGL does not Granger Cause R_V_AGL	309	0.0472* 0.4145	R_Y_SAB does not Granger Cause R_BVT R_BVT does not Granger Cause R_Y_SAB	309	0.6170 0.1635
R_W_ASA does not Granger Cause R_AGL R_AGL does not Granger Cause R_W_ASA	309	0.5925 0.1069	R_Z_SOL does not Granger Cause R_BVT R_BVT does not Granger Cause R_Z_SOL	309	0.3746 0.9707
R_X_BVT does not Granger Cause R_AGL R_AGL does not Granger Cause R_X_BVT	309	0.7283 0.5140	R_SOL does not Granger Cause R_SAB R_SAB does not Granger Cause R_SOL	309	0.7507 0.2720
R_Y_SAB does not Granger Cause R_AGL R_AGL does not Granger Cause R_Y_SAB	309	0.2570 0.3887	R_V_AGL does not Granger Cause R_SAB R_SAB does not Granger Cause R_V_AGL	309	0.0963^ 0.0828^
R_Z_SOL does not Granger Cause R_AGL R_AGL does not Granger Cause R_Z_SOL	309	0.9976 0.3774	R_W_ASA does not Granger Cause R_SAB R_SAB does not Granger Cause R_W_ASA	309	0.9036 0.3061
R_BVT does not Granger Cause R_ASA R_ASA does not Granger Cause R_BVT	309	0.3977 0.8750	R_X_BVT does not Granger Cause R_SAB R_SAB does not Granger Cause R_X_BVT	309	0.1603 0.5115
R_SAB does not Granger Cause R_ASA R_ASA does not Granger Cause R_SAB	309	0.9842 0.4590	R_Y_SAB does not Granger Cause R_SAB R_SAB does not Granger Cause R_Y_SAB	309	0.2454 0.9525
R_SOL does not Granger Cause R_ASA R_ASA does not Granger Cause R_SOL	309	0.2096 0.0834^	R_Z_SOL does not Granger Cause R_SAB R_SAB does not Granger Cause R_Z_SOL	309	0.7230 0.3141
R_V_AGL does not Granger Cause R_ASA R_ASA does not Granger Cause R_V_AGL	309	0.5946 0.1934	R_V_AGL does not Granger Cause R_SOL R_SOL does not Granger Cause R_V_AGL	309	0.5898 0.5189
R_W_ASA does not Granger Cause R_ASA R_ASA does not Granger Cause R_W_ASA	309	0.4160 0.2167	R_W_ASA does not Granger Cause R_SOL R_SOL does not Granger Cause R_W_ASA	309	0.1402 0.7951
R_X_BVT does not Granger Cause R_ASA R_ASA does not Granger Cause R_X_BVT	309	0.7942 0.2290	R_X_BVT does not Granger Cause R_SOL R_SOL does not Granger Cause R_X_BVT	309	0.4640 0.9303
R_Y_SAB does not Granger Cause R_ASA R_ASA does not Granger Cause R_Y_SAB	309	0.6249 0.1165	R_Y_SAB does not Granger Cause R_SOL R_SOL does not Granger Cause R_Y_SAB	309	0.9815 0.9624
R_Z_SOL does not Granger Cause R_ASA R_ASA does not Granger Cause R_Z_SOL	309	0.9238 0.0400*	R_Z_SOL does not Granger Cause R_SOL R_SOL does not Granger Cause R_Z_SOL	309	0.9132 0.4116
R_SAB does not Granger Cause R_BVT R_BVT does not Granger Cause R_SAB	309	0.8932 0.7964			

* Indicates statistical significance at the 95% level.
^ Indicates statistical significance at the 90% level.

