

Hedge fund return attribution and performance forecasting using the Kalman filter

by

Daniel Benjamin Thomson
28396332

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SUPERVISOR: DR GARY VAN VUUREN

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To

GARY VAN VUUREN, FOR THE LEAP YEAR

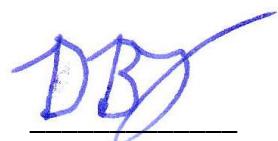
Preface

The theoretical work described in this dissertation was carried out whilst in the employ of Aviva Investors (London, UK). Some theoretical and practical work was carried out in collaboration with the Department of Risk Management at the School of Economics, North-West University (South Africa) under the supervision of Dr Gary van Vuuren.

These studies represent the original work of the author and have not been submitted in any form to another university. Where use was made of the work of others, this has been duly acknowledged in the text. Unless otherwise stated, all data were obtained from Bloomberg™, non-proprietary internet sources, and non-proprietary financial databases of Aviva Investors, London, UK. Discussions with personnel from this institution also provided invaluable insight into current investment trends and challenges faced in the investment risk and portfolio management arena.

The results associated with the work presented in Chapter 5 (apportioning of hedge fund returns between market timing and stock selection and a novel interpretation of results from the former) have been collated into journal article format and submitted to the *Journal of Applied Economics* (November 2016).

Additional work, relating to *economic forecasting* [and thus relevant to, but not directly associated with, other aspects of this dissertation] has been published in the *International Business and Economic Research* journal (Thomson & van Vuuren, 2016). A copy of this article appears in the Appendix.



DANIEL BENJAMIN THOMSON
11 November 2016

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Abstract

The practice of forecasting is fraught with difficulty: history is replete with examples of wildly inaccurate predictions. Nevertheless, attempts to guess future events as accurately as possible form the basis of risk management, economic policy and financial remuneration and rewards. Skilful prediction is, however, a non-trivial exercise, particularly in the fields of finance and economics. Apart from the normal impediment of insufficient historical data to establish the presence and persistence of patterns, prediction accuracy suffers from two additional obstructions. One, opinion and sentiment are often involved, both often (but not always) based on irrational suppositions and two, data are noisy, infrequently sampled, inconsistently recorded and often in short supply. These hindrances, coupled with a multitude of relevant variables (each of which may influence others via multifaceted interactions) can conceal real, but frequently buried, relationships.

The capital asset pricing model (CAPM), mean-variance framework is a metric commonly-used to identify investment performance. Quantities generated from the CAPM assume time-invariance of historical data and use rolling-window, ordinary least squares regression methods to forecast future returns. These quantities are of considerable significance to investors and fund managers since all rely on these to establish compensation and rewards for relevant parties. Problems associated with CAPM regression models diminish the significance of the outputs – sometimes rendering the results irrelevant and the interpretation of the results suspect. The Kalman filter, a variance reduction framework, estimates *dynamic* CAPM parameters. These time-varying parameters improve predictive accuracy considerably compared with ordinary least square (and other) estimates.

The institution and advance of hedge funds offers attractive investment possibilities because they engage in investment styles and opportunity sets which – because they are different from traditional asset class funds – generate different risk exposures (Fung & Hsieh, 1997 and Agarwal & Naik, 2000). Murguía & Umemoto (2004) showed that hedge funds provide unique investment opportunities and add value because of their ability to invest in different risk exposures, not because of the manager's ability to add value through stock selection or market timing. Individual hedge fund returns are apportioned into market timing and stock selection components to identify whether fund managers really do generate statistically significant abnormal profits and, if so, which component dominates the return profile. Compelling evidence is produced to support an alternative interpretation for measured return constituents. As far as the author is aware, this work represents the first time the Kalman filter has been used to extract a time series of the CAPM's dynamic variables for determining fund return component magnitudes. The Kalman filter output provided critical insight into the reassessment of the market timing return component.

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Chapter 1

Introduction

1.1 Problem statement

Is the Kalman filter – widely used in physics and engineering – an effective tool for:

- accurately forecasting hedge fund returns; and
- correctly apportioning the contribution of underlying components to these returns?

1.2 Dissertation structure

This dissertation is structured as follows: Chapter 2 presents literature on forecasting in financial markets, as well as common techniques identified to address the problem. Chapter 3 discusses the origin, development and subsequent introduction of the Kalman filter into mainstream physics and engineering disciplines. This chapter also covers the filter's gradual percolation into financial applications and its successes and failures in this field.

Chapter 4 applies the Kalman filter to the problem of forecasting hedge fund index returns and the determination of the accuracy of these forecasts using out of sample data and Chapter 5 presents results for the decomposition of hedge fund returns into timing and stock selection components – again using the Kalman filter – and proposes a novel interpretation of work conducted and reported in the literature.

Chapter 6 concludes the dissertation by summarising the findings of the entire study and proposing suggestions for future research.

1.3 Background

Forecasting economic and financial variables is a complex task. While forecasting approaches in all fields require sufficient historical data to establish whether patterns are present and persistent, forecasting in finance and economics suffers from two additional obstructions. One, sentiment is often involved and there are no guarantees this sentiment is rational and two, the data are often scant: infrequently sampled and inconsistently recorded. These problems, coupled with the multitude of variables which influence one another through complex interactions, can mask relationships which may be present.

1.3.1 Economic forecasting

Forecasting economic variables customarily involves an understanding of underlying macro-economic relationships. Forecasting a country's gross domestic product (GDP), for example, may require some knowledge of prevailing interest and foreign exchange (FX) rates, consumption per capita, unemployment rates and the national deficit or current account. These quantities are reported at different frequencies (second by second for interest and FX rates, quarterly for unemployment) by various sources (market data providers such as Bloomberg and Reuters and relevant exchanges, central banks for deficits and current accounts, government agencies for consumption) and the rigour and accuracy that accompanies each assembly may not be comparable. In addition, a variable such as the GDP could feasibly depend on other economic quantities such as business innovations, shifts in consumer preferences, governmental policy, or the discovery or depletion of natural resources such as oil fields.

These quantities are less tangible since some subjectivity could be involved in their determination or calculation. The combination of these effects gives rise to noisy data within which trends and sequences will be embedded, but will be mixed with other data that may be spurious or inaccurate. How to extract the 'true' underlying signal is a non-trivial exercise and one which has – and continues to – elicit considerable research.

An example of such research is a technique – borrowed from physics – and now used in mainstream economic forecasting analysis known as Fourier analysis. This approach has established its reliability for extracting information about underlying cycles embedded in economic data. The central ideas underpinning Fourier analysis are the identification of cycle frequencies in a noisy signal and the establishment of the most significant ones, i.e. those cycles which are clearly *not* noise, but rather those that represent cycles present in the signal which may be explained by other economic activity. An example of this may be an annual cycle: a country supplying electricity to another may witness an increase in electricity demand in winter and an associated lull in summer. This ebbing and waning demand will feed through to the country's GDP and could dominate these values if electricity is a major constituent of the commodities and services provided. Other examples could be the rise and fall of business cycles that mark economic progress, interest rate increases and decrease in response to business activity, and even long-term weather effects such as El Niño, currently

(2016) wreaking havoc on certain economies while benefitting others. The frequencies of these cycles may be obscured – and extracting them from GDP data requires sophisticated mathematical methods.

In finance and economics, the predominant method of analysing time-series data is usually to view these data in the time-domain, i.e., analysing changes of a series as it progresses through time. Fourier analysis identifies and isolates any potential cyclical signals and allows the practitioner to extract the frequency and amplitude of these components for further analysis. Changes in annual South African GDP, for example, exhibits possible cyclical behaviour, as shown in Figure 1.1, but this is difficult to determine without detailed mathematical techniques. Even after cyclicity has been established, the frequencies of the underlying cycles still need to be determined if they are to be of any use in forecasting. Again, determining these frequencies is a non-trivial task.

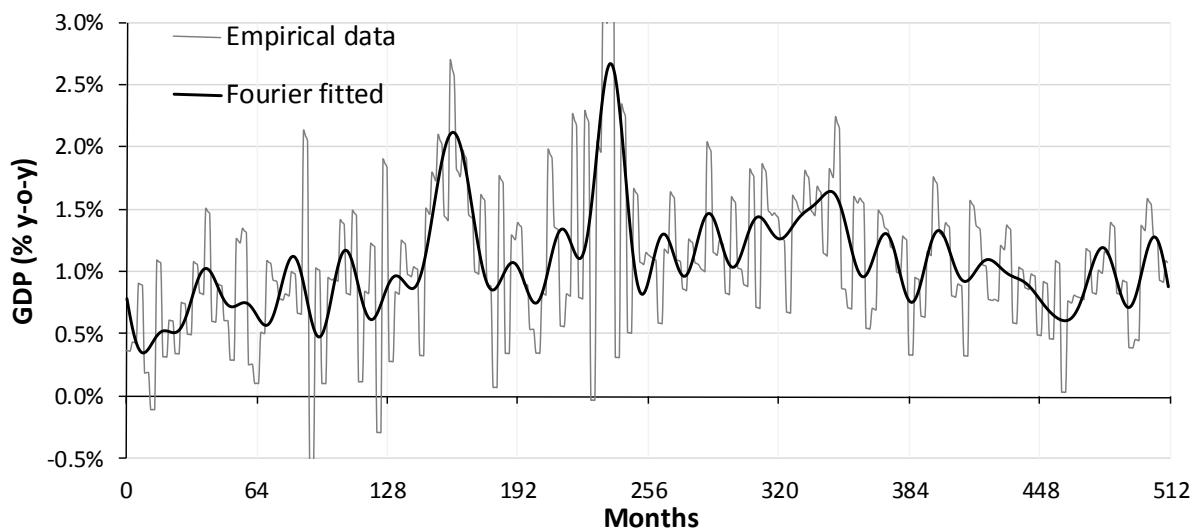


Figure 1.1: South African GDP data in the time domain measured monthly from January 1973 to September 2015, as well as the filtered signal comprising only principal cyclical components.

The problem in using only this approach to study financial datasets is that all realisations are recorded at a predetermined frequency. This frequency corresponds to whichever period the realisations are recorded at and the implicit assumption is made that the relevant frequency to study the behaviour of the variable matches with its sampling frequency (Masset, 2008). This can be construed as analysing inflation figures with a one-year time frame and presuming that the cycle repeats the next year as the cycle is presumed to be one-year long.

The realisations of financial and economic variables often depend on several frequency components rather than just one (see Figure 1.2) so the time-domain approach alone will not be able to process the information precisely (Masset, 2008). For example, data from Figure 1.1 that have been transformed into the frequency domain using Fourier analysis, (Figure 1.2) shows prominent cycle frequencies (as indicated by large amplitudes (< 0.05 cycles/month)), as opposed to noise (as indicated by small amplitudes (> 0.20 cycles/month)).

Spectral analysis methods that enable a frequency-domain representation of the data, such as Fourier series and wavelet methods, can identify at which frequencies the time series variable is active. The strength of the activity may be measured using Fourier analysis to construct a frequency spectrum (or periodogram) – a graphic representation of the intensity of a frequency component plotted against the frequency at which it occurs. This method is particularly attractive for the use of economic variables that exhibit cyclical behaviour as the cycle length may be identified using the Fourier transform (Baxter & King, 1999).

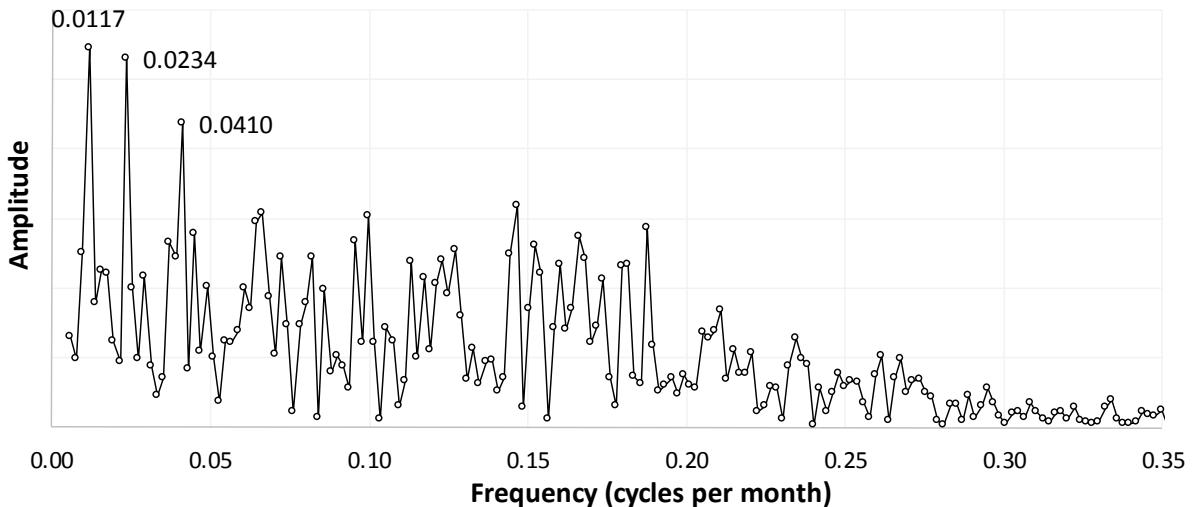


Figure 1.2: South African GDP data in the frequency domain showing prominent cycles present at the frequencies indicated (and associated dominant cycles of 7.1y, 3.6y and 2.0y respectively).

Understanding the business cycle of a region and having an idea of its current position (or phase in the cycle) enables participants in the economy to make informed decisions. Because business cycle information is so valuable, much research has been done to identify its behaviour and the South African business cycle is no exception (see Venter, 2005; Bosch & Ruch, 2012 and Du Plessis, *et al*, 2014). In fact, owing to South Africa's volatile political and

economic history, modelling its behaviour and identifying any signal periodicity provides a robust test to structural breaks and regime shifts of any technique (Aaron & Muellbauer, 2002 and Chevillon, 2009).

Having made the case for *economic* forecasting using techniques such as spectral decomposition, the forecasting of *financial* variables is considered in the next section.

1.3.2 Financial forecasting

Financial variable data are also often plagued with noise, recording anomalies and inadequate weighting techniques to distinguish between recently recorded and distantly recorded quantities. The CAPM mean variance framework, for example, is often used for identifying investment performance. Excess investment portfolio returns (i.e. returns over the relevant 'risk free' rate of interest) are regressed on excess *market* returns and the resulting linear relationship is interrogated statistically to identify and isolate (1) the intercept (or excess portfolio return when the excess market return is 0%, referred to as α) and (2) the slope of the regression line or gearing of the portfolio relative to the market (β). The latter quantity identifies a multiplicative factor applied to excess market returns to determine excess portfolio returns, so, for $\beta = 2$, and an excess market return of 1% should result in a portfolio return of 2%, all else being equal. These quantities are of considerable significance to investors and fund managers: both rely on these (and other) quantities to establish compensation and rewards for fund managers responsible for generating these returns. Historically, favourable measures are substantially rewarded while mediocre measures result in small to no compensation (or worse). Because there are consequences – positive and negative – attached to these figures, accurate, topical values are required by all parties.

Problems arise, however, when generating these quantities. Consider first the data requirements for "accurate" values. More data are always preferred over fewer; using three months of return values, for example, informs analysts of nothing valuable. Three months is not only woefully inadequate to establish fund manager investment performance consistency but also, the standard error associated with such a regression line's α and β will always be large enough to dominate any presumption of "accuracy". The alternative is no better: using too *many* market and portfolio excess return data – say 60 monthly values – now means one point used in the regression analysis occurred five-years in the past. Another occurred four years and 11 months ago, and so on, as shown in Figure 1.3.

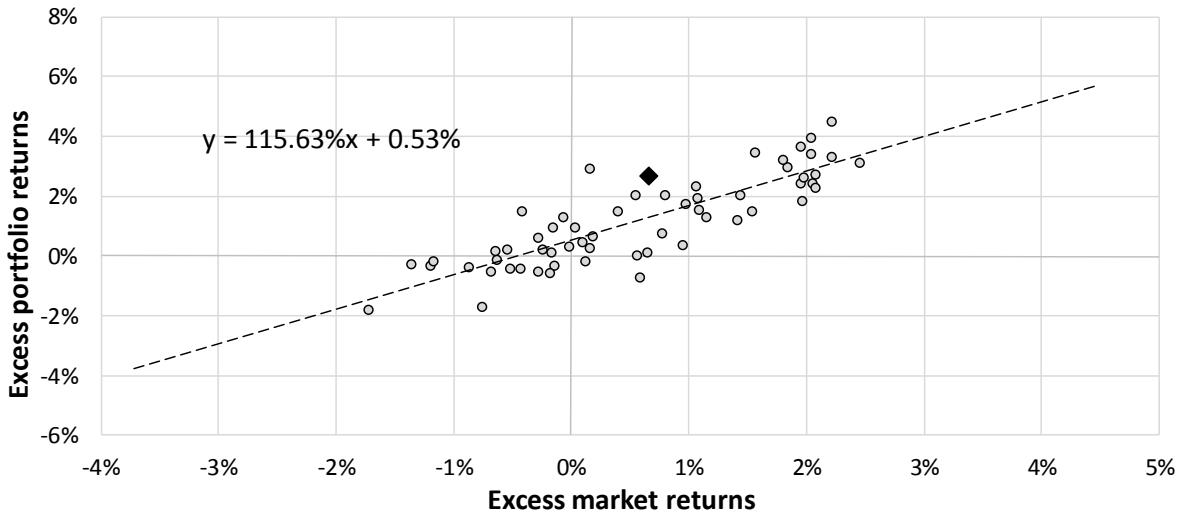


Figure 1.3: Simulated excess portfolio and market returns and associated regression line for $\alpha = 0.53\%$ and $\beta = 1.16$ using 60 months of equally weighted return data. The black diamond point (\blacklozenge) occurred 60 months ago yet still influences the slope and intercept as much as any other point including last month's value.

Should fund managers be rewarded or penalised for performance based on information gathered from that long ago in time? Not only will fund managers and investors be unlikely to wait for five years until the first "accurate" estimation of α and β may be determined, but values from five years ago will have been assembled from very different market conditions and possibly other managers of the fund in any case.

A possible solution to this problem may be accomplished using time weighting of excess returns. This is a commonly used technique in finance: the exponentially weighted moving average technique, used for the determination of volatility, for example, assigns high weights to recent return data and low weights to data collected longer ago in the past via an exponential weighting scheme (Figure 1.4). This results in volatility estimates which are more responsive to market moves and the absence of 'ghosting'.¹ Although this technique does result in more accurate (or, at least, more *recent* and thus more *relevant*) estimates of α and β , it has been shown that the effect is still insufficient. Other regression types are also possible, such as the LOESS (locally weighted scatterplot smoothing), but this is not suitable for CAPM α and β estimates: dynamic variables here result in highly unstable values.

¹ Volatility shocks that are abruptly incorporated into the standard deviation measure remain in the calculation, equally weighted for the entire duration of the trailing window used to calculate the volatility. When the trailing window passes, these shocks fall out of the calculation equally abruptly. This effect is known as ghosting (Dowd, 2002).

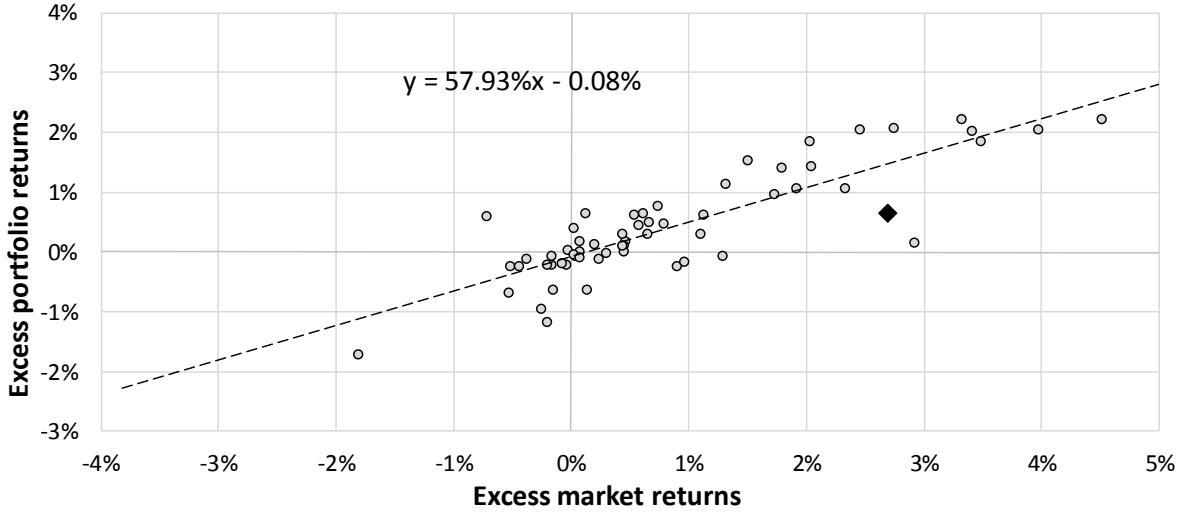


Figure 1.4: Identical simulated return data as those used to generate Figure 3, now exponentially weighted with $\lambda = 0.95$. The associated regression line gives $\alpha = -0.08\%$ and $\beta = 0.58$. Note the considerable difference with the values obtained from those determined in Figure 1.3 (the black diamond still represents the five-year-old value).

Hedge fund managers have historically generated meaningful, excess, skill based returns (α) through active management. These excess returns, whilst still significant, have decayed over time as the industry has grown. The α in hedge fund returns has consistently originated from security selection decisions while being reduced by market timing decisions. The benefits of taking risks to generate active skill based returns outweigh their costs. In secular equity bear markets, hedge funds have significantly outperformed on both an absolute as well as on a risk adjusted basis. In secular equity bull markets, hedge funds have sacrificed some upside, but have been less volatile and have outperformed on a risk-adjusted basis. Quantification of time-varying α has important implications for asset allocation and portfolio construction, as well as manager selection and remuneration.

1.4 Literature review

1.4.1 The Hodrick-Precott filter

A popular method of trend-extraction from financial data is the Hodrick-Prescott (HP) filter (Ley, 2006). The HP filter was first introduced by Hodrick and Prescott in 1980 (Hodrick & Prescott, 1980) in the context of estimating business cycles, but the research was only published in 1997 after the filter had gained popularity in macroeconomics (Hodrick & Prescott, 1997). The Basel Committee for Banking Supervision (BCBS) uses the HP filter to de-trend relevant macroeconomic ratio data and extract the requisite information for the evaluation

of excessive credit growth in various jurisdictions. This HP filter is of considerable importance to the countercyclical capital buffer as it will determine when the buffer should be instated in an overheating market.

The HP filter, still widely used in finance has been criticised for several limitations and undesirable properties (Ravn & Uhlig, 2002). The principal of trend extraction from business cycles using the HP filter with macro-economic data for duration of about four to six years was supported by Canova (1994 and 1998). However, spurious cycles and distorted estimates of the cyclical component when using the HP filter were obtained by Harvey & Jaeger (1993) who argued that this property may lead to misleading conclusions regarding the relationship between short-term movements in macroeconomic time series data. Cogley & Nason (1995) also found spurious cycles when using the HP filter on difference-stationary input data. Application of the HP filter to US time series data was found to alter measures of persistence, variability and co-movement dramatically (King & Rebelo, 1993). The HP filter, however, remains widely-used among macroeconomists for detrending data which exhibit short term fluctuations superimposed on business cycle-like trends (Ravn & Uhlig, 2002).

1.4.2 The Baxter-King filter

Baxter & King (1999) argued that a time series function X_t comprised three components: a trend τ , a cyclical component c , and a 'noise' (random) component, ϵ , such that $X_t = \tau_t + c_t + \epsilon_t$, where $t = 1, 2, \dots, T$. The Baxter King (BK) filter removes the trend and noise components, leaving only the cycle component $c_t = X_t - \tau_t - \epsilon_t$ where $t = 1, 2, \dots, T$.

Guay & St-Amant (2005) assessed the ability of the HP and BK filters to extract the business cycle component of macroeconomic time series using two different cycle definitions. The first was that the duration of a business cycle was between six and 32 quarters. The second definition distinguished between permanent and transitory components. Guay & St-Amant (2005) concluded that in both cases the filters performed adequately if the spectrum of the original series peaked at business-cycle frequencies. Low frequencies dominant in the spectrum distorted the business cycle.

