

DYNAMIC MODELLING OF BANKING ACTIVITIES

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**Thesis submitted in partial fulfillment of the requirements
for the degree Philosophiae Doctor in
Applied Mathematics
at the
North-West University (Potchefstroom Campus)**

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**November 2007
Potchefstroom**

Preface

One of the contributions made by North-West University (Potchefstroom Campus) to the activities of the applied mathematics community in South Africa has been the establishment of an active research group that has an interest in financial mathematics. Under the guidance of my supervisor, Prof. Mark A. Petersen, this group has recently made valuable contributions to the existing knowledge about the stochastic control of financial systems in pensions, insurance and banking.

The work in this thesis originated from our interest in the connections between concepts that arise in systems and (stochastic) control theory and financial models. In this regard, the interests of the group lie with the stochastic controllability of interest rate models, stochastic control of continuous- and discrete-time pension funds, the solvency of dividend equalization funds and the solvency, profitability and operational control of commercial banks.

The most important outcomes of this project were collected in 5 peer-reviewed international journal articles (3 appeared, another 2 have been submitted subsequently) and 6 peer-reviewed conference proceedings papers.

Summary

We investigate the discrete-time dynamics of banking items such as loan demand and supply, deposits, treasury securities, capital and bank value under the influence of macroeconomic factors. These models enable us to formulate an optimization problem subject to cash flow, loan demand, financing and balance sheet constraints. Furthermore, we consider the effect that regulation has on capital adequacy decisions in banking. Our investigation suggests that we are able to maximize the value of a bank for an investor via optimal choices of loan rates, treasury securities, deposits and profits.

With the drafting of new banking regulation via the Basel II capital accord, bank regulatory capital and its adequacy has become the subject of much debate. In this thesis, we model and simulate two of the main measures of capital adequacy, namely capital adequacy ratios (CARs) for unweighted and risk-weighted assets. In order to accomplish this, we consider the stochastic dynamics of items such as bank assets, liabilities, regulatory capital and CARs in a Lévy process setting.

Also, we demonstrate that bank capital dynamics is subject to changes in the demand for loans and is thus procyclical. A further conclusion is that macroeconomic shocks will affect the loan risk-weights via tighter capital constraints when the shock is negative and vice versa. In addition, we provide a descriptive example that illustrates economic aspects of the bank modelling and optimization discussed in the main body of the thesis.

Considering such ratios as the CARs in isolation is not very useful for economic analysis. Instead, an important issue related to CARs is their relationship with the economic cycle and consequent effect on financial stability in the banking industry. By way of addressing this topic, we provide computer simulations of such ratios for several countries including some of those that belong to the Organization for Economic Co-operation and Development (OECD). In order to investigate the cyclicity of CARs, we probe the relationship between the output gap (proxy for resource utilization) and the aforementioned ratio. Two of our conclusions are that bank regulatory capital is inclined to be procyclical while CARs tend to be acyclical in most of the countries studied. In addition, we provide a brief analysis of some of the modelling and computation issues arising from the dynamic banking models derived in the main body of the thesis.

Opsomming

Met die aanvaarding van die nuwe Basel II regulasies (vir implementering in Suid-Afrika in Januarie 2008) het die soeke na beter maniere om banke se belangrikste bedrywighede te modelleer net soveel meer belangrik geword. In hierdie proefskrif probeer ons om beter modelle te bou.

Ons begin die studie deur te kyk na die verliese wat banke ly ten opsigte van lenings wat kliënte nie kan betaal nie. Ons toets sekere afleidings oor die vraag na en die aanbod van lenings deur banke en kyk dan ook na die voorsorg wat banke tref om negatiewe gevolge te minimaliseer. Dit stel ons in staat om te kyk na hoe 'n mens waarde kan heg aan 'n sekere bank. Basel II gee sekere voorskrifte oor hierdie modelle en dit word hier in ag geneem.

Hierdie proefskrif beskou 'n manier om bank aktiwiteite soos byvoorbeeld die uitreik van lenings te modelleer deur te kyk na die sogenaamde Lévy proses. Hierdie proses word bestudeer omdat daar kritiek bestaan teen die algemeen gebruikte Brown se beweging wat beskou word as onvoldoende om realiteit te simuleer. Ons lei stogastiese differensiaal vergelykings af vir die bank se hoof balansstaat items om sodoende dan die kapitaal van die bank te simuleer. Basel II gee voorskrifte oor die vlak van kapitaal wat banke moet handhaaf vir tye waarin ekonomiese aktiwiteite afneem. Dit is dus vir ons belangrik om te kyk na die kapitaalberekenings proses siende dat dit ingevolge Basel II voorskrifte gebruik word om skokke te kan absorbeer.

Vervolgens kyk ons na die voorsorg wat getref word vir slegte skuld en die sikliese patroon van kapitaal van ontwikkelde lande sowel as die van Suid-Afrika. Ons vergelyk die verskil tussen die werklike produksie van lande soos gemeet deur die Bruto Binnelandse Produk (BBP) met dit wat hulle produksie potensiaal is. Hieruit kan ons belangrike gevolgtrekkings maak aangaande die siklusse wat kapitaal volg.

Ons beskou al die analise wat gedoen is in die tesis en kyk of dit aangepas kan word vir sekere uitsonderings.

Acknowledgements

Firstly, I would like to thank God for His grace in enabling me to complete this thesis.

I would like to acknowledge the emotional support provided by my immediate family, my parents and brother Pieter, Elize and Arno Fouché and fiancé, Marilie Liebenberg.

I am indebted to my supervisor, Prof. Mark A. Petersen of the School of Computer, Mathematical and Statistical Sciences at the North-West University (Potchefstroom Campus), for the guidance provided during the completion of this dissertation. Also, I would like to thank the remaining members of staff in the Department of Mathematics and Applied Mathematics at the North-West University (Potchefstroom Campus) for providing moral and logistical support during my studies.

Finally, I am grateful to the National Research Foundation (NRF) for providing me with a Prestigious Scholarship for Doctoral Studies throughout my PhD studies. Prior to that, during my masters studies, I also received NRF grantholder bursaries under projects with GUN No.'s 2053343 and 2074218. I would like to thank the Research Director of the School for Computer, Statistical and Mathematical Sciences at North-West University (Potchefstroom Campus), Prof. Koos Grobler, for the encouragement and additional financial support received.

Finally, I would like to express my gratitude towards Dr. Riaan Hattingh and Prof. Hendrik Nel from the South African Reserve Bank (SARB) for facilitating the delivery of data on South African financial variables.

Index of Abbreviations and Symbols

OECD - Organization for Economic Co-operation and Development
 LLP - Loan Loss Provision
 GDP - Gross Domestic Product
 PD - Probability Default
 LGD - Loss Given Default
 NPL - Non-Performing Loans
 TA - Total Assets
 VaR - Value-at-Risk
 GKW - Galtchouck-Kunita-Watanable
 TCR - Total Capital Ratio
 CRC - Credit Risk Charge
 $SIGN_{it}$ - One-Year-Ahead Changes of Earnings Before Taxes and Loan Loss Provisions
 ER_{it} - Positive Correlation Between Earnings Before Taxes and Loan Loss Provisions
 y'_{it} - Annual Growth Rate of GDP
 P_{it} - Ratio of Loan Loss Provisions to Total Assets at the End of Year t for Bank i
 Λ - Loans
 T - Treasuries
 R - Reserves
 K - Capital
 L - Lévy Process
 D - Deposits
 ϕ - Characteristic Function of a Distribution
 ψ - Lévy or Characteristic Exponent of L
 x - Variable
 γ - Drift of a Process
 X - Value Process
 Z - Standard Brownian Motion
 $Q(dt, dx)$ - Poisson measure
 dt - Lebesgue measure
 M - Martingale
 ξ - Doléans-Dade Exponential
 P - Total Provision
 A - Assets

r^Λ - Loan Rate
 c^d - Risk Premium
 c^a - Administrative Cost
 S^e - Expected Loan Losses
 S^u - Unexpected Loan Losses
 ν - *Lévy Measure*
 B - Borel Sets
 S - Aggregate Loan Losses
 T - Terminal Time
 P' - Net Loan Loss Provisioning
 ϱ - Net Instantaneous Return of a Value Process
 σ - Volatility of a Value Process
 μ - Mean of a Value Process
 π - Provisioning Strategy
 k_d - Depository Value
 D - Depository Contracts
 L_{it} - Provisions for Loan Losses-to-Total Assets Ratio
 n^T - Number of Treasuries
 n^R - Number of Reserves
 $\widehat{V}(\pi)$ - Provisioning Portfolio Value Process
 c^o - Cost Process
 Λ^c - Probability of Insolvency to Occur
 C_T - Cost of Insolvency
 N^L - Number of Loan Losses
 l - Unexpected Loan Losses sizes
 r^R - Loan Loss Reserve Rate
 P^π - Total Loan Loss Provisioning Under Strategy π .
 R^l - Loan Loss Reserve
 P'^π - Net Loan Loss Provisioning Under π
 r^R - Deterministic Rate of (Positive) Return on Reserves
 f^R - Fraction of the Reserves Consumed by Deposit Withdrawals
 σ^R - Volatility in the Level of Reserves
 G - Girsanov Parameter
 $M^{\mathbf{Q}}(dt, dx)$ - Compensated Jump Measure of L^R Under \mathbf{Q}_g
 W - Sum of Treasuries and Reserves
 D^c - Sum of Cohort Deposits
 w_{x+t} - Withdrawal Rate Function
 N^I - Number of Withdrawals

M^I - Compensated Counting Process

w^{un} - Unanticipated Deposit Withdrawals

$f(w^{un})$ - Probability Density Function

c^l - Cost of Liquidation

r_t^P - Penalty Rate on Deposit Withdrawals

$c^{w^{un}}$ - Cost of Deposit Withdrawals

ρ^r - Relative Risk Ratio

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Chapter 1

INTRODUCTION

1.1 RELATION TO PREVIOUS LITERATURE

1.2 PRELIMINARIES

1.3 OUTLINE OF THE THESIS

In this thesis, we mainly consider two important aspects of the modelling of banking activities, viz., the discrete-time modelling of bank valuation (see Chapter 2 for more details) and the Lévy-process driven modelling of bank regulatory capital and its adequacy (see Chapter 3 for a complete discussion). As far as the former is concerned, in the acquisition of bank equity, a bank valuation gives a stock analyst (possibly acting on behalf of a potential shareholder) an independent estimate of a fair price of the bank's shares. In this regard, a bank valuation determines the price that such a shareholder would pay for a share in a bank under a given set of circumstances. On the other hand, the investigation into the modelling of bank regulatory capital and its adequacy is motivated by new risk sensitive regulation in the form of Basel II. The modelling procedure involves some of the latest stochastic techniques related to Lévy processes.

As will be demonstrated in Chapter 2, a popular approach to the study of banking dynamics and valuation in discrete-time involves a financial system that is assumed to be imperfectly competitive. As a consequence, profits are ensured by virtue of the fact that the net loan interest margin is greater than the marginal resource cost of deposits and loans. Besides competition policy, the decisions related to capital structure play a significant role in bank behavior. Here, the relationship between bank capital and lending and macro-economic activity is of crucial importance. By way of addressing these issues, we present a two-period discrete-time bank model involving on-balance sheet items such as assets (loans, Treasuries and reserves), liabilities (deposits), bank capital (shareholder equity, subordinate debt and loan loss reserves) and off-balance sheet items such as intangible assets. In turn, the aforementioned models enable us to formulate an optimization problem that seeks to establish a maximal value of the bank by a stock analyst that acts in the interests of a potential shareholder by choosing an appropriate loan rate and loan supply. Under a cash flow constraint, the solution to this problem also yields a procedure for profit maximization in terms of the loan rate and deposits. Here profits are not only expressed as a function of assets and liabilities but also depend heavily on the capital held by the bank. Other constraints that impact our optimal valuation problem in a significant way are those involving total capital, loan demand and financing. In the discussion on bank valuation in Chapter 2, we note that loan portfolios decline in value as some of the individual loans become non-performing. Accordingly, the intrinsic value of the assets will differ from the value as represented on a bank's books. From time to time, banks will adjust the book value of the assets to reflect the changes in value. At some point prior to the classification of the loan as uncollectible, an adjustment is made to a contra asset account (called an *allowance for loan losses* account) to make allowance for a portion or for the entire loan. An offsetting expense called the *loan loss provision* (LLP) is charged against

net profit (net earnings or the bottom line). This offset will reduce reported income but has no impact on taxes, although when the assets are finally written off, a tax-deductible expense is created. The process of removal is often referred to as *writing off* the loan. When the loan is classified as uncollectible, the portion of the loan that is deemed as such will be removed from both the asset account and from the allowance for loan losses account. The allowance may consist of a *specific loss* component, which relates to specific loans, or an *inherent loss* component that may consist of a country risk allowance, an allowance for smaller-balance standardized homogeneous loans and another inherent loss component to cover losses in the loan portfolio that have not yet been individually identified. An important factor influencing the valuation and loan loss provisioning procedure is regulation and supervision. Measures of capital adequacy are generally calculated using the book values of assets and equity.

As in Chapter 3 of this thesis, more attention is being paid to financial modelling techniques that deviate from those that rely on the seminal Black-Scholes model (see, for instance, [51] and [52]). A battery of such techniques is available with some of the most popular and tractable of these being associated with Lévy process-based models. In this spirit, our contribution discusses the dynamics of banking items such as assets, capital and regulatory ratios that are driven by such processes. An advantage of Lévy-processes is that they are very flexible since for any time increment Δt any infinitely divisible distribution can be chosen as the increment distribution of periods of time Δt . In addition, they have a simple structure when compared with general semi-martingales and are able to take different important stylized features of financial time series into account. If there is a deviation from the Black-Scholes paradigm, one typically enters into the realm of incomplete market models. Most theoretical financial market models are incomplete, with academics and practitioners alike agreeing that "real-world" markets are also not complete. A specific motivation for modelling banking items in terms of Lévy processes is that they have an advantage over the more traditional modelling tools such as Brownian motion (see, for instance, [26], [32], [49] and [63]), since they describe the non-continuous evolution of the value of economic and financial indicators more accurately. Our contention is that these models lead to analytically and numerically tractable formulas for banking items that are characterized by jumps. This is an important consideration in the case where the models are applied practically. Lévy processes also improve the scope for the optimization of banking activities and risk, capital and asset management. Despite the issues raised in the above, there is a paucity of literature on the dynamic modelling of banks in a Lévy process framework. One of the main reasons for this is that the stochastic analysis of classes of processes that are more general than Brownian motion like, for instance, semi-martingales, is a subject that mainly resides in the domain of special-

ists. This is not surprising since the financial models driven by semi-martingales are usually highly complex. In this thesis, a main objective is to model and simulate some of the main measures of capital adequacy, namely Capital Adequacy Ratios (CARs) for un-weighted and risk-weighted assets. The former type of CAR is computed as the *regulatory capital-to-total assets ratio* while the latter may be represented by the *regulatory capital-to-risk-weighted assets*. In order to do this calculation, we consider the stochastic dynamics of items such as bank assets, liabilities, regulatory capital and CARs in a Lévy process setting. A discussion of the value of these ratios separate from other factors does not lend itself to a complete economic analysis. Instead, an important issue related to CARs are their relationship with the economic cycle and consequent effect on financial stability in the banking industry. By way of addressing this topic, we provide computer simulations of such ratios for several countries including those that belong to the Organization for Economic Co-operation and Development (OECD). In order to investigate the cyclicity of CARs, we probe the relationship between the output gap (proxy for resource utilization; measured as the difference between actual and potential output) and the aforementioned ratio. The output gap can be an important output for monetary policy decisions as it provides an indication of the intensity of resource utilization and of inflationary pressures. The policy outlook for a country depends importantly on both near- and long-term prospects for real output growth. It is an accepted fact that near-term prospects can be measured by potential output growth and the output gap. On the other hand, longer-term growth prospects are based on the full utilization of factors of production and the output gains that arise as these factors are more efficiently utilized, for example through structural reforms. For the sake of simplicity, transparency and comparability with the literature, this thesis computes the potential output via the Hodrick-Prescott (HP) filter (see, for instance, [39]), which is a common univariate filtering technique to decompose a time series into a trend and cyclical part.

The provisioning for loans and their associated write-offs will cause a decline in capital, and may precipitate increased capital requirements by bank authorities. Greater levels of regulation generally entail additional costs for the bank. Currently, this regulation takes the form of the Basel II Capital Accord (see [9] and [14]) that is to be implemented (mostly in developed countries) on a worldwide basis by 2007. The latter accord prescribes the minimum level of regulatory capital banks should maintain. As a consequence, bank regulatory capital and its adequacy has become the subject of much debate that has spawned renewed interest in the construction of mathematical models for such capital. Models of capital adequacy are generally based on the book values of assets and equity. Also, the impact of a risk-sensitive framework such as Basel II on the financial stability of banks is an important modelling issue. The 1996

Amendment's Internal Models Approach (IMA) determines the capital requirements on the basis of the institutions' internal risk measurement systems. Banks are required to report daily their value-at-risk (VaR) at a 99% confidence level over both a one day and two weeks (10 trading days) horizon. The minimum capital requirement is then the sum of a premium to cover *credit risk* and a premium to cover general *market risk*. The credit risk premium is made up of 8% risk weighted assets and the market risk premium is equal to a multiple of the average reported two-week VaRs in the last 60 trading days. The impact of a risk-sensitive framework such as Basel II on macro-economic stability of banks is an important issue. In order for a bank to determine their minimum capital requirements they will first decide on a planning horizon. This planning horizon is then divided into non-overlapping backtesting-periods, which is in turn divided into non-overlapping reporting periods. At the start of each reporting period the bank has to report its VaR for the current period and the actual loss from the previous period. The market risk premium for the current reporting period is then equal to the multiple m of the reported VaR. At the end of each backtesting period, the number of reporting periods in which actual loss exceeded VaR is counted and this determines the multiple m for the next backtesting period according to a given increasing scale.

1.1 RELATION TO PREVIOUS LITERATURE

In this section, we briefly comment on selected literature related to bank valuation, regulatory capital, Lévy processes, optimization, output and cyclicity.

1.1.1 Bank Valuation

The topic of bank valuation has enjoyed much attention over the years. The most common method of valuing a bank is related to the calculation of the present value of the bank's future cash flows. For instance, in [29] a regression model is derived to address the problem of valuing a bank. Similar to this is [31] where a regression model is derived for the change in market value for a specific bank. These papers, and others not mentioned explicitly, discuss activities that add value to the bank making it attractive for potential shareholders. Also, the extent of exposure to emerging markets plays a role in the valuation of the bank. Most of the studies considered, has a statistical background. The novelty of our contribution is that we use control laws to find the optimal bank value.

1.1.2 Basel I vs. Basel II

The first Basel Capital accord was adopted by about 100 countries after its release in 1988. It had two main objectives. One, it believed the framework would help strengthen the soundness and stability of the international banking system by encouraging international banking organizations to boost their capital positions. Secondly, it believed by adopting a standard approach to internationally active banks in different countries would reduce competitive inequalities. Although it achieved its goal the banking industry has evolved very rapidly. In June 2004 the Basel Committee published the Basel II accord to aid where the Basel I accord had failed. Some of the main additions of the Basel II are the inclusion of elements such as operational risk. Also, it adds another pillar to enable it to consider market discipline. The main advantages of the Basel II accord is that it is as state of the art as can be. It is also a dynamic system in the sense that it allows for best of practice decisions.

In this thesis we refer to the Basel II Accord as it was originally set out and not necessarily the specific application of the accord by a specific country. We note that loan loss provisioning is not the same in all countries considered here. Provisioning will include general provisioning. We are aware though that Japan for instance does not have the same regulatory system as South Africa.

1.1.3 Bank Capital

The most important role of capital is to mitigate the moral hazard problem that results from asymmetric information between banks, depositors and borrowers. In the presence of asymmetrical information about the LLP, bank managers may be aware of asset quality problems unknown to outside analysts. Provisioning for the assets may convey a clearer picture regarding the value of these assets and precipitate a (negative) market adjustment. In the absence of information asymmetry, there may be no new asset quality information released as a result of the LLP announcement. The Modigliani-Miller theorem forms the basis for modern thinking on capital structure (see [53]). In an efficient market, their basic result states that, in the absence of taxes, insolvency costs and asymmetric information, the bank's value is unaffected by how it is financed. In this framework, it does not matter if bank capital is raised by issuing equity or selling debt or what the dividend policy is. By contrast, in our contribution, in the presence of loan market frictions, the bank's value is dependent on its financial structure (see, for instance, [15], [28], [49] and [62] for banking). In this case, it is well-known that the bank's decisions about lending and other issues may be driven

by the capital adequacy ratio (CAR) (see, for instance, [25], [26], [54], [61] and [63]). Further evidence of the impact of capital requirements on bank lending activities are provided by [37] and [67].

A new line of research into credit models for monetary policy has considered the association between bank capital and loan demand and supply (see, for instance, [2], [18], [21], [23], [68], [69] and [70]). This credit channel is commonly known as the *bank capital channel* and propagates that a change in interest rates can affect lending via bank capital. We also discuss the effect of macro-economic activity on a bank's capital structure and lending activities (see, for instance, [36]). With regard to the latter, for instance, there is considerable evidence to suggest that macro-economic conditions impact the probability of default and loss given default on loans (see, for instance, [3], [36] and [44]). Of all the papers mentioned in this paragraph our contribution has the closest connection with [23]. Chapter 2 extends the said paper in six definite directions. Firstly, taking our lead from the requirements of Basel II, by contrast to [23], the risk weight for the assets appearing on and off the balance sheet may vary with time. Furthermore, we include both Treasuries and reserves as part of the provisions for deposit withdrawals whereas the aforementioned paper only discusses the role of Treasuries. Thirdly, we provide substantive evidence of the relationship between the business cycle and provisioning and profitability for OECD countries. Also, we include loan losses and its provisioning as an integral part of our analysis. In the fifth place, we recognize the important role that intangible assets play in determining bank profit. In essence this means that, unlike the aforementioned contributions, we consider both on- and off-balance sheet items in the computation of profit. Finally, we determine the value of a bank subject to capital requirements based on reported Value-at-Risk (VaR) measures, as in the Basel Committee's Internal Models Approach (see, for instance, [1] and [24]).

1.1.4 Optimization

As in Chapter 2, several discussions related to discrete-time optimization problems for banks have recently surfaced in the literature (see, for instance, [36], [49], [55] and [61]). Also, some recent papers using dynamic optimization methods in analyzing bank regulatory capital policies include [59] for Basel II and [4], [24] and [48] for Basel market risk capital requirements. In [61], a discrete-time dynamic banking model of imperfect competition is presented, where the bank can invest in a prudent or a gambling asset. For both these options, a maximization problem that involves the bank value for shareholders is formulated. On the other hand, [55] examines a problem related to the optimal risk management of banks in a continuous-time

stochastic dynamic setting. In particular, the authors minimize market and capital adequacy risk that involves the safety of the assets held and the stability of sources of capital, respectively (see, also, [56]). Further optimization problems involving banking activities were solved in a broader framework in [33], [34] and [57].

1.1.5 Lévy Processes

Our discussion in Chapter 3 extends aspects of the recent article [32] (see, also, [54] and [55]) by generalizing the description of bank behaviour in a continuous-time Brownian motion framework to one in which the dynamics of bank items may have jumps and be driven by Lévy processes. As far as information on these processes is concerned, Protter in [60, Chapter I, Section 4] and Jacod and Shiryaev in [43, Chapter II] are standard texts (see, also, [16] and [64]). Also, the connections between Lévy processes and finance are embellished upon in [65] (see, also, [45] and [46]).