Period 6					
Null Hypothesis	Obs.	Prob.	Null Hypothesis	Obs.	Prob.
R_ASA does not Granger Cause R_AGL R_AGL does not Granger Cause R_ASA	349	0.5788 0.3269	R_SOL does not Granger Cause R_BVT R_BVT does not Granger Cause R_SOL	349	0.1008 0.1465
R_BVT does not Granger Cause R_AGL R_AGL does not Granger Cause R_BVT	349	0.0651^ 0.0597^	R_V_AGL does not Granger Cause R_BVT R_BVT does not Granger Cause R_V_AGL	349	0.2678 0.8944
R_SAB does not Granger Cause R_AGL R_AGL does not Granger Cause R_SAB	349	0.6030 0.1567	R_W_ASA does not Granger Cause R_BVT R_BVT does not Granger Cause R_W_ASA	349	0.0078* 0.9109
R_SOL does not Granger Cause R_AGL R_AGL does not Granger Cause R_SOL	349	0.6534 0.1154	R_X_BVT does not Granger Cause R_BVT R_BVT does not Granger Cause R_X_BVT	349	0.6102 0.8669
R_V_AGL does not Granger Cause R_AGL R_AGL does not Granger Cause R_V_AGL	349	0.1522 0.9069	R_Y_SAB does not Granger Cause R_BVT R_BVT does not Granger Cause R_Y_SAB	349	0.3242 0.6827
R_W_ASA does not Granger Cause R_AGL R_AGL does not Granger Cause R_W_ASA	349	0.1999 0.8818	R_Z_SOL does not Granger Cause R_BVT R_BVT does not Granger Cause R_Z_SOL	349	0.6779 0.3621
R_X_BVT does not Granger Cause R_AGL R_AGL does not Granger Cause R_X_BVT	349	0.2305 0.2487	R_SOL does not Granger Cause R_SAB R_SAB does not Granger Cause R_SOL	349	0.3475 0.5840
R_Y_SAB does not Granger Cause R_AGL R_AGL does not Granger Cause R_Y_SAB	349	0.9166 0.7644	R_V_AGL does not Granger Cause R_SAB R_SAB does not Granger Cause R_V_AGL	349	0.1536 0.9341
R_Z_SOL does not Granger Cause R_AGL R_AGL does not Granger Cause R_Z_SOL	349	0.3098 0.5513	R_W_ASA does not Granger Cause R_SAB R_SAB does not Granger Cause R_W_ASA	349	0.0182* 0.4149
R_BVT does not Granger Cause R_ASA R_ASA does not Granger Cause R_BVT	349	0.1436 0.0032*	R_X_BVT does not Granger Cause R_SAB R_SAB does not Granger Cause R_X_BVT	349	0.8092 0.2579
R_SAB does not Granger Cause R_ASA R_ASA does not Granger Cause R_SAB	349	0.6403 0.5208	R_Y_SAB does not Granger Cause R_SAB R_SAB does not Granger Cause R_Y_SAB	349	0.5627 0.5053
R_SOL does not Granger Cause R_ASA R_ASA does not Granger Cause R_SOL	349	0.3539 0.6420	R_Z_SOL does not Granger Cause R_SAB R_SAB does not Granger Cause R_Z_SOL	349	0.4286 0.6671
R_V_AGL does not Granger Cause R_ASA R_ASA does not Granger Cause R_V_AGL	349	0.7241 0.7108	R_V_AGL does not Granger Cause R_SOL R_SOL does not Granger Cause R_V_AGL	349	0.9143 0.8307
R_W_ASA does not Granger Cause R_ASA R_ASA does not Granger Cause R_W_ASA	349	0.4442 0.3659	R_W_ASA does not Granger Cause R_SOL R_SOL does not Granger Cause R_W_ASA	349	0.9593 0.9002
R_X_BVT does not Granger Cause R_ASA R_ASA does not Granger Cause R_X_BVT	349	0.5357 0.4082	R_X_BVT does not Granger Cause R_SOL R_SOL does not Granger Cause R_X_BVT	349	0.2661 0.2322
R_Y_SAB does not Granger Cause R_ASA R_ASA does not Granger Cause R_Y_SAB	349	0.1573 0.9775	R_Y_SAB does not Granger Cause R_SOL R_SOL does not Granger Cause R_Y_SAB	349	0.9056 0.0161*
R_Z_SOL does not Granger Cause R_ASA R_ASA does not Granger Cause R_Z_SOL	349	0.6188 0.5493	R_Z_SOL does not Granger Cause R_SOL R_SOL does not Granger Cause R_Z_SOL	349	0.8584 0.2921
R_SAB does not Granger Cause R_BVT R_BVT does not Granger Cause R_SAB	349	0.2977 0.8370			

* Indicates statistical significance at the 95% level.
^ Indicates statistical significance at the 90% level.