1.4.3 The Kalman filter

The Kalman filter is a recursive procedure for computing the optimal estimator of the state vector at time $t + 1$, based on information available at time t (Kalman, 1960 and Arnold, et

al, 2005) which provides a linear estimation method for equations represented in a state space form. Estimation problems are transformed into state-space by defining state vectors. The Kalman filter output is governed by two equations: a measurement equation and a transition equation. The measurement equation unites unobserved data (x_t where t represents the time of measurement) and observed data (y_t) by the equation, $y_t = m \cdot x_t + v_t$, where $E[v_t] = 0$ and the variance of the error term, $Var[v_t]$, is known (r_t). The transition equation describes the evolution in time of unobserved data, such that $x_{t+1} = a \cdot x_t + w_t$, where $E[w_t] = 0$ and the variance of the error term, $Var[w_t]$, is unknown (q_t).

1.5 Specific objectives

The specific objectives of this research are:

1. to determine the accuracy of forecast values of various hedge fund indices using the Kalman filter using back-testing and statistical acceptance tests and
2. to minimise or eliminate noise from hedge fund return data using the Kalman filter and thus establish accurate, concomitant α and β CAPM values to accurately assess hedge fund performance and to decompose these returns to establish α due to market timing and α due to stock selection.

1.6 Research methods

This research, pertaining to the specific objectives, comprises two phases: a literature review and two empirical studies.

1.6.1 Literature review

The literature review focuses on the origin, development, history and applications of the CAPM, various mathematical filters used to clean data, regression techniques used to establish parameters for financial data forecasting and the Kalman filter (including all relevant model characteristics). The detailed mathematical exposition of the Kalman filter's operation follows in Chapter 3.

Information and data were sourced from journals, non-proprietary internet databases, working papers, textbooks and white papers. Proprietary data were sourced from paid sources, but the fund names were anonymised in accordance with proprietary agreements.

1.6.2 Empirical study

The empirical study comprises the research method, referring to the techniques developed in Microsoft Excel. The variables used refer to data being the various historical time series of nominal GDP values and hedge fund indices. All these data are available in the public domain and are refreshed either quarterly (GDP) or monthly (hedge fund returns).

Relevant acceptance tests will also be conducted out to confirm the effectiveness of each measurement technique. These may include timing tests to confirm whether the techniques would have aligned with historical stress events.

1.6.3 Data

Data in this study comprised several published, historical time series, mostly available from non-proprietary sources (e.g. internet databases). In some cases, proprietary information was used for specific hedge fund returns, but in these cases, the fund name was not disclosed – see Table 1.1. The investment style, domicile, operating market and other relevant information has, however, been preserved and used in the subsequent analysis.

Table 1.1: Data requirements, frequency and source.

| # | Topic | Data required | Frequency | Sources |
|---|---|---|-----------|---|
| 1 | Forecasting hedge fund indices returns using the Kalman filter | Hedge fund index return data Hedge fund styles | | EDHEC hedge fund database MSCI indices US treasury |
| 2 | Attribution of fund timing α and stock selection α using the Kalman filter Novel interpretation of return apportioning | Hedge fund returns Fund styles, domicile country, Relevant risk free rates, relevant market returns | Monthly | Proprietary hedge fund databases Selected Bloomberg time-series data |

1.6.4 Research output

The research output is indicated in Table 2.1 below. Topic 2 has been (separately) organised into article format and has been submitted to the *Journal of Applied Economics* (Nov 16). Earlier work on the role of Fourier analysis in forecasting the South African GDP has been published in the *International Business and Economics Research* journal and is reproduced in the Appendix (Thomson & van Vuuren, 2016).

Table 2.1: Research output.

| # | Topic | Models required | Research methodology |
|---|---|--|---|
| 1 | Forecast hedge fund return indices using the Kalman filter Compare and assess robustness and accuracy of forecasting techniques with Kalman filter results | CAPM Unconstrained Kalman filter | Run relevant hedge fund <i>index</i> data through Kalman filter and establish forecast accuracy Use rolling linear regression and weighted regression techniques to determine which measure provides the most robust results Using the resultant output, determine the accuracy of out-of-sample returns using relevant statistical tests |
| 2 | Isolate and apportion hedge fund performance components using the Kalman filter Offer novel interpretation of return apportioning | Linear regression Time-weighted linear regression | Run relevant hedge fund return data through Kalman filter and determine CAPM α s and β s Decompose results into timing α and stock selection α Determine if these values are reliable, robust and reproducible Interpret these values – investigate possible alternative interpretations |

1.6.5 Hedge fund index performance evaluation using the Kalman filter

In the CAPM, portfolio market risk is recognised through β while α summarises asset selection skill. Traditional parameter estimation techniques assume time-invariance and use rolling-window, ordinary least squares regression methods. The Kalman filter estimates dynamic α s and β s where measurement noise covariance and state noise covariance are known – or may be calibrated – in a state-space framework. These time-varying parameters result in superior predictive accuracy of fund return forecasts against ordinary least square (and other) estimates, particularly during the financial crisis of 2008/9 and are used to demonstrate increasing correlation between hedge funds and the market.

Two goals of this topic are sought:

- 1 to determine whether simple (or time-weighted) linear regression provide better forecast estimates of hedge fund index returns than forecasts estimated using time-varying coefficients from the Kalman filter, and
- 2 to assess these differences statistically and evaluate forecast success using each technique.

1.6.6 Apportioning hedge fund return components using the Kalman filter

Choosing a hedge fund is a highly subjective endeavour because each fund presents investors with a specific, and often unique, set of risks and potential rewards which can only be truly appreciated with a detailed qualitative analysis and review. Alternatively, subjectively reviewing a hedge fund is a labour-intensive process and many investors might look to a quantitative scoring system to help narrow the field of candidates to more manageable numbers. There are thousands of hedge funds globally, all vying for capital inflows and occasionally the multitude requires considerable pruning.

The goals of this topic are three-fold:

- 1 to use the Kalman filter to apportion hedge fund returns into a market timing contribution and a stock selection contribution,
- 2 to ascertain whether this apportioning methodology does indeed operate robustly and reliably, and
- 3 propose alternative explanations for (2) if this is not the case.

1.7 Conclusion

The conclusion presents a summary of the findings of both topics, providing details of recommendations for possible future research.

The next chapter presents a literature survey of the dissertation.

Chapter 2

Literature study

2.1. Forecasting

Forecasting *economic* variables customarily involves an understanding of underlying macroeconomic relationships and principles. Forecasting these quantities accurately and timely is important because predictable expectations encourage confidence in the economy, the direction the economy is headed and – ultimately – those in charge of the economic trajectory, namely central banks and government. Government treasuries manage the country's finances and allocate funding to departments and provinces, but it relies on *forecasts*, predictions of *future* revenues and cash flows to do so (Chevillon, 2009). These future values are by nature uncertain, yet the interpretation of and subsequent allocation of the revenue provides signals to the population as well as both global and local investors. The trustworthiness and correctness of these predictions are thus of critical importance: government policy relies on, and is shaped by, these numbers (SARB, 2015).

Forecasting a country's gross domestic product (GDP) requires knowledge of prevailing interest and foreign exchange (FX) rates, consumption per capita, unemployment rates and the national deficit or current account. These quantities are reported at different frequencies (second by second for interest and FX rates, quarterly for unemployment) by various sources (market data providers such as Bloomberg and Reuters and relevant exchanges, central banks for deficits and current accounts, government agencies for consumption) and the rigour and accuracy that accompanies each assembly may not be comparable. In addition, a variable such as the GDP could feasibly depend on other economic quantities such as business innovations, shifts in consumer preferences, governmental policy changes, or the discovery or depletion of natural resources. These quantities are less tangible since their estimation could involve some subjectivity. There are also often lags which are embedded in economic activity which stem from a variety of sources including human delays in recording the relevant data or from the non-immediate impact of certain macroeconomic variables on the economy (Aaron & Muellbauer, 2002). The combination of these effects gives rise to noisy data in which trends may be present, but will be embedded in – and mixed with – oth-

er data that may be spurious, inaccurate or in some sense "wrong". How to extract 'true' underlying signals is a non-trivial exercise.

Forecasting *financial* data which are frequently beset with noise, presents significant challenges to practitioners. A significant component of financial forecasting is the accurate predicting of fund returns (including asset management funds such as pension and mutual funds, but also hedge funds and exchange traded funds). Although heavily caveated with warnings that *past* performance is no guarantee of *future* performance, predicting returns (even with substantial uncertainty) dominates the financial industry (Dukes, Bowlin & MacDonald, 1987 and Rubio, Bermúdez & Vercher, 2016).

Hedge funds have expanded considerably since the early 1990s, and as they have grown and subsequently become available to retail investors, the need for regulation (driven by the national treasury) has amplified (Norton Rose Fulbright, 2016). Hedge funds that promise market outperformance have come under increased scrutiny so the assessment of forecast returns (and the *allocation* of those forecast returns between market timing skills and stock selection skills) has become critically important (Jain, *et al* 2011 and Avramov, Barras & Kosowski, 2013) for both fund managers and the investing community.

The traditional, widely-used methodology in finance to forecast returns is the CAPM developed by Treynor (1961).

2.2. The Capital Asset Pricing Model (CAPM)

The CAPM describes how the expected return on an asset or portfolio of assets is a linear function of the markets systematic risk component or market risk. Markowitz's (1952, 1959) work provided a direct foundation for the CAPM, with collective contributions from Treynor (1961), Sharpe (1964), Lintner (1965) and Mossin (1966). A detailed review of the principal concepts underlying the CAPM, its historical development and applications may be found in Fama & French (2004) and Perold (2004). Subsequent models, such as Arbitrage Pricing Theory introduced by Ross (1976) and later augmented by Chen, Roll & Ross (1986), introduced the notion of multivariate asset pricing models which estimated asset returns, in a manner which did not distinguish between the causality of macro and micro return predictors.

The CAPM relies on a selection of stringent assumptions. A fundamental notion is that investors hold well-diversified portfolios, implying that idiosyncratic risk can be diversified away and the only risk for which investors are compensated is attributable to a systematic, non-diversifiable risk component (represented by the market).² Other assumptions posit that investors:

1. aim to maximise economic utilities (asset quantities are given and fixed),
2. are rational and risk-averse,
3. are broadly diversified across a range of investments,
4. are price takers, i.e., they cannot influence prices,
5. can lend and borrow unlimited amounts under the risk-free rate of interest,
6. trade without transaction or taxation costs,
7. deal with securities that are highly divisible (all assets are perfectly divisible and liquid),
8. have homogeneous expectations, and
9. assume all information is available at the same time to all investors.

The allure of the CAPM is that it is described as offering powerful and intuitive predictions with respect to expected return-risk relationships, in a rational equilibrium market (French, 2004). Early cross-sectional tests and time-series regressions applied to the CAPM suggested that the relationship between asset returns and market returns were found to be approximately linear. The addition of other explanatory variables also led to no significant explanatory improvement (perhaps because of the immature nature of financial markets at the time), resulting in a premature conclusion that the market proxy portfolio was indicative of a “stand-alone” indicator of risk.

In the risk premium model of the CAPM mean-variance framework (see Equation 2.1), excess return on a security (or portfolio) is calculated as a combination of ‘abnormal return’ generated by the skill of the fund manager (either through timing or asset selection) and the product of the market risk premium and systematic risk. As such, the CAPM is frequently used to forecast investment performance. Excess investment portfolio returns (i.e. returns over a relevant ‘risk free’ interest rate) are regressed on excess *market* (usually an index)

² Idiosyncratic risk is the specific risk associated with a company or asset, while systematic risk refers to risk attributable to the market and its movements, which cannot be diversified away.

returns and the resulting (assumed) linear relationship is interrogated statistically (Das & Ghoshal, 2010) to identify and isolate:

- (1) the intercept (or excess portfolio return when the excess market return is 0%, usually referred to as α) and
- (2) the slope of the regression line or gearing of the portfolio relative to the market (known as β) as given in (2.1).

$$(r^P - r^f) = \alpha + \beta(r^M - r^f) + \varepsilon \quad (2.1)$$

where r^P is the security (or portfolio) return, r^f is the risk free rate, α is the abnormal rate of return on the security/portfolio, r^M is the market return, β is a measure of systemic risk given by

$$\beta = \frac{\sigma_P}{\sigma_M} \cdot \rho_{PM} \quad (2.2)$$

where σ_P is the portfolio volatility, ρ_{PM} is the correlation between the portfolio and market returns and σ_M is the market volatility. The noise term, ε , is assumed i.i.d. and $\sim N(0, \sigma_\varepsilon^2)$. For $\beta = 2$ and an excess market return of 1% should result in a portfolio return of 2%, all else being equal.

The CAPM pioneered the way in which assets are priced, however it is encumbered with several limitations. Firstly, the model makes a series of unrealistic assumptions and may be an inadequate representation of the behaviour of financial markets. Secondly, historical estimates of β s are problematic as they have been found to vary considerably through time (Mullins, 1982). Roll (1977) criticised the CAPM by suggesting that it is impossible to observe a strictly diversified market portfolio, and a market index serving as a proxy for such a portfolio would inherently have predictive errors.

Forecasts of α and β are of considerable significance to investors and fund managers: both rely on them (and other) quantities to establish compensation and rewards for fund managers responsible for generating these returns. Historically, favourable measures are substantially rewarded while mediocre measures result in small to no compensation (or worse). Because there are consequences – positive and negative – attached to these figures, accurate, topical forecasts values are required.

2.3. Measuring α and β

The standard method of determining CAPM α s and β s is via simple linear regression. Portfolio returns in excess of the risk-free rate are regressed on market returns in excess of the risk free rate. The intercept of the fitted regression line assigns a value to α , while β is the line's gradient.

For greater accuracy, more data are always preferred over less, so, using three months of returns (for example) informs the analyst of nothing valuable. Three months is woefully inadequate to forecast fund manager investment performance consistency and the standard error associated with such a regression line's α and β will always be substantial (thereby reducing accuracy). The alternative is no better: using too *many* data – say 60 monthly values – now means one point occurred five-years in the past; another occurred four years and 11 months ago, and so on (Figure 2.1) rendering them irrelevant for contemporary analysis.

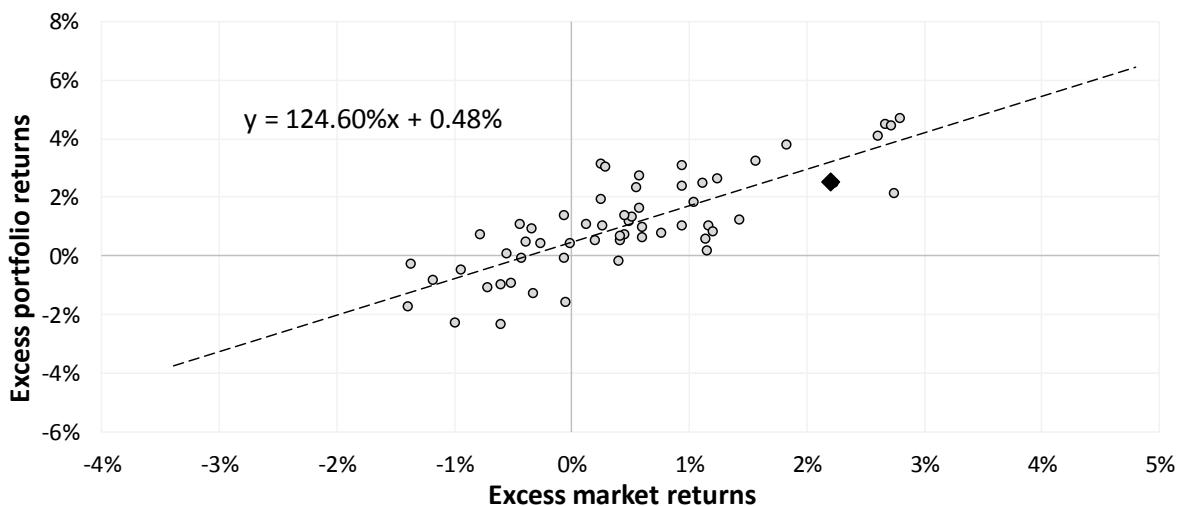


Figure 2.1: Simulated excess portfolio and market returns and associated regression line for $\alpha = 0.48\%$ and $\beta = 1.24$ using 60 months of equally weighted return data. The black diamond point occurred 60 months ago yet still influences the slope and intercept as much as any other point.

It is debatable that fund managers should be rewarded or penalised for performance based on information gathered from a period long ago in time. Managers and investors are unlikely to wait for five years until the first "accurate" forecast of α and β is determined, and even if so, values from five years ago will have been assembled from very different market conditions in any case.

A possible solution may be accomplished using *time weighting* of excess returns. The exponentially weighted moving average technique, traditionally used to measure volatility, as-

signs high weights to recent data and low weights to data collected longer ago in the past using an exponential weighting scheme as shown in Figure 2.2. This results in estimates which are more responsive to market moves. Although this technique results in more accurate (or, at least, more *recent* and thus more *relevant*) forecasts of α and β , it has been shown that the increased forecast accuracy is still insufficient.

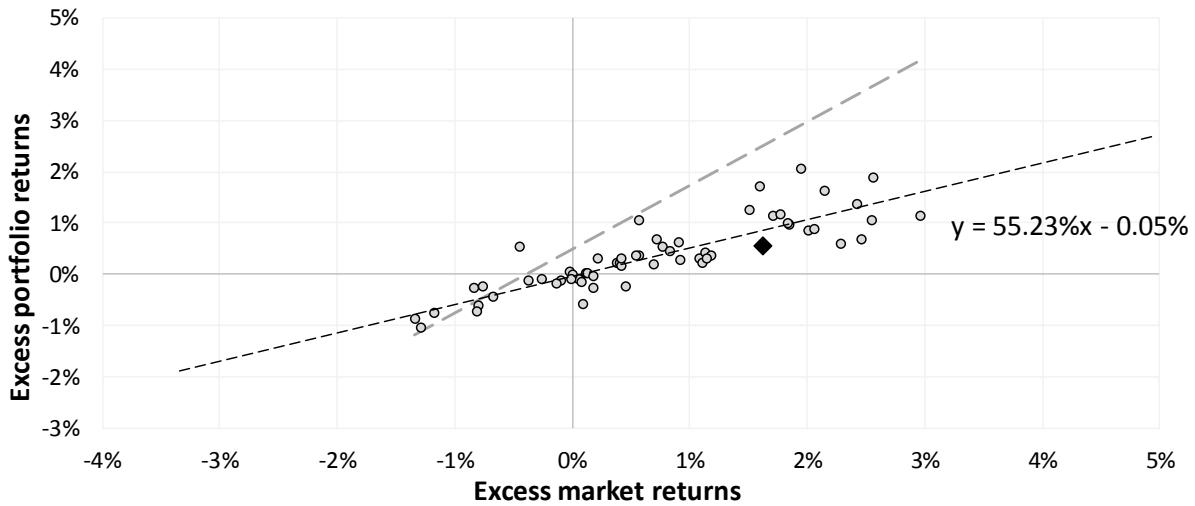


Figure 2.2: Identical simulated return data as those used to generate Figure 3, now exponentially weighted with $\lambda = 0.95$. The associated regression line gives $\alpha = -0.05\%$ and $\beta = 0.55$. Note the considerable difference with values obtained in Figure 2.1 (the black diamond still represents the five-year-old value). The grey dashed line indicates the original (unweighted) regression line from Figure 2.1.

Other types of regression are also possible, such as the LOESS (locally weighted scatterplot smoothing), but these are not suitable for CAPM α and β estimates: dynamic variables here result in highly unstable values.

2.4. Forecasting α s and β s

Corporate financial managers employ α and β forecasts to assist with capital structure decisions as well as appraise investment decisions (Choudhry & Hao, 2009 and Celik, 2013). Celik (2013) argued that the assumption of static α and β values could lead to erroneous assessment of fund manager performance. Although early empirical tests found the CAPM to be robust and reliable (Black, Jensen & Scholes, 1973; Fama & Macbeth, 1973; He & Ng, 1994 and Pettengill, Sundaram & Mathur, 1995), later studies questioned the non-stationarity of β and the risk premium (Fama & French, 1992 and Davis, 1994), the inadequacy of the market portfolio proxy (He & Ng, 1994) and joint hypothesis test problems associated with unobservable expected returns (Burnie & Gunay, 1993 and Pettengill, et al, 1995).

The descriptive accuracy of constant (as opposed to time-varying) β s was also questioned by Chan & Chen (1988) and later by Longstaff (1989), Ferson & Harvey (1991) and then Fama & French (1992). Since the capital (and hence risk) structure of all companies change with the macroeconomic environment, Jagannathan & Wang (1993) asserted that the constant β assumption was unreasonable and that a more appropriate evaluation would be to examine the relationship between returns and time-varying β s. Jagannathan & McGrattan (1995) blamed the focus on *constant* β values on the fact that the CAPM model had been originally developed to explain differences in risks across capital assets. Later work found considerable improvement in the description and accuracy of portfolio return behaviour if the constant β requirement was relaxed (Groenewold & Fraser, 1999; Black & Fraser, 2000; Fraser, Hamelink, Hoesli & Macgregor, 2000 and Prysiaznyuk & Kirdyaeva, 2010).

Other econometric methods have been employed to estimate time-varying β s (Brooks, Faff & McKenzie, 1998): two well-known approaches are GARCH models (various types are discussed in Choudhry & Hao, 2009) and the Kalman filter (e.g. Black, Fraser & Power and Well, 1994). The former construct the conditional β series using conditional variance information while the latter uses an initial set of priors to estimate the β series recursively, thereby generating a series of conditional α s and β s. Das & Ghoshal (2010) found that estimating dynamic β s in the CAPM using traditional (auto-regressive) methods, yielded suboptimal results while the Kalman filter was able to estimate an optimal dynamic β even where measurement noise and state noise covariances were unknown, but themselves dynamically determined.

Albrecht (2005) concluded that models which assumed time-varying α s and β s provided superior return forecasts to those that assumed static CAPM coefficients (such as OLS regression models). Albrecht (2005) applied dynamic exposure results – derived from the Kalman model – to fund returns and found that dynamic CAPM coefficients could be partially explained by the active adjustment of portfolio-weights, confirming that value generated by fund managers arises not only from asset selection skills, but also from dynamic portfolio management.

2.5. The Kalman filter

The Kalman filter (a recursive procedure for computing the optimal estimator of a state vector at time t , based on information available at time t (Kalman, 1960; Harvey & Jaeger, 1990

and Arnold, *et al*, 2005)) provides *contemporaneous* estimates of α and β via variance minimisation and thus improves forecast reliability considerably: knowledge of these values at the *current* time allows practitioners to forecast expected returns far more accurately. Application of the Kalman filter to finance is, however, still in its infancy.

The Kalman filter offers considerable flexibility in capturing funds' dynamic exposure behaviour and, thus this technique was chosen to estimate how the CAPM variables vary over time for hedge funds. While OLS multivariate regression may be suitable for a hedge fund characterised by slowly-varying exposures, the Kalman filter proves the superior approach for hedge funds during volatile periods (Roll & Ross, 1994 and Faff, Hillier & Hillier, 2000). The use of exponential weighting of excess returns in regression analysis provides more dynamic (or, at least, more *recent*, thus more *relevant*) forecasts of CAPM α and β , but it has been shown that these improvements are still insufficient (see e.g. Rapach & Wohar, 2006) when compared with results obtained from the Kalman filter.

2.6. Market timing versus security selection skills

The ability to assess hedge fund market timing performance is an onerous, but useful, task (Cavé, Hübner & Sougné, 2012). Hedge fund managers do not add much value beyond their given risk exposures – so justification for high fees and expenses of active money management are diminishing. Over the last ten years (2006 – 2016) hedge fund performance fees have declined by 30% as returns have generally disappointed (Griffin & Xu, 2009 and Keller, 2015). This trend, fuelled by increased competition, hikes in regulatory costs, and waning arbitrage opportunities due to widespread information access, is likely to continue with the result that hedge funds unable to generate superior returns and display true investment talent will vanish (Keller, 2015). Of course, this is only true in aggregate: individual, deft hedge fund managers may still generate abnormal returns. The accurate identification of these skills has become a critical compensation component (Mladina, 2015).