1.1.6 Output and Cyclicity

In Chapter 4, it will be important to be able to calculate the output gap and hence the potential output and trend output. In this regard, [35] reviews the methods used to estimate potential output in OECD countries and the resulting output gaps for the calculation of structural budget balances. The *split time trend* method for estimating trend output that was previously used for calculating structural budget balances is compared with two alternative methods, *smoothing actual GDP using a Hodrick Prescott filter* and estimating potential output using a *production approach*. The conclusion is that the product function approach for estimating potential output provides the best method for estimating output gaps and for calculating structural budget balances, with the results obtained by smoothing GDP providing a cross check. New tax and expenditure elasticities, along with the potential output gaps, are used to derive structural budget balances.

It is a widely accepted fact that certain financial variables (for instance, credit prices, asset prices, bond spreads, ratings from credit rating agencies, provisioning, profitability, capital, leverage and risk-weighted capital adequacy ratios, other ratios such as write-off/loan ratios and perceived risk) exhibit cyclical tendencies. In particular, “procyclicality” has become a buzzword in discussions around the new regulatory framework offered by Basel II. In the sequel, the movement in a financial indicator is said to be procyclical if it tends to amplify business cycle fluctuations. A consequence of procyclicality is that banks tend to restrict their lending activity during economic downturns because of their concern about loan quality and the probability of loan

defaults. This exacerbates the recession since credit constrained businesses and individuals cut back on their investment activity. On the other hand, banks expand their lending activity during boom periods, thereby contributing to a possible overextension of the economy that may transform an economic expansion into an inflationary spiral. In this thesis, our interest in cyclicalities extends to its relationship with provisioning and profitability. In particular, the fact that provisioning (profitability) behaves procyclically by falling (rising) during economic booms and rising (falling) during recessions (see, for instance, [17], [18], [19], [21], [22] and [23]) is incorporated in our models. The cyclicalities issue will be briefly discussed in Chapters 4 and 5.

1.2 PRELIMINARIES

In this section, we provide some preliminaries on the basic model of a bank as well as Lévy processes. In the sequel, we use the notational convention “subscript t or s ” to represent (possibly) random processes, while “bracket t or s ” are used to denote deterministic processes.

1.2.1 Preliminaries about Bank Valuation

The preliminaries in this subsection mainly apply to the discussion in Chapter 2. Throughout, we suppose that $(\Omega, \mathbf{F}, (\mathcal{F}_t)_{t \geq 0}, \mathbf{P})$ is a filtered probability space. As is well-known, the bank balance sheet consists of assets (uses of funds) and liabilities (sources of funds) that are balanced by bank capital (see, for instance [28]) according to the well-known relation

$$\text{Total Assets } (A) = \text{Total Liabilities } (\Gamma) + \text{Total Bank Capital } (K).$$

In period t , the main on-balance sheet items can specifically be identified as

$$A_t = \Lambda_t^m + W_t, \quad W_t = T_t + R_t; \quad \Gamma_t = D_t; \quad K_t = n_t E_{t-1} + O_t + R_t^l,$$

where Λ^m , T , R , D , n , E , O and R^l are the market value of loans, Treasuries, reserves, outstanding debt, number of shares, market price of the bank’s common equity, subordinate debt and loan loss reserves, respectively. The relation of the aforementioned banking items to retained earnings, E^r , and profit, Π , are extensively discussed in the sequel.

As far as profit, Π , is concerned, we closely follow the report [22] and use the basic fact that profits can be characterized as the difference between income and expenses that are reported in the bank's income statement. In our case, *income* is solely constituted by the return on intangible assets, $r_t^I I_t$, the return on loans, $r_t^\Lambda \Lambda_t$, and the return on Treasuries, $r_t^T T_t$. In this regard, r^I , r^Λ and r^T denote the rates of return on intangible assets, loans (that may include a component for provisions for expected loan losses) and Treasuries, respectively. Furthermore, we assume that the level of macro-economic activity is denoted by M_t . As *expenses*, in period t , we consider the cost of monitoring and screening of loans and capital, $c^\Lambda \Lambda_t$, interest paid to depositors, $r_t^D D_t$, the cost of taking deposits, $c^D D_t$, the cost of deposit withdrawals, $c^w(W_t)$, the value of loan losses, $L(M_t)$, and total loan loss provisions, $P(M_t)$. Here r^D and c^D are the deposit rate and cost of deposits, respectively. We assume all the aforementioned costs would sum to operating costs so that profit, Π , can be expressed as

$$\begin{aligned} \Pi_t = & r_t^\Lambda \Lambda_t + r_t^T T_t + r_t^I I_t - c^\Lambda \Lambda_t - \left(r_t^D + c^D \right) D_t \\ & - c^w(W_t) - L(M_t) - P(M_t). \end{aligned} \quad (1.1)$$

1.2.2 Preliminaries about Lévy Processes

In this subsection, for sake of completion, we firstly provide a general description of a Lévy process and its measure and then describe the Lévy decomposition that we consider.

In this regard, we assume that $\phi(\xi)$ is the characteristic function of a distribution. If for every positive integer n , $\phi(\xi)$ is also the n -th power of a characteristic function, we say that the distribution is *infinitely divisible*. For each infinitely divisible distribution, a stochastic process $L = (L_t)_{0 \leq t}$ called a *Lévy process* exists. This process initiates at zero, has independent and stationary increments and has $(\phi(u))^t$ as a characteristic function for the distribution of an increment over $[s, s+t]$, $0 \leq s, t$, such that $L_{t+s} - L_s$. Next, we provide important definitions and a useful result.

Definition 1.2.1 (Cádlág Stochastic Process): *A stochastic process X is said to be cádlág if it almost surely (a.s.) has sample paths which are right continuous, with left limits.*

Proposition 1.2.2 (Stopping Time): *Let X be an adapted cádlág stochastic process, and let Λ be a closed set. Then the random variable*

$$T(\omega) = \inf\{t > 0 : X_t(\omega) \in \Lambda \text{ or } X_{t-}(\omega) \in \Lambda\}$$

is a stopping time.

Proof. The proof is contained in [60] and will not be shown here. \square

Definition 1.2.3 (Random Partition): Let ς denote a finite sequence of finite stopping times

$$0 = T_0 \leq T_1 \leq \dots \leq T_k < \infty.$$

The sequence ς is called a random partition.

Every Lévy process is a semi-martingale and has a càdlàg version (right continuous with left hand limits) which is itself a Lévy process. We will assume that the type of such processes that we work with are always càdlàg. As a result, sample paths of L are continuous a.e. from the right and have limits from the left. The *jump* of L_t at $t \geq 0$ is defined by $\Delta L_t = L_t - L_{t-}$. Since L has stationary independent increments its *characteristic function* must have the form

$$\mathbf{E}[\exp\{-i\xi L_t\}] = \exp\{-t\Psi(\xi)\}$$

for some function Ψ called the *Lévy* or *characteristic exponent* of L . The *Lévy-Khintchine formula* is given by

$$\begin{aligned} \Psi(\xi) = & i\gamma\xi + \frac{c^2}{2}\xi^2 + \int_{|x|<1} \left[1 - \exp\{-i\xi x\} - i\xi x\right] \nu(dx) \\ & + \int_{|x|\geq 1} \left[1 - \exp\{-i\xi x\}\right] \nu(dx), \quad \gamma, c \in \mathbb{R} \end{aligned} \tag{1.2}$$

and for some σ -finite measure ν on $\mathbb{R} \setminus \{0\}$ with

$$\int \inf\{1, x^2\} \nu(dx) = \int \inf(1 \wedge x^2) \nu(dx) < \infty.$$

An infinitely divisible distribution has a *Lévy triplet* of the form

$$[\gamma, c^2, \nu(dx)]$$

where the measure ν is called the *Lévy measure*.

The Lévy-Khintchine formula given by (1.2), is closely related to the Lévy process, L . This is particularly true for the *Lévy decomposition* of L which we specify in the rest of this paragraph. From (1.2), it is clear that L must be a linear combination of a Brownian motion and a quadratic jump process X which is independent of the Brownian motion. We recall that a process is classified as *quadratic pure jump* if the continuous part of its quadratic variation $\langle X \rangle^c \equiv 0$, so that its quadratic variation becomes

$$\langle X \rangle_t = \sum_{0 < s \leq t} (\Delta X_s)^2,$$

where $\Delta X_s = X_s - X_{s-}$ is the jump size at time s . If we separate the Brownian component, Z , from the quadratic pure jump component X we obtain

$$L_t = X_t + cZ_t,$$

where X is the quadratic pure jump and Z is standard Brownian motion on \mathbb{R} . Next, we describe the Lévy decomposition of X . Let $Q(dt, dx)$ be the Poisson measure on $\mathbb{R}^+ \times \mathbb{R} \setminus \{0\}$ with expectation (or intensity) measure $dt \times \nu$. Here dt is the Lebesgue measure and ν is the Lévy measure as before. The measure $dt \times \nu$ (or sometimes just ν) is called the compensator of Q . The Lévy decomposition of X specifies that

$$\begin{aligned} X_t &= \int_{|x| < 1} x \left[Q((0, t], dx) - t\nu(dx) \right] + \int_{|x| \geq 1} x Q((0, t], dx) + t\mathbf{E} \left[X_1 - \int_{|x| \geq 1} x\nu(dx) \right] \\ &= \int_{|x| < 1} x \left[Q((0, t], dx) - t\nu(dx) \right] + \int_{|x| \geq 1} x Q((0, t], dx) + \gamma t, \end{aligned} \quad (1.3)$$

where

$$\gamma = \mathbf{E}\left[X_1 - \int_{|x|\geq 1} x\nu(dx)\right].$$

The parameter γ is known as the *drift of X* . In addition, in order to describe the Lévy decomposition of L , we specify more conditions that L must satisfy. The most important supposition that we make about L is that

$$\mathbf{E}[\exp\{-hL_1\}] < \infty, \quad \text{for all } h \in (-h_1, h_2), \quad (1.4)$$

where $0 < h_1, h_2 \leq \infty$. This implies that L_t has finite moments of all orders and in particular, $\mathbf{E}[X_1] < \infty$. In terms of the Lévy measure ν of X , we have, for all $h \in (-h_1, h_2)$, that

$$\begin{aligned} \int_{|x|\geq 1} \exp\{-hx\}\nu(dx) &< \infty; \\ \int_{|x|\geq 1} x^\alpha \exp\{-hx\}\nu(dx) &< \infty, \quad \forall \alpha > 0; \\ \int_{|x|\geq 1} x\nu(dx) &< \infty. \end{aligned}$$

The above assumptions lead to the fact that (1.3) can be rewritten as

$$X_t = \int_{\mathbb{R}} x \left[Q((0, t], dx) - t\nu(dx) \right] + t\mathbf{E}[X_1] = M_t + at,$$

where we have that

$$M_t = \int_{\mathbb{R}} x \left[Q((0, t], dx) - t\nu(dx) \right]$$

is a martingale and $a = \mathbf{E}[X_1]$.

In the specification of our model, we assume that the Lévy measure $\nu(dx)$ of L satisfies

$$\int_{|x|>1} |x|^3 \nu(dx) < \infty. \quad (1.5)$$

As in the above, this allows a decomposition of L of the form

$$L_t = cZ_t + M_t + at, \quad 0 \leq t \leq T, \quad (1.6)$$

where $(cZ_t)_{0 \leq t \leq \tau}$ is a Brownian motion with standard deviation $c > 0$, $a = \mathbf{E}(L_1)$ and the martingale

$$M_t = \int_0^t \int_{\mathbb{R}} x M(ds, dx), \quad 0 \leq t \leq T,$$

is a square-integrable. Here, we denote the *compensated Poisson random measure* on $[0, \infty) \times \mathbb{R} \setminus \{0\}$ related to L by $M(dt, dx)$. Subsequently, if $\nu = 0$ then we will have that $L_t = Z_t$, where Z_t is appropriately defined Brownian motion.

1.3 OUTLINE OF THE THESIS

The current chapter is introductory of nature. The rest of the thesis is structured as follows.

1.3.1 Outline of Chapter 2

Chapter 2 describes a discrete-time model for banks. We start by stating two problems, (see Problems 2.0.1 and 2.0.2), that will be solved in the chapter. In this regard, we will describe loans and their supply and demand as well as provisioning for loan losses and how this is measured. We will assume that the bank faces a Hicksian demand for loans. Next, in Section 2.3, we discuss related items such as Treasuries, reserves, risk-weighted assets. Also included are intangible assets which can be seen as the value of the brand of the bank. The final part of Chapter 2, Section 2.4, is dedicated to bank valuation with the goal of finding the optimal bank value for a stock analyst that is possibly acting on behalf of a potential shareholder. Many factors are taken into consideration including profit, retained earnings and capital constraints. The main result of this chapter is Theorem 2.4.2 where a solution to the optimal bank

valuation problem is given. Chapter 2 along with Chapter 3 is the main part of the thesis. We do our main analysis in these two chapters and we will frequently refer back to them.

1.3.2 Outline of Chapter 3

Chapter 3 describes assets (see Subsection 3.1), liabilities (see Subsection 3.2) and capital (see Subsection 3.3) as part of an effort to find a Lévy process driven model for a bank. The price process for assets is defined and applied to obtain equations for the asset portfolio of a bank (see Subsubsection 3.1.2). We consider the risk-weighted assets in Subsubsection 3.1.3. We next define liabilities for our thesis (Subsection 3.2). The regulatory capital of a bank is discussed in the next part, Subsection 3.3 where we look at the stochastic dynamics of bank regulatory capital. We derive equations for both the total assets, (see Theorem 3.3.1) and risk-weighted assets, (see Theorem 3.3.2) capital adequacy ratios. This is the main result of Chapter 3.

1.3.3 Outline of Chapter 4

In Chapter 4 we consider numerical and illustrative examples of provisioning, (see Subsection 4.2) and capital adequacy ratios, (see Subsection 4.3) for OECD countries as well as South Africa (in some cases). We compare the provisioning for loan losses to the output gap of the respective countries and explain why they can be seen as procyclical in Subsubsection 4.2.1.4. We will also do a simulation of the CAR in Japan using the model obtained in Chapter 3 in Subsubsection 4.3.1. This is followed by illustrative examples of the other OECD countries (see Subsubsection 4.3.2) and South Africa (see Subsubsection 4.3.3). The final section of Chapter 4, Section 4.4 contains a stylized illustration of bank management practice in relation to the analysis done in the sections prior to Section 4.4.

1.3.4 Outline of Chapter 5

Chapter 5 contains a brief discussion of the main issues involved in the thesis. We start in Section 5.1 by looking at the issues raised in Chapter 2. We discuss the assumptions made and consider special cases. Next, in Section 5.2 we analyze the results obtained in Chapter 3 to see what the implications are of the work that was done. Special attention is also given to simulation contained in Chapter 4 in Subsubsection 4.3.1. We conclude this chapter with a discussion of the illustrative example that is supplied at the end of Chapter 4 in Section 4.4.

1.3.5 Outline of Chapter 6

Chapter 6 contains the conclusion that we can draw from the study. We also point out what further research problems may be addressed by future students.

1.3.6 Outline of Chapter 7

The bibliography in Chapter 7 contains all the articles, books and other sources used throughout the thesis.

1.3.7 Outline of Chapter 8

Finally, Chapter 8 contains the tables of data that was used in Chapter 4 as well as the techniques used to measure the potential output of a country. Prior to this, we provide some more information on the calculation of operational risk.

Chapter 2

DISCRETE-TIME MODEL OF BANKING ACTIVITIES

2.1 LOANS AND THEIR DEMAND AND SUPPLY

2.2 LOAN LOSSES AND PROVISIONING

2.3 OTHER ASSETS

2.4 BANK VALUATION

In this chapter, we construct discrete-time models for bank loans and their supply, demand and losses. Furthermore, we discuss the provisioning for these loan losses and banking items related to them. The main problems that are solved in this chapter can be formulated as follows.

Problem 2.0.1 (Bank Valuation and Loan Losses): *How can we model the value of a bank and quantify losses from its lending activities? (Sections 2.1 and 2.4).*

Problem 2.0.2 (Optimal Bank Valuation Problem): *Which decisions about loan rates, deposits and Treasuries must be made in order to attain an optimal bank value for a stockholder (possibly acting in the interests of a potential shareholder)? (Theorem 2.4.2 in Section 2.4).*

2.1 LOANS AND THEIR DEMAND AND SUPPLY

In this section, we discuss bank loans and their supply and demand.

2.1.1 Bank Loans

We suppose that, after providing for liquidity, the bank lends funds in the form of t -th period loans, Λ_t , at the *interest rate on loans* or *loan rate*, r_t^Λ . Profit maximizing banks set their loan rates, r^Λ , as a sum of the risk-free Treasuries rate, r^\top , the *expected loan loss ratio*, $\mathbf{E}(d)$, and of the *risk premium*, k . Furthermore, expressing the expected losses, $\mathbf{E}(d)$, as a rate of return per unit time, we obtain the expression

$$r_t^\Lambda = r_t^\top + k + \mathbf{E}(d).$$

The sum $r_t^\top + k$ provides the remuneration for the cost of monitoring and screening of loans and of capital, c^Λ . The $\mathbf{E}(d)$ component is the amount of provisioning that is needed to match the average expected losses faced by the loans. The representation of the banks' interest setting shows that banks will experience positive returns in good times when the *actual rate of default*, r^d , is lower than the provisioning for expected losses, $\mathbf{E}(d)$, and will not be able to cover their expected losses when $r^d > \mathbf{E}(d)$. In the latter case, bank capital may be needed to cover these excess (and unexpected) losses. If this capital is not enough then the bank will face insolvency.

Next, we introduce the generic variable, M_t , that represents the level of macro-economic activity in the bank's loan market. We suppose that $M = \{M_t\}_{t \geq 0}$ follows the first-order autoregressive stochastic process

$$M_{t+1} = \mu^M M_t + \sigma_{t+1}^M,$$

where σ_{t+1}^M denotes zero-mean stochastic shocks to macro-economic activity.

2.1.2 Bank Loan Supply and Demand

In this subsection, we provide a brief discussion of loan demand and supply. Taking our lead from the equilibrium arguments in [68], we denote both these credit price processes by $\Lambda = \{\Lambda_t\}_{t \geq 0}$. In this case, the bank faces a *Hicksian demand for loans* given by

$$\Lambda_t = l_o - l_1 r_t^\Lambda + l_2 M_t + \sigma_t^\Lambda. \quad (2.1)$$

We note that the loan demand in (2.1) is an increasing function of M and a decreasing function of r_t^Λ . Further, we suppose that σ_t^Λ is the *random shock to the loan demand* with support $[\underline{\Lambda}, \bar{\Lambda}]$ that is independent of an exogenous stochastic variable, x_t , to be characterized below. Also, we assume that the *loan supply process*, Λ , follows the first-order autoregressive stochastic process

$$\Lambda_{t+1} = \mu_t^\Lambda \Lambda_t + \sigma_{t+1}^\Lambda, \quad (2.2)$$

where $\mu_t^\Lambda = r_t^\Lambda + k + \mathbf{E}(d) - c^\Lambda - r^d(M_t)$ and σ_{t+1}^Λ denotes zero-mean stochastic shocks to loan supply.

2.2 LOAN LOSSES AND PROVISIONING

The bank's investment in loans may yield substantial returns but may also result in loan losses. In line with reality, our dynamic bank model allows for loan losses for which provision can be made. *Total loan loss provisions*, P , mainly affects the bank in the following ways. Reported net profit will be less for the period in which the provision is taken. If the bank eventually writes off the asset, the write-off will reduce

taxes and thus increase the banks cash flows. Empirical evidence suggests that P is affected by macro-economic activity, M , so that the notation $P(M_t)$ for period t loan loss provisioning is in order (see, for instance, [17] and [19]). In this section, we discuss these issues in more detail.

2.2.1 Loan Losses

An initial observation is that loan losses are also dependent on macro-economic activity. As a consequence, for the *value of loan losses*, L , and the *default rate*, r^d , we set

$$L(M_t) = r^d(M_t)\Lambda_t, \quad (2.3)$$

where $r^d \in [0, 1]$ increases when macro-economic conditions deteriorate according to

$$0 \leq r^d(M_t) \leq 1, \quad \frac{\partial r^d(M_t)}{\partial M_t} < 0.$$

We note that the above description of the loan loss rate is consistent with empirical evidence that suggests that bank losses on loan portfolios are correlated with the business cycle under any capital adequacy regime (see, for instance, [17], [19], [22] and [47]).

2.2.2 Loan Loss Provisioning

As was mentioned in [17] (see, also, [22] and [47]), provisions for expected loan losses, $\mathbf{E}[d]\Lambda_t$, and capital, K , act as buffers against expected and unexpected loan losses, respectively. Next, we distinguish between total provisioning for loan losses, P , and loan loss reserves, R^l . Provisioning is a decision made by bank management about the size of the buffer that must be set aside in a particular time period in order to cover loan losses, L . However, not all of P may be used in a time period with the amount left over constituting *loan loss reserves*, R^l , so that for period t we have

$$R_t^l = P(M_t) - L(M_t).$$

The contribution [17] considers the following strategy to be optimal for banks to shield their profits from loan losses. The loan loss reserves, R^l , is built up in every period that $P > L$. On the other hand, when $P = L$ the bank is allowed to draw on R^l from the current period and for $L > P$ it has to access βK , where $\beta \in [0, 1]$ is the proportion of the bank capital, K , including loan loss reserves used to deal with unexpected losses. For the latter scenario, at some point the bank will face insolvency. As a consequence of adopting this strategy, our model for provisioning in period $t + 1$ is taken to be

$$P(M_{t+1}) = \begin{cases} \mathbf{E}[d]\Lambda_t, & \text{for } P > L \text{ Expected Losses} \\ \mathbf{E}[d]\Lambda_t + R_{t+1}^l, & \text{for } P = L \text{ Expected Losses} \\ \mathbf{E}[d]\Lambda_t + \beta K_{t+1}, & \text{for } L > P \text{ Expected + Unexpected Losses} \end{cases}$$

We note that our model determines the provisions for the period $t + 1$ in the t -th period which is a very reasonable assumption. Our suspicion is that provisioning, P , is a decreasing function of current macro-economic conditions, M , so that

$$\frac{\partial P(M_t)}{\partial M_t} < 0. \quad (2.4)$$

This claim has resonance with the idea of procyclicality where we expect the provisioning to decrease during booms, when macro-economic activity increases. By contrast, provisioning may increase during recessions because of an elevated probability of default and/or loss given default on loans. This suspicion is confirmed in Chapter 4 where empirical data from OECD countries comparing macro-economic activity (via the output gap) and provisioning (via the provisions-to-total assets ratio) is examined.

2.3 OTHER ASSETS

In this section, we discuss intangible assets, Treasuries, reserves and risk-weighted assets that are all categories of banking assets.

2.3.1 Intangible Assets

In the contemporary banking industry, shareholder value is often created by *intangible assets* which consist of patents, trademarks, brand names, franchises and economic goodwill (more specifically, core deposit customer relationships, customer loan relationships as differentiated from the loans themselves, etc.). Economic goodwill consists of the intangible advantages a bank has over its competitors such as an excellent reputation, strategic location, business connections, etc. In addition, such assets can comprise a large part of the bank's total assets and provide a sustainable source of wealth creation. Intangible assets are used to compute Tier 1 bank capital and have a risk weight of 100 % according to Basel II regulation (see Table 2.1 below). In practice, valuing these off-balance sheet items constitutes one of the principal difficulties with the process of bank valuation by a stock analyst. The reason for this is that intangibles may be considered to be risky assets for which the future service potential is hard to measure. Despite this, our model assumes that the measurement of these intangibles is possible (see, for instance, [38] and [71]). As we mentioned in Chapter 1, we denote the value of intangible assets, in the t -th period, by I_t and the return on these assets by $r_t^I I_t$.

2.3.2 Treasuries

Treasuries, T_t , coincide with securities that are issued by national Treasuries at a rate denoted by r^T . In essence, they are the debt financing instruments of the government. There are four types of Treasuries, viz., Treasury bills, Treasury notes, Treasury bonds and savings bonds. All of the treasury securities besides savings bonds are very liquid and are heavily traded on the secondary market.