Period 7					
Null Hypothesis	Obs.	Prob.	Null Hypothesis	Obs.	Prob.
R_ASA does not Granger Cause R_AGL R_AGL does not Granger Cause R_ASA	333	0.3561 0.0552^	R_SOL does not Granger Cause R_BVT R_BVT does not Granger Cause R_SOL	333	0.4111 0.9437
R_BVT does not Granger Cause R_AGL R_AGL does not Granger Cause R_BVT	333	0.6763 0.0146*	R_V_AGL does not Granger Cause R_BVT R_BVT does not Granger Cause R_V_AGL	333	0.4979 0.8881
R_SAB does not Granger Cause R_AGL R_AGL does not Granger Cause R_SAB	333	0.3549 0.0892^	R_W_ASA does not Granger Cause R_BVT R_BVT does not Granger Cause R_W_ASA	333	0.9769 0.6553
R_SOL does not Granger Cause R_AGL R_AGL does not Granger Cause R_SOL	333	0.8044 0.3262	R_X_BVT does not Granger Cause R_BVT R_BVT does not Granger Cause R_X_BVT	333	0.4534 0.9703
R_V_AGL does not Granger Cause R_AGL R_AGL does not Granger Cause R_V_AGL	333	0.7424 0.1092	R_Y_SAB does not Granger Cause R_BVT R_BVT does not Granger Cause R_Y_SAB	333	0.1047 0.7899
R_W_ASA does not Granger Cause R_AGL R_AGL does not Granger Cause R_W_ASA	333	0.9724 0.4032	R_Z_SOL does not Granger Cause R_BVT R_BVT does not Granger Cause R_Z_SOL	333	0.1820 0.4344
R_X_BVT does not Granger Cause R_AGL	333	0.3372	R_SOL does not Granger Cause R_SAB	333	0.9190

R_AGL does not Granger Cause R_X_BVT		0.1787	R_SAB does not Granger Cause R_SOL		0.3749
R_Y_SAB does not Granger Cause R_AGL	333	0.2901	R_V_AGL does not Granger Cause R_SAB	333	0.9752
R_AGL does not Granger Cause R_Y_SAB		0.3903	R_SAB does not Granger Cause R_V_AGL		0.5781
R_Z_SOL does not Granger Cause R_AGL	333	0.0480*	R_W_ASA does not Granger Cause R_SAB	333	0.2199
R_AGL does not Granger Cause R_Z_SOL		0.7142	R_SAB does not Granger Cause R_W_ASA		0.6604
R_BVT does not Granger Cause R_ASA	333	0.5239	R_X_BVT does not Granger Cause R_SAB	333	0.2647
R_ASA does not Granger Cause R_BVT		0.0759^	R_SAB does not Granger Cause R_X_BVT		0.0521^
R_SAB does not Granger Cause R_ASA	333	0.2065	R_Y_SAB does not Granger Cause R_SAB	333	0.2943
R_ASA does not Granger Cause R_SAB		0.7589	R_SAB does not Granger Cause R_Y_SAB		0.6442
R_SOL does not Granger Cause R_ASA	333	0.0398*	R_Z_SOL does not Granger Cause R_SAB	333	0.0851^
R_ASA does not Granger Cause R_SOL		0.4953	R_SAB does not Granger Cause R_Z_SOL		0.4768
R_V_AGL does not Granger Cause R_ASA	333	0.3535	R_V_AGL does not Granger Cause R_SOL	333	0.3688
R_ASA does not Granger Cause R_V_AGL		0.4230	R_SOL does not Granger Cause R_V_AGL		0.4870
R_W_ASA does not Granger Cause R_ASA	333	0.8027	R_W_ASA does not Granger Cause R_SOL	333	0.7173
R_ASA does not Granger Cause R_W_ASA		0.0607^	R_SOL does not Granger Cause R_W_ASA		0.2085
R_X_BVT does not Granger Cause R_ASA	333	0.2382	R_X_BVT does not Granger Cause R_SOL	333	0.8625
R_ASA does not Granger Cause R_X_BVT		0.4462	R_SOL does not Granger Cause R_X_BVT		0.2823
R_Y_SAB does not Granger Cause R_ASA	333	0.2380	R_Y_SAB does not Granger Cause R_SOL	333	0.2527
R_ASA does not Granger Cause R_Y_SAB		0.1489	R_SOL does not Granger Cause R_Y_SAB		0.7460
R_Z_SOL does not Granger Cause R_ASA	333	0.2107	R_Z_SOL does not Granger Cause R_SOL	333	0.1971
R_ASA does not Granger Cause R_Z_SOL		0.4419	R_SOL does not Granger Cause R_Z_SOL		0.4732
R_SAB does not Granger Cause R_BVT	333	0.1466			
R_BVT does not Granger Cause R_SAB		0.4051			