Hedge fund managers enjoy considerable flexibility in amending and adjusting both their asset allocation and leverage. If hedge fund managers possess and employ authentic anticipation skills, an evaluation of these market timing abilities would expose an important performance constituent. Several obstacles, however, make the determination of this performance measurement challenging, e.g.:

- many hedge fund strategies have insufficient liquidity to allow investors and hedge fund managers to take advantage of fortuitous market opportunities when they arise,
- the high prevalence of extinct funds in data samples can prevent the successful analysis of time series of returns to extract market timing skill information,
- monthly return frequency reporting makes the detection of true market timing behavior an arduous task (Bollen & Busse, 2001) and
- the clear majority of hedge funds report returns only a monthly basis which smooths the real return profile and obscures hedge funds' lack of liquidity (see Cavenaile, Cohen & Hubner, 2011).

To date (November 2016), only a few studies have specifically targeted the assessment of hedge fund market timing skills. Despite the variety of timing measures examined, results are mixed and far from conclusive, as shown in Table 2.1.

Table 2.1: Summary of existing literature on the market timing of hedge funds (in order of publication).

| Authors | Period | Database ³ | Fund universe ⁴ | Market timing measure | Market timing activity ² |
|-----------------------|--------|-----------------------|----------------------------|--|---|
| Fung, Xu & Yau (2002) | 94–00 | MAR | 115 GE | Henriksson-Merton (HM) (1981) | ✗ but superior security selection ability |
| Chen & Liang (2006) | 94–02 | TASS | 1 471 CA | Conditional multi-factor Treynor-Mazuy (TM) (1966) and HM (1981) | CA, MF, GM: ✓ (+ve market timing) ED, EM, FIA: ✓ (-ve market timing) |
| Cheng & Liang (2007) | 94–05 | CISDM, HFR, TASS | 221 CA, MF | Consistent with Jensen (1972) and Admanti, <i>et al</i> (1986) Joint tests for return & volatility timing | ✓ |
| French & Ko | 96–05 | TASS | 157 L/S | TM (1966) | ✗ but superi- |

³ TASS: Lipper hedge fund database, HFR: hedge fund research, CISDM: centre for international securities and derivatives markets, MAR: managed account reports.

⁴ L/S: long/short, GE: global equity, CA: convertible arbitrage, MF: managed futures, GM: global macro, ED: event driven, EM: emerging markets, FIA: fixed income arbitrage, FoF: fund of funds.

| (2007) | | | | | or security selection ability |
|------------------------------|------------|-----------|-------------------|--|--|
| Park (2010) | 94–08 | TASS | 6 114 ED, EM, MF | Covariance between factor loading & factor risk premium (Lo, 2008) | ✗ but superior security selection ability |
| Monarcha (2011) | 03–08 | TASS | 6 700 CA, L/S, GM | Dynamic style analysis (Monarcha, 2011) | MF, GM: ✓ ED: ✗ Good market timers deliver low α |
| Cavé, Hübner & Sougné (2012) | 1/06–12/08 | Bloomberg | 215 L/S MF & FoF | TM (1966), but regression α adjusted for cost of option-based replicating portfolio | ✓ +ve and -ve market timers |

Using time-variant models, Géhin & Vaissié (2005) decomposed hedge fund returns into three components:

- security selection (pure α),
- static fund exposures to risk factors (static β) plus factor timing (dynamic β), and
- manager skill, pure α + dynamic β (total α).

The contribution of each component was estimated using a dynamic Kalman filtering approach. Results indicated that about half of the return variability arose from α (25% from pure α and 24% from dynamic β), and the contribution of static β to performance was more than 99% on average, with a positive pure α of 4% and a negative dynamic β of -3%. Total α appeared to be driven by factor timing rather than selection skills, underlining the importance of correctly identifying time-varying β s (Géhin & Vaissié, 2005).

Griffin & Xu (2009) used holdings-based data to analyse hedge and mutual fund performance attribution using security selection, sector and style timing, and pervasive-style tilts. Scant evidence was found that either hedge funds or mutual funds were able to consistently add value in any of the three attribution areas. Although hedge funds outperformed mutual funds slightly in stock selection, that advantage was considerably offset by the difference in fees.

Ingersoll, Spiegel, Goetzmann & Welch (2007) argued that hedge fund performance measures may be manipulated by dynamically trading securities to curb the return distribu-

tion. Jagannathan & Korajczyk (1986) associated these option-like characteristics with spurious market timing effects explicitly and asserted that the mere separation between the regression intercept and the market timing coefficient in the HM (1981) and the TM (1966) models was insufficient to discriminate between authentic and false market timing skills.

To better identify reliable market timers, three types of corrections were proposed: a variance correction approach (Grinblatt & Titman, 1994); an approximation based on the squared benchmark returns (Bollen & Busse, 2004); and a synthetic option pricing approach (Merton, 1981). All three methods, however, are still subject to possible manipulations because a manager with access to a complete derivatives market may alter the market timing coefficient without affecting the regression intercept (α) proportionally. Although this prompted Ingersoll, *et al* (2007) to develop a manipulation-proof performance measure, its identification remained contingent on the characterisation of the investor's preferences.

Despite the frequent definition of hedge funds as *absolute* return investment vehicles, research has shown that they are exposed to traditional as well as alternative risk factors (Albrecht, 2005). Option-like components in hedge fund models indicate that hedge fund exposure is not static, but time-varying since fund managers regularly amend portfolio weights in an attempt to time the market. Static regression methods provide inferior descriptions of hedge fund returns and may incorrectly alter attribute return components from market exposure to manager skill or vice versa. Albrecht (2005) used recursive filtering models – such as the Kalman filter – to assess time-variation exposures and asserted that this would lead to more accurate results and better exposure forecasts. Results showed that these extended models performed better than ordinary regressions, both in and out-of sample. Evidence also suggested that a time-varying constituent may be attributable to the active amendment of portfolio-weights. This latter result indicated that besides asset selection, market-timing is an important hedge fund managers' skill.

Using Jiang's (2003) non-parametric market timing tests, Cuthbertson, Nitzsche & O'Sullivan (2010) examined a large database of UK equity and balanced mutual funds. The onerous data requirements required knowledge of timing over multiple frequencies (daily, monthly, quarterly) precludes hedge funds since they do not generally record or publish return data at the frequency required for this analysis. Results indicated that 1% of UK mutual funds possessed significant positive market timing skill, while 19% mis-timed the market. Little

other evidence of successful conditional market timing was found using the nonparametric approach after accounting for publicly available information and based on private, individual UK fund information (Cuthbertson, *et al*, 2010). These results supported results obtained from previous work (Fletcher, 1995; Leger, 1997 and Byrne, Fletcher & Ntozi, 2006).

Hübner (2011) exploited both the linear and the quadratic coefficients of the TM (1996) model to assess the replicating cost of the cheapest option portfolio with the same convexity as a hedge fund. The portfolio replication approach showed that market timing performance increased with fund convexity level, and the effect is larger and more significant for positive market timers.

Although the evidence on hedge fund market timing abilities is both sparse and generally negative (see, e.g. Fung, Xu & Yau, 2002 and Chen & Liang, 2007), hedge funds may not necessarily be attempting to time markets in the traditional sense. Rather than trying to tilt opportunistically toward or away from, say, equity at the expense of fixed income exposure, some hedge funds may attempt to time the point at which funds strike their net asset values (NAVs) and exploit valuation misalignments (Geczy, 2010). Although it is possible that some funds may have been able to time the market, at monthly horizons, hedge funds appear (in aggregate) not to have been able to do so (Argawal & Naik, 2002). Nothing about the hedge fund experience in the recent crisis or about the results of Chen and Liang (2007) suggests that hedge funds should be able to time markets (Geczy, 2010).

Hochberg & Mulhofer (2011) used a variation of a characteristic *timing* measure and a characteristic *selectivity* measure (originally developed for mutual funds by Daniel, Grinblatt, Titman & Wermers, 1997) to assess fund manager abilities in the US real estate market. The data required for this analysis are scant to non-existent in most markets, particularly hedge funds, but the US real estate market enjoys a broad and deep database of relevant information and thus affords researchers a rich source from which to test potential abnormal profit generation. Hochberg & Mulhofer (2011) found that (on average) portfolio managers exhibited little market timing ability and low correlation between characteristic timing and selectivity abilities.

Mwamba (2013) estimated outperformance, selectivity and market timing skills in hedge fund indices using the linear and quadratic CAPMs. Mwamba (2013) asserted that managers who generated abnormal returns may be identified by a statistically significant Jensen α .

These abnormal returns may be generated from selectivity skills or market timing skills. The TM (1966) "quadratic CAPM regression model" was used to measure selectivity and market timing skill coefficients and the Jensen (1968) "linear CAPM regression model" for the out-performance skill coefficient. Fund managers were found to outperform the market during periods of positive economic growth only, with both selectivity and market timing skills contributing (Mwamba, 2013).

Kang (2013) used a holdings-based measure built on Ferson & Mo (2012) to decompose hedge fund managers' overall performance into stock selection and three timing ability components: market return, volatility and liquidity. Kang (2013) found that hedge fund managers exhibited weak evidence for stock picking skills (attributed to conditioning information and equity capital constraints) and no timing skills (due to large timing performance fluctuations with market conditions).

2.7. Treynor/Mazuy (TM) and Henriksson/Merton (HM) models

The TM (1966) performance measure is often used as a proxy for selectivity and market timing skills. The idea is that for returns on portfolios managed by fund managers exhibiting forecasting power will not be *linearly* related to market returns since the manager will gain more than the market does when the market return is forecast to rise and less than the market does when the market is forecast to fall. These portfolio returns will thus be a *concave* function of market returns, giving rise to the quadratic model:

$$r^P - r^f = \alpha + \beta_1 \cdot (r^M - r^f) + \beta_2 \cdot (r^P - r^f)^2 + \varepsilon$$

where the terms have the same definitions as those in (2.1). Treynor & Mazuy (1966) demonstrated that the significance of β_2 provided evidence of portfolio over-performance. Admati, *et al* (1986) suggested that α can be interpreted as the selectivity component of performance and $\beta_2 \left[(r_t^M - r_t^f)^2 \right]$ as the performance timing component.

The HM (1981) model considers that the manager switches the portfolio's β depending on the sign of the market return. A good market timer increases the market exposure when the return is positive, and keeps it lower when negative. The HM model is defined as:

$$r^P - r^f = \alpha + \beta \cdot (r^M - r^f) + \gamma \cdot (-r^{M+})^2 + \varepsilon$$

Where the terms have their usual definitions, γ is the sensitivity coefficient for the negative market returns and $-r^{M+} = \max(-r^M, 0)$. The HM model translates the behavior of a manager who succeeds in switching his market β from a high level (β) when $r^M > r^f$ to a low level of ($\beta - \gamma$) otherwise.

The HM regression model exhibits heteroscedasticity which, if ignored, renders the HM test inferior in terms of size and power (Breen, Jagannathan & Ofer, 1986). TM model coefficients are biased due to strong correlations between various hedge fund risk factors and the quadratic term used to measure timing ability in the TM model (Comer, 2003).

A negative correlation between market timing and selectivity performance measures has been identified (Jagannathan & Korajczyk, 1986, Coggin, Fabozzi & Rahman, 1993, Goetzmann, Ingersoll & Ivkovich, 2000 and Jiang 2003). Jiang (2003) reported simulation results which showed a negative correlation between the two performance measures in the TM and HM models, even though none existed. The correlation between nonparametric timing measures and security selection measures in regression models is generally small and indistinguishable from zero for larger sample sizes (Cuthbertson, *et al*, 2010).

Jagannathan & Korajczyk (1986) suggest that a spurious negative correlation may arise due to the nonlinear pay-off structure of options and option-like securities in fund portfolios - holding a call option on the market yields a high pay-off in a rising market but in a steady or falling market the premium payment lowers return and appears as poor security selection.

The regression-based methods of TM (1966) and HM (1981) are not able to decompose overall fund abnormal performance into market timing and security selection components (Admati, *et al*, 1986, Grinblatt & Titman 1989). Jiang's (2003) nonparametric procedure was developed to overcome these limitations.

2.8. Decomposing hedge fund returns using the Kalman filter

Hedge fund managers have historically generated meaningful, excess, skill based returns (α) through active management. These excess returns, whilst still very significant, have decayed over time as the industry has grown. Hedge fund α s have consistently originated from security selection decisions while being reduced by market timing decisions (Griffin & Xu, 2009). The benefits of taking risks to generate active skill based returns outweigh their costs. In secular equity bear markets, hedge funds have significantly outperformed on both an abso-

lute as well as on a risk adjusted basis (Keller, 2015). In secular equity bull markets, hedge funds have sacrificed some upside, but have been less volatile and have outperformed on a risk adjusted basis. Quantification of time-varying α has important implications for manager selection, asset allocation and portfolio construction (Mladina, 2015).

Jain, Yongvanich & Zhou (2011) assessed the skills-based component (α) of US fund returns from data spanning 18 years (1993 – 2011) by regressing fund returns against S&P 500 index returns, but obtained spurious results. A rolling regression technique was then attempted, in which α and β were calculated over a fixed window of 36 months and then rolled forward by one month to obtain the next month's α and β . A time-weighted regression, in which decreasing weights are assigned to observations the longer ago they occurred in the past, was also explored. Although the last two techniques are common choices for estimating average α and β , Jain, *et al* (2011) found them to be inadequate at capturing the dynamic nature of the CAPM coefficients.

Jain, *et al*, (2011) assessed the skills-based component (α) of US fund returns from data spanning 18 years (1993 – 2011) by regressing fund returns against S&P 500 index returns using ordinary least squares regression (OLS). Spurious results were obtained and factor sensitivities were found to vary over time. A rolling regression technique was then attempted, in which α and β were calculated over a fixed window of 36 months (this being found to optimally balance variance and bias) and then rolled forward by one month to obtain the next month's α and β . A time-weighted regression, in which decreasing weights were assigned to observations the longer ago they occurred in the past, was also attempted. Although the last two techniques are common choices for estimating average α and β , these were found to be inadequate at capturing the dynamic nature of the CAPM coefficients, in particular when funds' strategic investment horizons were smaller than the window size.

Jain, *et al* (2011) then employed a Kalman filter to establish the dynamics of hedge fund exposures. Although the filter requires a substantial amount of data (roughly an order of magnitude more than standard OLS regressions) this limitation was partially ameliorated by imposing more structure to the model, e.g. by assuming no correlation between α and β . Both α and β retain their unique variances, but the covariance between α and β (off-diagonal elements in the process covariance matrix, Q – see (3.4)) is set to 0. Despite noisy data, Jain

et al (2011) were able to extract reasonable values of the hidden variables: fund market risk exposure (α) and market sensitivity (β).

Although the Kalman filter has been shown to be better than most models at capturing exposure dynamics, this flexibility is sometimes disadvantageous. If the model specification describing the underlying process is inaccurate, or if too few return data are used to generate forecasts, the filter fits the excess noise rather than the core signal (Punales, 2011). While obsolete data renders regression model results *and* Kalman filter model results inaccurate if substantial shifts in hedge fund risk profiles occur, the dynamic quality of the state space model allows it to adapt more quickly (Punales, 2011). Assumptions used in the formulation of the Kalman filter are thus more robust than the assumption of constant exposure employed in regression analysis (Tsay, 2010).

The inherent flexibility of the Kalman model is required to capture dynamic fund exposure behaviour and, thus the technique employed to estimate exposures may vary over time and across hedge funds. So, while OLS multivariate regression may be suitable for a hedge fund characterised by slowly-varying exposures, the Kalman filter proves the superior approach for hedge funds during volatile periods (Roll & Ross, 1994 and Faff, Hillier & Hillier, 2000).

Chapter 3

The Kalman filter

3.1. Introduction

The Kalman filter was first described and partially developed by Swerling (1959): further work was conducted by Kalman (1960) and then Kalman & Bucy (1961). Sometimes called the Stratonovich–Kalman–Bucy filter because it is a special case of a more general, non-linear filter developed earlier by Stratonovich (1959), the Kalman filter was popularised by Hungarian, Rudolf Kálmán, (after whom the filter is now named). Schmidt (1981) developed the first practical implementation of a Kalman filter after realising that it could be used for timing sensor outputs and for incorporating measurements. Schmidt (1981) applied Kalman's ideas to trajectory estimation for the US lunar (Apollo) program, and it has been subsequently employed in US Navy nuclear ballistic missile submarines navigation systems and in the guidance of cruise missiles (Kirubarajan, Bar-Shalom & Wang, 2001).

3.2. Filter function

The Kalman filter operates as an optimal estimator – that is, it infers relevant parameters using indirect, uncertain and frequently inaccurate observations – and it is also recursive, so new measurements may be processed as they arrive sequentially in time. This differs from batch processing which requires that all data must already be present before computations can proceed. "Filter" asserts that estimating the best estimates from noisy data requires *filtering* out the noise which accompanies and is embedded in the signal. The Kalman filter not only refines data measurements, but also projects these measurements onto the state estimate. These concepts will be expanded upon later in this chapter.

The Kalman filter identifies "optimal" solutions in the sense that, if all the noise (present in the uncertain and potentially inaccurate observations) is Gaussian, the Kalman filter minimises the mean square error of the estimated parameters recursively, so the ultimate solution is "optimal". If the noise is not Gaussian, non-linear estimators often perform better, but if only the mean and standard deviation of the noise are required inputs to the state estimate (i.e. the noise is $\approx N(0, \sigma)$), then the Kalman filter is the best linear estimator.

The Kalman filter enjoys widespread applications in engineering and physics because it provides practical results, it is relatively easy to formulate and implement because of its small computational requirements, it affords practitioners with a convenient format for online, real-time processing, it enjoys elegant recursive properties and it is not necessary to invert the measurement equations (discussed later).

3.3. Formulating the Kalman filter problem

To formulate the problems for which the Kalman filter is adept at solving, a description is required (in discrete-time) of a linear, dynamic system using a vector difference equation with additive white noise in which unpredictable disturbances are modelled. All these terms are discussed in greater detail in the example which follows in Sections 3.4 and 3.5. Knowledge of the state of the deterministic system (the smallest vector of input variables that summarises, in full, the system's history) allows a theoretical prediction of the future (and prior) dynamics and outputs of the deterministic system in the absence of noise.

The Kalman filter algorithm permits exact inferences of linear dynamical system inputs – making it a Bayesian model and similar in many respects to hidden Markov models. The Kalman filter's latent variables state space is, however, continuous (rather than discrete in the case of Markov models) and its latent as well as observed variables are Gaussian-distributed (frequently a multivariate Gaussian distribution).

3.4. Kalman filter application: physical system

In this example, the Kalman filter is derived from first principles by considering a simple physical example (that of targeting a moving aircraft with a missile) which makes use of a key property of Gaussian distributions, namely that the product of two Gaussian distributions is another Gaussian distribution.

The Kalman filter makes the assumption that the system state which it aims to describe at time t evolved from the prior state at time $t - 1$ according to:

$$x_t = F_t x_{t-1} + B_t u_t + w_t \quad (3.1)$$

where x_t is the state vector containing the terms of interest for the system (e.g., position, velocity, heading, acceleration) at time t , u_t is the vector which contains all control inputs (such as the throttle setting, the braking force and flight direction angles), F_t is the state

transition matrix which applies the effect of each system state parameter at time $t - 1$ on the system state at time t (e.g., the position, velocity and acceleration at time $t - 1$ all affect the position at time t), B_t is the control input matrix which applies the effect of each control input parameter in the vector u_t on the state vector (e.g., applies the effect of the throttle setting on the system velocity, position and acceleration) and w_t is a vector which contains the process noise terms for each of the parameters in the state vector. The process noise is assumed to be sampled from a multivariate normal distribution with zero mean and covariance matrix Q_t . System measurements can also be performed according to the model:

$$z_t = H_t x_t + v_t \quad (3.2)$$

where z_t is the measurements vector, H_t is the transformation matrix that maps the state vector parameters into the measurement domain and v_t is the vector containing the measurement noise terms for each observation in the measurement vector. Like the process noise, the measurement noise is assumed to be Gaussian white noise also with zero mean and covariance R_t .

In the derivation that follows, a simple one-dimensional tracking problem is considered, in this case, an aircraft moving in a horizontal straight line (Figure 3.1).⁵

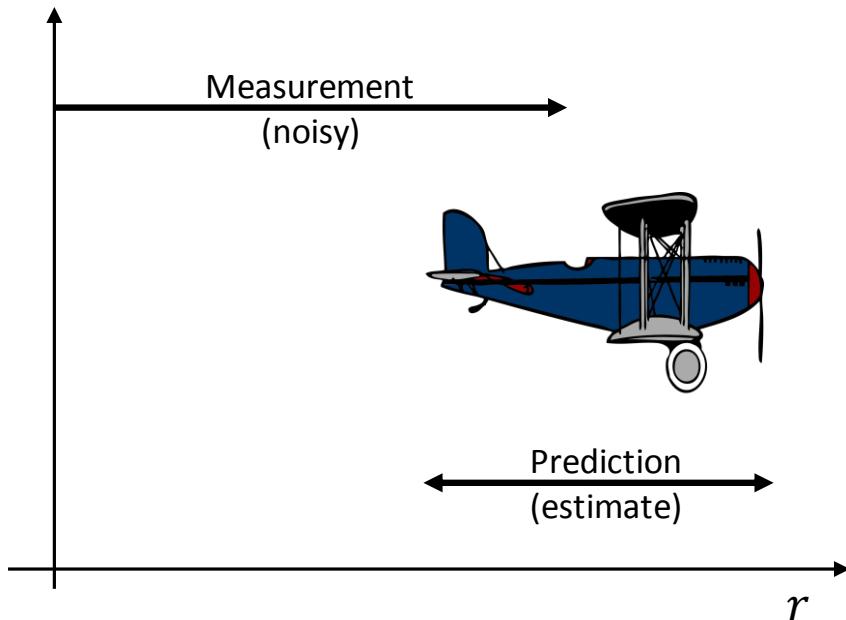


Figure 3.1: The one-dimensional system under consideration.

⁵ The aircraft could be moving in *any* manner: up and down, sinusoidally, vertically upwards, downwards towards the south-east, etc., all that would need to change in the description above is the estimate of the way in which the aircraft is believed to moved. The Kalman filter will determine the more precise way in which the craft moves.

The state vector x_t contains the position and velocity (\dot{x}_t) of the aircraft:

$$x_t = \begin{bmatrix} x_t \\ \dot{x}_t \end{bmatrix}$$

The pilot may apply a braking or accelerating input to the aircraft, which is a function of an applied force f_t and the mass of the aircraft, m . This control information is stored in the control vector, u_t :

$$u_t = \frac{f_t}{m}$$

Note that here, u_t is the acceleration from Newton's first law of motion, $F = ma$. The relationship between the force applied via the ailerons (to brake the aircraft) or throttle during the time period Δt (the time elapsed between time $t - 1$ and t) and the position and velocity of the aircraft is given by the following equations from Newton's laws of motion:

$$x_t = x_{t-1} + (\dot{x}_{t-1} \cdot \Delta t) + \frac{f_t(\Delta t)^2}{2m}$$

and

$$\dot{x}_t = \dot{x}_{t-1} + \frac{f_t \Delta t}{m}.$$

These linear equations may be written in matrix format as:

$$\begin{bmatrix} x_t \\ \dot{x}_t \end{bmatrix} = \begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_{t-1} \\ \dot{x}_{t-1} \end{bmatrix} + \begin{bmatrix} (\Delta t)^2 \\ 2 \end{bmatrix} \cdot \frac{f_t}{m}$$

By comparison with (1):

$$\mathbf{F}_t = \begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix}$$

and

$$\mathbf{B}_t = \begin{bmatrix} (\Delta t)^2 \\ 2 \\ \Delta t \end{bmatrix}$$

It is not possible to directly observe the true state of the system x_t , and the Kalman filter provides an algorithm to determine an estimate \hat{x}_t by combining models of the system and noisy measurements of certain parameters or linear functions of parameters.

The estimates of the parameters of interest in the state vector are therefore now provided by probability density functions (pdfs), rather than discrete values. The Kalman filter is based on Gaussian pdfs and to fully describe these, knowledge of their variances and covariances is required: these are stored in the covariance matrix, P_t . The variances associated with the corresponding terms in the state vector terms populate the diagonal of P_t , off-diagonal terms of P_t provide the covariances *between* terms in the state vector. For a one-dimensional linear system with measurement errors drawn from a Gaussian distribution with zero-mean, the Kalman filter has been shown to be the optimal estimator (Anderson & Moore, 2005). The filter equations permit the recursive calculation of \hat{x}_t by combining prior knowledge, predictions from systems models, and noisy measurements.

The algorithm proceeds via two distinct stages:

1. prediction and
2. measurement update.