2.3.3 Reserves

Bank reserves are the deposits held in accounts with the central bank of a country (for instance, the South African Reserve Bank in the case of South Africa) plus money that is physically held by banks (vault cash). Such reserves constitute money that is not lent out but is earmarked to cater for withdrawals by depositors. Since it is uncommon for depositors to withdraw all of their funds simultaneously, only a portion of total deposits may be needed as reserves. As a result of this description, we may introduce a *reserve-deposit ratio*, γ , for which

$$R_t = \gamma D_t. \quad (2.5)$$

The bank uses the remaining deposits to earn profit, either by issuing loans or by investing in assets such as Treasuries and stocks.

2.3.4 Risk-Weighted Assets

We consider risk-weighted assets (RWAs) that are defined by placing each on- and off-balance sheet item into a risk category. The more risky assets are assigned a larger weight in this study. Table 2.1 below provides a few illustrative risk categories, their risk weights and representative items.

Risk Category	Risk-Weight	DFI Items
1	0 %	Cash, Reserves, Bonds
2	20 %	Shares
3	50 %	Home Loans
4	100 %	Intangible Assets
5	100 %	Loans to Private Agents

Table 2.1: Risk Categories, Risk-Weights and Representative Items

As a result, RWAs are a weighted average of the various assets of the banks. In the sequel, we denote the risk weight on intangible assets, loans, Treasuries and reserves by ω^I , ω^A , ω^T and ω^R , respectively. With regard to the latter, we can identify a special risk weight on loans, $\omega^A = \omega(M_t)$, that is a decreasing function of current macro-economic conditions so that

$$\frac{\partial \omega(M_t)}{\partial M_t} < 0.$$

This is in line with the procyclical notion that during booms, when macro-economic activity increases, the risk weights will decrease. On the other hand, during recessions, risk weights may increase because of an elevated probability of default and/or loss given default on loans.

2.4 BANK VALUATION

In this section, we discuss bank regulatory capital, binding capital constraints, retained earnings and the valuation of a bank by a stock analyst on behalf of a potential shareholder.

2.4.1 Bank Regulatory Capital

In this subsection, we provide a general description of bank capital and then specify the components of total bank capital that we use in this particular chapter and related commentary.

2.4.1.1 General Description of Bank Capital

According to Basel II, three types of capital can be identified, viz., Tier 1, 2 and 3 capital, which we describe in more detail below. *Tier 1 capital* comprises ordinary share capital (or equity) of the bank and audited revenue reserves, e.g., retained earnings less current year's losses, future tax benefits and intangible assets (for more information see, for instance, [38] and [71]). Tier 1 capital or core capital acts as a buffer against losses without a bank being required to cease trading. *Tier 2 capital* includes unaudited retained earnings; revaluation reserves; general provisions for bad debts (e.g., loan loss reserves); perpetual cumulative preference shares (i.e., preference shares with no maturity date whose dividends accrue for future payment even if the bank's financial condition does not support immediate payment) and perpetual subordinated debt (i.e. debt with no maturity date which ranks in priority behind all creditors except shareholders). Tier 2 capital or supplementary capital can absorb losses in the event of a wind-up and so provides a lesser degree of protection to depositors. *Tier 3 capital* consists of subordinated debt with a term of at least 5 years and redeemable preference shares which may not be redeemed for at least 5 years. Tier 3 capital can be used to provide a hedge against losses caused by market risks if Tier 1 and Tier 2 capital are insufficient for this.

2.4.1.2 Specific Components of Total Bank Capital

For the purposes of our study, *regulatory capital*, K , is the book value of bank capital defined as the difference between the accounting value of the assets and liabilities. More specifically, Tier 1 capital is represented by period $t - 1$'s market value of the bank equity, $n_t E_{t-1}$, where n_t is the number of shares and E_t is the period t market

price of the bank's common equity. Tier 2 capital mainly consists of subordinate debt, O_t , that is subordinate to deposits and hence faces greater default risk and loan loss reserves, R_t^l . Subordinate debt issued in period $t - 1$ are represented by a one-period bond that pays an interest rate, r^O . Also, we assume that loan loss reserves held in period $t - 1$ changes at the rate, r^{R^l} . Tier 3 capital is not considered at all. In the sequel, we take the bank's total regulatory capital, K , in period t to be

$$K_t = n_t E_{t-1} + O_t + R_t^l. \quad (2.6)$$

For K_t given by (2.6), we obtain the balance sheet constraint

$$W_t = D_t - \Lambda_t + K_t. \quad (2.7)$$

2.4.1.3 Binding Capital Constraints

We define the *regulatory capital constraint* by the inequality

$$K_t \geq \rho a_t + mVaR, \quad (2.8)$$

where a_t is the sum of the risk-weighted assets, $\rho = 0,08$ and $mVaR$ is as described in Chapter 1. In this case, we have that

$$a_t = \omega^I I_t + \omega^\Lambda \Lambda_t + \omega^T T_t + \omega^R R_t$$

where ω^I , ω^Λ , ω^T and ω^R are the risk weights for intangible assets, loans, Treasuries and reserves, respectively. We assume that the risk weights associated with intangible assets, Treasuries, reserves and loans may be taken to be $\omega^I \neq 0$, $\omega^T = \omega^R = 0$ and $\omega^\Lambda = \omega(M_t)$, respectively, so that equation (2.8) becomes

$$K_t \geq \rho[\omega(M_t)\Lambda_t + \omega^I I_t] + mVaR \quad (2.9)$$

2.4.2 Profits and Retained Earnings

In this subsection, we discuss profits and its relation to retained earnings.

2.4.2.1 Profits

We assume that (2.3) holds. If we now add and subtract $r_t^T \gamma D_t$ from (1.1) and use the fact that $W_t = T_t + \gamma D_t$, we obtain

$$\begin{aligned} \Pi_t = & \left(r_t^A - c^A - r^d(M_t) \right) \Lambda_t + r_t^T W_t + r_t^I I_t - \left(r_t^D + c^D \right) D_t \\ & - c^w(W_t) - P(M_t) - r_t^T \gamma D_t. \end{aligned} \quad (2.10)$$

This is the cash flow constraint for a bank and will be used later. Furthermore, by considering (2.4) and (2.10), we suspect that profit, Π , is an increasing function of current macro-economic conditions, M , so that

$$\frac{\partial \Pi_t}{\partial M_t} > 0.$$

This is connected with procyclicality where we expect profitability to increase during booms, when macro-economic activity increases. By contrast, profitability may decrease during recessions because of, among many other factors, an increase in provisioning (see equation (2.10)). Importantly, examples of this phenomenon is provided in Chapter 4 where the correlation between macro-economic activity, provisioning and profitability is illustrated.

2.4.2.2 Profits and Its Relationship with Retained Earnings

To establish the relationship between bank profitability and retained earnings, a model of bank financing is introduced that is based on [2]. We know that *bank profits*, Π_t , are used to meet the bank's commitments that include *dividend payments on equity*, $n_t d_t$, *interest and principal payments on subordinate debt*, $(1 + r_t^O) O_t$, and *changes in loan-loss reserves*, $(1 + r_t^{R^l}) R_t^l$. The *retained earnings*, E_t^r , subsequent to these payments may be computed by using

$$\Pi_t = E_t^r + n_t d_t + (1 + r_t^O) O_t + (1 + r_t^{R^l}) R_t^l. \quad (2.11)$$

In standard usage, retained earnings refer to earnings that are not paid out in dividends, interest or taxes. They represent wealth accumulating in the bank and should be capitalized in the value of the bank's equity. Retained earnings are also defined

to include bank charter value income. Normally, *charter value* refers to the present value of anticipated profits from future lending.

In each period, banks invest in fixed assets (including buildings and equipment) which we denote by F_t . The bank is assumed to maintain these assets throughout its existence so that the bank must only cover the costs related to the *depreciation of fixed assets*, ΔF_t . These activities are financed through retaining earnings and the eliciting of additional debt and equity, so that

$$\Delta F_t = E_t^r + (n_{t+1} - n_t)E_t + O_{t+1} + R_{t+1}^l. \quad (2.12)$$

We can use (2.11) and (2.12) to obtain an expression for bank capital of the form

$$K_{t+1} = n_t(d_t + E_t) + (1 + r_t^O)O_t + (1 + r_t^{R^l})R_t^l - \Pi_t + \Delta F_t, \quad (2.13)$$

where K_t is defined by (2.6).

2.4.3 Bank Valuation by a Stock Analyst

If the expression for retained earnings given by (2.11) is substituted into (2.12), the *net cash flow generated by the bank for a shareholder* is given by

$$N_t = \Pi_t - \Delta F_t = n_t d_t + (1 + r_t^O)O_t + (1 + r_t^{R^l})R_t^l - K_{t+1} + n_t E_t. \quad (2.14)$$

In addition, we have the relationship

$$\text{Bank Value for a Shareholder} = \text{Net Cash Flow} + \text{Ex-Dividend Bank Value}$$

This translates to the expression

$$V_t = N_t + K_{t+1}, \quad (2.15)$$

where K_t is defined by (2.6). Furthermore, the stock analyst (acting in the interest of a shareholder) evaluates the expected future cash flows in j periods based on a *stochastic discount factor*, $\delta_{t,j}$ such that the value of the bank is

$$V_t = N_t + \mathbf{E}_t \left[\sum_{j=1}^{\infty} \delta_{t,j} N_{t+j} \right]. \quad (2.16)$$

2.4.4 Bank Valuation: Related Items

In this subsection, we consider deposits and provisioning for deposit withdrawals.

2.4.4.1 Deposits

The bank takes deposits, D_t , at a constant *marginal cost*, c^D , that may be associated with cheque clearing and bookkeeping. It is assumed that deposit taking is not interrupted even in times when the *interest rate on deposits* or *deposit rate*, r_t^D , is less than the *interest rate on Treasuries* or *bond rate*, r_t^T . We suppose that the dynamics of the deposit rate process, $r^D = \{r_t^D\}_{t \geq 0}$, is determined by the first-order autoregressive stochastic process

$$r_{t+1}^D = \mu^{r^D} r_t^D + \sigma_{t+1}^{r^D},$$

where $\sigma_{t+1}^{r^D}$ is zero-mean stochastic shocks to the deposit rate.

2.4.4.2 Provisioning for Deposit Withdrawals

We have to consider the possibility that unanticipated deposit withdrawals will occur. By way of making provision for these withdrawals, the bank is inclined to hold Treasuries and reserves that are both very liquid. In our contribution, we assume that the *unanticipated deposit withdrawals*, u , originates from the *probability density function*, $f(u)$, that is independent of time. For sake of argument, we suppose that the unanticipated deposit withdrawals have a uniform distribution with support $[\underline{D}, \overline{D}]$ so that the *cost of liquidation*, c^l , or additional external funding is a quadratic function of the sum of Treasuries and reserves, W . In addition, for any t , if we have that

$$u > W_t,$$

where $W_t = T_t + R_t$, then bank assets are liquidated at some *penalty rate*, r_t^p . In this case, the *cost of deposit withdrawals* is

$$c^w(W_t) = r_t^p \int_{W_t}^{\infty} [u - W_t] f(u) du = \frac{r_t^p}{2\bar{D}} [\bar{D} - W_t]^2.$$

2.4.5 Optimal Bank Valuation

In this subsection, we make use of the modelling of the preceding discussion to solve an optimal bank valuation problem.

2.4.5.1 Statement of the Optimal Bank Valuation Problem

In the sequel, suppose that the bank's performance criterion, J , at t is given by

$$J_t = \Pi_t + l_t \left[K_t - \rho[\omega(M_t)\Lambda_t + \omega^I I_t] - mVaR \right] - c_t^{dw} \left[K_{t+1} \right] + \mathbf{E}_t \left[\delta_{t,1} V \left(K_{t+1}, x_{t+1} \right) \right], \quad (2.17)$$

where l_t is the Lagrangian multiplier for the total capital constraint (2.9), K_t is defined by (2.6), $\mathbf{E}_t[\cdot]$ is the expectation conditional on the bank's information at time t and x_t is the deposit withdrawals in period t with probability distribution $f(x_t)$. Also, c_t^{dw} is the deadweight cost of total capital consisting of debt and equity. We are now in a position to formally state the optimal valuation problem for banks that we solve in the sequel.

Problem 2.4.1 (Statement of the Optimal Bank Valuation Problem): *Suppose that the total capital constraint and the performance criterion, J , are given by (2.9) and (2.17), respectively. The optimal bank valuation problem is to maximize the value of the bank given by (2.16) from the point of view of a stock analyst, by choosing the loan rate, deposits and regulatory capital for*

$$V(K_t, x_t) = \max_{r_t^\Lambda, D_t, K_t} J_t, \quad (2.18)$$

subject to the loan demand, balance sheet, cash flow and financing constraints given by (2.1), (2.7), (2.10) and (2.13), respectively.

2.4.5.2 Solution to the Optimal Bank Valuation Problem for Expected Losses

In this subsection, we find a solution to Problem 2.4.1 when the capital constraint is binding. In this regard, the main result can be stated and proved as follows.

Theorem 2.4.2 (Solution to the Optimal Bank Valuation Problem): *Suppose that J and V are given by (2.17) and (2.18), respectively and $P(M_t) = \mathbf{E}(d)\Lambda_{t-1}$. When the capital constraint given by (2.9) is binding (i.e., $l_t > 0$), a solution to the optimal bank valuation problem stated in Problem 2.4.1 yields an optimal bank loan supply and loan rate of the form*

$$\Lambda_t^* = \frac{K_t - mVaR}{\rho\omega(M_t)} - \frac{\omega^I I_t}{\omega(M_t)} \quad (2.19)$$

and

$$r_t^{\Lambda^*} = \frac{1}{l_1} \left(l_0 + l_2 M_t + \sigma_t^\Lambda - \frac{K_t - mVaR}{\rho\omega(M_t)} + \frac{\omega^I I_t}{\omega(M_t)} \right), \quad (2.20)$$

respectively. In this case, the corresponding optimal deposits, provisions for deposit withdrawals and profits are given by

$$D_t^* = \bar{D} + \frac{\bar{D}(1-\gamma)}{r_t^p} \left[r_t^r - \frac{(r_t^D + c^D)}{1-\gamma} \right] + \frac{K_t - mVaR}{\rho\omega(M_t)} - \frac{\omega^I I_t}{\omega(M_t)} - K_t,$$

$$W_t^* = \bar{D} + \frac{\bar{D}(1-\gamma)}{r_t^p} \left[r_t^r - \frac{(r_t^D + c^D)}{1-\gamma} \right]$$

and

$$\begin{aligned}
\Pi_t^* = & \left(\frac{K_t - mVaR}{\rho\omega(M_t)} - \frac{\omega^I I_t}{\omega(M_t)} \right) \left\{ \frac{1}{l_1} \left(l_0 - \frac{K_t - mVaR}{\rho\omega(M_t)} + \frac{\omega^I I_t}{\omega(M_t)} + l_2 M_t + \sigma_t^\Lambda \right) \right. \\
& \left. - \left(c^\Lambda + (r_t^D + c^D + r_t^T \gamma) + r^d(M_t) \right) \right\} \\
& + (r_t^D + c^D + r_t^T \gamma) K_t + \bar{D} \left(r_t^T - (r_t^D + c^D + r_t^T \gamma) \right) \\
& + \left(r_t^T - \frac{(r_t^D + c^D)}{1 - \gamma} \right) \left[\frac{\bar{D}(1 - \gamma)(r_t^T - (r_t^D + c^D + r_t^T \gamma))}{r_t^p} \right] - c^w(W_t) - P(M_t) + r_t^I I_t,
\end{aligned}$$

respectively.

Proof. An immediate consequence of the prerequisite that the total capital constraint from (2.9) is binding, is that loan supply is closely related to the capital adequacy constraint and is given by (2.19). Also, the dependence of changes in the loan rate on macro-economic activity may be fixed as

$$\frac{\partial r_t^\Lambda}{\partial M_t} = \frac{l_2}{l_1}.$$

Equation (2.19) follows from (2.9) and the fact that the capital constraint is binding which leads to equality in (2.9). In (2.20) we substituted the optimal value for Λ_t into control law (2.1) to get the optimal default rate. We obtain the optimal W_t using the following steps. Firstly, we rewrite (2.7) to make deposits the dependent variable so that

$$D_t = W_t + \Lambda_t - K_t.$$

Next, we note that the first order conditions are given by

$$\frac{\partial \Pi_t}{\partial r_t^\Lambda} \left[1 + c_t^{dw} - \mathbf{E}_t \left\{ \int_{\underline{\Lambda}}^{\bar{\Lambda}} \delta_{t,1} \frac{\partial V}{\partial K_{t+1}} dF(\sigma_{t+1}^\Lambda) \right\} \right] + l_1 \rho^r \omega(M_t) \Lambda_t l_t = 0; \quad (2.21)$$

$$\frac{\partial \Pi_t}{\partial D_t} \left[1 + c_t^{dw} - \mathbf{E}_t \left\{ \int_{\underline{\Lambda}}^{\bar{\Lambda}} \delta_{t,1} \frac{\partial V}{\partial K_{t+1}} dF(\sigma_{t+1}^\Lambda) \right\} \right] = 0; \quad (2.22)$$

$$l_t(mVaR + \rho[\omega(M_t)\Lambda_t + \omega^I I_t]) \leq K_t; \quad (2.23)$$

$$-c_t^{dw} + \mathbf{E}_t \left\{ \int_{\underline{\Lambda}}^{\bar{\Lambda}} \delta_{t,1} \frac{\partial V}{\partial K_{t+1}} dF(\sigma_{t+1}^\Lambda) \right\} = 0. \quad (2.24)$$

Here $F(\cdot)$ is the cumulative distribution of the shock to the loans. Using (2.24) we can see that (2.22) becomes

$$\frac{\partial \Pi_t}{\partial D_t} = 0.$$

Looking at the form of Π_t given in (2.10) and the equation

$$c^w(W_t) = \frac{r_t^p}{2\bar{D}} [\bar{D} - W_t]^2$$

it follows that

$$\Pi_t = (r_t^\Lambda - c^\Lambda - r^d(M_t))\Lambda_t + r_t^T W_t + r_t^I I_t - (r_t^D + c^D)D_t - \frac{r_t^p}{2\bar{D}} [\bar{D} - W_t]^2 - P(M_t) - r_t^T \gamma D_t.$$

Therefore

$$\frac{\partial \Pi_t}{\partial D_t} = -(r_t^D + c^D) + \frac{r_t^p}{2\bar{D}^2} [\bar{D} - W_t]^2 - \frac{r_t^p}{\bar{D}} [\bar{D} - W_t] - r_t^T \gamma = 0.$$

This would then give us the optimal value for D_t . Using (2.7) and all the optimal values calculated to date, we can find optimal deposits, and the same goes for optimal profits.

□

Chapter 3

A LÉVY PROCESS-DRIVEN BANKING MODEL

3.1 ASSETS

3.2 LIABILITIES

3.3 MODELLING OF BANK REGULATORY CAPITAL

In this chapter, we construct Lévy process-driven stochastic dynamic models of bank assets, A , (uses of funds), liabilities, Γ , (sources of funds) and regulatory capital. The main problems that are solved in this chapter can be formulated as follows.

Problem 3.0.3 (Capital-to-Total Assets Ratio): *Can we deduce a Lévy-process driven model of the capital-to-total assets ratio?* (Theorem 3.3.1 in Subsection 3.3.3).

Problem 3.0.4 (Capital-to-Risk-Weighted Assets Ratio): *Can we deduce a Lévy-process driven model of the capital-to-risk-weighted assets ratio?* (Theorem 3.3.2 and Corollary 3.3.3 in Subsection 3.3.3).

Throughout this chapter, we discuss a bank with a planning horizon equal to $B \in \mathbb{Z}_+$ back-testing periods which, in turn, is divided into n non-intersecting reporting periods of equal length β .

3.1 ASSETS

In this subsection, we discuss bank asset price processes and unweighted and risk-weighted assets. In order to model the uncertainty associated with these items we consider the filtered probability space $(\Omega_1, \mathbf{G}, (\mathcal{G}_t)_{0 \leq t \leq T}, \mathbf{P}_1)$.

3.1.1 Bank Asset Price Processes

The bank's investment portfolio is constituted by $m + 1$ assets including loans and Treasuries. We pick the first asset to be the riskless Treasuries, \mathbb{T} , that earns a constant, continuously-compounded interest rate of $r^{\mathbb{T}}$. Profit maximizing banks set their rates of return on assets as a sum of the risk-free Treasuries rate, $r^{\mathbb{T}}\bar{\mathbf{1}}$, risk premium, μ , and the default premium, $\mathbf{E}(d)$. Here the unitary vector and risk premium are given by

$$\bar{\mathbf{1}} = (1, 1, \dots, 1)^T \text{ and } \mu = (\mu_1, \mu_2, \dots, \mu_m)^T,$$

respectively. Also, we have that the default premium is defined by

$$\mathbf{E}(d) = (\mathbf{E}(d_1), \mathbf{E}(d_2), \dots, \mathbf{E}(d_m))^T, \text{ where } \begin{cases} \mathbf{E}(d_i) \neq 0 & i\text{-th asset is a loan,} \\ \mathbf{E}(d_i) = 0 & i\text{-th asset is not a loan.} \end{cases}$$

The sum $r_t^T \bar{1} + \mu$ covers, for instance, the cost of monitoring and screening of loans and cost of capital. The $\mathbf{E}(d)$ component corresponds to the amount of provisioning that is needed to match the average expected losses faced by the loans. The m assets besides Treasuries are risky and their price process, S (reinvested dividends included), follows a geometric Lévy process with drift vector, $r^T \bar{1} + \mu + \mathbf{E}(d)$ and diffusion matrix, σ , as in

$$S_t = S_0 + \int_0^t I_s^S \left(r^T \bar{1} + \mu + \mathbf{E}(d) \right) ds + \int_0^t I_s^S \sigma dL_s + \sum_{0 < s \leq t} \Delta S_s 1_{\{|\Delta S_s| \geq 1\}}, \quad (3.1)$$

where I_t^S denotes the $m \times m$ diagonal matrix with entries S_t and L is an m -dimensional Lévy process. Also, ΔS_s is the jump of the process S at time $t > 0$ and $1_{\{|\Delta S_s| \geq 1\}}$ is the indicator function of $\{|\Delta S_s| \geq 1\}$. We suppose, without loss of generality, that $\text{rank}(\sigma) = m$ and the bank is allowed to engage in continuous frictionless trading over the planning horizon, $[0, T]$.

3.1.2 Bank Asset Portfolio

In the sequel, we suppose that ρ is the m -dimensional stochastic process that represents the *current value of risky assets*. In this case, the dynamics of the *current value of the bank's entire asset portfolio*, A , over any reporting period may be given by

$$dA_t = A_t \left\{ r^T + \rho^T \left(\mu + \mathbf{E}(d) \right) \right\} dt + A_t \sigma dL_t - r^T D_t dt \quad (3.2)$$

where the face value of the deposits, D , is as described in Subsection 3.2 below, and $r^T D_t dt$ represents the interest paid to depositors.

3.1.3 Risk-Weighted Assets

The charge to cover credit risk equals the sum of the bank's long and short trading positions multiplied by asset specific risk weights. As a result, if we let $\omega \in [0, 1]^m$ denote the $m \times 1$ vector of asset risk weights, then the *capital charge to cover credit risk* at time t equals

$$\omega^T \left(\rho_t^+ + \rho_t^- \right), \quad (3.3)$$

where for any ρ we denote by ρ^+ the $m \times 1$ vector with components

$$\rho_i^+ = \max[0, \rho_i]$$

and by ρ^- the $m \times 1$ vector with components

$$\rho_i^- = \max[0, -\rho_i].$$

3.2 LIABILITIES

The bank has liabilities represented by deposits. For simplicity, we assume that the face value (or outstanding value) of deposits, D , is fixed over the planning horizon and that there are no equity issues or dividend payments over this period. Deposits are fully insured and earn the risk-free Treasuries rate, $r^T \geq 0$, which is paid out continuously to depositors. The cost to insure deposits is included in the cost of deposits.