* Indicates statistical significance at the 95% level.
^ Indicates statistical significance at the 90% level.

Period 8

Null Hypothesis	Obs.	Prob.	Null Hypothesis	Obs.	Prob.
R_ASA does not Granger Cause R_AGL	317	0.4412	R_SOL does not Granger Cause R_BVT	317	0.4436
R_AGL does not Granger Cause R_ASA		0.0046*	R_BVT does not Granger Cause R_SOL		0.1193
R_BVT does not Granger Cause R_AGL	317	0.2952	R_V_AGL does not Granger Cause R_BVT	317	0.0513^
R_AGL does not Granger Cause R_BVT		0.7532	R_BVT does not Granger Cause R_V_AGL		0.8204
R_SAB does not Granger Cause R_AGL	317	0.9224	R_W_ASA does not Granger Cause R_BVT	317	0.0502^
R_AGL does not Granger Cause R_SAB		0.9753	R_BVT does not Granger Cause R_W_ASA		0.2995
R_SOL does not Granger Cause R_AGL	317	0.9529	R_X_BVT does not Granger Cause R_BVT	317	0.3283
R_AGL does not Granger Cause R_SOL		0.0002*	R_BVT does not Granger Cause R_X_BVT		0.2457
R_V_AGL does not Granger Cause R_AGL	317	0.0311*	R_Y_SAB does not Granger Cause R_BVT	317	0.2139
R_AGL does not Granger Cause R_V_AGL		0.4897	R_BVT does not Granger Cause R_Y_SAB		0.0620*
R_W_ASA does not Granger Cause R_AGL	317	0.8826	R_Z_SOL does not Granger Cause R_BVT	317	0.4781
R_AGL does not Granger Cause R_W_ASA		0.9901	R_BVT does not Granger Cause R_Z_SOL		0.2714
R_X_BVT does not Granger Cause R_AGL	317	0.9932	R_SOL does not Granger Cause R_SAB	317	0.3672
R_AGL does not Granger Cause R_X_BVT		0.9556	R_SAB does not Granger Cause R_SOL		0.0542^
R_Y_SAB does not Granger Cause R_AGL	317	0.2615	R_V_AGL does not Granger Cause R_SAB	317	0.0101*
R_AGL does not Granger Cause R_Y_SAB		0.1101	R_SAB does not Granger Cause R_V_AGL		0.4767
R_Z_SOL does not Granger Cause R_AGL	317	0.8698	R_W_ASA does not Granger Cause R_SAB	317	0.8905
R_AGL does not Granger Cause R_Z_SOL		0.0905^	R_SAB does not Granger Cause R_W_ASA		0.9019
R_BVT does not Granger Cause R_ASA	317	0.1304	R_X_BVT does not Granger Cause R_SAB	317	0.7553
R_ASA does not Granger Cause R_BVT		0.9752	R_SAB does not Granger Cause R_X_BVT		0.6493
R_SAB does not Granger Cause R_ASA	317	0.2548	R_Y_SAB does not Granger Cause R_SAB	317	0.2762
R_ASA does not Granger Cause R_SAB		0.5284	R_SAB does not Granger Cause R_Y_SAB		0.7035
R_SOL does not Granger Cause R_ASA	317	0.6053	R_Z_SOL does not Granger Cause R_SAB	317	0.7865
R_ASA does not Granger Cause R_SOL		0.5134	R_SAB does not Granger Cause R_Z_SOL		0.1850
R_V_AGL does not Granger Cause R_ASA	317	0.6131	R_V_AGL does not Granger Cause R_SOL	317	0.8247
R_ASA does not Granger Cause R_V_AGL		0.9476	R_SOL does not Granger Cause R_V_AGL		0.9457
R_W_ASA does not Granger Cause R_ASA	317	0.5772	R_W_ASA does not Granger Cause R_SOL	317	0.8280
R_ASA does not Granger Cause R_W_ASA		0.4769	R_SOL does not Granger Cause R_W_ASA		0.5786
R_X_BVT does not Granger Cause R_ASA	317	0.5938	R_X_BVT does not Granger Cause R_SOL	317	0.8357