The standard Kalman filter equations for the *prediction* stage are:

$$\hat{x}_{t|t-1} = F_t \hat{x}_{t-1|t-1} + B_t u_t \quad (3.3)$$

$$P_{t|t-1} = F_t P_{t-1|t-1} F_t^T + Q_t \quad (3.4)$$

where Q_t is the process noise covariance matrix associated with noisy control inputs. Note that (3.3) has been derived above, while (3.4) is derived as follows using the fact that the variance associated with the prediction $\hat{x}_{t|t-1}$ of an unknown true value x_t is given by:

$$P_{t|t-1} = E \left[(x_t - \hat{x}_{t|t-1})(x_t - \hat{x}_{t|t-1})^T \right]$$

Taking the difference between (3.3) and (3.1) gives:

$$\begin{aligned} x_t - \hat{x}_{t|t-1} &= F(x_{t-1} - \hat{x}_{t|t-1}) + w_t \\ \rightarrow P_{t|t-1} &= E(F(x_{t-1} - \hat{x}_{t|t-1}) + w_t) \times (F(x_{t-1} - \hat{x}_{t|t-1}) + w_t)^T \\ &= FE \left[(x_{t-1} - \hat{x}_{t-1|t-1}) \times (x_{t-1} - \hat{x}_{t-1|t-1})^T \right] \times F^T \\ &\quad + FE[(x_{t-1} - \hat{x}_{t-1|t-1}) w_t^T] \\ &= E[w_t x_{t-1} - \hat{x}_{t-1|t-1}^T] F^T + E[w_t w_t^T] \end{aligned}$$

Because the state estimation errors and process noise are uncorrelated (a common assumption which has been proved empirically for most physical systems):

$$\begin{aligned}
E[(x_{t-1} - \hat{x}_{t-1|t-1})w^T] &= E[w_t(x_{t-1} - \hat{x}_{t-1|t-1}^T)] = 0 \\
\rightarrow P_{t|t-1} &= FE \left[(x_{t-1} - \hat{x}_{t-1|t-1})(x_{t-1} - \hat{x}_{t-1|t-1})^T \right] F^T + E[w_t w_t^T] \\
\rightarrow P_{t|t-1} &= FP_{t-1|t-1}F^T + Q_t
\end{aligned}$$

The measurement update equations are:

$$\hat{x}_{t|t} = \hat{x}_{t|t-1} + K_t(z_t - H_t \hat{x}_{t|t-1}) \quad (3.5)$$

$$P_{t|t} = P_{t|t-1} - K_t H_t P_{t|t-1} \quad (3.6)$$

where

$$K_t = P_{t|t-1} H_t^T (H_t P_{t|t-1} H_t^T + R_t)^{-1} \quad (3.7)$$

For this example, the Kalman filter will be derived by considering a simple one-dimensional tracking problem, specifically that of an aircraft moving along a path in a horizontal straight line. At every measurement time, knowledge of the best possible estimate of the location of the aircraft (or the location of the signalling beacon mounted on the aircraft) is desired. Information may come from two sources:

1. predictions based on the last known position and velocity of the aircraft and
2. measurements from a radar (or similar) system located at a fixed point on earth.

The information from the predictions and measurements are combined to provide the best possible estimate of the location of the aircraft (Figure 3.2).

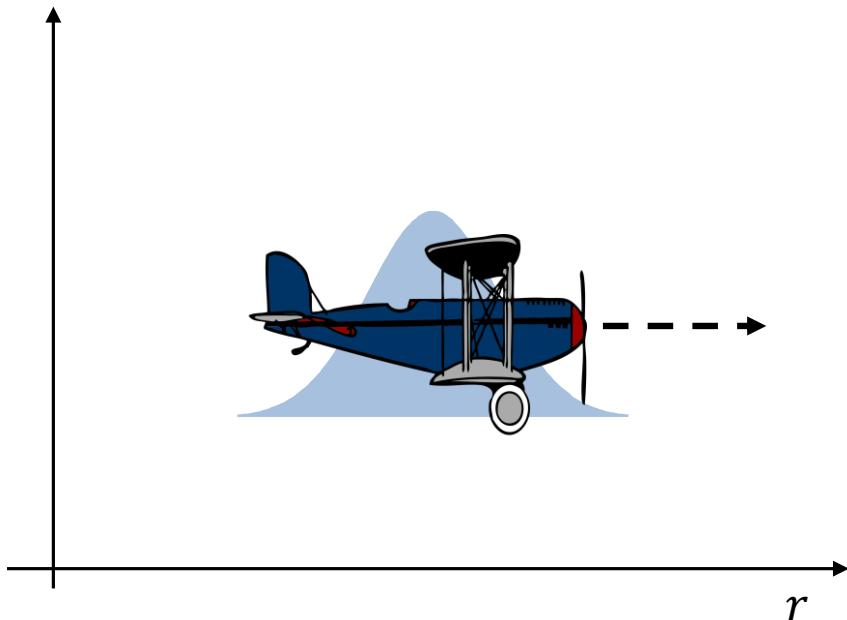


Figure 3.2: The initial knowledge of the system at time $t = 0$. The red Gaussian distribution represents the pdf providing the initial confidence in the estimate of the position of the aircraft. The arrow pointing to the right represents the known initial velocity of the aircraft.

The initial state of the system (at time $t = 0$) is assumed to be known to a reasonable accuracy, as shown in Figure 3.3.

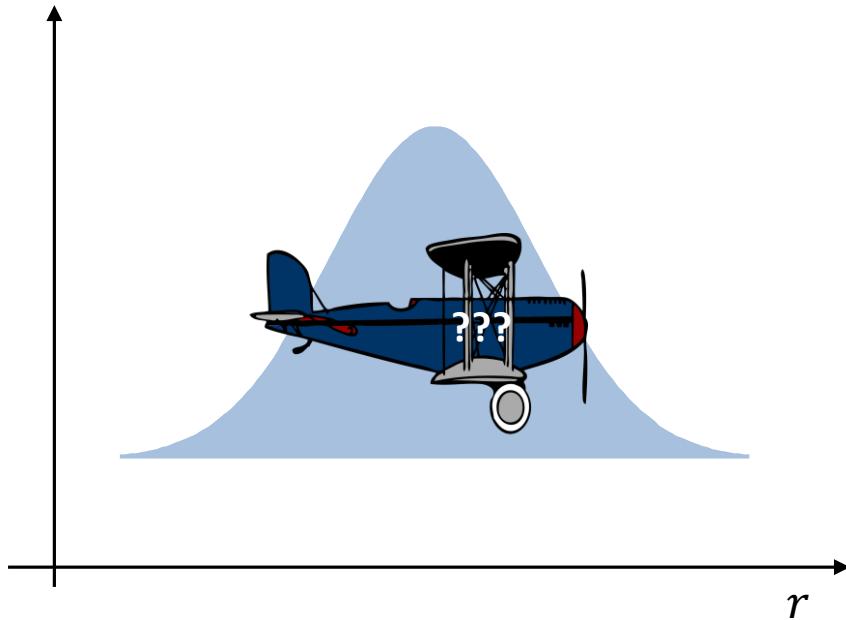


Figure 3.3: The prediction of the location of the aircraft at time $t = 1$ and the level of uncertainty in that prediction is shown. The confidence in the knowledge of the position of the aircraft has decreased, as it is not certain whether the aircraft has undergone any accelerations or decelerations in the intervening period from $t = 0$ to $t = 1$.

The location of the aircraft is given by a Gaussian pdf. At the next measurement time ($t = 1$), the new position of the aircraft, based on known limitations such as its position and velocity at $t = 0$, its maximum possible acceleration and deceleration, etc., may be estimated. In practice, some knowledge of the control inputs on the brake or accelerator by the driver may be known – this is true in satellite positioning and most missile navigation systems. A prediction of the new position of the aircraft (Figure 3.3) by a new Gaussian pdf with a different, re-calculated mean and variance.

This step is represented mathematically by (3.1). The variance has increased [see (3.2)], representing diminished certainty in the accuracy of the position estimate (as compared to $t = 0$), due to uncertainty associated with any process noise, e.g. that coming from accelerations or decelerations undertaken between times $t = 0$ to time $t = 1$.

At $t = 1$, a measurement of the location of the aircraft using radar, for example, is represented by the blue Gaussian pdf in Figure 3.4 – is also made.

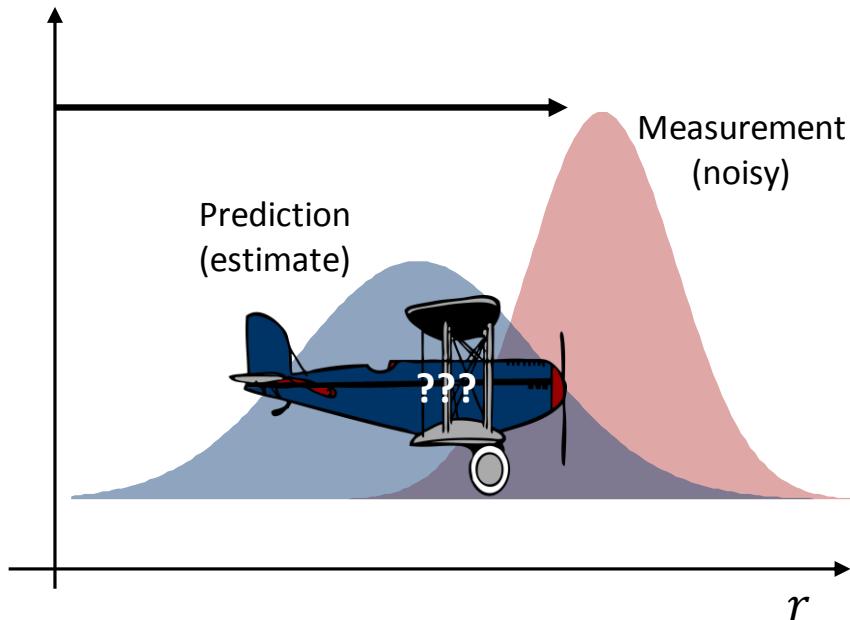


Figure 3.4: The measurement of the location of the aircraft at time $t = 1$ and the level of uncertainty in that noisy measurement, represented by the blue Gaussian pdf. The combined knowledge of this system is provided by the product of these two pdfs.

The best estimate that may be made of the location of the aircraft is provided by combining knowledge from the prediction and the measurement: achieved by calculating the product of the two corresponding pdfs. The green shaded pdf shows the result (Figure 3.5). A crucial property of the Gaussian function has been exploited, namely that the product of two Gaussian functions is another Gaussian function. This is critical as it permits a limitless number of Gaussian pdfs to be multiplied, time after time, yet the resulting Gaussian function does not increase in complexity or number of terms. After each time measurement, the new pdf is fully represented by yet another Gaussian function and it is this key point that leads to the Kalman filter's elegant recursive properties. The stages described above are mathematically recursive and are used to derive the Kalman filter measurement update equations. The prediction pdf represented by the red Gaussian pdf in Figure 3.4 is given by

$$y_1(r; \mu_1, \sigma_1) \triangleq \frac{1}{\sqrt{2\pi\sigma_1^2}} e^{-\frac{(r-\mu_1)^2}{2\sigma_1^2}} \quad (3.8)$$

The measurement pdf represented by the blue Gaussian function in Figure 3.4 is given by

$$y_2(r; \mu_2, \sigma_2) \triangleq \frac{1}{\sqrt{2\pi\sigma_2^2}} e^{-\frac{(r-\mu_2)^2}{2\sigma_2^2}} \quad (3.9)$$

The information provided by these two pdfs is fused by obtaining the product of the two, i.e., considering the prediction and the measurement together (Figure 3.5). The new pdf representing the fusion of the information from the prediction and measurement, and the best current estimate of the system, is the product of these two Gaussian functions:

$$\begin{aligned} y_{\text{fused}}(r; \mu_1, \sigma_1, \mu_2, \sigma_2) &= \frac{1}{\sqrt{2\pi\sigma_1^2}} e^{-\frac{(r-\mu_1)^2}{2\sigma_1^2}} \times \frac{1}{\sqrt{2\pi\sigma_2^2}} e^{-\frac{(r-\mu_2)^2}{2\sigma_2^2}} \\ &= \frac{1}{2\pi\sqrt{\sigma_1^2\sigma_2^2}} e^{-\left(\frac{(r-\mu_1)^2}{2\sigma_1^2} + \frac{(r-\mu_2)^2}{2\sigma_2^2}\right)} \end{aligned} \quad (3.10)$$

The quadratic terms in this new function can be expanded and the expression rewritten in Gaussian form:

$$y_{\text{fused}}(r; \mu_{\text{fused}}, \sigma_{\text{fused}}) = \frac{1}{\sqrt{2\pi\sigma_{\text{fused}}^2}} e^{-\frac{(r-\mu_{\text{fused}})^2}{2\sigma_{\text{fused}}^2}} \quad (3.11)$$

where

$$\mu_{\text{fused}} = \frac{\mu_1\sigma_2^2 + \mu_2\sigma_1^2}{\sigma_1^2 + \sigma_2^2} \quad (3.12)$$

and

$$\sigma_{\text{fused}} = \frac{\sigma_1^2\sigma_2^2}{\sigma_1^2 + \sigma_2^2} = \sigma_1^2 - \frac{\sigma_1^4}{\sigma_1^2 + \sigma_2^2} \quad (3.13)$$

(3.12) and (3.13) represent the measurement update steps of the Kalman filter algorithm. Note that it is the unique properties of the Gaussian distribution that allow for the "fusion" of means and standard deviations in (3.12) and (3.13). No other distribution allows this.

To present a more general case, an extension to this example is required. It has been assumed that the predictions and measurements were made in the same coordinate frame and in the same units. This has resulted in a particularly concise pair of equations representing the prediction and measurement update stages. However, a real function is usually required to map predictions and measurements into the same domain. The position of the aircraft is predicted directly as a new distance in standard international (S.I.) units of meters, but time of flight measurements are recorded in S.I. units of seconds.

To determine the product of the prediction and measurement pdfs, one must be converted into the domain of the other. Standard practice maps the predictions into the measurement domain via the transformation matrix H_t .

Using (3.8) and (3.9) and, instead of allowing y_1 and y_2 to both represent distances in meters along the aircraft trajectory, assume the distribution y_2 represents the time of flight in seconds for a radio signal propagating from a transmitter positioned at $x = 0$ to the aircraft's antenna. The spatial prediction pdf, y_1 is then converted into the measurement domain by scaling by c , the speed of light. (3.8) and (3.9) therefore are rewritten as:

$$y_1(s; \mu_1, \sigma_1, c) \triangleq \frac{1}{\sqrt{2\pi\left(\frac{\sigma_1}{c}\right)^2}} e^{-\frac{(s-\frac{\mu_1}{c})^2}{2\left(\frac{\sigma_1}{c}\right)^2}} \quad (3.14)$$

and

$$y_2(s; \mu_2, \sigma_2, c) \triangleq \frac{1}{\sqrt{2\pi\sigma_2^2}} e^{-\frac{(s-\mu_2)^2}{2\sigma_2^2}} \quad (3.15)$$

where both distributions are now defined in the measurement domain, radio signals propagate along the time s axis, and the measurement units are seconds.

Using the same derivation as before, the fused mean becomes:

$$\begin{aligned} \frac{\mu_{\text{fused}}}{c} &= \frac{\mu_1}{c} + \frac{\left(\frac{\sigma_1}{2}\right)^2 \left(\mu_2 - \frac{\mu_1}{c}\right)}{\left(\frac{\sigma_1}{c}\right)^2 + \sigma_2^2} \\ \rightarrow \mu_{\text{fused}} &= \mu_1 + \left(\frac{\frac{\sigma_1^2}{c}}{\left(\frac{\sigma_1}{c}\right)^2 + \sigma_2^2} \right) \cdot \left(\mu_2 - \frac{\mu_1}{c} \right) \end{aligned} \quad (3.16)$$

Substituting

$$H = \frac{1}{c}$$

and

$$K = \frac{H\sigma_1^2}{H^2\sigma_1^2 + \sigma_2^2}$$

gives

$$\mu_{\text{fused}} = \mu_1 + K \cdot (\mu_2 - H\mu_1) \quad (3.17)$$

In a similar way, the fused variance estimate becomes:

$$\begin{aligned} \frac{\sigma_{\text{fused}}^2}{c^2} &= \left(\frac{\sigma_1}{c}\right)^2 - \frac{\left(\frac{\sigma_1}{c}\right)^4}{\left(\frac{\sigma_1}{c}\right)^2 + \sigma_2^2} \\ \rightarrow \sigma_{\text{fused}}^2 &= \sigma_1^2 - \left(\frac{\frac{\sigma_1^2}{c}}{\left(\frac{\sigma_1}{c}\right)^2 + \sigma_2^2} \right) \frac{\sigma_1^2}{c} \\ &= \sigma_1^2 - KH\sigma_2^2 \end{aligned} \quad (3.18)$$

Figure 3.5 shows this fusion graphically.

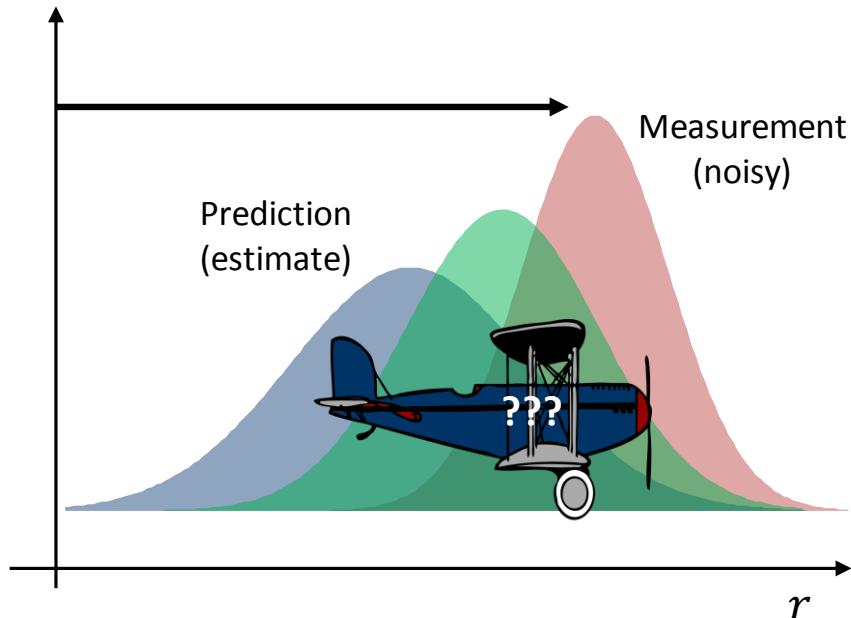


Figure 3.5: The new pdf (green) generated by multiplying the pdfs associated with the prediction and measurement of the aircraft's location at time $t = 1$. This new pdf provides the best estimate of the location of the aircraft, by fusing data from prediction and measurement.

Terms from this scalar derivation may be compared with standard vectors and matrices used in the Kalman filter algorithm:

- $\mu_{\text{fused}} \rightarrow \hat{x}_{t|t}$: state vector following data fusion
- $\mu_1 \rightarrow \hat{x}_{t|t-1}$: state vector before data fusion, i.e., the prediction
- $\sigma_{\text{fused}}^2 \rightarrow P_{t|t}$: covariance matrix (confidence) following data fusion
- $\sigma_1^2 \rightarrow P_{t|t-1}$: covariance matrix (confidence) before data fusion
- $\mu_2 \rightarrow z_t$: measurement vector

- $\sigma_2^2 \rightarrow R_t$: uncertainty matrix associated with a noisy set of measurements
 $H \rightarrow H_t$: transformation matrix used to map state vector parameters into measurement domain

Kalman gain
$$K = \frac{H\sigma_1^2}{H^2\sigma_1^2 + \sigma_2^2} \rightarrow K_t = P_{t|t-1}H_t^T(H_tP_{t|t-1}H_t^T + R_t)^{-1}$$

It is then a trivial matter to determine how the standard Kalman filter equations relate to (3.17) and (3.18) above:

$$\begin{aligned}\mu_{\text{fused}} &= \mu_1 + \left(\frac{H\sigma_1^2}{H^2\sigma_1^2 + \sigma_2^2} \right) \cdot (\mu_2 - H\mu_1) \\ \rightarrow \hat{x}_{t|t} &= \hat{x}_{t|t-1} + K_t(z_t = H_t\hat{x}_{t|t-1}) \\ \sigma_{\text{fused}}^2 &= \sigma_1^2 - \left(\frac{H\sigma_1^2}{H^2\sigma_1^2 + \sigma_2^2} \right) H\sigma_1^2 \\ \rightarrow P_{t|t} &= P_{t|t-1} - K_t H_t P_{t|t-1}.\end{aligned}$$

The mechanics of the Kalman filter function are non-trivial. Even for an experienced physicist, diagrams are often required to explain the vagaries of Bayesian recursion and estimation updates. For an enlightening description of the way the Kalman filter operates, also see, for example, Bzarg (2016).

3.5. Kalman filter application: financial system

Prior to the credit crisis of 2008/9, hedge funds were characterised by high returns compared with conventional funds and indices, large capital inflows and low market correlation (Tran, 2006). Hedge funds also charged high fees from their opaque investment strategies (usually a percentage of profits and a steep – up to 20% – management fee), and operated largely outside the heft of market regulations (Poloner, 2010). The substantial out-performance of hedge funds prior to the crisis offered investors substantial returns by enhancing their portfolio risk-return trade-off.

The financial crisis, however, altered the hedge fund universe considerably and perhaps permanently. Since the crisis began in 2008, hedge fund investment performance has substantially deteriorated: large portfolio losses have fed a spiral of diminishing investment and investor withdrawals (Ben-David, Franzoni & Moussawi, 2011). Diversification benefits of hedge funds have continuously deteriorated as the correlation between hedge fund return and conventional asset class returns have slowly, but inexorably, increased (Guesmi, Jebri,

Jabri & Teulon, 2014). This dismal return profile no longer justifies the high fees enjoyed by hedge funds prior to the crisis (Authers, 2014). Investors have abandoned (and continue to abandon) hedge funds in favour of cheaper, higher return investments elsewhere (e.g. Schlaikier, 2014 and Marois, 2014), often with lower risk profiles.

In the light of the waning popularity of hedge funds, financial performance measures have acquired renewed significance (Heuson, Hutchinson & Kumar, 2014 and Bussière, Hoerova & Klaus, 2014). Hedge funds must now establish robust, consistent performance, with reasonable certainty, before yield-seeking investors participate in (traditionally) opaque investment strategies. Fortunately, several such measures exist, among them the Sharpe ratio (which measures excess fund returns relative to the risk-free rate per unit of risk), the Information ratio (excess fund return relative to a benchmark per unit of tracking error, or relative risk), the Treynor ratio (excess fund return relative to the risk free rate per unit of systemic market risk, β) and several others. A detailed summary of hedge fund performance measures can be found in Aragon & Ferson (2006) and more recently in Le Sourd (2009).

The capital asset market model (CAPM) – an equilibrium model – has profoundly influenced the way investors understand and react to the asset price/risk relationship (Sharpe, 1964; Mossin, 1966 and Fama & French, 2004). The CAPM formulation asserts that fund excess returns over a benchmark (usually the market) include an abnormal portfolio return, α and a single risk measure of risk, β , which affects systematic differences in security returns. The CAPM coefficients (α and β) were originally assumed to be static – or relatively static – and most formulations still make this assumption (as evidenced by regression tests conducted to assess fund performance: these implicitly assume constant coefficients). Evidence indicates, however, that CAPM coefficients are dynamic, changing as market factors change and as portfolio components are altered by fund managers (e.g. Glabadanidis, 2009). Measuring time-varying coefficients is non-trivial, yet it is important to establish whether they are static or behave dynamically (and, if dynamic, the question of whether this time-dependence is due to changing β s of the underlying assets or to the change of portfolio weights via the fund manager becomes important). The latter represents an active strategy justifying the high fees, as does the accurate, dynamic measurement of manager stock-selection skill (α) (Albrecht, 2005).

Various techniques have been developed to estimate time-varying coefficients such as time-weighted least-square estimators (including non-parametric weights) and GARCH-based estimators (Nieto, Orbe & Zarraga, 2014). Bali & Engle (2010) introduced and Engle (2014) tested a new method for formulating dynamic β 's using conditional covariance matrices of both the exogenous and dependent variables. Called the Dynamic Conditional Beta (DCB), the approach was successfully applied to asset pricing and global systemic risk estimation (Engle, 2014).