3.3 MODELLING OF BANK REGULATORY CAPITAL

In this subsection, we discuss bank regulatory capital, K (see, for instance [28]), and its stochastic dynamics as well as capital adequacy.

3.3.1 Description of Bank Regulatory Capital

Let us define the bank's regulatory capital, K , as

$$K_t = A_t - D_t, \tag{3.4}$$

where A is the current value of the asset portfolio and D is the face value of the deposits. The bank is required to maintain this capital above a minimum level equal to the sum of the charge to cover general market risk plus a charge to cover credit risk plus a charge to cover operational risk (see, for instance, [11] and Appendix 8.1).

We suppose that the bank reports its current VaR at the beginning of each reporting period as well as its recorded profits and losses from the previous reporting period. The charge to cover market risk equals the VaR reported at the beginning of the current reporting period times a multiple k . As a consequence of the above, if $VaR \geq 0$ denotes the VaR reported to regulators at the beginning of the current reporting period and k is the currently-applicable multiple, the bank must satisfy the constraint

$$K_t \geq kVaR + \omega^T \left(\rho_t^+ + \rho_t^- \right) + \max \left[\sum_{k=1}^8 \beta_k g_k, 0 \right], \quad (3.5)$$

at all times during the reporting period. The reported VaR can differ from the true VaR since the bank's future trading strategy, and hence the bank's true VaR, are unobservable by regulators. In inequality (3.5), the term

$$\max \left[\sum_{k=1}^8 \beta_k g_k, 0 \right]$$

is the charge to cover operational risk under the standardized approach from Basel II (see Appendix 8.1 for more details).

3.3.2 Stochastic Dynamics of Bank Regulatory Capital

From this point forward we do not consider the operational risk (compare the last term in (3.5)), since it may be considered to be constant over all reporting periods. The bank incurs a non-financial cost, c , at the end of each reporting period in which the actual loss exceeds the reported VaR. This cost is meant to capture additional regulatory actions that can be undertaken in response to exceptions (besides the increase in the reserve multiple k) or reputation losses. For simplicity, we refer to these costs simply as reputation costs and assume that they are proportional to the amount by which the actual loss exceeds the reported VaR, that is,

$$c = \lambda(K_b - K_e - VaR)^+,$$

where $\lambda \geq 0$ is the proportional cost. Also, K_b and K_e is the value of the bank's regulatory capital at the beginning and the end of the reporting period, respectively. At the end of each back-testing period, the number $i = 0, 1, \dots, n$, of reporting periods

in which the actual loss exceeded the reported VaR is computed, and the capital reserve multiple, k , for the next back-testing period is set equal to $k(i)$, for given positive numbers

$$k(0) \leq k(1) \leq k(2) \leq \dots \leq k(n).$$

Clearly, reputation costs and the revision of the capital reserve multiple, k , at the end of each back-testing period represent incentives to not under-report the true VaR. On the other hand, capital requirements provide an incentive to not over-report.

Besides the market risk represented by the normal fluctuations in the value of its assets (as given in (3.2)), the bank is subject to unhedgeable credit risk. In this regard, at the end of every reporting period, there is a small probability, p , that a rare event will occur resulting in the loss of an amount equal to qK_b , where $q \in [0, 1]$. These rare events can force the bank into default if they result in the value of the bank's capital becoming negative (implying that the bank will be unable to repay its deposits). While these shocks are unhedgeable, the bank can control the probability of default by controlling the probability of losses in the market value of its assets exceeding $(1 - q)K_b$ in any given period (that is, by avoiding very risky investment strategies). Since the market value of deposits is fixed and there are no new equity issues, it follows from (3.2) that the bank's regulatory capital satisfies

$$dK_t = K_t \left\{ r^T + \rho^T \left(\mu + \mathbf{E}(d) \right) \right\} dt + K_t \rho^T \sigma dL_t \quad (3.6)$$

in the absence of rare events.

3.3.3 Capital Adequacy Ratios

In this subsection, we deduce the dynamics of the capital-to-total assets ratio and the capital-to-risk-weighted ratio.

3.3.3.1 Stochastic Dynamics of the Capital-to-Total Assets Ratio

Next we derive a stochastic differential equation (SDE) for the capital-to-total assets ratio. We start by recalling that the current value of assets, A , and bank regulatory capital, K , are given by equations (3.2) and (3.6), respectively. In this case, we easily obtain a SDE for the dynamics of the capital-to-total assets ratio as in the following

result.

Theorem 3.3.1 (Computation of the Capital-to-Total Assets Ratio): *Let (3.2) and (3.6) be given. Then we have that the dynamics of the capital-to-total assets ratio has the form*

$$d\kappa_t^T = \kappa_t^T \left\{ \left(-2 \left[r^T + \rho^T \left(\mu + \mathbf{E}(d) \right) \right] - 2(\rho^T)^2 \sigma^4 - A_t^{-1} r^T D_t \right) dt + 2\rho^T \sigma dL_t \right\}. \quad (3.7)$$

Proof. The proof is a straightforward application of Ito's formula. For (3.2) and (3.6), we define the capital-to-total assets ratio as

$$\kappa^T = \frac{K}{A}.$$

Next, in the proof, we apply Ito's formula to find the dynamics of the capital-to-total assets ratio as given above. We set $U = A_t^{-1}$ and then we have

$$d\kappa_t^T = \frac{\partial \kappa_t^T}{\partial K_t} dK_t + \frac{\partial \kappa_t^T}{\partial U} dU + \frac{1}{2} \frac{\partial^2 \kappa_t^T}{\partial K_t^2} (dK_t)^2 + \frac{\partial^2 \kappa_t^T}{\partial K_t \partial U} dK_t dU + \frac{1}{2} \frac{\partial^2 \kappa_t^T}{\partial U^2} (dU)^2.$$

Also, we have that

$$\frac{\partial \kappa_t^T}{\partial K_t} = U = A_t^{-1}, \quad \frac{\partial \kappa_t^T}{\partial U} = K_t; \quad \frac{\partial^2 \kappa_t^T}{\partial K_t^2} = 0; \quad \frac{\partial^2 \kappa_t^T}{\partial K_t \partial U} = 1; \quad \frac{\partial^2 \kappa_t^T}{\partial U^2} = 0.$$

So it follows that

$$d\kappa_t^T = A_t^{-1} dK_t + K_t dA_t^{-1} + dK_t dA_t^{-1}.$$

In addition, we have

$$dA_t^{-1} = -A_t^{-2} dA_t + A_t^{-3} dA_t \cdot dA_t.$$

We now use the above equations to obtain an expression for $d\kappa_t^T$ as in (3.7). \square

3.3.3.2 Dynamics of the Capital-to-Risk-Weighted Assets Ratio

For sake of argument, our study considers a simplified version of the capital-to-risk-weighted assets ratio, κ , of the form

$$\kappa = \frac{K}{A^r}. \quad (3.8)$$

In other words, in the calculation of the capital-to-risk-weighted assets ratio, the total risk charge is only constituted by the credit risk charge with the capital charges for market and operational risk not being included (see, for instance, [11] and Appendix 8.1). By way of justifying this simplification, we may consider the capital charges for market and operational risk to be invariant over the reporting period and hence of lesser importance dynamically.

Next, we derive an expression for the capital-to-risk-weighted assets ratio by considering the SDE for bank regulatory capital given by (3.6). By considering Definition 1.2.1, Proposition 1.2.2 and Definition 1.2.3 in Subsection 1.2.2, we may deduce the following result.

Theorem 3.3.2 (Computation of Capital-to-Risk-Weighted Assets Ratio):

Let the dynamics of bank regulatory capital be as in (3.6). Furthermore, assume that T_i represents stopping times and that ς is a random partition of the reporting period. Then the capital-to-risk-weighted assets ratio, κ , may be represented by

$$d\kappa_t^{-1} = A_t^r \{r^\top + \rho^T (\mu + \mathbf{E}(d))\} dt + A_t^r \rho^T \sigma dL_t. \quad (3.9)$$

Proof. We multiply equation (3.6) with $\frac{\omega^T}{A_t^r}$ to get

$$\frac{\omega^T}{A_t^r} dK_t = \frac{\omega^T}{A_t^r} K_t (r^\top + \rho^T \mu) dt + \frac{\omega^T}{A_t^r} K_t \rho^T \sigma dL_t.$$

But we have that the capital-to-risk-weighted assets ratio is equal to $\kappa_t = \frac{K_t}{A_t^r}$ so the above equation simplifies to

$$\frac{\omega^T}{A_t^r} dK_t = \omega^T \kappa_t (r^T + \rho^T (\mu + \mathbf{E}(d))) dt + \omega^T \kappa_t \rho^T \sigma dL_t,$$

where κ_t is the capital-to-risk-weighted assets ratio. Now divide by ω^T and multiply by A_t^r to get

$$dK_t = \kappa_t A_t^r (r^T + \rho^T (\mu + \mathbf{E}(d))) dt + \kappa_t A_t^r \rho^T \sigma dL_t.$$

If we now simplify even more by first obtaining κ_t on the left hand side we have

$$\kappa_t^{-1} dK_t = A_t^r (r^T + \rho^T (\mu + \mathbf{E}(d))) dt + A_t^r \rho^T \sigma dL_t.$$

We now take integrals on both sides to obtain

$$\kappa_t^{-1} + \kappa_0^{-1} K_0 + \sum_i \kappa_{T_i}^{-1} (K^{T_{i+1}} - K^{T_i}) = \int A_t^r (r^T + \rho^T (\mu + \mathbf{E}(d))) dt + \int A_t^r \rho^T \sigma dL_t.$$

If we now write this in the form of an SDE we obtain (3.10). □

In our discussion, it is realistic to assume that the credit risk charge in (3.3) may be set equal to the total risk-weighted assets, A_t^r . As a consequence, the following corollary of Theorem 3.3.2 is immediate.

Corollary 3.3.3 (Computation of Capital-to-Risk-Weighted Assets Ratio):

If the hypothesis of Theorem 3.3.2 holds and

$$A_t^r = \omega^T (\rho_t^+ + \rho_t^-),$$

then

$$d\kappa_t^{-1} = \omega^T (\rho_t^+ + \rho_t^-) \left\{ \left[r^T + \rho^T (\mu + \mathbf{E}(d)) \right] dt + \rho^T \sigma dL_t \right\}. \quad (3.10)$$

Chapter 4

NUMERICAL AND ILLUSTRATIVE EXAMPLES

4.1 DATA

4.2 NUMERICAL EXAMPLES: BANK PROVISIONING

4.3 NUMERICAL EXAMPLES: BANK REGULATORY CAPITAL

4.4 ILLUSTRATION OF BANK MANAGEMENT PRACTICE

In this chapter, we consider numerical and illustrative examples of the models for banking activities derived in Chapters 2 and 3.

4.1 DATA

In this section, we declare the sources of data that we use. The data on the provisions for loan losses to total assets ratio was sourced from OECD countries including Australia, Finland, Italy, Japan, Norway, Spain, Sweden, the United Kingdom (U.K.) and the United States of America (U.S.A.). We also used data for the capital-to-risk-weighted asset ratio, the capital-to-total assets ratio and the output gap (see Section 8.2 for more details on output gap) for the OECD countries mentioned earlier. Furthermore, we sourced data for the capital-to-risk-weighted assets ratio and capital-to-total assets ratio for South Africa from the SARB and calculated the output gap. The data for the OECD countries was obtained from the International Monetary Fund (IMF) and OECD websites while data for South African financial variables was delivered via the South African Reserve Bank (see [41] and [66], respectively).

4.2 NUMERICAL EXAMPLES: BANK PROVISIONING

This section illustrates some of the properties of the bank provisioning model proposed in Chapter 2. In this regard, we firstly provide evidence to support the fact that the output gap (see Section 8.2 for more information) and the provisions for loan losses-to-total assets ratio are negatively correlated. In essence this means that provisions for loan losses are procyclical. Here the *output gap* is defined as the amount by which a country's output, or GDP, falls short of what it could be given its available resources. Of course, GDP is often used as a proxy for macro-economic activity. In addition, we investigate the correlation between output gap and provisions in relation to profitability.

The specific countries for which data was accessed are Australia, Finland, Italy, Japan, Norway, Spain, Sweden, the United Kingdom and the United States of America. The figures show that provisions typically do not increase until after economic growth has slowed considerably and often not until the economy is clearly in recession. This is best observed in the figures for Australia, Sweden, Norway and Spain.

4.2.1 Procyclicality of Provisions for Loan Losses

In this subsection, we look at empirical evidence that provisions for loan losses is procyclical.

4.2.1.1 Provisioning for Australia, Norway, Spain and Sweden

This subsection provides historical evidence that provisions for loan losses were procyclical in Australia, Norway, Spain and Sweden.

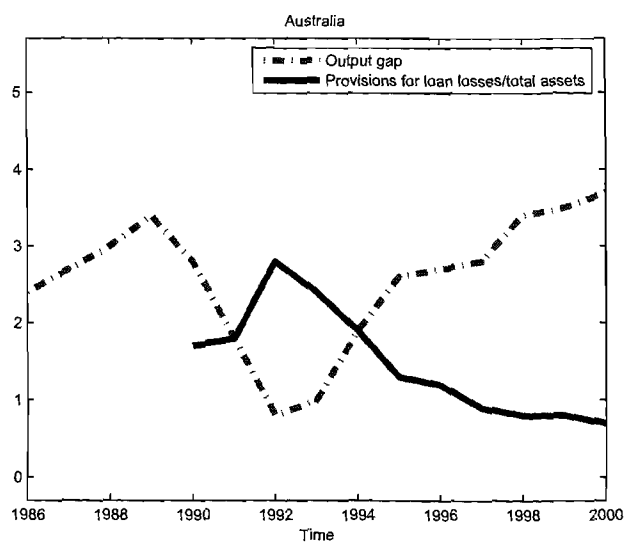


Figure 4.1: Output Gap vs Provisions for Loan Losses-to-Total Assets Ratio for Australia

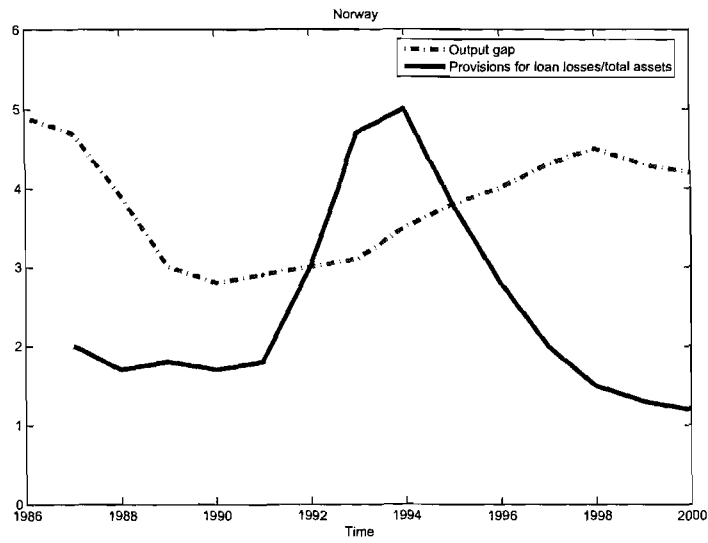


Figure 4.2: Output Gap vs Provisions for Loan Losses-to-Total Assets Ratio for Norway

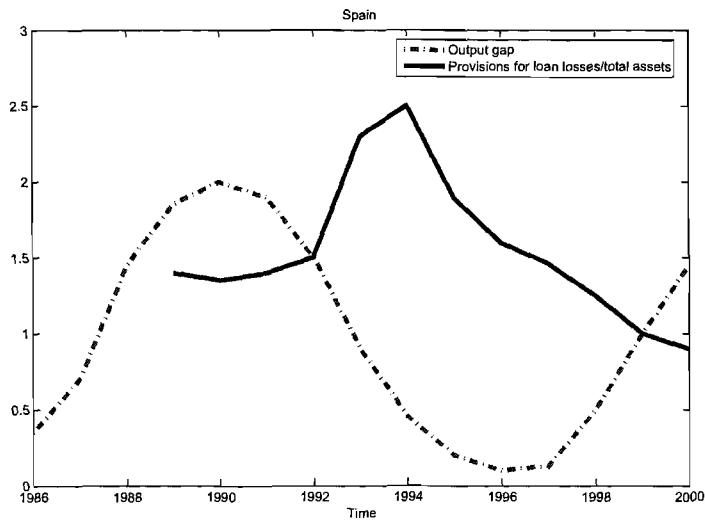


Figure 4.3: Output Gap vs Provisions for Loan Losses-to-Total Assets Ratio for Spain

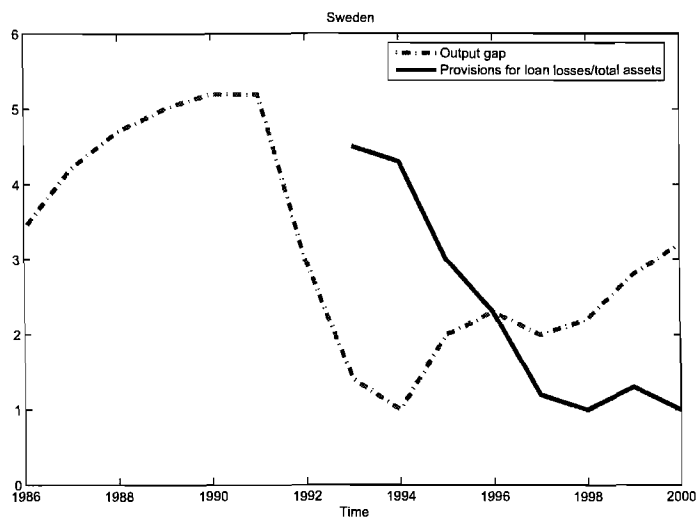


Figure 4.4: Output Gap vs Provisions for Loan Losses-to-Total Assets Ratio for Sweden

4.2.1.2 Provisioning for Finland, Italy, Japan and United Kingdom

Below we provide historical evidence that provisions for loan losses in Finland, Italy, Japan and the United Kingdom were procyclical.

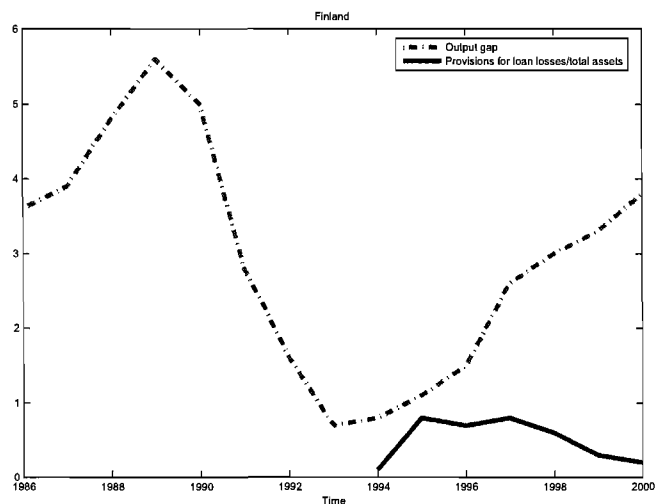


Figure 4.5: Output Gap vs Provisions for Loan Losses-to-Total Assets Ratio for Finland

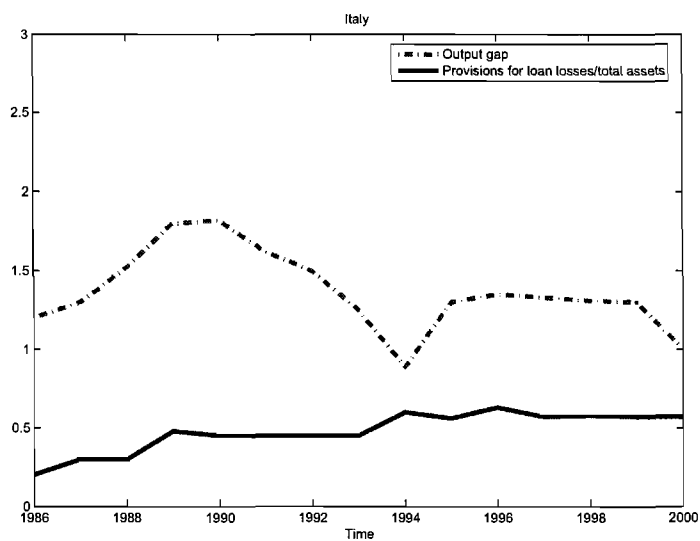


Figure 4.6: Output Gap vs Provisions for Loan Losses-to-Total Assets Ratio for Italy

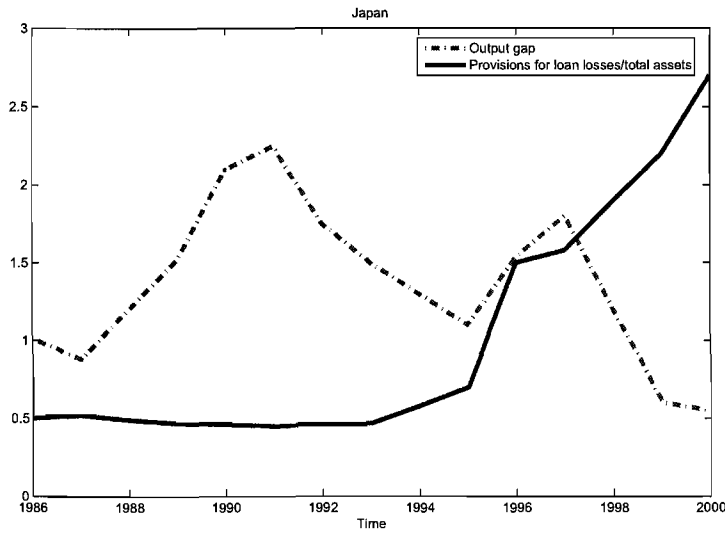


Figure 4.7: Output Gap vs Provisions for Loan Losses-to-Total Assets Ratio for Japan

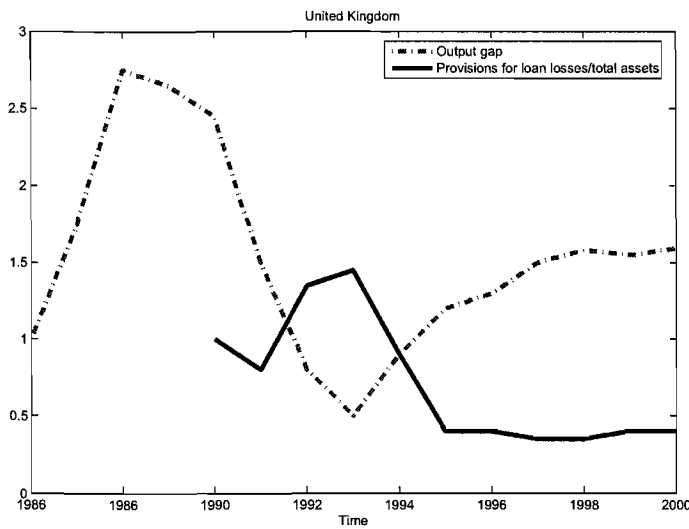


Figure 4.8: Output Gap vs Provisions for Loan Losses-to-Total Assets Ratio for the United Kingdom

4.2.1.3 Provisioning for the United States

This subsection provides historical evidence that in the USA provisions for loan losses were procyclical.

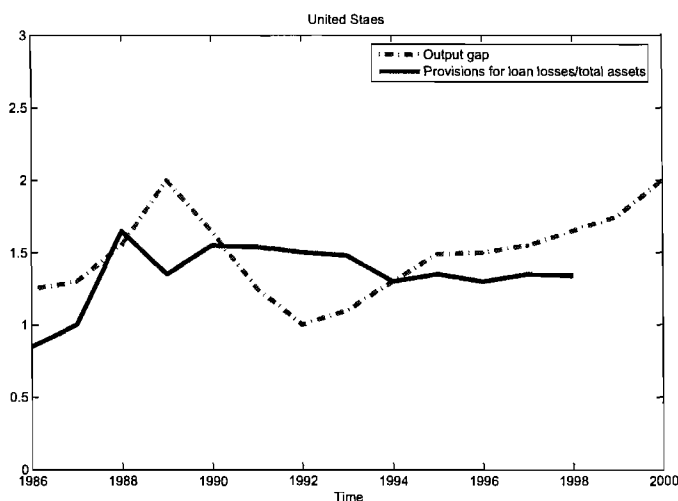


Figure 4.9: Output Gap vs Provisions for Loan Losses-to-Total Assets Ratio for the U.S.A.