Null Hypothesis	Obs.	Prob.	Null Hypothesis	Obs.	Prob.
R_ASA does not Granger Cause R_X_BVT		0.9158	R_SOL does not Granger Cause R_X_BVT		0.2096
R_Y_SAB does not Granger Cause R_ASA	317	0.7263	R_Y_SAB does not Granger Cause R_SOL	317	0.2286
R_ASA does not Granger Cause R_Y_SAB		0.3254	R_SOL does not Granger Cause R_Y_SAB		0.5561
R_Z_SOL does not Granger Cause R_ASA	317	0.2005	R_Z_SOL does not Granger Cause R_SOL	317	0.8777
R_ASA does not Granger Cause R_Z_SOL		0.2929	R_SOL does not Granger Cause R_Z_SOL		0.4620
R_SAB does not Granger Cause R_BVT	317	0.8177			
R_BVT does not Granger Cause R_SAB		0.3504			

* Indicates statistical significance at the 95% level.
^ Indicates statistical significance at the 90% level.

Period 9					
Null Hypothesis	Obs.	Prob.	Null Hypothesis	Obs.	Prob.
R_ASA does not Granger Cause R_AGL	333	0.2999	R_SOL does not Granger Cause R_BVT	333	0.1531
R_AGL does not Granger Cause R_ASA		0.0093*	R_BVT does not Granger Cause R_SOL		0.8193
R_BVT does not Granger Cause R_AGL	333	0.8347	R_V_AGL does not Granger Cause R_BVT	333	0.5758
R_AGL does not Granger Cause R_BVT		0.1442	R_BVT does not Granger Cause R_V_AGL		0.5039
R_SAB does not Granger Cause R_AGL	333	0.0788^	R_W_ASA does not Granger Cause R_BVT	333	0.7494
R_AGL does not Granger Cause R_SAB		0.0318*	R_BVT does not Granger Cause R_W_ASA		0.2397
R_SOL does not Granger Cause R_AGL	333	0.0812^	R_X_BVT does not Granger Cause R_BVT	333	0.2423
R_AGL does not Granger Cause R_SOL		0.0018*	R_BVT does not Granger Cause R_X_BVT		0.6973
R_V_AGL does not Granger Cause R_AGL	333	0.0941^	R_Y_SAB does not Granger Cause R_BVT	333	0.4625
R_AGL does not Granger Cause R_V_AGL		0.2609	R_BVT does not Granger Cause R_Y_SAB		0.3613
R_W_ASA does not Granger Cause R_AGL	333	0.7418	R_Z_SOL does not Granger Cause R_BVT	333	0.8173
R_AGL does not Granger Cause R_W_ASA		0.4204	R_BVT does not Granger Cause R_Z_SOL		0.1667
R_X_BVT does not Granger Cause R_AGL	333	0.7517	R_SOL does not Granger Cause R_SAB	333	0.3301
R_AGL does not Granger Cause R_X_BVT		0.4090	R_SAB does not Granger Cause R_SOL		0.2924
R_Y_SAB does not Granger Cause R_AGL	333	0.9216	R_V_AGL does not Granger Cause R_SAB	333	0.5540
R_AGL does not Granger Cause R_Y_SAB		0.9285	R_SAB does not Granger Cause R_V_AGL		0.3343