Each of the above technique has drawbacks. Rolling-window ordinary least squares techniques require no parameterisation, but require window lengths to be pre-selected resulting in unstable β estimates (Lin, Chen & Boot, 1992) and although GARCH(1,1) models describe volatility clustering and other important features of returns such as excess kurtosis, standard GARCH models do not capture important volatility properties (Prysyazhnyuk & Kirdyaeva, 2010). Engle's (2014) DCB approach does not observe the vector of conditional means nor conditional covariances directly, so models are required for each of these, requiring yet more assumptions.

The Kalman filter, originally developed to assist with aeronautics, may provide a solution. A principal advantage of the Kalman filter is that it can be applied in real time, i.e. for any observed value of the time series the forecast for the next observation can be calculated, making the method highly practical and important in the financial field (see, e.g., Jain, Yongvanich & Zhou, 2011). The filter uses only historical information, but it reacts rapidly to changing conditions. These expedient properties confirm the usefulness and applicability of the Kalman filter to the changing nature of hedge fund portfolios and the importance of it to detect crises or large market changes. Indeed, the Kalman filter is suited to take into consideration the multiple investment style variations of actively managed hedge funds (Swinkels & Sluis, 2006), but its use has been rather limited in the literature due to the limited size of hedge fund databases. In addition, due to the complexity of understanding and implementing the Kalman filter, it has not been widely used over the traditional regression analysis in most of the statistical inference problems.

Jain, *et al* (2011) quantified the market timing component of returns using a conceptually simple approach by exploring market exposure traded at every point in time. Jain, *et al* (2011) argued that, if the market exposure were increased (on average) in bullish markets,

and decreased in bear markets, positive returns would be generated through market timing. A comparison of the returns from a manager's *average market exposures* to the returns from the manager's *time-varying market exposures* could therefore assist in the identification of value added from market timing.

Jain, et al (2011) expanded (2.1) to accomplish this:

$$\begin{aligned}
 (r^P - r^f) &= \alpha + \beta(r^M - r^f) + \varepsilon \\
 &= \alpha + (\beta + \bar{\beta} - \bar{\beta})(r^M - r^f) + \varepsilon \\
 &= \alpha + \bar{\beta}(r^M - r^f) + (\beta - \bar{\beta})(r^M - r^f) + \varepsilon
 \end{aligned} \tag{3.19}$$

where the terms have their usual meanings and $\bar{\beta}$ is defined as the manager's average β exposure. Note that the second line of (3.19) above is generated by adding 0 (via the addition and subtraction of $\bar{\beta}$ – the long run average – as determined from standard regression analysis).

The term $(\beta - \bar{\beta})$ is the manager's active β exposure based on timing decisions (i.e. the difference between an 'instantaneous' β and a 'long run', or average β defined here as $\bar{\beta}$). Using (3.19), a manager's return at each point in time thus comprises four components:

1. an expected α based on security selection decisions: α
2. a return based on the manager's average exposure to the market: $\bar{\beta}(r^M - r^f)$
3. a return based on the manager's market timing decisions: $(\beta - \bar{\beta})(r^M - r^f)$
4. a residual component with zero mean, corresponding to the risk generated from security selection decisions: ε .

The Kalman filter (Kalman, 1960) is a special case of a Bayesian updating scheme that maximises the likelihood of correctly estimating unknown parameter values (Koch, 2006). The Kalman filter assumes linear state equations, linear observation equations and Gaussian noises. The filter addresses the general problem of attempting to estimate the state $[x \in \Re^n]$ of a discrete, time-controlled process governed by the linear stochastic difference equation:

$$x_t = Fx_{t-1} + Bu_{t-1} + w_{t-1} \tag{3.20}$$

with a measurement:

$$z_t = Hx_t + v_t \quad (3.21)$$

where $z \in \Re^n$.

The random variables w and v represent *process* white noise and *measurement* white noise respectively. These are assumed to be independent of each other (i.e. 0 correlation between them) with normal probability distributions:

$$\begin{aligned} w(\cdot) &\sim N(0, Q) \\ v(\cdot) &\sim N(0, R). \end{aligned}$$

More detailed mathematics underling the principles of the Kalman filter are provided in Chapter 4, which describes the technique used to forecast hedge fund returns using the CAPM formulation. This chapter also presents the results obtained and discusses these observations in the light of the Kalman filter exposition described here.

Chapter 4

Hedge fund index performance forecasting using the Kalman filter

4.1 Introduction

Hedge funds enjoyed high returns (in comparison with conventional funds and indices), large capital inflows and low market correlation (Tran, 2006) prior to the credit crisis of 2008/9 which altered the financial milieu considerably. Before the credit crisis, hedge funds charged high fees (a considerable profit percentage and management fees of up to 20%) without full disclosure of their investment strategies, and they conducted their affairs outside the regulatory market environment (Poloner, 2010). This has only recently begun to change (Michaels, 2016). The superior, pre-crisis, return performance invoked by hedge funds enhanced investor's risk-return trade-off with little regulatory pressure and negligible investment policy disclosure.

The financial crisis transformed the financial milieu – including the hedge fund universe – considerably; perhaps even permanently. The combination of quantitative easing, extremely low interest rates (by both historical and absolute standards) and low inflation have altered traditional economics and finance. Hedge fund performance has deteriorated because of these and other factors, such as crowded markets squeezing out arbitrage opportunities and the flight to quality. Hedge fund portfolio losses have led directly to reduced investment and substantial withdrawals (Ben-David, Franzoni & Moussawi, 2011). The much-lauded diversification benefits – once feted by hedge funds – have deteriorated as the correlation between hedge fund returns and returns from the markets in which they operate have gradually but relentlessly increased (Guesmi, Jebri, Jabri & Teulon, 2014). The difference between traditional asset class returns, such as mutual funds, and hedge fund returns has diminished and turned negative in 2011. This trend (Figure 4.1) shows no sign of changing (November 2016) so the high fees once commanded by hedge funds are no longer justified (Authers, 2014 and Uhlfelder, 2016). Investors have abandoned (and continue to abandon) hedge funds in favour of cheaper, higher return/lower risk profiles elsewhere (e.g. Schlaikier, 2014 and Marois, 2014 and Uhlfelder, 2016).

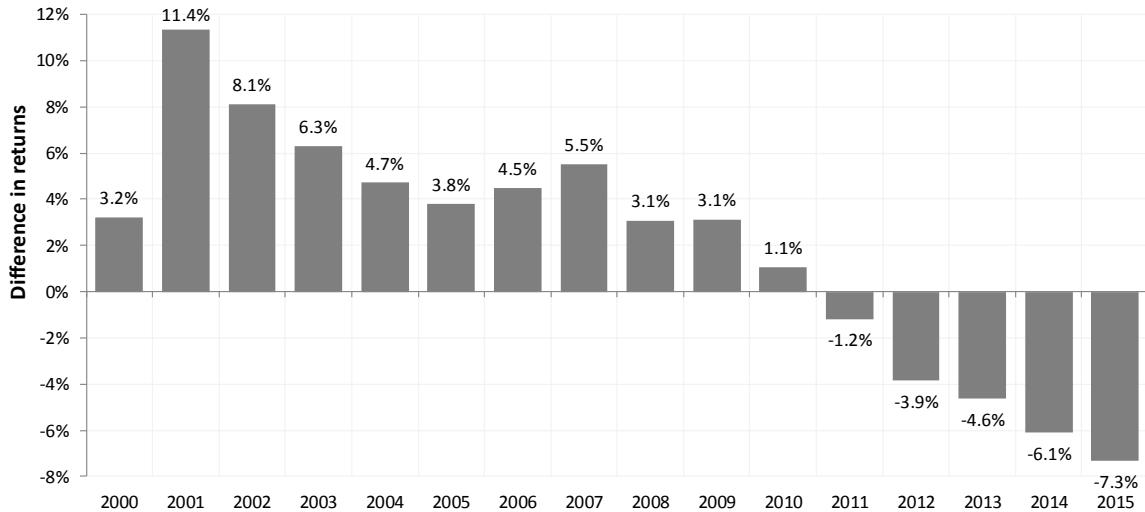


Figure 4.1: Difference between hedge fund returns and conventional asset class average returns from 2000 to 2015.

The diminishing esteem for hedge funds has led to the search for more accurate, more robust and more relevant performance measures (Heuson, Hutchinson & Kumar, 2014). The *correct* and *consistent* identification of superior return performance has become the goal of researchers and investors alike (Bussière, Hoerova & Klaus, 2014): the post-crisis investor generation have become more demanding of transparency and accountability (AIMA, 2014), with the impact on hedge funds being that robust, consistent performance must be established with reasonable certainty, before participation is sought. Metrics such as the Sharpe ratio, the Information ratio, the Treynor ratio and the Omega ratio (AIMA, 2014) have been used to some effect.

The remainder of the work in the Chapter addresses an update of Yacumakis and van Vuuren's (2014) work using hedge fund indices. Additional work includes time-weighted regression and an analysis on the forecast versus empirical goodness-of-fit results made using standard regression, time-weighted regression and the Kalman filter.

Estimating time-dependent α and β coefficients for the CAPM has become an important endeavor and several methodologies have been suggested including GARCH-based estimators (Nieto, Orbe & Zarraga, 2014) and time-weighted, least-square estimators, (including non-parametric weights) (Engle, 2014). A technique known as Dynamic Conditional Beta (DCB) which employs conditional covariance matrices of both the dependent and exogenous variables has been used to establish time-varying β s (Bali & Engle, 2010 and Engle, 2014) and has enjoyed some success.

No method comes without its downside. GARCH(1,1) models, for example, accurately describe the observed clustering of volatility as well as excess kurtosis – a common feature of fund returns – but standard GARCH models fail to capture other influential properties that describe volatility (Prysyazhnyuk & Kirdyaeva, 2010). While no parameterisation is required for rolling-window, OLS approaches, optimal time windows must be selected before analysis commences, with the result that the calculated β s are unstable (Lin, Chen & Boot, 1992). The DCB technique ignores the conditional mean vector as well as conditional covariances, with the result that additional quantitative models are needed for each – occasioning ever more input assumptions.

The Kalman filter – which was originally developed to provide aeronautical navigation as described in Chapter 3 – offers a different, but parsimonious and accurate solution to the problem of time-dependent variable estimation. The Kalman filter offers the advantage of real-time application. Time series observations are used to forecast future observations (as in most forecast applications), but in the Kalman filter case, the state variables (which define the forecast framework) are 'optimal', being determined via minimisation of the variance between prediction and observation differences by rapidly reacting to changing conditions. This makes the Kalman approach to the problem of time-dependent variable estimation extremely practical to financial practitioners (see, e.g., Jain, Yongvanich & Zhou, 2011). The advantageous properties of the Kalman filter affirm its applicability to the variable nature of hedge fund returns and – because of its sensitivity to rapidly changing market conditions – its importance in detecting the onset of crises or large changes in the market.

The Kalman filter has been shown to be suitable for many actively-managed, hedge fund investment styles (Swinkels & Sluis, 2006 and van Vuuren & Yacumakis, 2015). Use of the Kalman filter has, however, been limited due to the proprietary nature of hedge fund returns, the limited size number of hedge fund databases and the fledgling nature of financial Kalman filter applications. Although the concepts which underpin the mechanics of the filter are not overly complex, some distributional analytics are non-trivial, and third party-software is often perceived as being a "black box" (i.e. opaque) application if the essential mathematics is not fully-understood. These factors could account for the filter not being in widespread use – at least not in favour of standard regression analysis.

This chapter explores the effectiveness of the Kalman filter in the determination of time-varying CAPM variables. This effectiveness is assessed by examining the accuracy (or lack thereof) of predicted hedge fund returns using *Kalman* CAPM variables and those determined using *standard* techniques such as OLS regression.

4.2 Data and methodology

Data used in this study were monthly returns of global hedge fund indices representing a range of various investment styles as listed in Table 4.1 below as well as well as a hedge fund index (HFI) which represents time series return data for the entire industry.

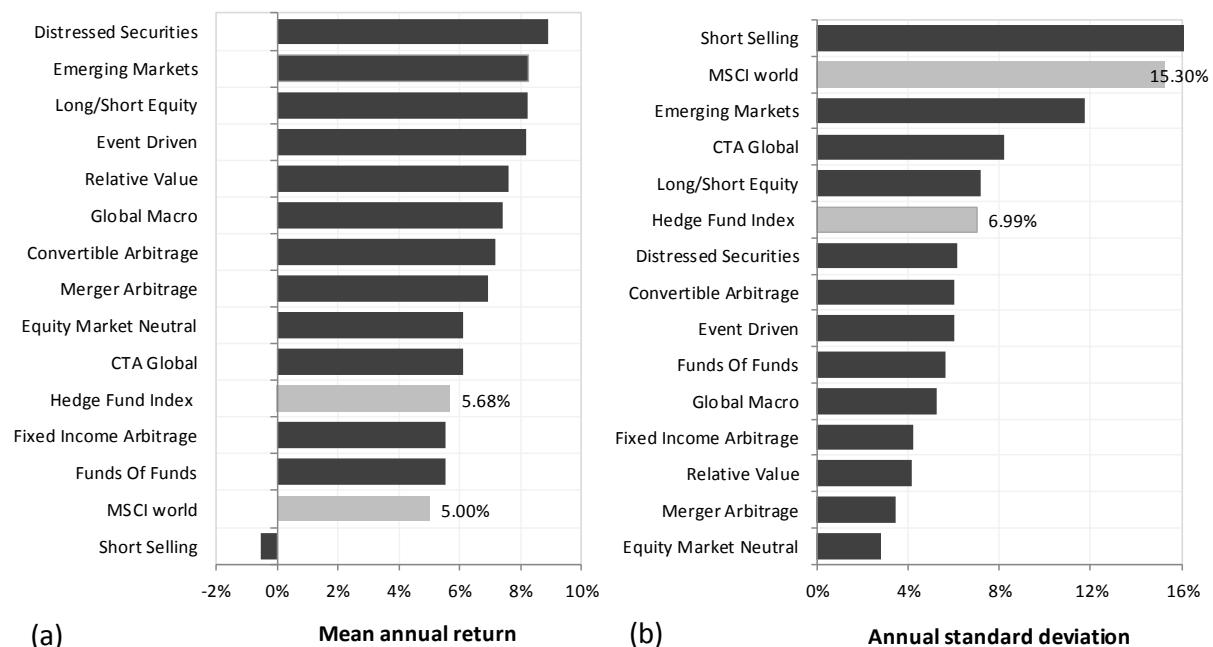
Table 4.1: Representative investment styles and description used in the study.

| Investment strategy | Strategy |
|---|--|
| Convertible Arbitrage | Exploit mispricing between convertible bonds and underlying securities |
| Commodity trading advisors /managed futures | Use precise trading rules to capture price movements, and focus on transitory patterns |
| Distressed Securities | Focuses on arbitrage opportunities of securities in companies that are in or near bankruptcy |
| Emerging Markets | Tactical trading strategies that profit by forecasting the overall direction of the market or market components. Unique risks accompany emerging market investments: e.g. political hazards and inadequate legal systems |
| Equity Market Neutral | Uses the combination of buys and short-sales (sometimes augmented by options and futures positions) to offset correlations between portfolio returns and market returns |
| Event Driven | Seek profits from arbitrage from short-duration events such as corporate restructuring, stock buybacks, bond upgrades, earnings surprises and spin-offs |
| Fixed Income Arbitrage | Exploit mispricing among fixed income securities |
| Global Macro | Bets are based on major macroeconomic forecasts such as changes in interest rates, currency movements and stock market performance |
| Long/Short Equity | Strategy is the same as equity market neutral but without explicit promises of market neutrality – this increases investment flexibility to choose net-long or net-short ($\beta > 0$ or $\beta < 0$) market exposure |
| Merger Arbitrage | Because there is a likelihood of a merger not completing, the target |

| | |
|----------------|---|
| | company's share price carries a "bid premium" (a discount to the proposed takeover price) until the merger does complete. |
| Relative Value | Takes advantage of perceived mispricing among related financial assets, such as company debt and equity securities |
| Short Selling | Concentrates on the short side by sacrificing market-neutrality. Tends to be most effective when markets are declining |
| Funds of Funds | Managed portfolio of other hedge funds |

These data were procured from the Hedge Fund Research (HFR) Global Hedge Fund database spanning 20 years, i.e. from January 1997 to November 2016 inclusive (HFR, 2016).

Annualised descriptive statistics are summarised for all fund indices in Figures 4.2a through d as well as – for comparison – the Morgan Stanley Capital International (MSCI) world equity index and the HFI. Annualised returns are calculated using $(1 + \bar{r}_m)^{12} - 1$, where \bar{r}_m is the average monthly return over the full 20-year period, annual volatility is $\sigma_m \cdot \sqrt{12}$, where σ_m is the monthly return standard deviation, annual skew is $s_m / \sqrt{12}$, where s_m is the monthly return skew, and annual excess kurtosis is $(k_m + 3 \cdot 11)/12 - 3$, where k_m is the monthly return kurtosis for each index. These are standard time-scaling techniques for the first four moments of return distributions (Gabrielsen, Zagaglia, Kirchner & Liu, 2012) which are particularly suited to this work as hedge fund return data are generally negatively-skewed and leptokurtic (see Figures 4.2c and d).



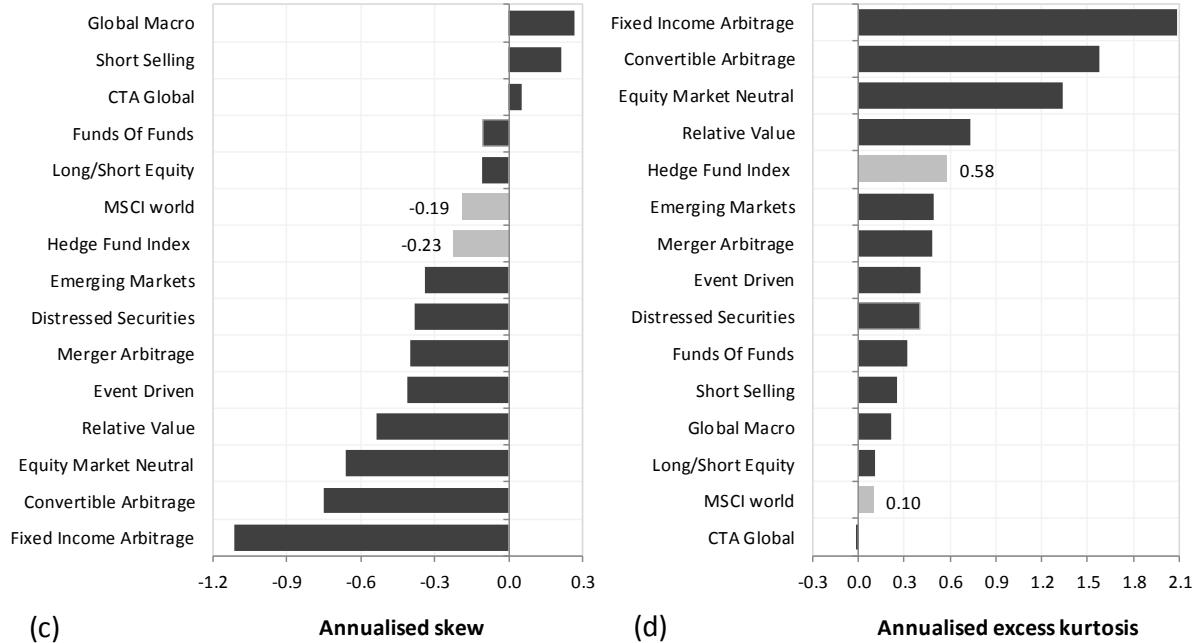


Figure 4.2: Ordered hedge fund return statistics for various investment types: (a) mean annual return, (b) annualised standard deviation, (c) annualised skew and (d) annualised excess kurtosis over the 20-year period Jan 97 to Nov 16.

Figure 4.3a shows the monthly HFI returns (left axis) for Jan 97 to Nov 16 as well as the cumulative return profile (right axis). MSCI world index returns are included for comparison on the same timescale. The reduction in variance of returns is evident, particularly post the credit crisis. Although the HFI has outperformed the MSCI equity index by almost 2:1 over the 20-year period, the cumulative return has remained flat since Jan 13, demonstrating the declining fortunes of hedge funds globally.

Monthly volatilities were calculated using an exponentially weighted moving average (EW-MA) approach with exponential weighting constant $\lambda = 0.950$. The λ (Hendricks, 1996) is calibrated using representative global monthly market data as at November 2016.

Figure 4.3b shows the monthly volatility profile of both indices over the same time period. The large volatility experienced by hedge funds at the start of the period was caused by the collapse of Long Term Capital Management (LTCM) followed by a lengthy decline in volatility until the credit crisis in 2008. Although the crisis resulted in steep volatility increases for most asset classes, hedge fund volatility increased threefold (1% to 3%, albeit from a low base) while the MSCI equity index volatility increased only twofold (from 3% to 6%). This lack of "hedging" by hedge funds has been partially responsible for the declining interest in this asset class subsequent to the crisis.

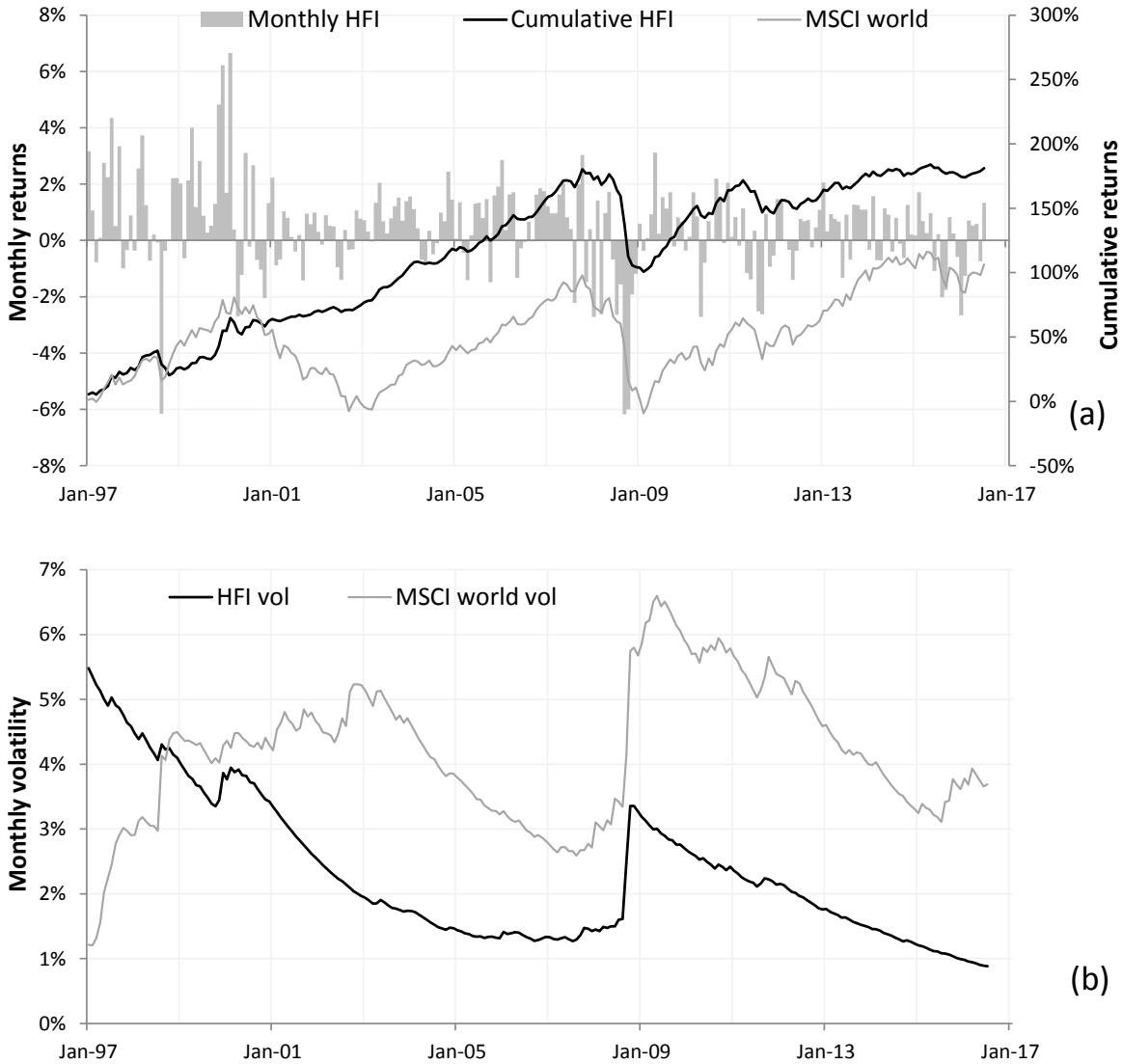


Figure 4.3: (a) Monthly + cumulative HFI returns and monthly MSCI world cumulative returns for comparison and (b) EWMA volatilities for the HFI and MSCI.