4.2.1.4 Discussion of Provisioning for the 9 OECD Countries

In this subsection, we provide a brief discussion of some of the outstanding features of the data for provisioning for loan losses provided in Subsection 4.2.1.

The data for Australia from Figure 4.1 shows that provisions failed to increase substantially in the early 1990's, when credit and asset prices were growing rapidly and the financial imbalances were developing. Moreover, the peak in provisions did not occur until at least two years after the economy started to slow down.

The data for Norway from Figure 4.2 exhibits a similar behavior as the data for Finland from Figure 4.5 and the data for Spain from Figure 4.3. In these cases, provisions failed to increase substantially in the late 1980's, when credit and asset prices were growing rapidly and the financial imbalances were developing. In each of these figures the peak for provisions did not occur until the recession. However, one

of the differences between these figures is the amount by which provisions increased when the economy had clearly slowed down.

In the data for Finland from Figure 4.5 we see a similar situation as in the data for Italy from Figure 4.6 where provisions failed to overlap the output gap during the recession. Again this can be linked with Japanese banking problems. Although in countries like the United States (see Figure 4.9) and Australia and Norway (see Figure 4.1 and 4.2), the provisions overlapped the output gap during the recessions, the situation in Italy (see Figure 4.6) is totally different. It seems that even after the banking problems that Japan experienced had been resolved, in Italy the situation changed slightly.

From the data for Japan from Figure 4.7, we can conclude that the level of provisioning only increased substantially during the second half of the 1990's, long after the problems in the Japanese banking system had been widely recognized.

The (low) positive correlation between provisions and the business cycle in the United States (see Figure 4.9) appears to be driven by the surge in provisions in the second half of the 1980's. This phenomenon seems to reflect the delayed cleaning of the balance sheets following the developing countries' debt crisis of the early 1980's.

4.2.2 Correlations Between Profitability and Provisions for Loan Losses

As has been suggested in Subsection 4.2.1, bank provisions are strongly procyclical with a negative correlation with the business cycle. For instance, Figure 4.9 shows that provisions typically do not increase until after economic growth has slowed down considerably and often not until the economy is in complete recession. As is shown in Table 4.1 below, this pattern appears to be strongest in those countries that experienced banking system problems in the 1990's.

In the main, the behavior of provisions translates into a procyclical pattern in bank profitability, which further encourages procyclical lending practices. This is borne out by the fact that if we substitute $r_t^\Lambda = r^T + k + \mathbf{E}(d)$ into equation (2.10) we obtain

$$\begin{aligned} \Pi_t = & \left([r^T + k + \mathbf{E}(d)] - c^\Lambda - r^d(M_t) \right) \Lambda_t + r_t^T W_t + r_t^I I_t - \left(r_t^D + c^D \right) D_t \\ & - c^w(W_t) - P(M_t) - r_t^T \gamma D_t. \end{aligned}$$

Our claim is thus that profit and provisions are negatively correlated. However, from

Countries	Provisions	Profitability
Australia	- 0,88	0,71
Finland	—	0,81
Germany	- 0,21	- 0,42
Italy	- 0,21	0,25
Japan	- 0,43	0,22
Norway	- 0,35	0,54
South Africa	- 0,85	0,74
Spain	- 0,41	0,84
Sweden	- 0,83	0,60
United Kingdom	- 0,38	0,12
United States	0,14	0,24

Table 4.1: Correlations between Provisions and Profitability

Table 4.1 we also conclude that the profitability of German banks is not procyclical. This may be due to their ability to smooth profits through hidden reserves. The procyclical nature of bank profits has arguably also contributed to the bank equity prices being positively correlated with the business cycle, although the correlation is typically weaker than that for profitability, reflecting the forward-looking nature of the equity market.

4.3 NUMERICAL EXAMPLES: BANK REGULATORY CAPITAL

The following sections are based on the results obtained in Chapter 3. Capital adequacy ratios on their own, as computed in Subsection 3.3.3, only tell a partial story. One of the most important issues related to CARs are their effect on financial stability in the banking industry. In this regard, it is a generally accepted fact that cyclicity is at the root of financial instability in banking. In particular, under Basel II, capital requirements are likely to increase in recessions. Yet if capital requirements show this tendency - when building reserves from decreasing profits is difficult or raising fresh capital is likely to be extremely costly - banks would have to reduce their loans and the subsequent credit crunch would add to the downturn. This would make the recession deeper, thus setting in motion an undesirable vicious circle that might ultimately have an adverse effect on the stability of the banking system. This is why capital

requirements are said to be procyclical despite actually increasing (decreasing) during a downturn (upturn). The implications of this link between financial stability and macro-economic stability in terms of the soundness of bank's merit being taken into account in the final design of Basel II.

In order to test the validity of the claims in the previous paragraph, we rely on illustrative data from some member countries of the Organization for Economic Cooperation and Development (OECD) as supplied on the website [58] as well as South Africa. The specific countries for which simulation parameters for regulatory capital-to-risk-weighted assets ratio and the capital-to-total assets ratio was sourced from Australia, Finland, Italy, Japan, Norway, South Africa, Spain, Sweden, the United Kingdom and the United States of America. In particular, the computer simulations in Subsection 4.3.1 below, provide evidence to support the fact that capital adequacy ratios are generally negatively correlated with the economic cycle. We also see this behavior in the examples given in Subsections 4.3.2 and 4.3.3 of other OECD countries and also in the South African case.

4.3.1 Simulation of Capital-to-Risk-Weighted Assets Ratio vs Output Gap for Japan

In this subsection, we simulate the capital-to-risk-weighted assets and compare it with the output gap for Japan. We assume for simplicity that the capital-to-risk-weighted assets ratio does not have jumps so that we are only looking at the Brownian motion part of equations (3.7) and (3.10). There are a few methods for simulating stochastic differential equation (SDE). We use a numerical simulation of stochastic differential equations based on the Euler-Maruyama Method (EMM). If we have a SDE of the form

$$dX_t = f(X_t)dt + g(X_t)dW_t, 0 \leq t \leq T,$$

where T is the time horizon, we can apply the EMM. To apply the method to the above equation over $[0, T]$ we first discretize the interval. Let $\Delta t = \frac{T}{L}$ for some positive integer L (number of subintervals) and $t_j = j\Delta t$. The EMM takes the form

$$X_j = X_{j-1} + f(X_{j-1})\Delta t + g(X_{j-1})(W(t_j) - W(t_{j-1})), j = 1, 2, \dots, L + 1.$$

In order for us to simulate the SDE for the capital-to-risk-weighted assets ratio in (3.10) we need to make certain choices for the parameters in this equation. The parameters that need to be chosen is the Treasury rate, r^T the weights of total risky assets, ρ , the risk premium, μ , the expected default rate, $\mathbf{E}(d)$ and the volatility of the process, σ . We chose the Treasury rate as 0.095 which is the actual Treasury rate of Japan when the simulation was done. The risk weight was chosen as 0.8. This means that the risk-weighted assets make up 80 % of total assets. The risk premium was set at 0.01 and the expected default rate as 0.025. The volatility was determined by calculating the actual standard deviation of the actual data on the capital-to-risk-weighted assets ratio of Japan. It has to be mentioned here that these are not the only choices that can be made for these parameters.

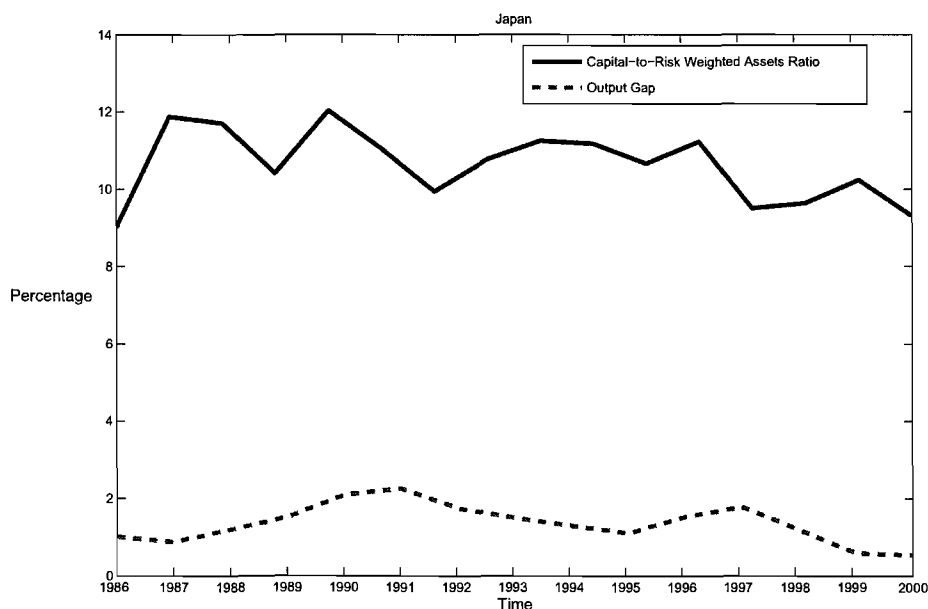


Figure 4.10: Simulated Capital-to-Risk-Weighted Assets Ratio for Japan

4.3.2 Illustrations of Capital-to-Risk-Weighted Assets Ratio and Capital-to-Total Assets Ratio vs Output Gap for Other OECD Countries

In this subsection, we illustrate the capital-to-risk-weighted assets ratio and the capital-to-total assets ratio versus output gap for Australia (period 1990-2000), Finland (period 1992-2000), Italy (period 1987-2000), Norway (period 1992-2000), Spain

(period 1986-2000), Sweden (period 1992-2000), the United Kingdom (period 1990-2000) and the United States of America (period 1990-2000). This enables us to characterize and discuss the cyclicity of capital adequacy ratios as in Subsubsections 3.3.3.1 and 3.3.3.2.

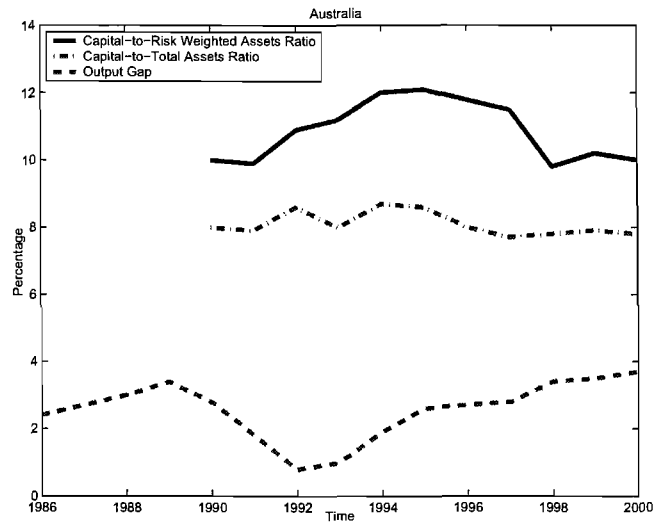


Figure 4.11: Capital-to-Risk-Weighted Assets Ratio vs Capital-to-Total Assets Ratio for Australia

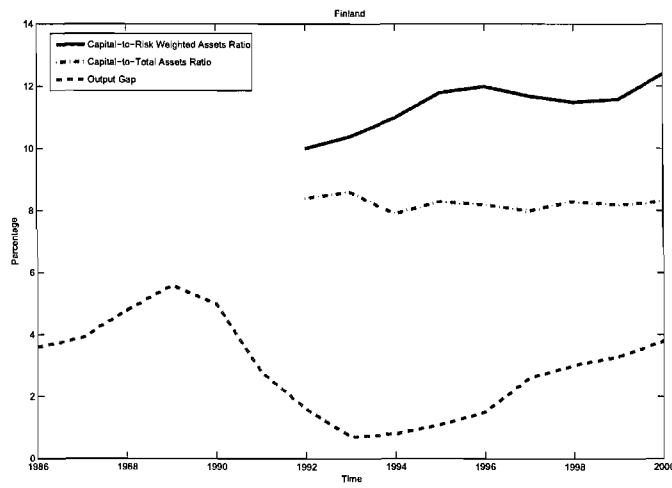


Figure 4.12: Capital-to-Risk-Weighted Assets Ratio vs Capital-to-Total Assets Ratio for Finland

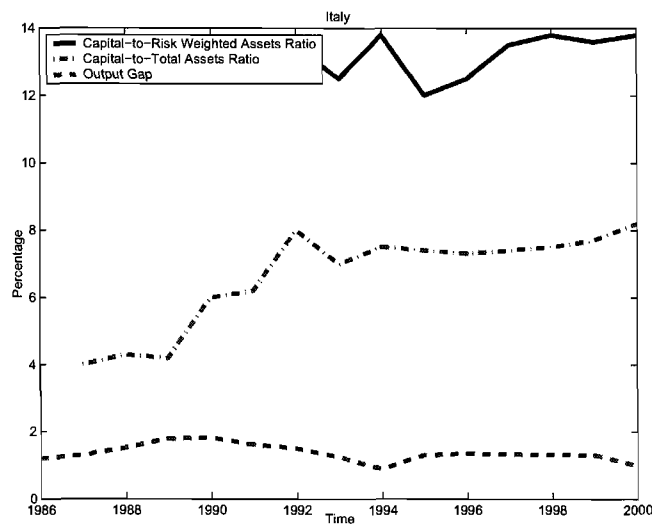


Figure 4.13: Capital-to-Risk-Weighted Assets Ratio vs Capital-to-Total Assets Ratio for Italy

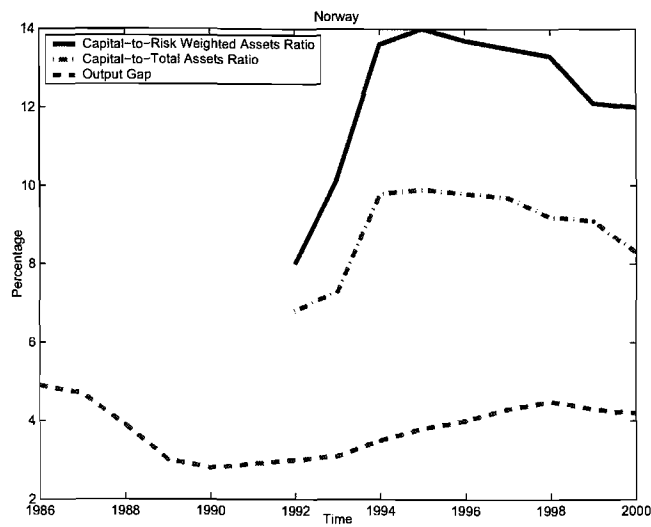


Figure 4.14: Capital-to-Risk-Weighted Assets Ratio vs Capital-to-Total Assets Ratio for Norway

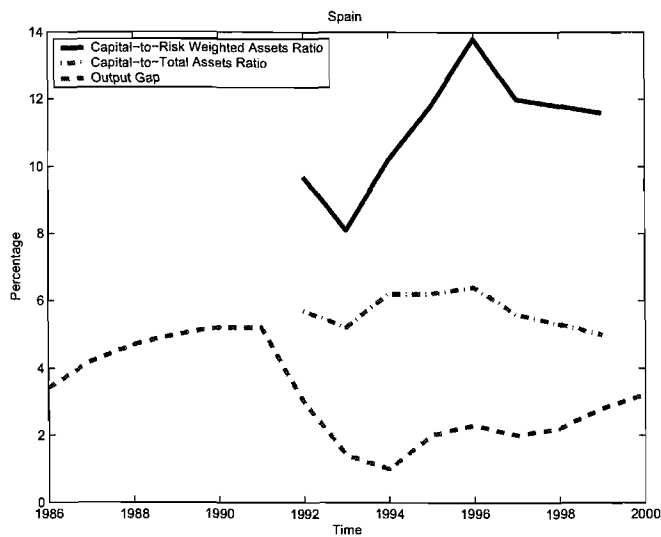


Figure 4.15: Capital-to-Risk-Weighted Assets Ratio vs Capital-to-Total Assets Ratio for Spain

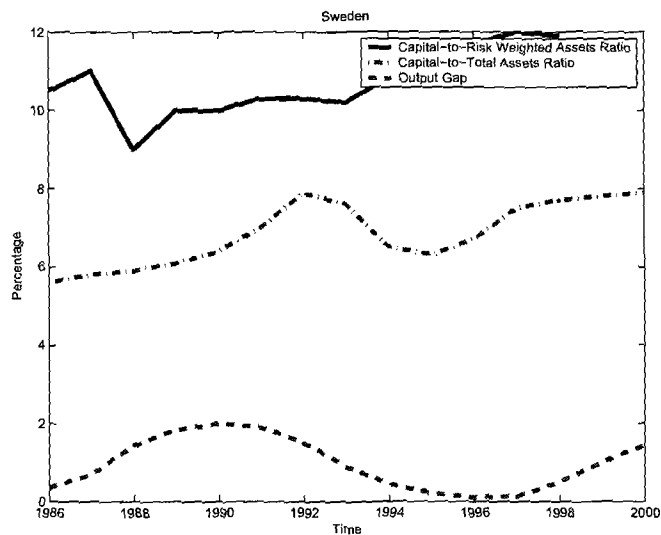


Figure 4.16: Capital-to-Risk-Weighted Assets Ratio vs Capital-to-Total Assets Ratio for Sweden

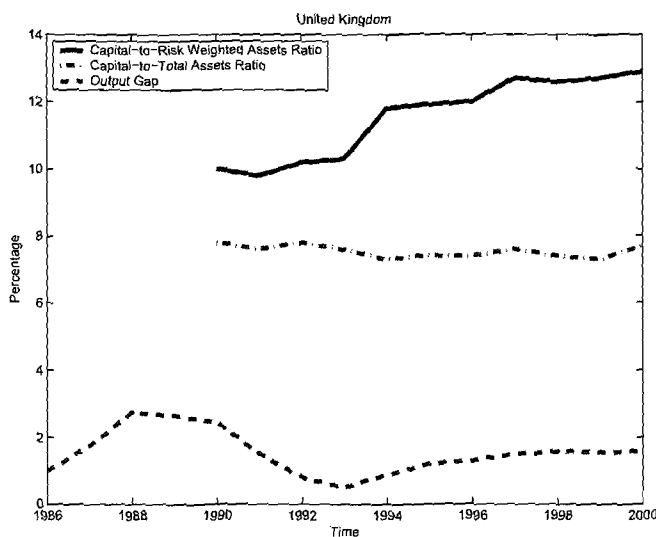


Figure 4.17: Capital-to-Risk-Weighted Assets Ratio vs Capital-to-Total Assets Ratio for the UK

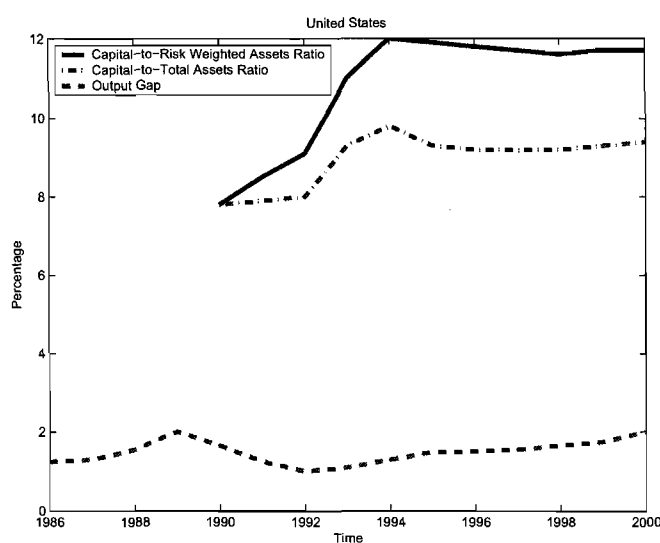


Figure 4.18: Capital-to-Risk-Weighted Assets Ratio vs Capital-to-Total Assets Ratio for the USA

4.3.3 Illustration of Capital-to-Risk-Weighted Assets Ratio and Capital-to-Total Assets Ratio vs Output Gap for South Africa

In this subsection, as an example of the situation in a non-OECD country, we illustrate the capital-to-risk weighted assets ratio and the capital-to-total assets ratio versus output gap for the South African situation. In the appendix, we plotted the actual GDP output vs. the potential GDP output for South Africa from 1970 to 2006 using data sourced from the South African Reserve Bank (SARB). This was done to calculate the output gap. The minimum capital adequacy changed to 10% in 2000.

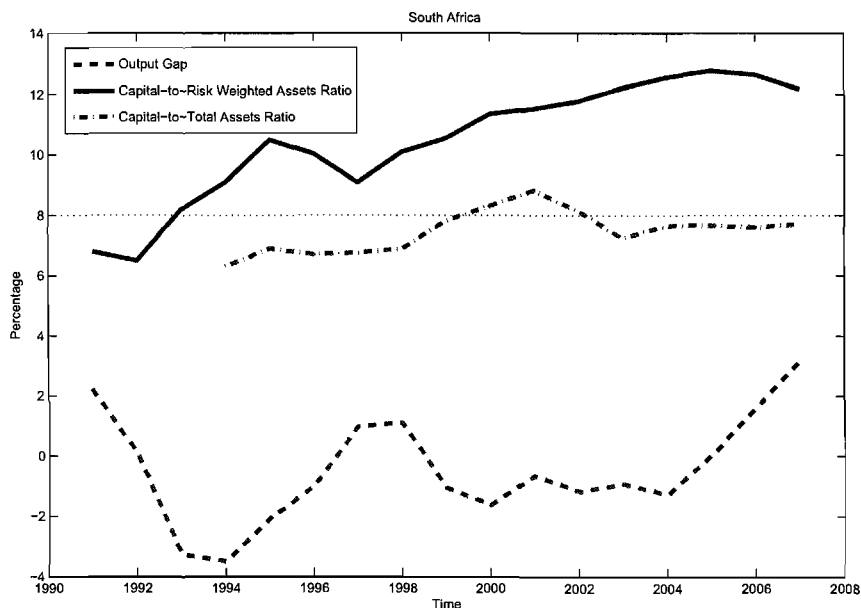


Figure 4.19: Capital-to-Risk-Weighted Assets Ratio and Capital-to-Total Assets Ratio vs Output Gap for SA

4.4 ILLUSTRATION OF BANK MANAGEMENT PRACTICE

In this section, we provide an illustration of some of the features of bank management practice referred to in the above sections. Our analysis has several connections with the arguments about risk management and regulatory policy in [26] and [63] (see also [28], [30] and [50]). Firstly, we illustrate issues related to credit risk by considering the probability of default of granted loans. The second feature of our example involves the implications of and the interactions between the three pillars of Basel II regulation for bank management that includes a consideration of a regulatory capital constraint. In this regard, we highlight the dynamic interaction between a regulator (who acts in the interest of the public) and bank owner (who, by assumption, acts in the interest of the shareholder). Here, we emphasize that information provided by the bank manager about the level of bank capital, K , is important for the decision by the supervisor on whether to allow the bank to continue to function or enforce bank closure. Finally, throughout the example, risk incentives, risk shifting and other constraints on banking behavior are referred to. With regard to the latter, realistic constraints associated

with the eliciting of additional debt and equity, profitability, incentive compatibility and financing are brought to bear on bank management practice.

4.4.1 Setting the Scene

Throughout the ensuing illustration, the bank capital, K , will consist solely of equity capital and subordinate debt. Also, the bank assets are constituted by loans to private agents and intangible assets, $\Lambda_t + I_t$. Furthermore, we follow a procedure that can be identified with the three-pillared approach of the Basel II capital accord (see, for instance, [26]). Pillar 1 (minimum capital requirement) involves the application of a quantitative minimum capital requirement based on public information that determines whether a bank will continue to operate or not. This pillar is related to the likely prompt corrective action that will be taken by supervisors in the event of banks becoming significantly or critically undercapitalized. In this regard, Table 4.2 below makes a distinction between the capitalization states of banks with respect to several benchmark regulatory ratios.