R_Z_SOL does not Granger Cause R_AGL	333	0.4311	R_W_ASA does not Granger Cause R_SAB	333	0.9838
R_AGL does not Granger Cause R_Z_SOL		0.0018*	R_SAB does not Granger Cause R_W_ASA		0.8633
R_BVT does not Granger Cause R_ASA	333	0.2488	R_X_BVT does not Granger Cause R_SAB	333	0.7886
R_ASA does not Granger Cause R_BVT		0.4387	R_SAB does not Granger Cause R_X_BVT		0.9257
R_SAB does not Granger Cause R_ASA	333	0.4988	R_Y_SAB does not Granger Cause R_SAB	333	0.4313
R_ASA does not Granger Cause R_SAB		0.9600	R_SAB does not Granger Cause R_Y_SAB		0.5077
R_SOL does not Granger Cause R_ASA	333	0.0856^	R_Z_SOL does not Granger Cause R_SAB	333	0.6914
R_ASA does not Granger Cause R_SOL		0.3436	R_SAB does not Granger Cause R_Z_SOL		0.5674
R_V_AGL does not Granger Cause R_ASA	333	0.1006	R_V_AGL does not Granger Cause R_SOL	333	0.2517
R_ASA does not Granger Cause R_V_AGL		0.3578	R_SOL does not Granger Cause R_V_AGL		0.6451
R_W_ASA does not Granger Cause R_ASA	333	0.3040	R_W_ASA does not Granger Cause R_SOL	333	0.9082
R_ASA does not Granger Cause R_W_ASA		0.8582	R_SOL does not Granger Cause R_W_ASA		0.3678
R_X_BVT does not Granger Cause R_ASA	333	0.3366	R_X_BVT does not Granger Cause R_SOL	333	0.6799
R_ASA does not Granger Cause R_X_BVT		0.7248	R_SOL does not Granger Cause R_X_BVT		0.9766
R_Y_SAB does not Granger Cause R_ASA	333	0.5818	R_Y_SAB does not Granger Cause R_SOL	333	0.2629
R_ASA does not Granger Cause R_Y_SAB		0.6803	R_SOL does not Granger Cause R_Y_SAB		0.4199
R_Z_SOL does not Granger Cause R_ASA	333	0.9401	R_Z_SOL does not Granger Cause R_SOL	333	0.4195
R_ASA does not Granger Cause R_Z_SOL		0.8734	R_SOL does not Granger Cause R_Z_SOL		0.6424
R_SAB does not Granger Cause R_BVT	333	0.8630			
R_BVT does not Granger Cause R_SAB		0.1302			

* Indicates statistical significance at the 95% level.
^ Indicates statistical significance at the 90% level.

Period 10					
Null Hypothesis	Obs.	Prob.	Null Hypothesis	Obs.	Prob.
R_ASA does not Granger Cause R_AGL	317	0.0394*	R_SOL does not Granger Cause R_BVT	317	0.3355
R_AGL does not Granger Cause R_ASA		0.6108	R_BVT does not Granger Cause R_SOL		0.4188
R_BVT does not Granger Cause R_AGL	317	0.2670	R_V_AGL does not Granger Cause R_BVT	317	0.2090