4.3 Kalman filter specification

The Kalman filter (Kalman, 1960) – as discussed in Chapter 3 – is a Bayesian updating scheme that maximises the likelihood of correctly estimating unknown parameter values (Koch, 2006). The filter addresses the general problem of determining the state $[x \in \Re^n]$ of a time-controlled, discrete process which is governed by the linear stochastic difference equation:

$$x_t = Fx_{t-1} + Bu_{t-1} + w_{t-1}$$

with a measurement $[z \in \Re^n]$:

$$z_t = Hx_t + v_t.$$

The random variables w and v , which are assumed to be independent (i.e. 0 correlation between them) and normally distributed:

$$w(\cdot) \sim N(0, \mathbf{Q})$$

$$v(\cdot) \sim N(0, \mathbf{R})$$

represent *process* white noise and *measurement* white noise respectively.

In practice, the process noise covariance \mathbf{Q} and measurement noise covariance \mathbf{R} matrices (here *variance* matrices because the correlation is zero) change with each time step. In this case, they are assumed to be constant (Koch, 2006) – a common assumption – and these values are obtained using maximum likelihood methods (Tommaso & Alessandra, 2012).

The 2×1 (in this case) state transition matrix, F , links the state at the previous time step $t - 1$ to the current state at step t , assuming no driving function nor process noise. The 2×2 control matrix B relates the optional control input $u \in \mathbb{R}^l$ to the state x . The 2×1 matrix H in the measurement relates the state to the measurement z_k . In practice F and H might change with each time step, but here again they are both assumed to be constant.

The mechanical process to be followed is:

PREDICT

$$\text{Project state 1 time step ahead} \quad \hat{x}_{t|t-1} = F_t \hat{x}_{t-1|t-1} + B_t u_t \quad (4.1)$$

$$\text{Project error covariance 1 step ahead} \quad P_{t|t-1} = F_t P_{t-1|t-1} F_t^T + Q_t \quad (4.2)$$

UPDATE

$$\text{Compute Kalman gain} \quad K_t = P_{t|t-1} H_t^T (H_t P_{t|t-1} H_t^T + R_t)^{-1} \quad (4.3)$$

$$\text{Update estimate with measurement } y_t \quad \hat{x}_{t|t} = \hat{x}_{t|t-1} + K_t (y_t - H_t \hat{x}_{t|t-1}) \quad (4.4)$$

$$\text{Update error covariance} \quad P_{t|t} = (I - K_t H_t) P_{t|t-1} \quad (4.5)$$

where \hat{x} is the estimated state, F is the state transition matrix (i.e., transition between states), u represents the control variables, B is the control matrix (i.e., mapping control to state variables), P is the state variance matrix (i.e., error of estimation), Q is the process variance matrix (i.e., error due to process), y represents the measurement variables, H is the measurement matrix (i.e., mapping measurements onto the state), K is the Kalman gain and R is the measurement variance matrix (i.e., measurement error).

Subscripts represent:

- $t|t$: the current time period
- $t - 1|t - 1$: the previous time period, and
- $t|t - 1$: intermediate steps.

The observation equation is the CAPM:

$$r_t^P = \alpha_t + \beta_t \cdot r_t^M + \epsilon_t \quad \epsilon_t \sim N(0, \sigma_\epsilon^2) \quad (4.6)$$

where r^P is the security (or portfolio) excess return (c.f. (1)), α is the abnormal rate of return on the security/portfolio, β is the systematic risk as defined earlier in (2.1), r^M is the excess market return and ϵ is a noise term, assumed i.i.d. and $\sim N(0, \sigma_\epsilon^2)$.

The form that the transition equation takes on relies on the underlying (assumed) stochastic process followed by time-varying α s and β s. Thus, the transition equation could employ autoregressive, mean-reverting (AR(1)) models, random walk processes, different distributional assumptions, etc. It has been established that the random walk model provides the most robust characterisation of time-varying β s, while AR(1) forms of transition equation experience convergence problems (symptomatic of transition equation misspecification) for some return series (Faff, et al., 2000).

The random walk model (RW) assumes both α and β evolve through time via a random walk process, i.e. the current market exposure is assumed to be a normally-distributed random variable with mean equal to the last period's exposure and with system noises also assumed to be normally distributed and uncorrelated.

State variables $x_t \in \Re^2$ are time-varying coefficients:

$$x_t = \begin{bmatrix} \alpha_t \\ \beta_t \end{bmatrix}$$

at each time t . Both state variables are assumed to evolve via the random walk process and the state equation is:

$$\begin{bmatrix} \alpha_{t+1} \\ \beta_{t+1} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \alpha_t \\ \beta_t \end{bmatrix} + \begin{bmatrix} \gamma \\ \delta \end{bmatrix} \quad (4.7)$$

where

$$\begin{bmatrix} \gamma \\ \delta \end{bmatrix} \sim N \left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{bmatrix} \sigma_\gamma^2 & 0 \\ 0 & \sigma_\delta^2 \end{bmatrix} \right)$$

and the measurement equation is:

$$r_t^P = [1 \quad r_t^M] \begin{bmatrix} \alpha_t \\ \beta_t \end{bmatrix} + \epsilon_t \quad (4.8)$$

which is merely the matrix representation of the CAPM, i.e. (2.1). The variances σ_γ^2 and σ_δ^2 are determined via maximum likelihood techniques.

The data employed for this analysis are not *hedge fund* return data, but rather hedge fund *index* return data. Using indices rather than individual hedge funds detracts from specific performance evaluation, so the determination of index α s makes sense only if the indices are investable. If not, hedge fund index α s are not performance measures, but rather an indication of return related to the index characteristics. Specific, individual hedge fund data were used for apportioning returns into market timing and stock selection. The results obtained from this analysis are presented in Chapter 5.

Hedge fund returns were grouped by geography (Europe and Asia, US and Canada). This grouping facilitated the use of a single risk free rate time series and a single market return index for each region. Monthly, annualised hedge fund *excess* returns were run through the Kalman model as specified above and the resulting α s and β s extracted and interrogated.

Rolling 36-month OLS regression α s and β s (using non-weighted *and* weighted excess market returns and excess fund returns – i.e. the annualised risk free rate was subtracted from both returns time series) were also calculated and plotted on the same time scale as those derived from the Kalman filter results for comparison.

4.4 Results and discussion

4.4.1 Diversification and hedge funds

Because of their historical low correlation with conventional asset classes, hedge funds were generally considered to be good diversifiers (e.g. Brown, Gregoriou & Pascalau, 2011 and Teo, 2013). Although dependent on the precise investment strategy invoked, hedge funds traditionally built hedging strategies by being long in securities which were expected to outperform the market when the market was in a bull phase and by being short in those securi-

ties believed to perform badly when the market was falling. Pursuing such a stratagem – if successful – eliminated (or diminished) systematic investment risk (Chan, Getmansky, Haas & Lo, 2006). This scheme worked well pre 2000, when the market was not crowded with participants and when arbitrage opportunities abounded, but post 2000 broad correlations between hedge fund returns and the market have *increased*, giving rise to concerns (Amenc & Goltz, 2007) and criticisms (Markowitz, 2014) about their value as an effective *hedging* asset class.

Figure 4.4 shows the correlation between representative hedge fund strategy indices and the MSCI world index from Jan 97 to Nov 16. Correlations decreased dramatically across all strategies during the 9/11 New York attacks. This may be an indication of hedge fund agility; the speed at which these funds can enter and exit markets and hedge positions effectively – although this decrease was short-lived, persisting only for a few months. The same is observed at the onset of the credit crisis: a transitory large decrease in correlations with the market and then the resumption to pre-crisis levels. The most important observation, however, is that high correlations prior to both the 9/11 event and the credit crisis signifies diminished diversification when it may be most required. The shaded regions in Figure 4.4 indicate the two market events (9/11 and the credit crisis), both periods of heightened correlation with the market and both preceding considerable market perturbations. Also, since 2002, correlations have increased inexorably although they have decreased across all strategies in 2015 before increasing again recently (Nov 16).

Note that diversification observations using only correlation coefficients, especially when the returns are generated from hedge fund *indices* (not from funds themselves), is not necessarily representative of hedge *fund* performance.

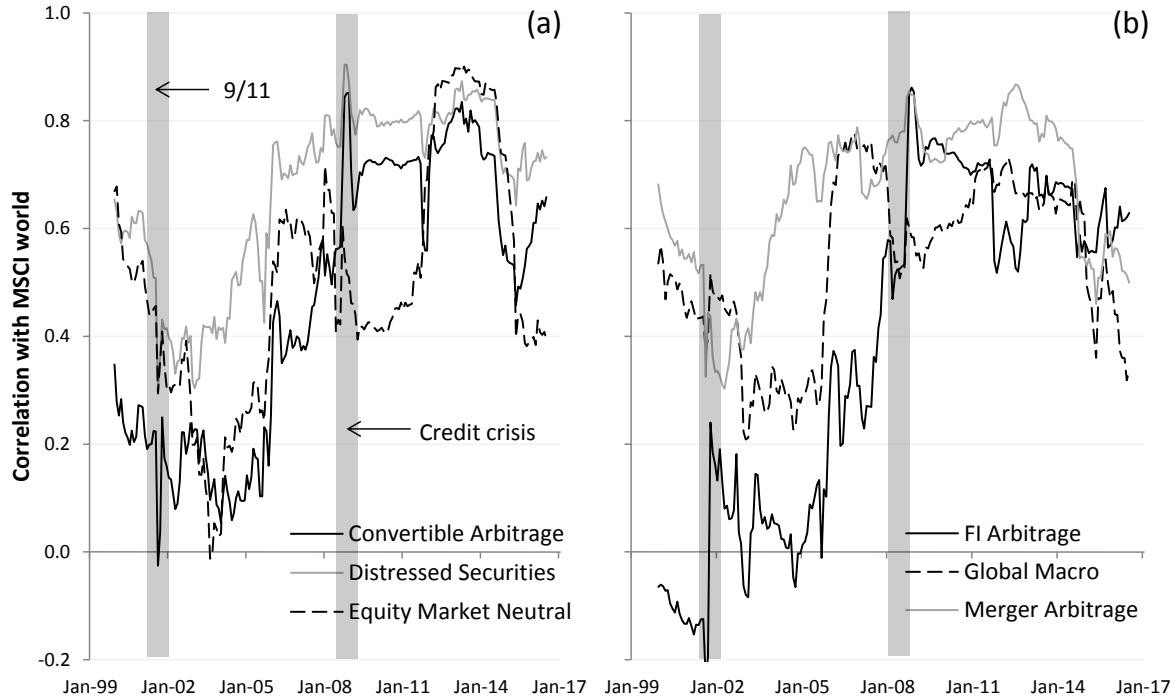


Figure 4.4: Correlation of (a) convertible arbitrage and distressed securities returns with MSCI world index returns and (b) fixed income arbitrage and global macro with MSCI world index returns.

4.4.2 Comparison of regression and Kalman α and β

The CAPM asserts that portfolio returns in excess of the risk-free rate are the combination of abnormal return generated by manager skill, α , and the interaction between excess market return and systemic risk factor, β . Figure 4.5 depicts the evolution of α for the Convertible Arbitrage Fund index from Jan 99 to Nov 16, as calculated from a 36-month linear unweighted regression: these results are compared with those obtained using the Kalman filter. Index excess returns are shown on the secondary axis in Figure 4.5. Similarities between the α values obtained from the two approaches are evident (e.g. between 2003 and 2005), but the regression α is slower to react to changes in the market compared with the Kalman approach. This is to be expected: regression weighs each component equally, while Kalman is far more adaptive, filtering out signal noise and capturing a *contemporaneous* α rather than an OLS α averaged over 36 months.

One example of this slow reaction time for regression α can be seen in Figure 4.5 over the period Jan 2009 to Jan 2011. Regression α in early 2006 reflects poor performance data measured over the crisis since it employs three years of equally-weighted monthly data. The index's market outperformance after the end of the crisis improves α more slowly over this

period than that observed for the Kalman α as the poor performance leaves the regression data sample a single month at a time. The Kalman approach is more responsive to improving (or deteriorating) performance, and at times the differences between the two techniques are more than double. These differences could prove substantial in the determination of fund manager compensation.

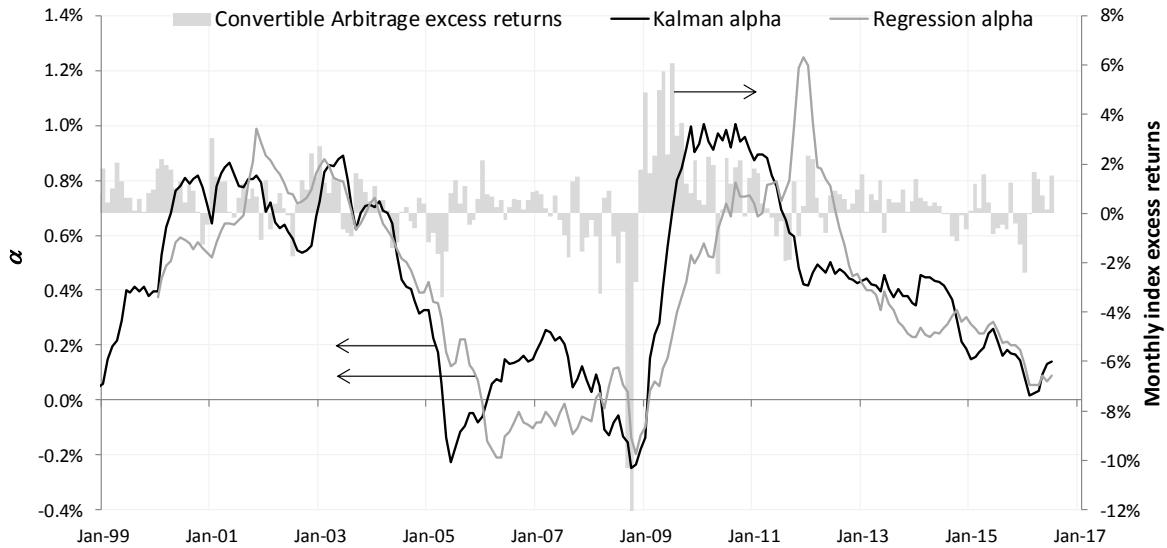


Figure 4.5: Comparison of Kalman filter and regression α estimates for the convertible arbitrage strategy with fund returns on the same timescale for comparison.

This characteristic is also observed for the β coefficient, as shown in Figure 4.6 for the fund of funds investment strategy. In this example, β s calculated for the fund of funds style using the Kalman filter lead the regression values, responding more quickly to market changes. A clear example is represented in the shaded period, when the Kalman β doubled (from 0.125 to 0.250) before the regression β s showed any signs of rallying. This slow responsiveness of regression β s will have proven prove detrimental to fund managers using CAPM to estimate this market metric.

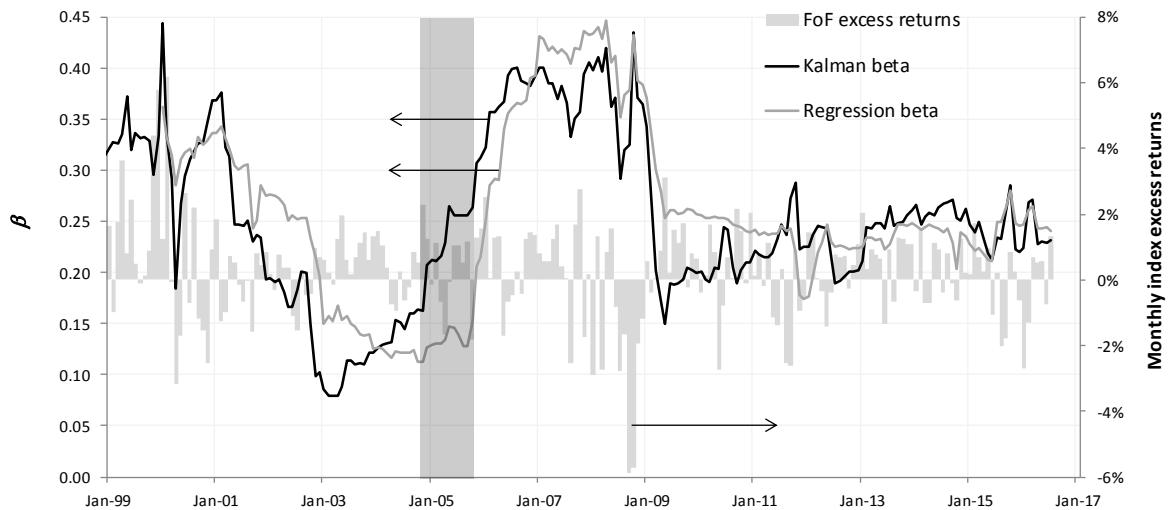
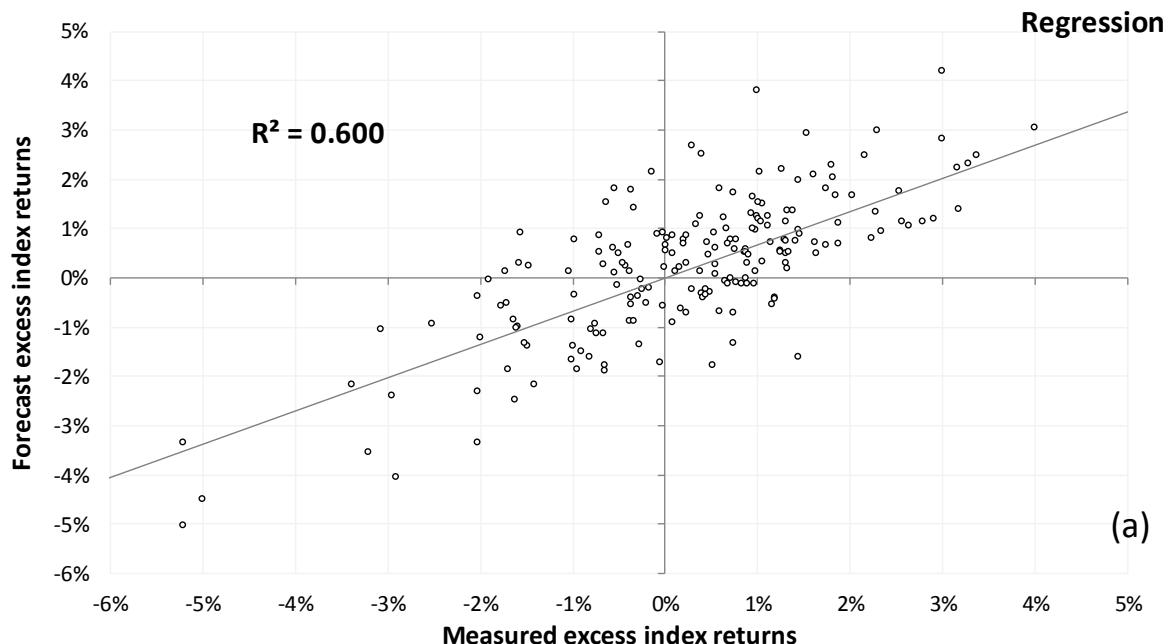


Figure 4.6: Comparison of Kalman filter and regression β estimates for the fund of funds investment style.

The superiority of the Kalman α and β measurements over the regression estimates of these parameters may be demonstrated by the forecast excess index values obtained using the three methods. Using (2.1), forecast excess index returns may be estimated using α s and β s obtained from (a) the Kalman approach, (b) the standard, equally-weighted regression methodology and (c) the time-weighted regression technique. The comparison of these estimates with the actual, measured values may then be evaluated: results using long-short index returns are given in Figure 4.7, showing that the Kalman technique produces superior forecast estimates than either regression approach. Higher R^2 values for the Kalman forecast fit were observed for all hedge fund strategies.



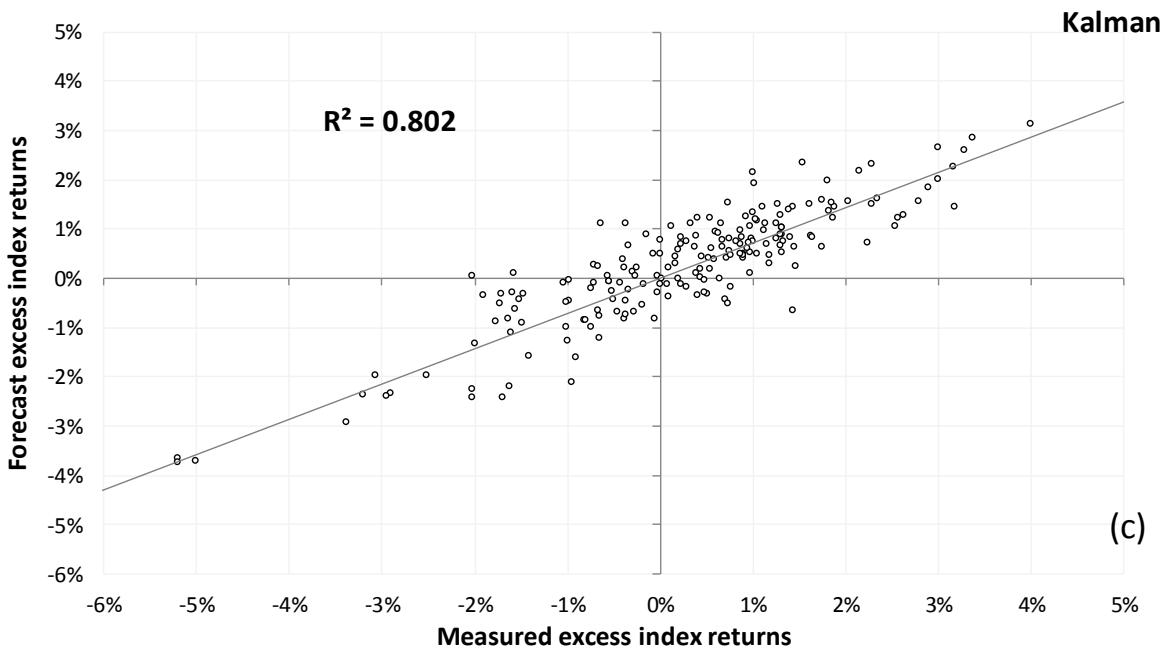
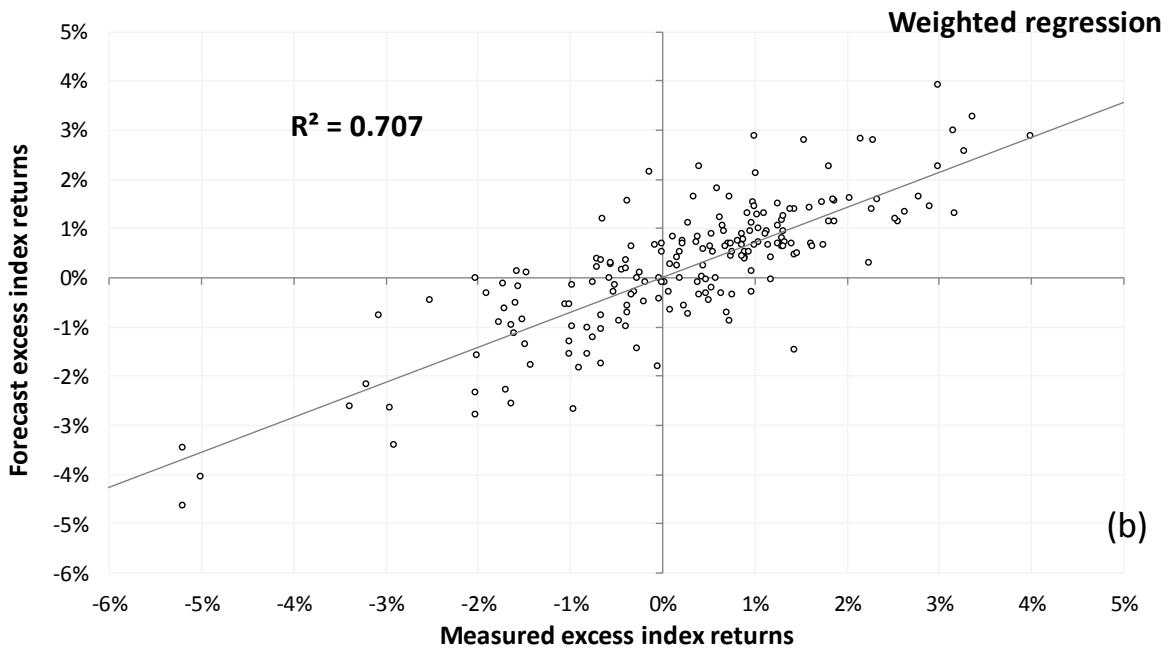


Figure 4.7: Comparison of excess equity long/short index return forecasts with realised returns using (a) standard linear regression (b) time-weighted linear regression and (c) the Kalman filter. Note the improvement of R^2 from (a) to (c).