CATEGORIES	ρ		T1CAR		TCAR		TE
Well-Capitalized	$\geq 0,1$	and	$\geq 0,06$	and	$\geq 0,06$		–
Adequately Capitalized	$\geq 0,08$	and	$\geq 0,04$	and	$\geq 0,04$		–
Undercapitalized	$\geq 0,06$	and	$\geq 0,03$	and	$\geq 0,03$		–
Significantly Undercapitalized	$< 0,06$	or	$\geq 0,03$	or	$\geq 0,03$	and	$> 0,02$
Critically Undercapitalized							$\leq 0,02$

Table 4.2: Categories of Banking Benchmark Regulatory Ratios

In Table 4.2, we have that TCAR, T1CAR and TE are the abbreviations for the total CAR (also known as the leverage ratio), Tier 1 CAR and tangible equity, respectively. Here the TCAR and T1CAR is the regulatory capital-to-total assets ratio (see, discussion and historical data in Subsection 5.2.3 and Section 8.2, respectively) and T1 capital-to-total assets ratio, respectively. As is the case for the TCAR and T1CAR, the CAR, ρ , in the first column of Table 4.2 gives an indication (in terms of the level of capitalization of the bank) of significant values for the benchmark CAR. In Pillar 2 (supervisory review), the bank decides on whether the bank capital held is sufficient to invest in a certain credit risk-type or whether it is necessary to elicit cap-

ital by issuing debt and raising equity. At this stage, we distinguish between failed, capital-constrained and capital-unconstrained banks. Pillar 3 (market discipline) is for market participants to evaluate the bank and once they react negatively and there is a loss of confidence, for example, the regulator might react.

4.4.2 Pillar 1 - Minimum Capital Requirement

Most banks consider the level of capital to be the binding constraint in deciding on whether to issue a loan or not. In order for a loan to be granted, the risk adjusted rate of return on a particular loan must exceed the return on capital. In the sequel, suppose that $A_t^r = \omega^A \Lambda_t + \omega^I I_t$, and also $\omega^A = \omega^I = 1$. Therefore, $A_t^r = \Lambda_t + I_t$. The Basel II capital accord contains the *total capital constraint* that, in our case, relates the CRC, A^r , to capital, K , via the inequality

$$K(t) \geq \rho\{\Lambda_t + I_t\} \text{ or, equivalently, } \kappa(t) \geq \rho, \quad (4.1)$$

where ρ denotes a CAR regulatory benchmark. The setting of a regulatory benchmark is an attempt to encourage banks to hold a RC-to-CRC ratio, ρ , of at least 8 % (see, for instance, [14], [15] and [27]). Several approaches to the management of bank closure and its relationship with the minimum capital requirement (Pillar 1) described in Basel II exist. In the event of closure, the loans to private agents and intangible assets, $\Lambda_t + I_t$, are liquidated at a cost of $\lambda\{\Lambda_t + I_t\}$, where λ is determined exogenously. The implementation of Pillar 1 via the liquidation cost approach will mainly be impacted by the level of bank capital and the CAR. In this regard, the supervisor has to decide whether capital, K , as described by (3.5), covers the cost of liquidation, $\lambda\{\Lambda_t + I_t\}$. If we have, for $t \in [0, t_1)$, that

$$K(t) \geq \lambda\{\Lambda_t + I_t\} \text{ or, equivalently, } \kappa(t) = \frac{K(t)}{\Lambda_t + I_t} \geq \lambda, \quad (4.2)$$

then the bank exceeds the minimum capital requirement and is able to continue operating. If condition (4.2) fails, closure may occur since it is unlikely that the bank will be able to re-capitalize itself.

4.4.3 Pillar 2 - Supervisory Review

In this subsection, we consider the interaction between the capital adequacy management of a bank with respect to its fundamental function of granting loans and supervisory constraints on, for instance, eliciting debt and equity (see [30] for more details). Having exceeded a minimum regulatory capital requirement in the first stage, under supervisory constraint, the bank may now acquire a new credit risk type, $\Lambda_t^1 + I_t^1$. The return on $\Lambda_t^1 + I_t^1$ is reliant on whether the profit, Π , is either

$$\Pi(t) = \Pi_p(t) \geq 0 \text{ or } \Pi(t) = \Pi_n(t) < 0. \quad (4.3)$$

The *social value* of acquisition $\Lambda_t^1 + I_t^1$ is $(1 - \psi)\Pi_p(t) - \psi\Pi_n(t)$, where ψ is the *anterior probability of bank failure*. In this regard, Π_p and Π_n from (4.3) correspond to the favored (low ψ) and the unfavored (high ψ) bank operational states, respectively. Here we suppose that the bank has no direct costs associated with failure (see [62] for more details). Next, the bank assesses whether its level of capital is high enough to invest in $\Lambda_t^1 + I_t^1$ by determining whether the $\Lambda_t^1 + I_t^1$ -capital constraint

$$\kappa(t) = \frac{K(t)}{\Lambda_t^1 + I_t^1} \geq 1 \quad (4.4)$$

is satisfied (compare (4.4) to the total capital constraint (4.1) described above). If inequality (4.4) holds, there is no need to elicit additional debt or equity. If, on the other hand, we have

$$\frac{\lambda\{\Lambda_t + I_t\}}{\Lambda_t^1 + I_t^1} \leq K(t) < 1$$

the bank has to acquire additional financing from, for instance, debt- and shareholders. In this regard, the market may impose certain restrictions on the amount of debt and equity that the bank can raise. This *eliciting constraint* is intended to discourage the bank from investing in riskier assets which have a higher default probability (see, for instance, [40]). For sake of argument, we suppose that $\Lambda_t^1 + I_t^1$ is replaced by another credit-risk type, $\tilde{\Lambda}_t^1 + \tilde{I}_t^1$, with a higher return $\tilde{\Pi}_p(t) > \Pi_p(t)$ and a higher probability of failure $\tilde{\psi} > \psi$. Let K^* be the additional debt and equity elicited with G being defined by

$$\text{Gross Return to Share- and Debt-holders} = \begin{cases} G, & \text{in favored operational state;} \\ 0, & \text{otherwise.} \end{cases}$$

Next, we introduce two constraints that is commensurate with prudent bank management practice. In order to make $\tilde{\Lambda}_t^1 + \tilde{I}_t^1$ less attractive in the *absence* of elicited debt and equity, we require that the returns on $\Lambda_t^1 + I_t^1$ and $\tilde{\Lambda}_t^1 + \tilde{I}_t^1$ satisfy the *profitability constraint* given by

$$(1 - \psi)\Pi_p(t) \geq (1 - \tilde{\psi})\tilde{\Pi}_p(t).$$

On the other hand, to discourage the shifting of risk in the *presence* of debt and equity, $G > 0$ must satisfy the *incentive compatibility constraint* expressed as

$$0 < G(t) \leq \Pi_p(t) - \frac{1 - \tilde{\psi}}{\tilde{\psi} - \psi} \left[\tilde{\Pi}_p(t) - \Pi_p(t) \right]. \quad (4.5)$$

The aforementioned debt and equity satisfy *individual rationality* if, at equilibrium, it guarantees an outcome such that the profit for the debt- and shareholders exceeds a certain level. This concept is useful when the debt- and shareholders have the option to terminate their involvement. In our case, such rationality leads to

$$(1 - \psi)G(t) \geq \beta K^*(t), \quad (4.6)$$

where the market requirement $\beta > 1$ is the gross return on capital. Inequalities (4.5) and (4.6) together suggest that the maximum additional amount of debt and equity, \tilde{K}^* , the bank may raise, may be expressed as

$$K^*(t) \leq \frac{1 - \psi}{\beta} \left[\Pi_p(t) - \frac{1 - \tilde{\psi}}{\tilde{\psi} - \psi} \left\{ \tilde{\Pi}_p(t) - \Pi_p(t) \right\} \right] := \tilde{K}^*(t).$$

In essence, Stage 2 distinguishes between failed, capital-constrained and capital-unconstrained bank types whose classification depends on how their level of capital compares with regulatory benchmarks (see Figure 1).

4.4.3.1 Failed Bank

A bank for whom the capital adequacy ratio, κ , induced by $\Lambda_t^1 + I_t^1$, is subject to the condition

$$K(t) < 1 - \frac{\tilde{K}^*(t)}{\Lambda_t^1 + I_t^1}$$

cannot raise enough debt and equity to invest in $\Lambda_t^1 + I_t^1$ and fails. If this happens, bank managers receive the market value of $K(t) - \lambda\{\Lambda_t + I_t\}$ which is positive because the bank exceeded the minimum capital requirement from the first pillar. The government has to cover certain costs when a bank fails. For instance, if upon failure, the resulting operational state is favorable, the opportunity costs of total profits that have been lost in the liquidation process is charged to the taxpayer but credit is given for the *net social value at closure*

$$\{\Lambda_t^1 + I_t^1\} - \lambda\{\Lambda_t + I_t\}. \quad (4.7)$$

The liquidator may also incur a deadweight loss, l_{dw} , where, for instance, external income with positive returns are foregone when a favored bank is closed or legal disputes arise from a bank that was deemed to be viable after closure.

4.4.3.2 Capital-Constrained Bank

The bank that has insufficient capital, κ , to invest in $\Lambda_t^1 + I_t^1$, may face a *financing constraint*,

$$1 > K(t) \geq 1 - \frac{K^*(t)}{\Lambda_t^1 + I_t^1},$$

and may be subject to an implicit capital requirement from the market of the form

$$K(t) \geq 1 - \frac{K^*(t)}{\Lambda_t^1 + I_t^1} \geq 1 - \frac{\tilde{K}^*(t)}{\Lambda_t^1 + I_t^1}.$$

Such a bank satisfies the minimum market capital requirement, but is capital-constrained and must issue debt and raise equity to invest in $\Lambda_t^1 + I_t^1$ in loans. If the constrained

bank is allowed to operate and it fails, the supervisor must deal with both the operating loss, Π_n , (which the bank's owners do not bear because of assumed limited liability) and the cost of liquidation, $\lambda\{\Lambda_t + I_t\}$. Compared with (4.7), if the bank fails it follows that the net social value at closure is given by

$$K(t) + K^*(t) - \lambda\{\Lambda_t + I_t\}$$

and the loss is $-\Pi_n - \{\Lambda_t^1 + I_t^1\}$. In the constrained case, the total social investment, including debt and equity is $\Lambda_t^1 + I_t^1$.

4.4.3.3 Capital-Unconstrained Bank

Finally, a bank with a $\Lambda_t^1 + I_t^1$ -induced CAR, κ , that satisfies

$$K(t) \geq 1 \tag{4.8}$$

is unconstrained and may invest in $\Lambda_t^1 + I_t^1$ without raising additional debt and equity. In that case, the excess bank capital

$$K(t) - \{\Lambda_t^1 + I_t^1\} \tag{4.9}$$

may be invested in a riskless asset (such as treasuries) that provides a zero net return. If the bank is unconstrained, the *aggregate social value* is always more than the constrained case by the amount of excess capital given by (4.9). However, if the unconstrained bank invests any excess capital in a riskless asset with zero net return then all consequences are exactly larger by (4.9).

4.4.4 Pillar 3 - Market Discipline

Pillar 3 aims to strengthen market discipline by insisting on enhanced disclosure by banks. The *disclosure requirements* will enable market roleplayers to access information about the bank's capital adequacy and risk exposures. If the bank successfully complies with the conditions above, in the third stage the bank acquires information that is relevant to its continued operational management. Market participants evaluate the bank and if they react negatively the regulator might react. The newly acquired information may be in the form of a signal

$$I^* = \begin{cases} Y, \\ N. \end{cases} \quad (4.10)$$

This signal depends on the observed value of I^* and results in a *posterior probability of bank failure* that may be expressed as $p_i = P(\text{failure}|I^* = i)$, $i = Y$ or N . Depending on its incentives, the bank discloses the correct or incorrect value of I^* to the supervisor. Since by definition

$$\psi = p_Y P(I^* = Y) + p_N P(I^* = N)$$

we have that $p_Y \leq \psi \leq p_N$. Our illustrative example of bank management practice is concluded by briefly mentioning the role that the *information signal*, I^* , in (4.10), can play in the interaction between the supervisor and bank manager. Before a decision about corrective action is made, the supervisor requests the value of I^* from the bank manager. If the supervisor's decision rule is incentive-compatible it must not impose a penalty on the disclosure of the authentic value of I^* .

Chapter 5

ANALYSIS OF THE MAIN ISSUES

5.1 ANALYSIS OF BANK VALUATION ISSUES

5.2 ANALYSIS OF BANK CAPITAL ISSUES

5.3 NUMERICAL AND ILLUSTRATIVE EXAMPLES

In this chapter, we analyze some of the main modelling and data issues raised in Chapters 2, 3 and 4. More specifically, in accordance with the dictates of the Basel II capital accord, the models of bank items constructed in Chapters 2 and 3 are related to the methods currently being used to assess the riskiness of bank portfolios and their minimum capital requirement (see [9] and [14]).

5.1 ANALYSIS OF BANK VALUATION ISSUES

In this section, we specifically analyze aspects of the bank model presented in Chapter 2.

5.1.1 Loans and Their Demand and Supply

Section 2.1 of Chapter 2 provides us with a description of the main components of a bank's lending activities. Banks respond differently to shocks that affect loan demand, Λ , when the minimum capital requirements are calculated by using risk-weighted assets. In the Hicksian case, these responses are usually sensitive to macro-economic conditions that are related to the term $l_2 M_t$ in (2.1). Loan defaults are independent of the capital adequacy paradigm that is chosen. In this regard, empirical evidence (compare Section 4.2) supports the opinion that better macro-economic conditions reduce the loan default rate and thus the loan marginal cost.

5.1.2 Loan Losses and Provisioning

With regard to Section 2.2, great concern has been expressed about the rapid growth in business loans at commercial banks with excessively easy credit standards. This is a global phenomenon. Some analysts claim that competition for lenders has greatly increased, causing banks to reduce loan rates and ease credit standards in order to issue new credit. This phenomenon is evident in especially the American subprime lending crisis which resulted in major instability in global markets in 2007. Others are of the opinion that as economic expansion continues and past loan losses have been forgotten, banks exhibit a greater propensity for risk. Be that as it may, the acceleration in loan growth could lead eventually to a surge in loan losses (see, for instance, equation (2.3)) resulting in reduced bank profits and precipitating a new round of bank failures. As the experience of the early 1990s has shown globally, such a slump in banking could not only threaten a deposit insurance fund but also slow an economy by entrenching credit crunches. The view that faster loan growth

leads to higher loan losses should not be taken lightly; nor should it be accepted without question. If loan growth increases because banks become more willing to lend, credit standards might fall and loan losses might eventually rise. But loan growth can increase for reasons other than a shift in loan supply (see, for instance, equation (2.2)). In this case, businesses may decide to shift their financing from the capital markets to banks, or an increase in productivity may enhance the return on investments. In such cases, faster loan growth need not lead to higher loan losses.

Next, we explain the association between loan growth and loan losses. An obvious factor that plays a role in this is the business cycle (compare Section 4.2). Loan growth tends to increase during booms, while loan losses tend to increase during recessions. Thus, as a result of the business cycle, periods of rapid loan growth naturally tend to precede periods of high loan losses. The question is whether faster loan growth lead to higher loan losses even after controlling for the state of the economy. Using the simple model in (2.10), we can identify when such a relationship between loan growth and loan losses is likely to exist. In particular, the model suggests how data on loan growth, credit standards, and loan losses can be used to test the view that faster loan growth leads to higher loan losses. Below, we explain why faster loan growth might lead to higher loan losses. Most of the reasons usually given for this phenomenon involve supply shifts, i.e., increases in the bank's willingness to lend. In the presence of such a shift, banks typically seek to increase their lending activities in two ways. Firstly, they reduce the interest rate charged on new loans, r_t^Λ , as referred to in (2.2). Secondly, they lower their minimum credit standards for new loans by, for example, reducing the amount of collateral the borrower must have to back his loan, accept borrowers with weaker credit histories or require less proof that the borrower will have enough cash flow to service his debts. Such a reduction in credit standards increases the chances that some borrowers will eventually default on their loans. Thus, assuming banks lower credit standards as well as reduce loan rates, increases in lending due to supply shifts will tend to lead to higher loan losses in the future.

A geometric Lévy-process driven analogue of (2.2) will be of the form

$$dM_t = M_t \left\{ \mu^M dt + \sigma_t^M dL_t \right\},$$

where σ_t^M is volatility in macro-economic activity and $L = \{L_t\}_{t \geq 0}$, is a Lévy process. This enables us to deal with the more realistic situation where discontinuities in the macro-economic process occur. In this case, we can also find a analogue of the demand

for loans, Λ_t given by (2.1), of the form

$$d\Lambda_t = \Lambda_t \left\{ l_1 r_t^\Lambda dt + \left[l_2 M_t + \sigma_t^\Lambda \right] dL_t \right\}.$$

The properties of the aforementioned analogues of M and Λ are the subject of an ongoing investigation. Also, the formula for, $L(M_s)$, presented in (2.3) can be expressed in terms of profit, Π , as $L(\Pi_s)$, by virtue of the evidence from empirical studies that suggest that a strong correlation between M_s and Π_s exists (see the discussion on the procyclicality of bank profitability in, for instance, [17] and [19]).

5.1.3 Other Assets

As is evidenced by Subsection 2.3.1, we consider intangible assets to be part of our model for profitability. In reality, valuing this off-balance sheet item constitutes one of the principal difficulties with the process of bank valuation (see, for instance, [38] and [71]). However, analysts should continually update their valuation procedures for measuring intangible assets for the following reasons. Firstly, the nature and structure of intangibles are not static. Secondly, accounting and other disciplines are developing new methodologies to value such assets. Finally, the valuation models use a causal framework that links the nature and structure of intangible assets to opportunities for future wealth generation.

5.1.4 Bank Valuation

In this subsection, we analyze intangible assets, total bank capital, binding capital constraints, retained earnings and bank value by a stock analyst for a shareholder.

5.1.4.1 Bank Capital

Despite the analysis in Subsection 2.4.1 of Section 2.4, bank capital is notoriously difficult to define, monitor and measure. For instance, in our model of bank capital, we regard intangible assets as influencing the computation of Tier 1 capital that appears on the balance sheet. However, in contributions such as [38] and [71], intangible assets are considered to be off-balance sheet items. With regard to the latter, the measurement of equity depends on how all of a bank's financial instruments and other assets are valued. The description of the shareholder equity component of bank capital, E , is largely motivated by the following two observations. Firstly, it is meant

to reflect the nature of the book value of equity. Our intention is also to recognize that the book and market value of equity is highly correlated.

Under Basel II, bank capital requirements have replaced reserve requirements (see Section 2.3) as the main constraint on the behavior of banks (see, for instance, [20]). A first motivation for this is that bank capital has a major role to play in overcoming the moral hazard problem arising from asymmetric information between banks, creditors and debtors. Also, bank regulators require capital to be held to protect the depositor and the taxpayer against the costs of financial distress, agency problems and the reduction of market discipline caused by the safety net. Subsection 2.4.1.3 suggests that a close relationship exists between bank capital holding and macro-economic activity in the loan market. As was mentioned before, Basel II dictates that a macro-economic shock will affect the loan risk weights in the CAR. In general, a negative (positive) shock results in the tightening (loosening) of the capital constraint from (3.5). As a consequence, in terms of a possible binding capital constraint, banks are free to increase (decrease) the loan supply when macro-economic conditions M_t improve (deteriorate). On the other hand, if the risk weights are constant, a shock does not affect the loan supply but rather results in a change in the loan rate when the capital constraint binds. It is not always true that Basel II risk-sensitive weights lead to an increase (decrease) in bank capital when macro-economic activity in the loan market increases (decreases). A simple explanation for this is that macro-economic conditions do not necessarily only affect loan demand but also influences the total capital constraint from (3.5). Furthermore, banks do not necessarily need to raise new capital to expand their loan supply, since a positive macro-economic shock may result in a decrease in the RWAs with a commensurate increase in CARs (compare the minimum capital constraint as expressed in (2.8) and (3.5)). Similarly, banks are not compelled to decrease their capital when the loan demand decreases since the capital constraint usually tightens in response to a negative macro-economic shock. A further complication is that an improvement in the latter conditions may result in an increase in the loan demand and, as a consequence, an increase in the probability that the capital constraint may be binding. Banks may react to this situation by increasing capital to maximize profits (compare the definition of the return on equity (ROE) measure of profitability). Our main conclusion is that bank capital is procyclical because it is dependent on fluctuations in loan demand which, in turn, is reliant on macro-economic activity.

5.1.4.2 Profits and Retained Earnings

As far as the bank valuation is concerned, an interesting scenario from Subsection 2.4.2 to consider, is when $\Delta F_t = 0$ in (2.14). This provides another expression for profit of the form

$$N_t = \Pi_t = E_t^r + n_t d_t + (1 + r_t^O)O_t + (1 + r_t^{R^l})R_t^l.$$

If, in addition, $(1 + r_t^O)O_t = O_{t+1}$ and $(1 + r_t^{R^l})R_t^l = R_{t+1}^l$ then we may conclude that

$$\Pi_t = E_t^r + n_t d_t + O_{t+1} + R_{t+1}^l.$$

In turn, this results in the inequalities

$$\Pi_t > n_t d_t \Rightarrow n_{t+1} E_t < n_t E_t \text{ and } \Pi_t < n_t d_t \Rightarrow n_{t+1} E_t > n_t E_t.$$

Essentially, under the assumption that $\Delta F_t = 0$, the first statement implies that if the profit exceeds the dividends in period t then there may be a decline in the period $t+1$ shareholder equity when compared with period t equity. The opposite is true for the second statement. Furthermore, the interplay between Basel II and bank valuation by the financial market can be seen by combining (2.15) and (2.16) that results in

$$K_{t+1} = \mathbf{E}_t \left[\sum_{j=1}^{\infty} \delta_{t,j} N_{t+j} \right].$$

5.1.4.3 Bank Valuation by a Stock Analyst

Bank value (as described in Subsection 2.4.3) is alternatively defined to be equal to the market value of the investors equity (stock market capitalization if a company is quoted) plus the market value of the net financial debt.

5.1.4.4 Bank Valuation: Related Items

In some quarters, the deposit rate , r^D , described in Subsection 2.4.4 is considered to be a strong approximation of the central bank monetary policy. Since such policy is

usually affected by macro-economic activity, M , we expect the aforementioned items to share an intimate connection. However, in our analysis, we assume that the shocks σ_{t+1}^D and σ_{t+1}^M to r^D and M , respectively, are uncorrelated. Essentially, this means that a precise monetary policy is lacking in our bank model. This interesting relationship is the subject of further investigation.

5.1.4.5 Optimal Bank Value for a Shareholder

In this subsection, we discuss some of the issues related to the optimal bank valuation problem presented in Subsection 2.4.5.

Problem 2.4.1 in Subsection 2.4.5 (see, also, Problem 2.0.2) addresses a very important issue in bank operations that is related to the optimal implementation of financial economic principles under regulatory constraint. In this regard, our investigation is largely motivated by the need to maximize profits. As far as the optimal loan rate and general interest rate decisions are concerned, market, credit (see, for instance, [42] and [55]) and interest rate risk are the main risks to be taken care of. An increase in the required CAR might either increase or decrease the market risk borne by the bank.

If we substitute the optimal bank dividends given by (2.24) in Subsection 2.4.5 into the optimal decisions for the loan rate and deposits represented by (2.21) and (2.22), respectively, we can obtain a time-independent solution for the optimal bank valuation problem. This leads to a significant reduction in the technical difficulty of the procedure.