R_AGL does not Granger Cause R_BVT		0.1175	R_BVT does not Granger Cause R_V_AGL		0.3977
R_SAB does not Granger Cause R_AGL R_AGL does not Granger Cause R_SAB	317	0.1896 0.5251	R_W_ASA does not Granger Cause R_BVT R_BVT does not Granger Cause R_W_ASA	317	0.5647 0.9154
R_SOL does not Granger Cause R_AGL R_AGL does not Granger Cause R_SOL	317	0.3722 0.0139*	R_X_BVT does not Granger Cause R_BVT R_BVT does not Granger Cause R_X_BVT	317	0.7033 0.1950
R_V_AGL does not Granger Cause R_AGL R_AGL does not Granger Cause R_V_AGL	317	0.5117 0.0965^	R_Y_SAB does not Granger Cause R_BVT R_BVT does not Granger Cause R_Y_SAB	317	0.3570 0.4777
R_W_ASA does not Granger Cause R_AGL R_AGL does not Granger Cause R_W_ASA	317	0.8139 0.4657	R_Z_SOL does not Granger Cause R_BVT R_BVT does not Granger Cause R_Z_SOL	317	0.4483 0.4525
R_X_BVT does not Granger Cause R_AGL R_AGL does not Granger Cause R_X_BVT	317	0.2890 0.9775	R_SOL does not Granger Cause R_SAB R_SAB does not Granger Cause R_SOL	317	0.2795 0.9264
R_Y_SAB does not Granger Cause R_AGL R_AGL does not Granger Cause R_Y_SAB	317	0.3020 0.1489	R_V_AGL does not Granger Cause R_SAB R_SAB does not Granger Cause R_V_AGL	317	0.8272 0.3958
R_Z_SOL does not Granger Cause R_AGL R_AGL does not Granger Cause R_Z_SOL	317	0.9447 0.7333	R_W_ASA does not Granger Cause R_SAB R_SAB does not Granger Cause R_W_ASA	317	0.5428 0.2925
R_BVT does not Granger Cause R_ASA R_ASA does not Granger Cause R_BVT	317	0.1754 0.0145*	R_X_BVT does not Granger Cause R_SAB R_SAB does not Granger Cause R_X_BVT	317	0.8295 0.1230
R_SAB does not Granger Cause R_ASA R_ASA does not Granger Cause R_SAB	317	0.7677 0.2237	R_Y_SAB does not Granger Cause R_SAB R_SAB does not Granger Cause R_Y_SAB	317	0.3373 0.2089
R_SOL does not Granger Cause R_ASA R_ASA does not Granger Cause R_SOL	317	0.3362 0.9214	R_Z_SOL does not Granger Cause R_SAB R_SAB does not Granger Cause R_Z_SOL	317	0.9594 0.8159
R_V_AGL does not Granger Cause R_ASA R_ASA does not Granger Cause R_V_AGL	317	0.9690 0.6031	R_V_AGL does not Granger Cause R_SOL R_SOL does not Granger Cause R_V_AGL	317	0.6775 0.4536
R_W_ASA does not Granger Cause R_ASA R_ASA does not Granger Cause R_W_ASA	317	0.3545 0.4614	R_W_ASA does not Granger Cause R_SOL R_SOL does not Granger Cause R_W_ASA	317	0.8302 0.9043
R_X_BVT does not Granger Cause R_ASA R_ASA does not Granger Cause R_X_BVT	317	0.8509 0.7739	R_X_BVT does not Granger Cause R_SOL R_SOL does not Granger Cause R_X_BVT	317	0.9137 0.9850
R_Y_SAB does not Granger Cause R_ASA R_ASA does not Granger Cause R_Y_SAB	317	0.3688 0.6663	R_Y_SAB does not Granger Cause R_SOL R_SOL does not Granger Cause R_Y_SAB	317	0.8597 0.9003
R_Z_SOL does not Granger Cause R_ASA R_ASA does not Granger Cause R_Z_SOL	317	0.8142 0.4350	R_Z_SOL does not Granger Cause R_SOL R_SOL does not Granger Cause R_Z_SOL	317	0.1220 0.4665
R_SAB does not Granger Cause R_BVT R_BVT does not Granger Cause R_SAB	317	0.8814 0.1912			

* Indicates statistical significance at the 95% level.

^ Indicates statistical significance at the 90% level.