4.4.3 Constituents of β

The constituents of β were defined in (2.2) as:

$$\beta = \frac{\sigma^P}{\sigma^M} \cdot \rho_{PM}$$

Two factors may be identified which influence the systemic risk coefficient: the ratio of index to market return *volatilities*, and the *correlation* between index and market returns.

Figure 4.8a shows the Kalman β for the equity market neutral index, 4.8b the individual volatility components and 4.8c the components of (2.2) and how these evolve over time.

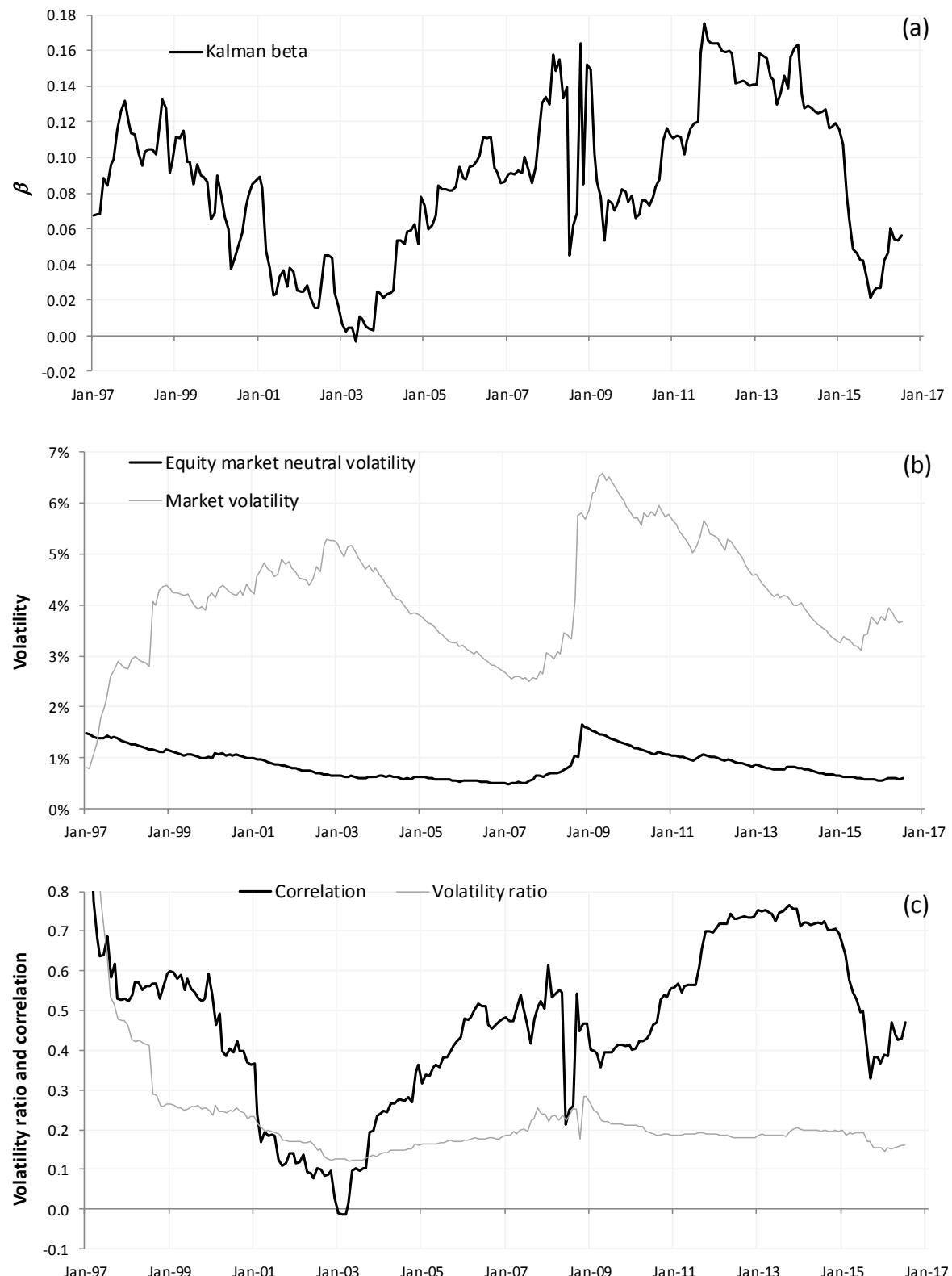


Figure 4.8: (a) Kalman β , (b) market and fund EWMA volatility and (c) volatility ratio and correlation between fund and market for the equity market neutral fund strategy.

The interaction between these variables demonstrates how the correlation component dominates the β determination. Most hedge fund index volatilities were *lower* than that observed for the market in absolute terms for the period under investigation: Figure 4.8b is a representative example. However, the volatilities (of index and market) generally move in unison, i.e. they are highly correlated, so the ratio of $(\frac{\sigma^P}{\sigma^M})$ does not deviate far from its mean (0.20 in this case – see Figure 4.8c). The only remaining component of β is the correlation between index and market returns. Figure 4.8c illustrates a close resemblance between the principal features of index β (Figure 4.8a) and the correlation between index and market returns (Figure 4.8c). Only when the correlation is flat, i.e. unchanging, can the *volatility ratio* influence the shape of the β profile.

4.5 Conclusions

Since the early 2000s, broad hedge fund performance has deteriorated with measured α s diminishing steadily since 2000 and correlations between fund returns and market remaining stubbornly high (as revealed by high β s). This combination has resulted in decreasing diversification, thereby violating the very principal underlying 'hedge' funds. Arbitrage opportunities that were previously exploited to great effect have shrunk and lacklustre performance compared with simple buy-and-hold 'strategies' have precipitated substantial asset outflows as well as the implementation of enhanced regulatory pressure and reduced management fees. These features of the hedge fund industry have been attributed to crowding of the available milieu by an ever-increasing number of market participants. This overcrowding has led to diminished arbitrage opportunities.

Because manager compensation for hedge funds is strongly linked to execution, accurate and timely measurement of fund performance has become critically important to hedge fund management, investors and regulators.

Traditional CAPM α s and β s are usually used to gauge fund performance, but the standard regression methodology obscures their measurement due to the rapidly-changing modern market. The Kalman filter provides an elegant solution. If properly calibrated, the filter tracks the underlying processes far more accurately and more timeously, as evidenced by the higher R^2 values for forecast excess fund (or index) returns versus measured excess re-

turns using a Kalman filter rather than weighted or unweighted regression methodologies. Because the Kalman filter reacts faster to market changes and filters out noise to leave behind the "pure" signal, its estimates of CAPM coefficients are demonstrably superior.

Further potential work could be to develop a Kalman filter-based model for asset prices that could be used in conjunction with a control theory oriented state space model for portfolio optimisation. CAPM with time-varying coefficients estimated using the Kalman filter could be connected to a formulation of a stochastic receding horizon control framework. Together, these applications could provide a methodology for estimating an asset pricing model and performing portfolio optimisation.

The next chapter focuses on the problem of isolating manager market timing skills and stock selection skills (Jain, *et al.*, 2011), using the Kalman filter. The filter is ideally suited to this type of research as it reacts quickly to changing market conditions and can thus identify and distinguish between these contributory factors and thereby ensure more efficient allocation of manager rewards and performance allocation.

Chapter 5

Hedge fund returns attribution using the Kalman filter

5.1. Introduction

Abnormal profit generation⁶ through active trading has long enjoyed a prominent research focus. Jensen (1968) explored the superior performance generation ability of mutual fund managers through systematic asset selection and investment timing. Jensen (1968) found that fund managers were unable (both on average and on an individual basis) to predict security prices well enough to outperform a simple buy-and-hold strategy. Subsequent research on the systematic ability of portfolio managers to generate abnormal profits has yielded mixed results at best, and generally negative (e.g. Hochberg & Mulhofer, 2011; Nasypbek & Rehman, 2011 and Tupitsyn & Lajbcygier, 2015). The evidence, then, has shown that stock markets are highly informationally efficient as originally suggested by Fama (1970) and more recently demonstrated by Shamshir & Mustafa (2014).

The institution and advance of hedge funds offers attractive investment possibilities because they engage in investment styles and opportunity sets which – because they are different from traditional asset class funds – give rise to different risk exposures (Fung & Hsieh, 1997 and Agarwal & Naik, 2000). Despite the recent surge in number and costs of compliance and regulatory requirements (Pasquali, 2015), hedge funds still enjoy the freedom to trade in multiple markets, employ substantial leverage and take both long and short market positions. Portfolio structuring exposes hedge funds to risk factors different from and beyond general market exposures, a feature which has given rise to the impression that hedge fund managers can provide consistent market outperformance or *αs* (Sun, Wang & Zheng, 2016). The evidence suggests otherwise: hedge fund managers, although able to expand investment opportunities beyond the scope of traditional investments, have not escaped the realities of investment risk-return parameters. Murguía & Umemoto (2004) showed that hedge funds provide unique investment opportunities and add value because of their ability to invest in different risk exposures, *not* because of the manager's ability to add value through stock selection or market timing.

⁶ The ability to predict future security prices accurately and thereby increase portfolio returns and minimise portfolio risk through efficient diversification.

Investment skill manifests itself in two important ways: security selection and market timing. Because these attributes are indicative of different skill-sets and compensated in different ways, their segregation is important (Cao, Goldie, Liang & Petrasek, 2016). The isolation of these contributions to returns is, however, fraught with difficulties. Most performance models – such as regression techniques – assume static sensitivity parameters which exclude market timing measures (Admati, Bhattacharya, Pfleiderer & Ross, 1986). *Dynamic* measures are required. Techniques such as the Kalman filter, which recursively isolate time-dependent parameters from noisy signals, have been shown to exhibit the desired characteristics required for the identification of investment skill (e.g. Jain, Yongvanich & Zhou, 2011 and van Vuuren & Yakumakis, 2015).

The principal purpose of this chapter is to address the problem of apportioning of market α into stock selection and market timing components. This has not been done before and the use of the use of a rolling-time Kalman filter to do this is also completely new. In addition, the new interpretation of the results (Section 5.5.3) is entirely novel: it involves a new interpretation of results and may have been overlooked by the original authors of the apportioning innovation (Jain, et al, 2011).

5.2. Forecasting and apportioning hedge fund returns

Forecasting accurate α s and β s is important for market participants in the evaluation of fund manager performance. Although early empirical tests found the CAPM to be robust and reliable (Black, Jensen & Scholes, 1973; Fama & Macbeth, 1973; He & Ng, 1994 and Pettengill, Sundaram & Mathur, 1995), later studies questioned the non-stationarity of β and the risk premium (Fama & French, 1992; Davis, 1994 and more recently Celik, 2013).

While OLS multivariate regression is suitable for funds characterised by slowly-varying exposures (such as passively-managed funds and low-risk unit trusts), the flexibility of the Kalman filter allows practitioners to capture dynamic fund exposure behaviour for funds which exhibit large and frequent exposure changes, such as some hedge fund strategies (Roll & Ross, 1994 and Faff, Hillier & Hillier, 2000).

Earlier work to determining dynamic hedge fund α s and β s (Yakumakis & van Vuuren, 2016) used return *indices* for various investment styles (sourced from Credit Suisse, 2015). These return indices effectively represent a weighted-average of all funds employing that particu-

lar style which could result in spurious, irrelevant results. In this research, the decision to use fund-by-fund performance was inspired using work conducted by Cuthbertson & Nitzsche (2013) who investigated German mutual equity fund performance using data spanning 20 years (monthly from 1990 – 2009). Although the research was directed at establishing false discovery rates, it employed model selection and performance measurement to do so. Using a Fama-French three factor (3F) model and ignoring market timing, Cuthbertson & Nitzsche (2013) found at most 0.5% of funds exhibited truly positive α -performance and about 27% exhibited truly negative- α performance. The inclusion of market timing variables led to a fivefold increase in the number of statistically significant positive α s and a 33% reduction in the number of statistically significant negative α s. The wide range of values exhibited by the distribution of fund performance α s implied that the distribution's extreme tails may contain funds with abnormally 'good' or 'bad' security selection. This is of considerable importance to investors who will endeavour to invest in funds in the right tail of the performance distribution and avoid funds whose returns reside in the extreme left tail. Investing in *average* fund performance is less important, or unimportant, emphasising the significance of fund-by-fund performance (rather than the weighted average of all funds).

Adapting an approach suggested by Jain, Yongvanich & Zhou (2011), the contribution of this work is to decompose individual hedge fund returns to identify whether fund managers generate statistically significant abnormal profits, to ascertain the degree of apportionment of these profits between skilful market timing and superior stock selection and to investigate whether the current approach for distinguishing between these components is valid.

5.3. Data

Two decades of monthly returns were used, on a rolling monthly basis. Relevant dynamic variables were monitored and assessed as they evolved through time. These were compared with values obtained from standard (and weighted) regression techniques. This is the first time the Kalman filter has been used to explore the time series of these dynamic variables to evaluate the relative magnitudes of fund return components.

The data employed were monthly returns of 100 globally-sourced hedge funds which employed various investment styles. These data were procured from the HFR database over 20 years, i.e. from January 1995 to January 2015 inclusive (HFR, 2015). Returns were grouped by geography (US, Europe and Asia): relevant risk free rates and associated market indices

(as prescribed by the fund strategy) were also collected from non-proprietary databases. Constituents of the data sample – distinguished by style – are shown in Figure 5.1 and a brief description of these styles is given in Table 5.1.

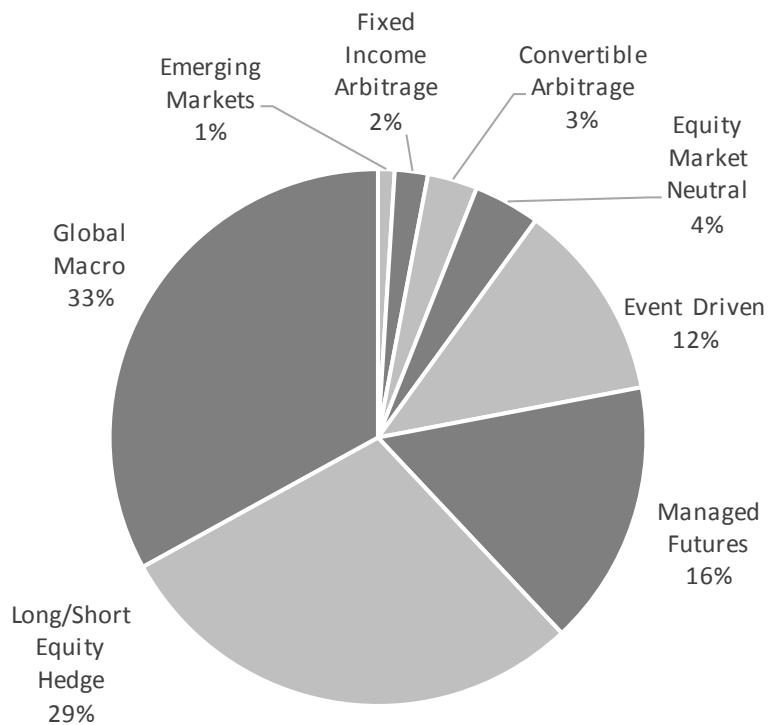


Figure 5.1: Hedge fund styles in data sample.

Table 5.1: Representative investment styles and description used in the study.

| Investment strategy | Strategy |
|---|--|
| Convertible Arbitrage | Exploit mispricing between convertible bonds and underlying securities |
| Commodity trading advisors /managed futures | Use precise trading rules to capture price movements, and focus on transitory patterns |
| Distressed Securities | Focuses on arbitrage opportunities of securities in companies that are in or near bankruptcy |
| Emerging Markets | Tactical trading strategies that profit by forecasting the overall direction of the market or market components. Unique risks accompany emerging market investments: e.g. political hazards and inadequate legal systems |
| Equity Market Neutral | Uses the combination of buys and short-sales (sometimes augmented by options and futures positions) to offset correlations between portfolio returns and market returns |

| | |
|------------------------|---|
| Event Driven | Seek profits from arbitrage from short-duration events such as corporate restructuring, stock buybacks, bond upgrades, earnings surprises and spin-offs |
| Fixed Income Arbitrage | Exploit mispricing among fixed income securities |
| Global Macro | Bets are based on major macroeconomic forecasts such as changes in interest rates, currency movements and stock market performance |
| Long/Short Equity | Strategy is the same as equity market neutral but without explicit promises of market neutrality – this increases investment flexibility to choose net-long or net-short ($\beta > 0$ or $\beta < 0$) market exposure |
| Merger Arbitrage | Because there is a likelihood of a merger not completing, the target company's share price carries a "bid premium" (a discount to the proposed takeover price) until the merger does complete. |
| Relative Value | Takes advantage of perceived mispricing among related financial assets, such as company debt and equity securities |
| Short Selling | Concentrates on the short side by sacrificing market-neutrality. Tends to be most effective when markets are declining |
| Funds of Funds | Managed portfolio of other hedge funds |

5.4. Strategy analysis

5.4.1. Distinction between 'best' and 'worst' funds by performance

The performance of different hedge fund strategies was examined for which there were enough funds using the relevant strategy to make statistical inferences. Figure 5.2 shows key strategies ranked by overall cumulative performance over the observation period (1 the best, 100 the worst).

The results demonstrate that the performance of global macro and event driven strategies was markedly superior⁷ to long/short equity hedge and managed futures strategies. Managed futures strategies use leverage and time series momentum strategies (Hurst, Ooi & Pedersen, 2013) and switch in and out of positions quickly. This investment style's returns performed the worst over the period. Global macro funds, which take advantage of global trends where and when they arise, exhibit more sedate investment styles, and do not switch rapidly between positions, preferring rather to take longer term views of developing global macro events.

⁷ These differences were all tested for statistical significance: all were found to be significant at the 90%, 95% and 99% levels of significance.

In Figure 5.2 (a box-and-whiskers plot) the central horizontal line represents the mean rank while the cross indicates the median rank. The distance between the central line and the horizontal edges of the boxes represent the 2nd and 3rd quartiles respectively and the "whiskers" represent the first and fourth quartile extremes.

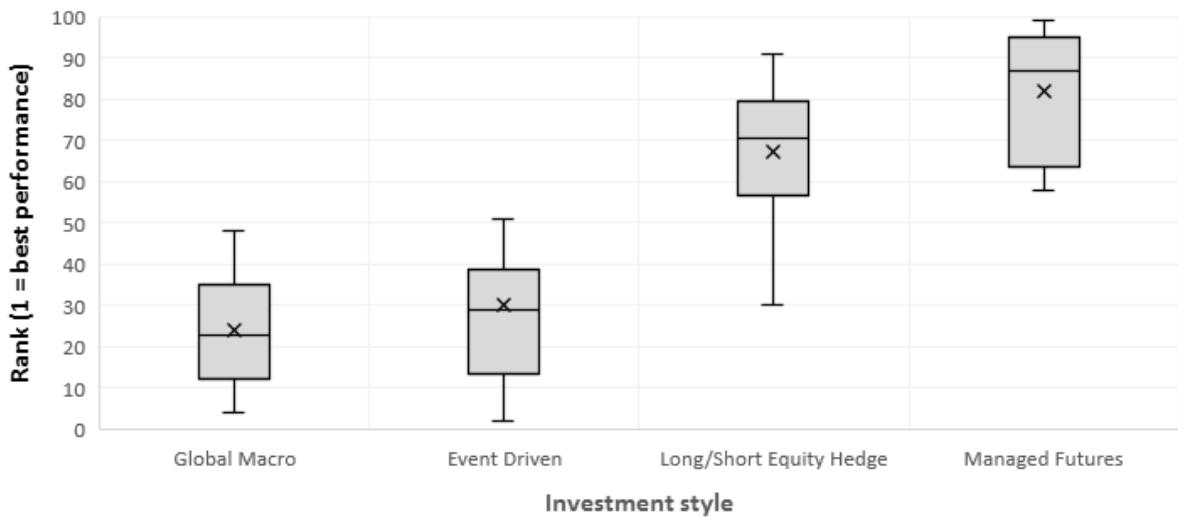


Figure 5.2: Investment style performance summary.

The average inception date for funds using the four styles shown in Figure 5.2 is shown in Figure 5.3. Managed futures are the oldest funds and global macro the youngest. Although the average difference between these is only roughly five years, this difference could also (partially) explain the superior returns generated from the latter. Global macro funds are younger, so it is possible that funds adopting this style have not yet completely saturated the market and squeezed out arbitrage opportunities from competitors. The reverse could be true for managed futures: being 'older', whatever arbitrage opportunities that once existed could have now been exhausted.

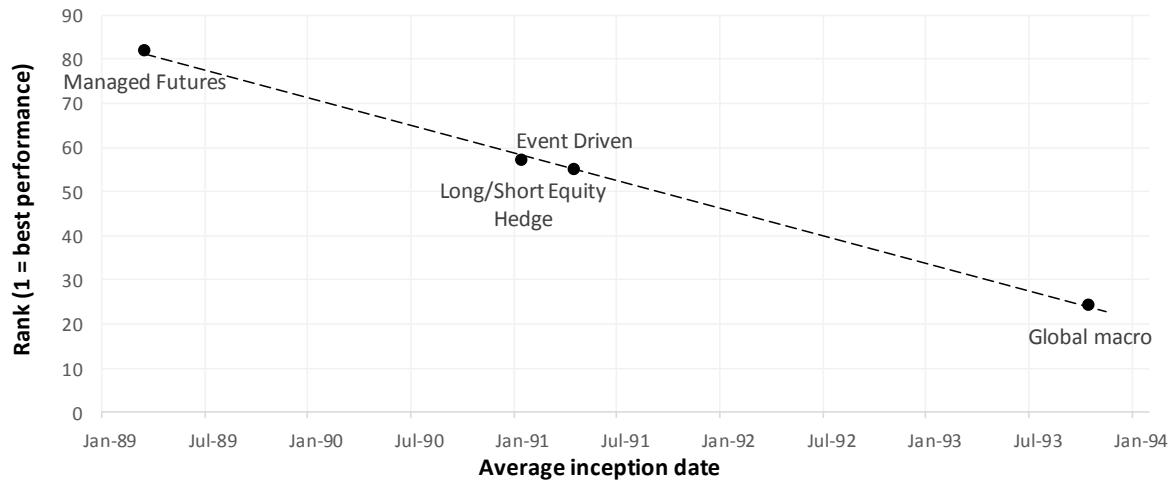


Figure 5.3: Average inception date for the four investment styles indicated in Figure 5.2.

5.4.2. Fund return analysis

Fund returns from various strategies were analysed and the top and bottom five (ranked by cumulative performance over the 20-year time span) were selected to demonstrate the extremes of the hedge fund performance spectrum. The average values of the first four moments extracted from these funds are summarised in Table 5.2

Table 5.2: Average annualised (between Jan 1995 – Jan 2015) summary statistics of top/bottom five hedge funds ranked by cumulative return.

| RETURN | | VOLATILITY | | SKEWNESS | | KURTOSIS | |
|--------|----------|------------|----------|----------|----------|----------|----------|
| Top 5 | Bottom 5 | Top 5 | Bottom 5 | Top 5 | Bottom 5 | Top 5 | Bottom 5 |
| 25.6% | 3.1% | 2.7% | 12.4% | 18.3% | -26.7% | 19.3% | -7.7% |
| 24.4% | 2.9% | 5.3% | 19.1% | 14.8% | -12.4% | 14.4% | -15.4% |
| 23.8% | 5.0% | 3.6% | 21.8% | 4.3% | -13.9% | 16.3% | -13.6% |
| 24.0% | -2.3% | 5.7% | 16.7% | 12.8% | -17.3% | 9.9% | -0.6% |
| 22.3% | -7.4% | 5.4% | 20.1% | 7.4% | -19.5% | 3.5% | -11.6% |

Source: Authors' calculations.