Since we have identified several situations where the capital constraint does not bind (i.e., $l_t = 0$), it would be interesting to consider an analogue of Theorem 2.4.2 that explores this possibility. In this case, a solution to the analogue of the optimal bank valuation problem stated in Problem 2.4.1 in terms of optimal bank loan rate and loan supply is of the form

$$r_t^{\Lambda*n} = \frac{1}{2l_1} \left(l_0 + l_2 M_t + \sigma_t^\Lambda \right) + \frac{1}{2} \left(c^\Lambda + (r_t^D + c^D) + r^d(M_t) + r_t^T(1 + \gamma) \right)$$

and

$$\Lambda_t^{*n} = \frac{1}{2} \left(l_0 + l_2 M_t + \sigma_t^\Lambda \right) - \frac{l_1}{2} \left(c^\Lambda + (r_t^D + c^D) + r^d(M_t) + r_t^T(1 + \gamma) \right),$$

respectively. In this case, the corresponding W_t , deposits and profits are given by

$$W_t^{*n} = \bar{D} + \frac{\bar{D}(1 - \gamma)}{r_t^p} \left(r_t^T - \frac{r_t^D + c^D}{1 - \gamma} \right),$$

$$D_t^{*n} = \bar{D} + \frac{\bar{D}(1 - \gamma)}{r_t^p} \left(r_t^T - \frac{(r_t^D + c^D)}{1 - \gamma} \right) + \Lambda_t^* - K_t$$

and

$$\begin{aligned} \Pi_t^{*n} = & \frac{1}{2} \left(l_0 + l_2 M_t + \sigma_t^\Lambda \right) - \frac{l_1}{2} \left(c^\Lambda + (r_t^D + c^D) + r^d(M_t) + r_t^T(1 + \gamma) \right) \times \\ & \left\{ \frac{1}{2l_1} \left(l_0 + l_2 M_t + \sigma_t^\Lambda \right) \right. \\ & \left. + \frac{1}{2} \left(c^\Lambda + (r_t^D + c^D) + r^d(M_t) + r_t^T(1 + \gamma) \right) \right. \\ & \left. - \left(c^\Lambda + (r_t^D + c^D + r_t^T \gamma) + r^d(M_t) \right) \right\} + (r_t^D + c^D + r_t^T \gamma) K_t \\ & + \bar{D} \left(r_t^T - (r_t^D + c^D + r_t^T \gamma) \right) + \left(r_t^T - \frac{(r_t^D + c^D)}{1 - \gamma} \right) \left[\frac{\bar{D}(1 - \gamma)(r_t^T - (r_t^D + c^D + r_t^T \gamma))}{r_t^p} \right] \\ & - c^w(W_t) - P(M_t), \end{aligned}$$

respectively.

5.2 ANALYSIS OF BANK CAPITAL ISSUES

In this section, we provide a brief analysis of the main issues arising from Chapter 3.

5.2.1 Assets

The drift term in (3.1) of Subsection 3.1 is an interesting one. Under the CAPM model, μ can be quantified as

$$\mu = \beta(r_t^m - r_t^T),$$

where r^m is the rate of return of the market portfolio. The representation of the banks' interest setting shows that banks will experience positive returns in good times when the actual rate of default, r^d , is lower than the provisioning for expected losses, $\mathbf{E}(d)$, and will not be able to cover their expected losses when $r^d > \mathbf{E}(d)$. In the latter case, bank capital will be needed to cover these excess (and unexpected) losses. If this capital is not enough then the bank will face insolvency. Our arguments in Subsection 3.1 (compare (3.1)) will work equally well if we make use of the SDE

$$dS_t = \left(r^T \bar{1} + \mu + \mathbf{E}(d) \right) dt + \sigma dL_t.$$

The supposition that the bank can trade continuously and without frictions is a simplification in the case of a bank that holds a high proportion of loans in its investment portfolio. If we incorporate illiquidity into the current model then the analysis will become considerably more complex. However, since the use of loan securitization by banks is on the increase, it is reasonable to consider the frictionless case. Furthermore, the unhedgeable capital shocks found in our model can be considered to be indicative of the risk associated with totally illiquid assets.

The discussion in Subsection 3.1.2 suggests that the current value of the bank asset portfolio, A , from (3.2) is allowed to evolve without any restriction on time. In practice, it is permitted to do this until it becomes less than a critical asset value, A^c , that is chosen by the shareholders and initiates the default process. Default by a bank results in exogenously determined social costs. Regulatory responses may have a role to play in reducing such costs, given the level of default risk of banks that may find themselves in market equilibrium. On the other hand, instruments such as deposit interest rate controls only affect default risks directly by creating profit buffers for banks. In reality, such banks are given an incentive to choose high risk investment strategies. In our contribution, we are also interested in a prescribed asset value, A^e , set by the regulator, that is instrumental in determining a threshold for bank closure and reorganization. Most banking models omit the possibility that the regulator

can choose the level of the closure rule, A^e , at which the bank will be closed and reorganized. This principle has a definite impact on the risk-taking by the bank's shareholders and managers. Usually, A^e is reliant on a bankruptcy/reorganization cost criterion which has the property that the regulator's closure costs are lower if the bank's asset value, A , is lower at the time of its closure at A^e . In essence, this may mean that the regulator waits for the bank's asset quality to deteriorate while receiving income from monitoring costs. In a more favorable scenario, the bank closure/reorganization cost function leads to the regulator requiring a higher bank asset value base at the time of closure. As a result, the need for continuous monitoring of the bank asset value, A , and the default risk on bank deposits would not arise.

Banks are among the most heavily regulated of all financial institutions. In particular, the computation of risk weighted assets as in Subsection 3.1.3 has become an essential part of the prudent regulation and supervision in the banking industry. As from June 1999, the Basel Committee on Banking Supervision (BCBS) released several proposals (see, for instance, [6], [7] and [8]) to reform the original 1988 Basel Capital Accord (see [5]). These efforts culminated in the Basel II Capital Accord (see, for instance, [9], [10], [13] and [14]) which is based on three pillars (see [26] and [63] for a discussion on the interaction between these pillars). Pillar 1 intends to provide a stronger link between the management of capital requirements and actual risk. An important factor in the establishment of this link is the computation of time-dependent risk-weighted assets. Pillar 2 focuses on strengthening the supervisory process, particularly in assessing the quality of risk management in banks and in evaluating whether these banks have adequate procedures to determine how much capital they need. Pillar 3 involves the improvement of market discipline through increased disclosure of details about the bank's credit exposures, its amount of reserves and capital, the bank owners and the effectiveness of its internal ratings system. Since bank management has become increasingly complicated and supervisors (acting as representatives of the depositors' interests) find it more difficult to monitor banking activities, the recourse to market discipline appears to be justified. In this regard, monitoring of banks by professional investors and financial analysts as a complement to banking supervision is being encouraged. However, the manner in which market discipline and the other two pillars are to be managed in concert with each other is a subject of much debate.

We note that the risk-weighted assets computed for the first capital accord were invariant over time while Basel II requires such assets to be time-varying. In this regard, from Subsection 3.1.3 we recall that (3.3) provides an expression for the capital charge to cover credit risk. This definition is aligned with Basel II (see, for instance, [12]) that prescribes that

$$\begin{aligned} \text{Credit Risk Charge} &= 0.08 \times \text{Asset Positions Sum} \\ &\times \text{Asset Specific Weights (Ranging from 0 to 1.5)}. \end{aligned}$$

In our situation, we have that this corresponds to risk weights ranging from 0 to $0.08 \times 1.5 = 0.12$. As a matter of interest, unrated corporate claims that include equity are assigned a weight of 100 % (i.e., $0.08 \times 1 = 0.08$ in our framework). On the other hand, for example a risk weight of zero is assigned for investment in a money market account.

5.2.2 Liabilities

In our thesis, we commented on the face value of the deposits (outstanding debt) that we consider to be the only liability. Much more can be said about the deposits introduced in our model. For instance, our study can be expanded to include stochastic counting processes for deposits taken and withdrawn as well as a consideration of the cost of such activities. Although this will improve the model that we have derived here, it will also add a great deal of complexity. However, this is a topic for future investigation.

5.2.3 Modelling of Bank Regulatory Capital

In this subsection, we describe bank regulatory capital, discuss the binding capital constraints and consider the stochastic dynamics of bank regulatory capital.

5.2.3.1 Description of Bank Regulatory Capital

We note that the definition of bank regulatory capital given by (3.4) differs from the market value of the bank's equity because the value of the bank's default option is not included in the value of the bank's assets.

5.2.3.2 Stochastic Dynamics of Bank Regulatory Capital

In Subsection 3.3.2, we discussed the dynamics of bank regulatory capital. We derived a Lévy driven process for the capital which excludes rare events. In future, a study can be made on the effect of rare events on the bank's capital.

5.2.3.3 Capital Adequacy Ratios

Subsection 3.3.3 suggests that CARs are an important tool to determine whether the bank is at risk or not. For instance, we derived a stochastic differential equation for the capital-to-total assets ratio. This gives us a good representation of the dynamics of the ratio. We can use this to simulate the CAR, but it has to be said that this is not very simple.

Furthermore, Subsection 3.3.3.2 provides an expression for the capital-to-risk-weighted assets ratio, κ . Essentially, banks strive to maintain κ in excess of some CAR regulatory benchmark with supervisory intervention resulting if this is not the case. The exact value of the regulatory ratio, κ , may vary quite considerably from institution to institution (see, for instance, [62] and [63]). In fact, subject to an appropriate choice for some CAR regulatory benchmark, κ^r , some banks may consider that equality in (3.5) implies an optimal choice of the investment in loans. Despite the fact that more than 100 countries will be Basel II-compliant by the end of the year 2007, limitations in this regulatory framework have become apparent (see, for instance, [32], [55], [27] and [42]). For instance, Basel II gives a precise description of the bank regulatory capital and TRCs to be used in the computation of κ in (3.8), but neglects to provide complete details of reference processes and thresholds for bank closure, shirking, corrective action and continuance in relation to κ . This is subject to legal requirements which differ vastly between different jurisdictions.

5.3 NUMERICAL AND ILLUSTRATIVE EXAMPLES

In this subsection, we provide some comments about the historical evidence supporting our modelling choices and examples in Chapter 4.

5.3.1 Data

The data for the OECD countries was easily obtained from the website of the International Monetary Fund (IMF). On the website one can get data for the GDP as well as for the output gap. Thus there was no need to calculate these values. For South Africa the capital-to-risk-weighted assets ratio as well as the capital-to-total assets ratio was obtained from the South African Reserve Bank. The output gap had to be calculated as is explained in Section 8.2.

5.3.2 Numerical Examples: Bank Provisioning

In this subsection, we discuss the procyclicality of provisions for loan losses and correlations between profitability and provisions for loan losses.

5.3.2.1 Procyclicality of Provisions for Loan Losses

By considering data from OECD countries, it seems clear from the figures in Subsection 4.2.1 that provisions for loan losses are strongly procyclical since they are negatively correlated with the business cycle. This then, in turn, encourages procyclical lending practices amongst banks. The financial imbalances caused by this results in financial instability when favorable economic conditions are reversed.

5.3.2.2 Correlations between Profitability and Provisions for Loan Losses

We have shown in Subsection 4.2.2 of Chapter 4 that provisions have a procyclical effect on bank profitability. Although this was true for most countries, it was not the case for Germany. By way of explanation of the latter phenomenon, we suspect that Germany has not been affected so much due to the fact that they smoothed their income using reserves. In general, however, from the empirical data presented in Subsection 4.2.2, we can confirm that our suspicions about the procyclicality effects of provisions on profitability are in fact correct. The effect of provisions are also felt in equity although not to the same extent as in profitability.

5.3.3 Numerical Examples: Bank Regulatory Capital

In this subsection, we consider the simulation of the capital-to-risk-weighted assets ratio vs output gap for Japan and other OECD countries. We follow the same programme for capital adequacy in South Africa.

5.3.3.1 Simulation of Capital-to-Risk-Weighted Assets Ratio vs Output Gap for Japan

In Subsection 4.3.1, we do a numerical simulation of a stochastic differential equation driven by Brownian motion based on the Euler-Maruyama Method (EMM). Methods for the numerical simulation of models involving Lévy processes are more complex and have not been considered here although they make for interesting future research.

5.3.3.2 Illustrations of Capital-to-Risk-Weighted Assets Ratio and Capital-to-Total Assets Ratio vs Output Gap for other OECD Countries

If we observe the figures in Subsection 4.3.2 more closely, we can clearly see that for Australia, Finland, Norway, Spain, Sweden and the United Kingdom the capital-to-risk-weighted assets ratio is negatively correlated with the output gap. Thus if the output gap is decreasing the capital-to-risk-weighted assets ratio is increasing. This is seen as an indication that the capital-to-risk-weighted assets ratio is procyclical. For the other countries included in this article that belong to the OECD the effect is not as clear from the figures but they respond in a similar way. In subsection 4.3.3 we see that this is in fact the same situation in South Africa. This shows us that the assumption that the Basel II Accord might have a procyclical effect on the economy, once it is implemented, was in fact very real. We now compare the capital-to-risk-weighted assets ratio with the capital-to-total assets ratio. We can see that for Italy, Spain, Sweden and the United Kingdom that the path of the two ratio's differ somewhat. The other OECD countries have very similar paths for their capital adequacy ratio's. The countries that have different paths for their capital adequacy ratio's indicate that their regulatory capital is procyclical. For these countries their regulatory capital reflects the economic cycle. If we now consider the case of South Africa we can see that their regulatory capital is procyclical.

Section 4.3 suggests that a close relationship exists between bank regulatory capital holding and macro-economic activity. Actually, Basel II dictates that a macro-economic shock will affect the loan risk-weights in the CAR. In general, a negative (positive) shock results in the tightening (loosening) of the capital constraint given by (3.5). As a consequence, in terms of a possible binding capital constraint, banks are free to increase (decrease) the loan supply when macro-economic conditions improve (deteriorate). On the other hand, if the risk-weights are constant, a shock does not affect the loan supply but rather results in a change in the loan rate when the capital constraint binds. It is not always true that Basel II risk-sensitive weights lead to an increase (decrease) in bank regulatory capital when macro-economic activity in the loan market increases (decreases). A simple explanation for this is that macro-economic conditions do not necessarily only affect loan demand but also influences the total capital constraint from (3.5). The role of rating agencies is of extreme importance as they will determine the risk weights to be used as the output gap and probability of default of borrowers change. Furthermore, banks do not necessarily need to raise new capital to expand their loan supply, since a positive macro-economic shock may result in a decrease in the RWAs with a corresponding increase in CARs. Similarly, banks are not compelled to decrease their capital when the loan demand decreases

since the capital constraint usually tightens in response to a negative macro-economic shock. A further complication is that an improvement in economic conditions may result in an increase in the loan demand and, as a consequence, an increase in the probability that the capital constraint will be binding. Banks may react to this situation by increasing capital to maximize profits (compare the definition of the return on equity (ROE)). Our main conclusion is that bank regulatory capital is procyclical because it is dependent on fluctuations in loan demand which, in turn, is reliant on macro-economic activity.

The illustrations in Section 4.3 suggest that the relationship between the economic cycle and bank regulatory capital is not as obvious as suggested in the previous paragraph. While it is clear that the level of bank regulatory capital is positively correlated with the economic cycle, there does not appear to be a robust relationship between measured capital ratios and the economic cycle. To some extent, the task of detecting any relationship is made difficult by the introduction of the Basel Capital Accord in 1988, which some have argued caused a structural change in capital ratios in some countries (for a survey of the impact of the Basel Capital Accord see, for instance, [8]). The analysis is further complicated by the fact that government support schemes have influenced CARs. Nevertheless, long-run historical time series do not suggest a strong economic cycle effect, with the main stylized fact being a steady decline in capital ratios over the 20th century before a slight increase over the past decade or so. Further, the cycle in the capital-to-risk-weighted assets ratio was much more pronounced than the cycle in the capital-to-total assets ratio. This reflects the fact that, in the aftermath of banking crises, risk-weighted assets fell more strongly than total assets, as banks shifted their portfolios away from commercial lending (which has a relatively high risk weight) towards residential mortgages and public sector securities (both of which have relatively low risk weights). For instance, empirical evidence shows that the evolution of capital ratios in the United States generally decreases in the time preceding the recession that began in the late 1990's and then increases during the recession. We conclude from Figure 4.3.3 that as the output gap decreases the capital-to-risk-weighted assets ratio increases and vice versa. This is broadly consistent with the data from most of the OECD countries. With regard to cyclicity, there are two important explanations to the conclusion that CARs tend to be acyclical. The first is that, to the extent that provisions underestimate expected losses in expansions, measured capital ratios overstate true capital ratios in expansions. This effect can be significant. For example, if the ratio of provisions to total assets is 1 percentage point below where it should be, then the measured capital ratio is likely to overstate the true capital ratio by at least 10%. If adjustments were made to capital for under-provisioning in economic booms, it is likely that, all else being equal, mea-

sured CARs would fall during expansions and increase during downswings. A second explanation is that there has been a pronounced cycle in aggregate capital ratios over the 1990s in those countries that experienced problems earlier in the decade. In the years immediately after the crisis, when conditions were relatively depressed, banks made a concerted effort, not only to rebuild their CARs, but also to substantially increase them above previous levels. Then, starting in the mid-1990s, when economic expansions were firmly entrenched, some of the increase in CARs was unwound. This pattern is evident in Australia, Sweden and Norway and to a lesser extent in Finland.

5.3.3.3 Illustration of Capital-to-Risk-Weighted Assets Ratio and Capital-to-Total Assets Ratio vs Output Gap for South Africa

For South Africa, from 2008 onward, the banks have to hold their 8% capital of risk-weighted assets in terms of Pillar 1. In addition they have to hold another 2% for Pillar 2 of the Basel II Accord. This brings the total to 10%. As from the Financial Stability Review of September 2007 of the South African Reserve Bank (see [66]) we find that banks are well capitalised. Against the minimum regulatory capital-adequacy requirement of 10 per cent (in terms of Basel I), the capital-adequacy ratio for the banking sector was 12,2 per cent in June and July 2007. The asset quality of banks, as measured by the ratio of gross overdues to total gross loans, remained at 1.2 per cent in June and July 2007. However, gross overdues are growing at a high rate and are monitored closely. For South Africa banks will need to monitor the output gap closely to see what the impact will be on the risk-weighted assets. Also, care should be taken to the rating agencies as these are the agents that decide on the risk weights when the output gap changes.

5.3.4 Illustration of Bank Management Practice

In this subsection, we provide some comments about the illustrative example of bank management in Section 4.4.

5.3.4.1 Setting the Scene

The illustration in Section 4.4 mainly deals with capital requirements but is also loosely related to asset requirements that are formulated by the bank's shareholders and regulators. In this regard, the value of the bank's asset portfolio, A , is allowed to evolve without any restriction on time until it becomes less than a critical asset value, A^s , that is chosen by the shareholders and initiates the default process. In

addition, we can consider a related prescribed asset value, α^r , set by the regulator, that is instrumental in determining a threshold for bank closure and reorganization. The problem of determining and characterizing A^s and α^e and their interrelationship is sometimes called the *asset threshold problem*.

5.3.4.2 Pillar 1 - Minimum Capital Requirement

Pillar 1 of Basel II intends to provide a stronger link between the management of capital requirements and actual risk. Another way of stating this is to align economic capital with regulatory capital.

5.3.4.3 Pillar 2 - Supervisory Review

Pillar 2 focusses on strengthening the supervisory process, particularly in assessing the quality of risk management in banking institutions and in evaluating whether these institutions have adequate procedures to determine how much capital they need.

5.3.4.4 Pillar 3 - Market Discipline

Pillar 3 involves the improvement of market discipline through increased disclosure of details about the bank's credit exposures, its amount of reserves and capital, the bank owners and the effectiveness of its internal ratings system. Since bank management has become increasingly complicated and supervisors (acting as representatives of the depositors' interests) find it a bit more difficult to monitor banking activities, the recourse to market discipline appears to be justified. In this regard, monitoring of banks by professional investors and financial analysts as a complement to banking supervision is being encouraged. However, the manner in which market discipline and the other two pillars are to be managed in concert with each other is a subject of much debate.

Chapter 6

CONCLUSIONS AND FUTURE DIRECTIONS

6.1 CONCLUDING REMARKS

6.2 FUTURE DIRECTIONS

In this chapter, we provide a few brief concluding remarks and comment about possible topics for future research.

6.1 CONCLUDING REMARKS

The first chapter was introductory of nature. Chapter 2 described a discrete-time model for banks. We started by stating two problems, (see Problems 2.0.1 and 2.0.2), that was solved in the chapter. In this regard, we described loans and their supply and demand as well as provisioning for loan losses and how this was measured. We assumed that the bank faces a Hicksian demand for loans. Next, in Section 2.3, we discussed related items such as Treasuries, reserves, risk-weighted assets. Also included were intangible assets which can be seen as the value of the brand of the bank. The final part of Chapter 2, Section 2.4, was dedicated to bank valuation with the goal of finding the optimal bank value for a stock analyst that is possibly acting on behalf of a potential shareholder. Many factors were taken into consideration including profit, retained earnings and capital constraints. The main result of this chapter was Theorem 2.4.2 where a solution to the optimal bank valuation problem was given. Chapter 2 along with Chapter 3 was the main part of the thesis. We did our main analysis in these two chapters and we frequently referred back to them.

Chapter 3 described assets (see Subsection 3.1), liabilities (see Subsection 3.2) and capital (see Subsection 3.3) as part of an effort to find a Lévy-process driven model for a bank. The price process for assets was defined and applied to obtain equations for the asset portfolio of a bank (see Subsubsection 3.1.2). We considered the risk-weighted assets in Subsubsection 3.1.3. We next defined liabilities for our thesis (Subsection 3.2). The regulatory capital of a bank was discussed in the next part, Subsection 3.3, where we looked at the stochastic dynamics of bank regulatory capital. We derived equations for both the total assets, (see Theorem 3.3.1) and risk-weighted assets, (see Theorem 3.3.2) capital adequacy ratios. This was the main result of Chapter 3.

In Chapter 4 we considered numerical and illustrative examples of provisioning, (see Subsection 4.2) and capital adequacy ratios, (see Subsection 4.3) for OECD countries as well as South Africa (in some cases). We compared the provisioning for loan losses to the output gap of the respective countries and explained why they can be seen as procyclical in Subsubsection 4.2.1.4. We also did a simulation of the CAR in Japan using the model obtained in Chapter 3 in Subsubsection 4.3.1. This was followed

by illustrative examples of the other OECD countries (see Subsubsection 4.3.2) and South Africa (see Subsubsection 4.3.3). The final section of Chapter 4, Section 4.4 contained a stylized illustration of bank management practice in relation to the analysis done in the sections prior to Section 4.4.

Chapter 5 contained a brief discussion of the main issues involved in the thesis. We started in Section 5.1 by looking at the issues raised in Chapter 2. We discussed the assumptions made and considered special cases. Next, in Section 5.2 we analyzed the results obtained in Chapter 3 to see what the implications were of the work that was done. Special attention was also given to the simulation contained in Chapter 4 in Subsection 4.3.1. We concluded this chapter with a discussion of the illustrative example that is supplied at the end of Chapter 4 in Section 4.4.

6.2 FUTURE DIRECTIONS

Future modelling research should consider that portfolio values also decline as interest rates decline. In this regard, it has to be borne in mind that interest rates usually decline with downturns in business. Our thesis assumes that the quality of the supply of loans is constant regardless of the position in the business cycles. Our discussion of the bank's lending responses to business cycles largely ignores the nexus between the proximity to its capital constraints and its forthrightness in recognizing loan losses. This relationship may not be constant over time. The manuscript does not establish any solid basis for comparison of extant valuation practices nor demonstrate the superiority of the preferred model to another model. In particular, we need to learn more about the inadequacies of current practice as a basis for substantiating the need for our modelling paradigm. Moreover, there is no empirical support preferred to demonstrate the superiority of the model to any other model or the accuracy of its performance relative to any market benchmark. Hence, even if one could utilize the model in some valuation project to estimate the value of the bank's common shareholder's equity, we have no idea of how close we might be to the truth with respect to market values. Since fair market values are of great interest to valuers, a test of reasonable congruence with such values would seem to be prerequisite to the use of the valuation model for purposes of business valuation. Although large empirical samples would be preferred, smaller sample analysis using recent actual acquisitions might also be helpful. All these facts have to be incorporated in future modelling programmes.

We would also like to investigate whether shifts in loan supply are responsible for faster loan growth that, in turn, leads to higher loan losses. This factor also determines if supply shifts have caused loan growth and loan losses to be positively related in the past. In the main, there is not much current support for this hypothesis. Data on loans and defaults show that banks experiencing unusually rapid loan growth tend to experience unusually big increases in default rates several years later. The worst loans are made at the top of the cycle. However, we should be cautious since evidence on business loan growth and business credit standards suggest that changes in loan growth are not always due to loan supply shifts.

Another research topic will involve complex models of bank items driven by Lévy processes (see, for instance, Protter in [60, Chapter I, Section 4]). Such processes have an advantage over the more traditional modelling tools such as Brownian motion in that they describe the non-continuous evolution of the value of economic and financial items more accurately. For instance, because the behavior of bank loans, wealth, capital and CARs are characterized by jumps, the representation of the dynamics of these items by means of Lévy processes is more realistic. As a result of this, recent research has strived to replace the existing Brownian motion-based bank models (see, for instance, [26], [32], [34], [49], [57] and [63]) by systems driven by more general processes. Also, a study of the optimal capital structure should ideally involve the consideration of taxes and costs of financial distress, transformation costs, asymmetric bank information and the regulatory safety net. Another research area that is of ongoing interest is the (credit, market, operational, liquidity) risk minimization of bank operations within a regulatory framework (see, for instance, [42], [55] and [56]). Another risk that becomes important is *interest rate risk* at the point of loan issuing. For instance, an alternative optimization problem would be to maximize the risk-free rate of interest in order to provide a shareholder with an incentive to invest money.

The reliability, transparency and quality of dynamic modelling are critical to the efficient allocation of resources by role-players in the banking industry. For instance, regulators can benefit greatly by the employment of sound modelling techniques. In this thesis, we are able to specifically add to the debate about the mathematical modelling and simulation of bank capital. We also discussed some of the economic issues arising from the stochastic dynamic models mentioned earlier.

Several interesting questions related to dynamic modelling and simulation of bank capital adequacy remain open. Amongst these is the removal of the assumption that the bank can only trade continuously. A way of increasing the complexity of the CAR models would be to lift the restriction that the deposits are constant over the planning horizon. While we do not explicitly impose short-sale constraints, it would be

interesting to see its effect on our results. Another factor that would affect the complexity of our model of bank capital would be illiquidity. A further interesting topic for future investigation would involve the influence of rare events on the definition of bank regulatory capital. This thesis leaves open the problem of risk management associated with the holding of capital in our framework. The aforementioned problem should be an interesting one since a Lévy process setting usually provides fertile ground for addressing risk issues.

Chapter 7

BIBLIOGRAPHY

Bibliography

- [1] Alexander GJ, Baptista AM, Does the Basle Capital Accord reduce bank fragility? An assessment of the value-at-risk approach, *Journal of Monetary Economics* 2006; **53**: 1631-1660.
- [2] Altug S, Labadie P, *Dynamic Choice and Asset Markets*. San Diego CA: Academic Press, 1994.
- [3] Anandarajan A, Hasan I, Lozano-Vivas A, Loan loss provision decisions: An empirical analysis of the Spanish depository institutions, *Journal of International Accounting, Auditing and Taxation* 2005; **14**: 55-77.
- [4] Basak S, Shapiro A, Value-at-risk based management: Optimal policies and asset prices, *The Review of Financial Studies* 2001; **14**: 371-405.
- [5] Basel Committee on Banking Supervision, *International Convergence of Capital Measurement and Capital Standards*, Bank for International Settlements, <http://www.bis.org/publ/bcbs04A.pdf> [July 1988].
- [6] Basel Committee on Banking Supervision, *Amendment to the Capital Accord to Incorporate Market Risks* (no. 24), Bank for International Settlements, <http://www.bis.org/publ/bcbs04A.pdf> [January 1996].
- [7] Basel Committee on Banking Supervision, *Amendment to the Capital Accord to Incorporate Market Risks*, Bank for International Settlements [September 1997].
- [8] Basel Committee on Banking Supervision, *Credit Risk Modelling: Current Practices and Applications*, Bank for International Settlements [1999].

- [9] Basel Committee on Banking Supervision, *Second Consultative Paper*, Bank for International Settlements, <http://www.bis.org/bcbs/bcbscp2.htm> [January 2001].
- [10] Basel Committee on Banking Supervision, *The New Basel Capital Accord*, Bank for International Settlements, <http://www.bis.org/publ/bcbzca.htm> [January 2001].
- [11] Basel Committee on Banking Supervision, *The Internal Ratings Based Approach*, Bank for International Settlements, <http://www.bis.org/publ/bcbzca.htm> [May 2001].
- [12] Basel Committee on Banking Supervision, *Working Paper on the Regulatory Treatment of Operational Risk*, Bank for International Settlements, <http://www.bis.org/publ/bcbzca.htm> [September 2001].
- [13] Basel Committee on Banking Supervision, *Third Consultative Paper*, Bank for International Settlements, <http://www.bis.org/bcbs/bcbscp3.htm> [April 2003].
- [14] Basel Committee on Banking Supervision, *International Convergence of Capital Measurement and Capital Standards; A Revised Framework*, Bank for International Settlements, <http://www.bis.org/publ/bcbs107.pdf> [June 2004].
- [15] Berger AN, Herring RJ, Szegö G, The role of capital in financial institutions, *Journal of Banking and Finance* 1995; **19**: 393-430.
- [16] Bertoin J, *Lévy Processes*, Cambridge Tracts in Mathematics 121, Cambridge: Cambridge University Press; 1996.
- [17] Bikker JA, Metzmakers PAJ, Bank provisioning behaviour and pro-cyclicality, *International Financial Markets, Institutions and Money* 2005; **15**, 141-157.
- [18] Bliss R, Kaufman G, Bank pro-cyclicality, credit crunches and asymmetric monetary policy effects: a unifying model, *Federal Reserve Bank of Chicago* 2002; Working Paper 2002-2018.
- [19] Borio C, Furfine C, Lowe P, Pro-cyclicality of the financial system and financial stability: issues and policy options, *Bank for International Settlements*, Working Paper 2001; www.bis.org/publ/biospap01a.pdf.
- [20] Bosch T, Mukuddem-Petersen J, Petersen MA, Schoeman IM, Optimal auditing in the banking industry, *Optimal Control, Applications and Methods* 2007; doi:10.1002/OCA.828.

- [21] Catarineu-Rabell E, Jackson P, Tsomocos DP, Pro-cyclicality and the new Basel Accord - bank's choice of loan rating system, *Bank of England* 2003; Working Paper, no. 181.
- [22] Cavallo M, Majnoni G, Do banks provision for bad loans in good times ? Empirical evidence and policy implications, In: Levich, R., Majnoni, G., Reinhart, C. (Eds.), *Ratings, Rating Agencies and the Global Financial System*, Boston, Dordrecht, London: Kluwer Academic Publishers, 2002.
- [23] Chami R, Cosimano TF, Monetary policy with a touch of Basel, *International Monetary Fund* 2001; Working Paper WP/01/151.
- [24] Cuoco D, Liu H, An analysis of VaR-based capital requirements, *Journal of Financial Intermediation* 2006; **15**: 362-394.
- [25] Dangl T, Zechner J, Credit risk and dynamic capital structure choice, *Journal of Financial Intermediation* 2004; **13**: 183-204.
- [26] Decamps J-P, Rochet J-C, Roger B, The three pillars of Basel II: Optimizing the mix, *Journal of Financial Intermediation* 2004; **13**: 132-155.
- [27] Dewatripont M, Tirole I, *The Prudential Regulation of Banks*. MIT Press, 1994.
- [28] Diamond DW, Rajan RG, A theory of bank capital, *The Journal of Finance* 2000; **55**: 2431-2465.
- [29] Djankov S, Jindra J, Klapper LF, Corporate valuation and the resolution of bank insolvency in East Asia, *Journal of Banking & Finance* 2005; **29**: 2095-2118.
- [30] Estrella A, Bank capital and risk: Is voluntary disclosure enough ? *Journal of Financial Services Research* 2004; **26**: 145-160.
- [31] Fissel GS, Goldberg L, Hanweck GA, Bank portfolio exposure to emerging markets and its effects on bank market value, *Journal of Banking & Finance* 2006; **30**, 1103-1126.
- [32] Fouche CH, Mukuddem-Petersen J, Petersen MA, Continuous-time stochastic modeling of capital adequacy ratios for banks, *Applied Stochastic Models in Business and Industry* 2006; **22**: 41-71.
- [33] Gideon F, Mukuddem-Petersen J, Mulaudzi MP, Petersen MA, Optimal provisioning for bank loan losses in a robust control framework, *Optimal Control, Applications and Methods* 2007; doi:10.1002/OCA.975.

- [34] Gideon F, Mukuddem-Petersen J, Petersen MA, Minimizing banking risk in a Lévy process setting, *Journal of Applied Mathematics* 2007; doi:10.1155/2007/32824.
- [35] Giorno C, Richardson P, Roseveare D, van den Noord P, *Estimating Potential Output, Output Gaps and Structural Budget Balances*. Organization for Economic Co-Operation and Development (OECD) Economics Department, Working Paper # 152 OECD/GD(95)1, 1995.
- [36] Hackbarth D, Miao J, Morellec E, Capital structure, credit risk and macroeconomic conditions, *Journal of Financial Economics* 2006; **82**:519-550.
- [37] Hancock D, Laing AJ, Wilcox JA, Bank capital shocks: Dynamic effects and securities, loans and capital, *Journal of Banking and Finance* 1995; **19**:132-155.
- [38] Hand JRM, Lev B, *Intangible Assets: Values, Measures and Risks*, Oxford University Press, 2003.
- [39] Hodrick R, Prescott E, Postwar U.S. business cycle: An empirical investigation, *Journal of Money, Credit and Banking* 1997; **29**:1-16.
- [40] Holmstrom B, Tirole J, Financial intermediation, loanable funds and the real sector, *Quarterly Journal of Economics* 1997; **112**: 663-691.
- [41] *The International Monetary Fund* www.imf.org
- [42] Jackson P, Perraudin W, Regulatory implications of credit risk modeling, *Journal of Banking and Finance* 2000; **24**: 1-14.
- [43] Jacod J, Shiryaev AN, *Limit Theorems for Stochastic Processes (2nd Ed.)*. Springer: Berlin; 2003.
- [44] Korajczyk R, Levy A, Capital structure choice: Macroeconomic conditions and financial constraints, *Journal of Financial Economics* 2003; **68**: 75-109.
- [45] Kleinert H, *Path Integrals in Quantum Mechanics, Statistics, Polymer Physics and Financial Mathematics*. World Scientific Publication Co.: Singapore; 2004.
- [46] Kyprianou AE, Schoutens W, Wilmott P (Editors), *Exotic Option Pricing and Advanced Lévy Models*, Wiley: Chichester; 2005.
- [47] Laeven L, Majnoni G, Loan loss provisioning and economic slowdowns: too much, too late ? *Journal of Financial Intermediation* 2003; **12**: 178-197.

- [48] Leippold M, Trojani F, Vanini P, Equilibrium impact of Value-at-Risk regulation, *Manuscript: M. Leippold*, 14 July 2003; Swiss Banking Institute, University of Zurich.
- [49] Leland H, Risky debt, bond covenants and optimal capital structure, *Journal of Finance* 1994; **49**:1213-1252.
- [50] Mella-Barral P, Perraudin W, Strategic debt service, *Journal of Finance* 1997; **2**: 531-556.
- [51] Merton RC, The pricing of options and corporate liabilities, *Journal of Political Economy* 1973; **81**: 637-654.
- [52] Merton RC, An analytic derivation of the cost of deposit insurance and loan guarantees: An application of modern option pricing theory, *Journal of Banking and Finance* 1977; **1**: 3-11.
- [53] Modigliani F, Miller M, The cost of capital, corporation finance and the theory of investment, *American Economic Review* 1958; **48**: 261-297.
- [54] Mukuddem-Petersen J, Petersen MA, Stochastic behavior of risk-weighted bank assets under Basel II capital accord, *Applied Financial Economics Letters* 2005; **1**: 133-138.
- [55] Mukuddem-Petersen J, Petersen MA, Bank management via stochastic optimal control, *Automatica* 2006; **42**: 1395-1406.
- [56] Mukuddem-Petersen J, Petersen MA, Optimizing asset and capital adequacy management in banking, *Journal of Optimization Theory and Applications* 2007; doi:10.1007/s10957-007-9322-x.
- [57] Mukuddem-Petersen J, Petersen MA, Schoeman IM, Tau AB, Maximizing banking profit on a random time interval, *Journal of Applied Mathematics* 2007; doi:10.1155/2007/29343.
- [58] Organization for Economic Co-operation and Development (OECD), *Main Economic Indicators*, July 2007.
- [59] Pelizzon L, Schaefer S, Pillar 1 vs Pillar 2 under risk management, *NBER Conference October 2004: Risks of Financial Institutions*, Mark Carey and Rene Stulz editors, 2005.
- [60] Protter P, *Stochastic Integration and Differential Equations (2nd Ed.)*, Springer, Berlin; 2004.

- [61] Repullo R, Capital requirements, market power, and risk-taking in banking, *Journal of Financial Intermediation* 2004; **13**: 156-182.
- [62] Rochet J-C, Capital requirements and the behaviour of commercial banks, *European Economic Review* 1992; **36**: 1137-1178.
- [63] Rochet J-C, Rebalancing the three pillars of Basel II, *Economic Policy Review* 2004; **10**: 7-25.
- [64] Sato K, *Lévy Processes and Infinitely Divisible Distributions*, Cambridge Studies in Advanced Mathematics 68, Cambridge: Cambridge University Press; 2000.
- [65] Schoutens W, *Lévy Processes in Finance: Pricing Financial Derivatives*, Chichester: Wiley; 2003.
- [66] *The South African Reserve Bank*, www.resbank.co.za, 2007.
- [67] Thakor AV, Capital requirements, monetary policy and aggregate bank lending, *Journal of Finance* 1996; **51**: 279-324.
- [68] Tsomocos D, Equilibrium analysis, banking, contagion and financial fragility, *Bank of England*, Working Paper no. 175, 2003.
- [69] van den Heuvel S, The bank capital channel of monetary policy, *Mimeo*, University of Pennsylvania, 2001.
- [70] van den Heuvel S, Does bank capital matter for monetary transmission ?, *Federal Reserve Bank of New York Economic Policy Review* 2002; **8**, 259-265.
- [71] Whitwell GJ, Lukas BA, Hill P, Stock analyst's assessments of the shareholder value of intangible assets, *Journal of Business Research* 2007; **60**: 84-90.

Chapter 8

APPENDICES

8.1 APPENDIX A: OPERATIONAL RISK

8.2 APPENDIX B: OUTPUT GAP

In this chapter, we briefly discuss operational risk and some background about the output gap and how it was calculated. We also provide some output gap data used to obtain certain graphical representations.

8.1 APPENDIX A: OPERATIONAL RISK

In this section, we explain how the charge for operational risk is obtained where the charge for credit risk and market risk were covered in the introduction. The charge to cover operational risk equals the sum of the charges for each of eight business lines (corporate finance, trading and sales, retail banking, commercial banking, payment and settlement, agency services, asset management and retail brokerage). More specifically, the capital charge for operational risk, under the Standardized Approach outlined in Basel II, may be expressed as

$$\max \left[\sum_{k=1}^8 \beta_k g_k, 0 \right],$$

where

g_{1-8} : Three-Year Average of Gross Income for Each of Eight Business Lines;

β_{1-8} : Fixed Percentage Relating Level of Required Capital to Level of Gross Income for Each of Eight Business Lines.

The β -values for operational risk are provided in the document [11].

8.2 APPENDIX B: OUTPUT GAP

This section relates to Chapter 4 where we compared the output gap to provisions for loan losses. In this appendix, we show how to compute the output gap and provide a graphical representation of the potential output and the real output in the South African context. In order to accomplish the former, we demonstrate a method that can be used to calculate potential output in order to determine the output gap.

8.2.1 Computing the Output Gap

The output gap is measured as the percentage difference between actual GDP and estimated potential GDP. Symbolically this means that

$$\text{Output Gap} = \frac{\text{Actual Output} - \text{Potential Output}}{\text{Potential Output}} \times 100.$$

In other words, the output gap involves measuring the position of output in relation to potential. Potential outputs are difficult to estimate and subject to margins of substantial error. Potential output is measured to capture the level of output that an economy can produce based on the available production factors (labour and capital) and the efficiency with which they are combined (total factor productivity). There are various methods of estimating potential output. In the literature, usually a choice from four methods is made. These methods may be listed as

Smoothing Split Time Trend Method; Smoothing Actual GDP via the Hodrick-Prescott Filter Method; Potential Output Method Using a Production Function Approach and Cyclically Adjusted Budget Balances.

We used the Hodrick-Prescott (HP) filter in our analysis. The HP filter derives a trend output such that it minimizes a weighted average of the gap between actual output, Y_t and trend output Y_t^* , and the rate of change in trend output, or its smoothness, over the whole period.

$$\min \frac{1}{T} \sum_{t=1}^T (\ln Y_t - \ln Y_t^*)^2 + \frac{\lambda}{T} \sum_{t=2}^{T-1} [(\ln Y_{t+1}^* - \ln Y_t^*) - (\ln Y_t^* - \ln Y_{t-1}^*)]^2$$

where T is the number of observations, and λ is the factor that determines the smoothness of the trend. A major disadvantage of the HP filter is that, since it is a two-sided symmetric filter, the estimated trend output series suffers from end-point biases. The method also fails to take account of structural breaks in the output series. The computation of potential output is usually based on a production function approach, taking into account the capital stock, changes in labour supply, factor productivities and underlying “non-accelerating wage rate of unemployment” (NAWRU) for South Africa. As regards the latter, the particular idea of potential output (from a supply perspective) considered in the sequel refers to the maximum level of output that is consistent with stable inflation which incorporates the role of NAWRU. The aforementioned approach coincides with the emphasis on the labour market and the

control of inflation as a key medium term priority.

From the viewpoint of macroeconomic analysis, a limitation of the smoothing methods are that they are largely mechanistic and bring to bear no information about the structural constraints and limitations on production through the availability of factors of production or other endogenous influences. Thus, trend output growth projected by time series methods may be inconsistent (too high or too low) with what is known or being assumed about the growth in capital, labour supply or factor productivity or maybe unsustainable because of inflationary pressures. The preferred potential output method attempts to overcome these shortcomings whilst adjusting for the limiting influence of demand pressure on employment and inflation. This is accomplished within a structural framework in which consistent judgement can also be exercised on some of the key elements. For the sake of implementability, this paper relies on the Hodrick-Prescott (HP) filter, a common univariate filtering technique to decompose a time series into a trend and cyclical part. The simplicity and transparency of the HP filter come at a cost as regard the endpoint biases.

8.2.2 Actual Output versus Potential Output for South Africa

In this subsection, we consider historical data for the actual and potential GDP in South Africa for the period 2000-2006. From this data, we can compute the output gap via the method of smoothing actual GDP by means of the Hodrick-Prescott (HP) filter. The following figure presents the actual output vs. the potential output for South Africa.

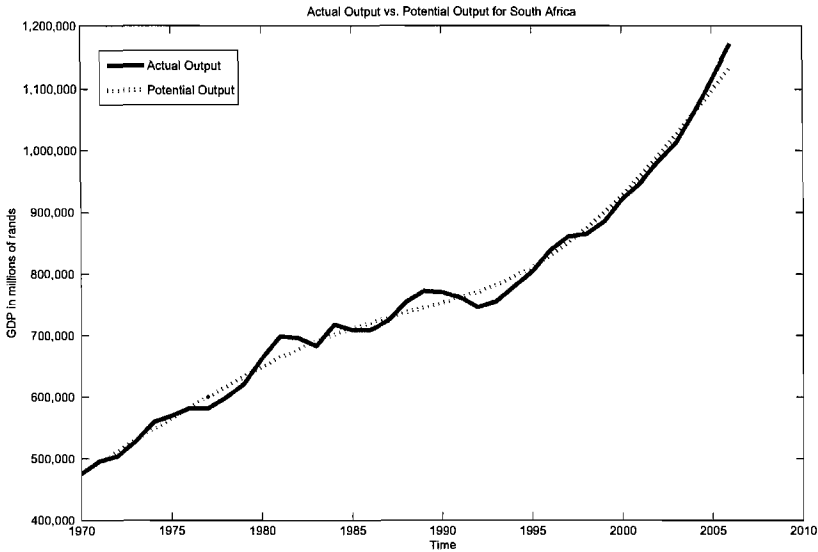


Figure 8.1: Actual Output vs Potential Output for South Africa
Data Source: South African Reserve Bank

From Figure 8.1, it is clear that the actual GDP and potential GDP has increased dramatically in the period 2000 to 2006.

8.2.3 Output Gap Tables

This subsection contains the observed and calculated output gaps of the OECD countries and South Africa.

Year	United States	Japan	Italy	Australia	Finland
1986	1.25	1.01	1.20	2.40	3.60
1987	1.30	0.88	1.30	2.70	3.90
1988	1.55	1.20	1.52	3.00	4.80
1989	2.00	1.52	1.80	3.40	5.60
1990	1.65	2.10	1.82	2.80	5.00
1991	1.25	2.25	1.62	1.80	2.80
1992	1.00	1.75	1.50	0.80	1.60
1993	1.10	1.50	1.25	1.00	0.70
1994	1.30	1.30	0.89	1.90	0.80
1995	1.49	1.10	1.30	2.60	1.10
1996	1.50	1.54	1.35	2.70	1.50
1997	1.55	1.80	1.33	2.80	2.60
1998	1.65	1.20	1.31	3.40	3.00
1999	1.75	0.60	1.30	3.50	3.30
2000	2.00	0.55	1.00	3.70	3.80

Table 8.1: Output Gap for United States, Japan, Italy, Australia and Finland

Year	Norway	Spain	Sweden	United Kingdom
1986	4.90	3.40	0.35	1.00
1987	4.70	4.20	0.70	1.75
1988	3.90	4.70	1.45	2.75
1989	3.00	5.00	1.85	2.65
1990	2.80	5.20	2.00	2.45
1991	2.90	5.20	1.90	1.50
1992	3.00	3.00	1.50	0.80
1993	3.10	1.40	0.90	0.50
1994	3.50	1.00	0.45	0.90
1995	3.80	2.00	0.20	1.20
1996	4.00	2.30	0.10	1.30
1997	4.30	2.00	0.13	1.50
1998	4.50	2.20	0.50	1.58
1999	4.30	2.80	1.00	1.55
2000	4.20	3.20	1.45	1.59

Table 8.2: Output Gap for Norway, Sweden, Spain and the United Kingdom

Year	Output Gap	Year	Output Gap
1970	-0.26	1989	3.62
1971	0.26	1990	2.22
1972	-1.61	1991	0.07
1973	-0.56	1992	-3.26
1974	2.11	1993	-3.49
1975	0.64	1994	-2.06
1976	-0.15	1995	-1.00
1977	-3.08	1996	0.97
1978	-2.91	1997	1.11
1979	-1.88	1998	-1.06
1980	2.03	1999	-1.62
1981	5.07	2000	-0.64
1982	2.57	2001	-1.17
1983	-1.10	2002	-0.92
1984	2.30	2003	-1.26
1985	-0.35	2004	0.01
1986	-1.61	2005	1.59
1987	-0.73	2006	3.15
1988	2.28	-	-

Table 8.3: Output Gap for South Africa