A summary of the annualised descriptive statistics for the top five and bottom five funds is provided in Figures 5.4a — d as well as the Morgan Stanley Capital International (MSCI) world equity index for comparison.

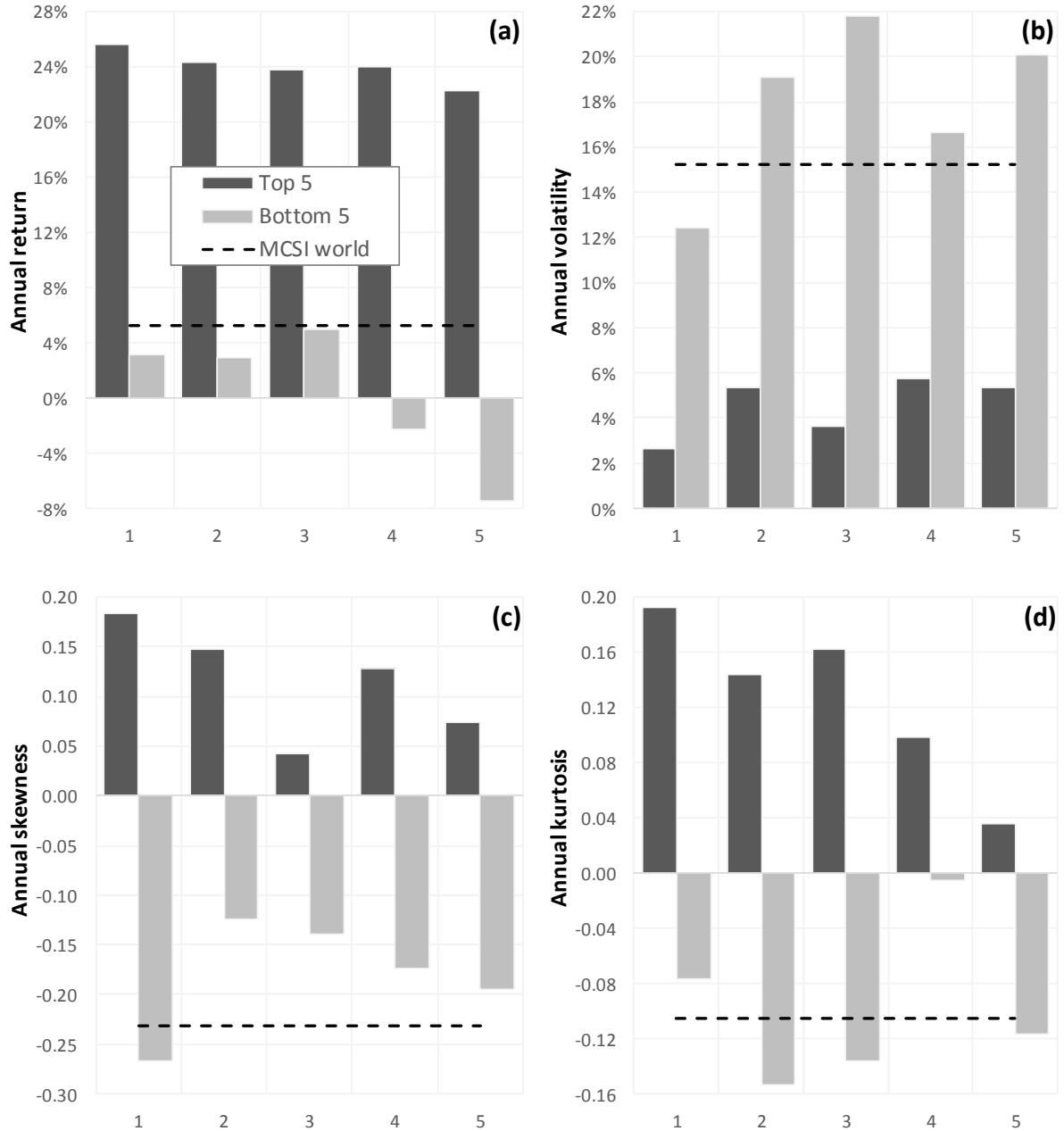


Figure 5.4: Average annualised top five and bottom five hedge fund return statistics (ranked by cumulative return from Jan 95 to Dec 14) by (a) returns, (b) volatility, (c) skew and (d) kurtosis. The MSCI world index equity return statistics are shown as dotted lines for comparison.

Annualised returns are calculated using $(1 + \bar{r}_m)^{12} - 1$, where \bar{r}_m is the average monthly return over the full period, annual volatility is $\sigma_m \cdot \sqrt{12}$, where σ_m is the monthly standard deviation of all monthly returns, annual skew is $s_m / \sqrt{12}$, where s_m is the skew of all monthly returns, and annual excess kurtosis is $(k_m + 3 \cdot 11)/12 - 3$, where k_m is the kurtosis of all monthly returns for the relevant index. These are standard scaling techniques for the first four moments of return distributions (Meucci, 2010).

The top five hedge funds annual returns are considerably higher than the bottom fives', by almost an order of magnitude. There is selection bias in the sample,⁸ but since both the top five and the bottom five hedge funds survived the full period, this demonstrates the extreme variation in performance tolerated by the investors.

Annual volatilities are also considerably different for funds at the extremes. The top five funds exhibit annual volatility lower than that of the bottom five by – again – roughly an order of magnitude. This translates into severely reduced return performance for the bottom five funds.

The top five funds all exhibit positive annual skewness, while the bottom five's skewness is always negative. This explains the low annual returns, and it is interesting to note that the MSCI world index – over the period of investigation – also showed high, negative skewness.

Annual kurtosis for the top five funds is also positive, while for the bottom five, the kurtosis is always negative (as it is for the MSCI world index). Lee, Phoon & Wong (2006) found that hedge fund return data are generally negatively-skewed and leptokurtic; the global macro fund style appears to oppose this trend. Global macro returns are positively skewed – leading to high annual average returns, but tend to cluster close to the mean (leptokurtic) while having some large outliers on the positive side of the return distribution. Managed futures funds on the other hand, have wide dispersion of returns – thus platykurtic – and negatively skewed leading to significant underperformance in comparison with global macro funds.

Figure 5.5a shows the average rolling cumulative return for the top five hedge funds (ranked by annual return, left axis) showing the substantial difference between cumulative performance between best and worst performing funds. MSCI equity world index returns are included for comparison on the same timescale. Average monthly rolling volatilities were calculated using an exponentially weighted moving average (EWMA) approach with $\lambda = 0.950$.⁹ Figure 5.5b shows the monthly volatility profile of both indices over the same period with the MSCI equity world index values included for comparison.

⁸ These hedge funds have, by definition, survived the full 20-year timespan: they were actively selected from a much larger sample of hedge fund returns on the basis that they had been in existence since December 1993 and remained active at the end of 2015.

⁹ The weighting constant which determines the weighting allocation: the further λ is from 1, the heavier the weighting for more recent data and the lighter the weighting for data which occurred at a time longer ago in the past (Hendricks, 1996). The λ used in this analysis was calibrated using representative global market data as at December 2015.

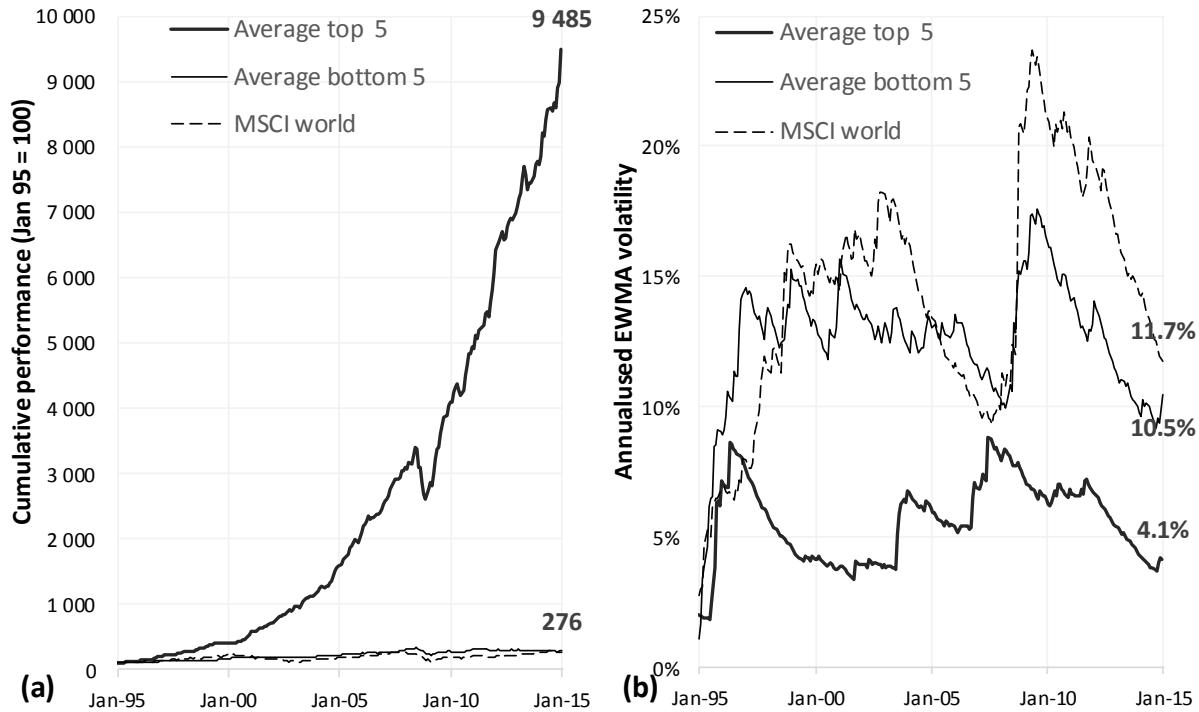


Figure 5.5: Average top five and bottom five hedge funds ranked by cumulative return (from Jan 95 to Dec 14) for (a) cumulative performance and (b) annualised EWMA volatility. MSCI world index equity returns and volatilities are shown as dotted lines for comparison.

5.2.1 Return apportioning methodology

As discussed in Chapter 2, Jain, *et al* (2011) quantified hedge fund returns' market timing component using a simple, but analytically tractable, approach. Because market exposure increased (on average) in bullish markets and decreased in bear markets, positive returns are generated through market timing. A comparison of returns from *average market exposures* to the returns from *time-varying market exposures* could therefore help identify the market timing value-added component.

To recap, Jain, *et al* (2011) expanded (2.1) to accomplish this:

$$\begin{aligned}
 (r^P - r^f) &= \alpha + \beta(r^M - r^f) + \varepsilon \\
 &= \alpha + (\beta + \bar{\beta} - \bar{\beta})(r^M - r^f) + \varepsilon \\
 &= \alpha + \bar{\beta}(r^M - r^f) + (\beta - \bar{\beta})(r^M - r^f) + \varepsilon
 \end{aligned} \tag{5.1}$$

where the terms have their usual meanings and $\bar{\beta}$ is defined as the manager's average β exposure. The term $(\beta - \bar{\beta})$ is the manager's instantaneous active β exposure based on tim-

ing decisions (i.e. the difference between an 'instantaneous' β and a 'long run', or average β defined here as $\bar{\beta}$). Using (5.1), a manager's return at time t comprises four components:

1. an expected α based on security selection decisions: α
2. a return based on the manager's average exposure to the market: $\bar{\beta}(r^M - r^f)$
3. a return based on the manager's market timing decisions: $(\beta - \bar{\beta})(r^M - r^f)$
4. a residual component with zero mean, corresponding to the risk generated from security selection decisions: ε .

For the Kalman filter setup, the observation equation is the CAPM model, i.e. (5.1) while the form of transition equation depends on the form of stochastic process that the time-varying α s and β s are assumed to follow. The transition equation may use an autoregressive, mean-reverting (AR(1)) model, a random walk or any other process that could describe the progression (see, for example, Racicot & Théoret (2013) who investigate the possibility that these variables behave procyclically). The random walk model was used as this provides the most robust characterisation of time-varying β s (Faff, *et al*, 2000 and see Ang, Gorovyyz, and van Inwegenx, 2010).

The assumption that α and β both evolve via a random walk means that current market exposure is a normally-distributed random variable with mean the exposure of the previous period, i.e. at $t - 1$. Uncorrelated system noises are also normally distributed. The state variables $x_t \in \Re^2$ are the time-varying coefficients:

$$x_t = \begin{bmatrix} \alpha_t \\ \beta_t \end{bmatrix}$$

at each time t . The state equation is:

$$\begin{bmatrix} \alpha_{t+1} \\ \beta_{t+1} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \alpha_t \\ \beta_t \end{bmatrix} + \begin{bmatrix} \gamma \\ \delta \end{bmatrix} \quad (5.2)$$

where

$$\begin{bmatrix} \gamma \\ \delta \end{bmatrix} \sim N \left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_\gamma^2 & 0 \\ 0 & \sigma_\delta^2 \end{bmatrix} \right)$$

and the measurement equation is:

$$r_t^P - r_t^f = [1 \quad r_t^M - r_t^f] \begin{bmatrix} \alpha_t \\ \beta_t \end{bmatrix} + \varepsilon_t \quad (5.3)$$

The data used for this research were *hedge fund* returns, rather than hedge fund index returns. Components and associated return contributions of (5.1) are given in Table 5.3.

Table 5.3: Component description, contribution source and key metric origin.

| Component | Contribution to overall returns from: | Key metric |
|--|--|--|
| α_t | Security selection | Time-varying α from Kalman Filter |
| $\bar{\beta}(r_t^M - r_t^f)$ | Average market exposure | $\bar{\beta}$ from standard multiple regression using 36 months of return data |
| $(\beta_t - \bar{\beta})(r_t^M - r_t^f)$ | Market timing decisions | Time-varying β from Kalman filter, $\bar{\beta}$ from standard multiple regression |
| ε_t | Risk generated from security selection | Calibrated using log likelihood methods and generated using the Kalman filter |

The procedure, then, is:

- assemble monthly, hedge fund returns, risk free rates and associated market returns for the period under examination,
- calibrate the Kalman model to estimate ε_t ,
- calculate hedge fund and market excess returns using relevant risk free rates,
- run the excess returns through the Kalman filer to determine (time-varying) α s and β s, i.e. α_t and β_t and
- estimate rolling 36-month (weighted and unweighted) OLS regression β s, i.e. $\bar{\beta}$.

This procedure generates all the necessary components of (5.1), so the portfolio return contribution due to security selection (α_t) and timing decisions $(\beta_t - \bar{\beta})(r_t^M - r_t^f)$ may be determined (Table 5.3).

5.5. Results and discussion

5.5.1 Comparison of Kalman and regression α s and β s

Figure 5.6 shows the CAPM α s using unweighted, rolling 36-month linear regression and the α s from the Kalman filter over the same timespan (also the CAPM β s and Kalman time-varying β s using the same techniques). Although the two techniques show broadly similar profiles, the Kalman estimates are dynamic and forecast excess portfolio returns more accurately than regression techniques. This has been demonstrated in prior research (see, e.g.

van Vuuren & Yacumakis, 2015 and see Figure 4.7). Values obtained from the Kalman filter are less volatile, and produce more accurate forecasts of expected portfolio returns, than those obtained from the unweighted regression.

During the non-volatile market period from Jan 03 to Jan 08, the α values obtained from the two techniques are broadly similar, but at other times, such as between Jan 00 and Jan 02 and since Jan 10, the values have been considerably different – often different signs. These differences can have significant impact on manager performance.

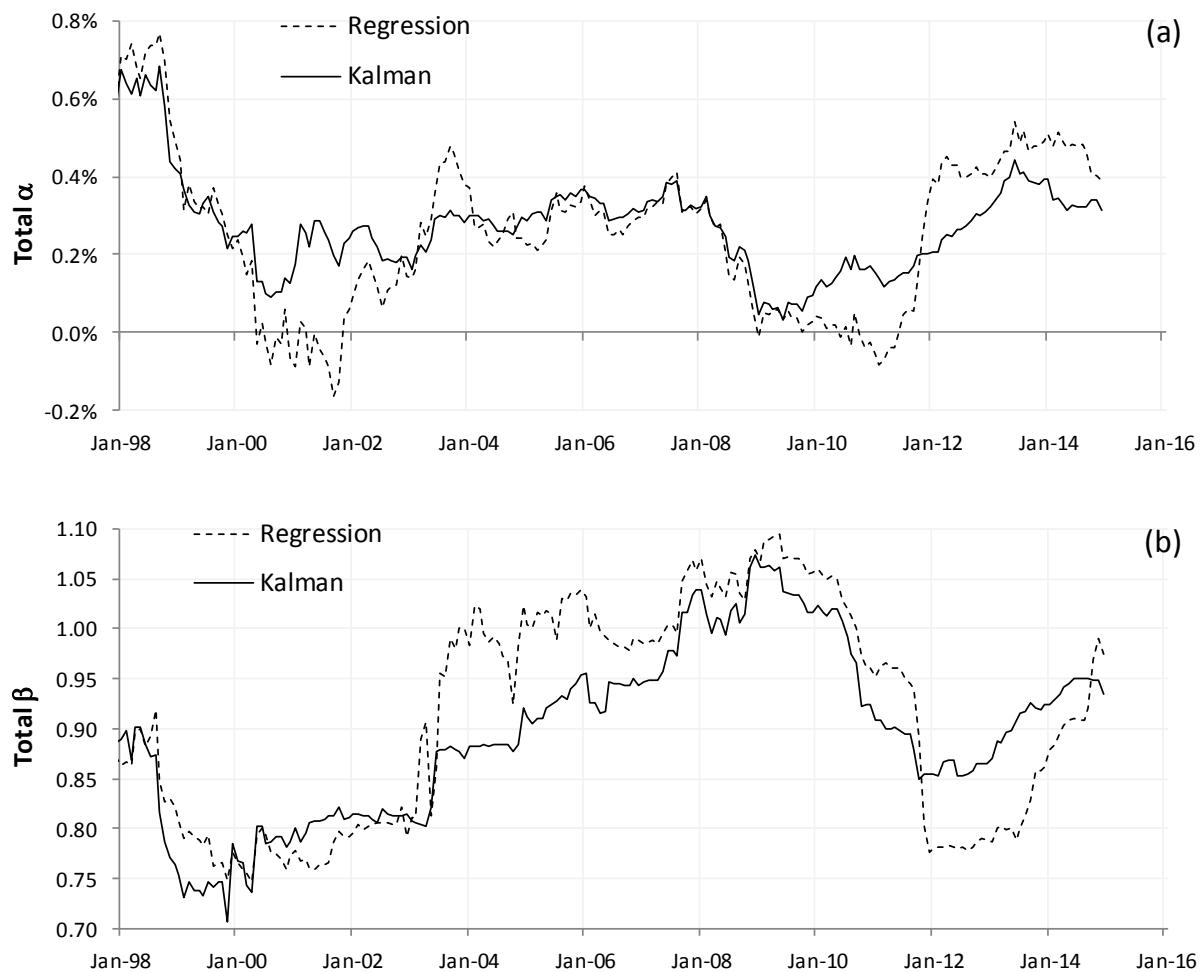


Figure 5.6: (a) Comparison of CAPM α s and (b) CAPM β s using a rolling, unweighted 36-month linear regression and the Kalman filter for a global macro hedge fund.

5.5.2 Apportioning of market timing and stock selection α s

Characteristic α s for market timing and stock selection (from 5.1), averaged over the best five and worst five funds (Table 5.2) are presented in Figures 5.7 and 5.8 below.

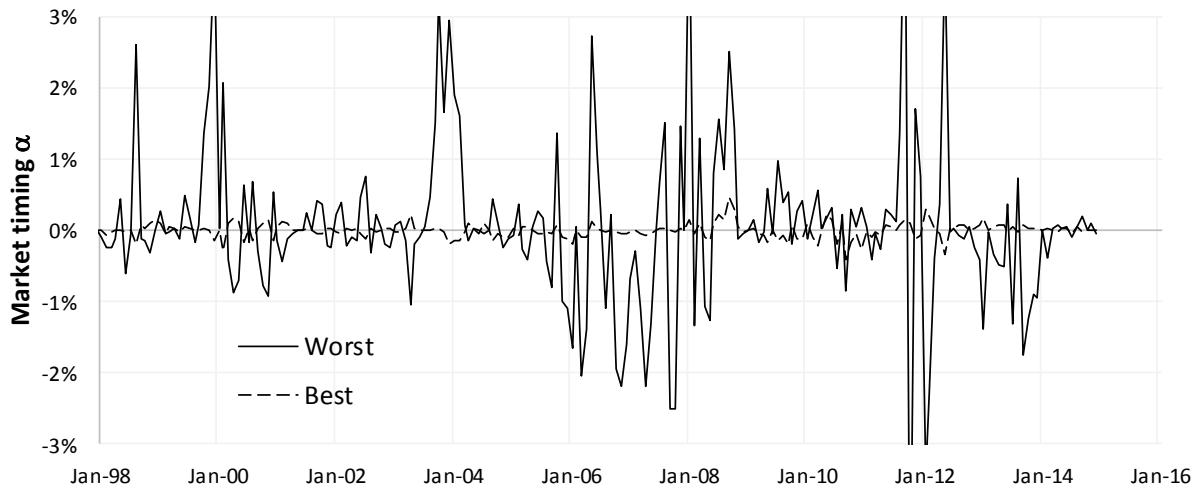


Figure 5.7: Characteristic market timing α s for best and worst funds.¹⁰

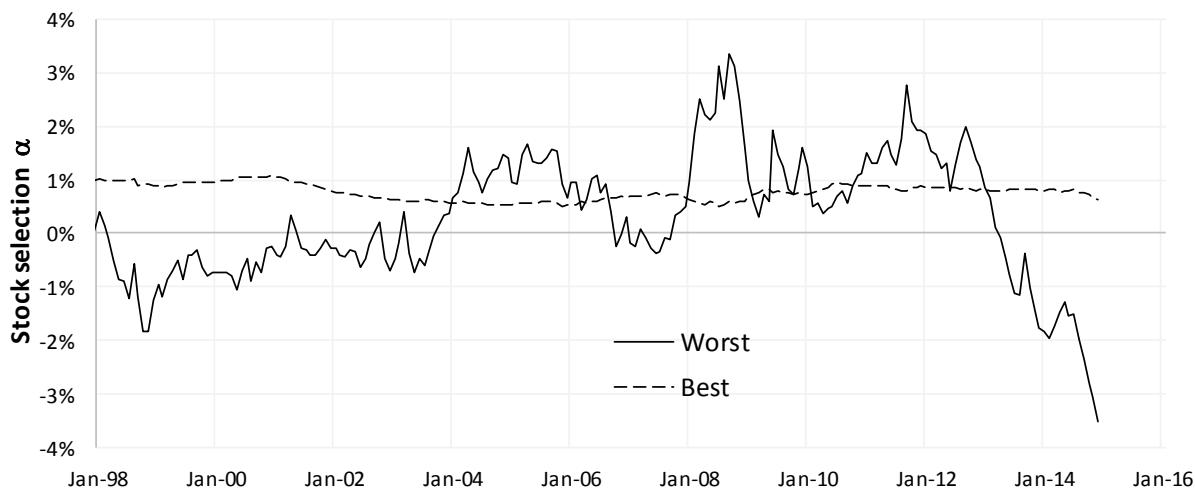


Figure 5.8: Characteristic stock selection α s for best and worst funds, shown on the same vertical scale as Figure 5.7.

Time-dependent α s from market timing are consistently low (approximately 0%) in the best funds, with very low variability (monthly $\sigma \sim 0.15\%$), while in the worst performing funds, market timing α s are highly variable (monthly $\sigma \sim 1.50\%$). Stock selection α s for the best performing funds are small, but consistently positive, whilst for the worst funds, these also exhibit high variability, with a mean of $\sim 0\%$.

Managers responsible for the best performing hedge funds, then, do not attempt to time the market, but instead focus on superior stock selection to generate abnormal profits. Managers from the worst funds alter investment choices (both from a timing and an asset

¹⁰ The data in Figures 5.6 through 5.9 are plotted from Jan-98 (rather than Jan-95) as the Kalman filter results require about 36 months to sufficiently reduce the variance between forecasted and measured values.

selection point of view) dramatically and frequently with the result that profits one month are negated the next, as shown in Figure 5.9.

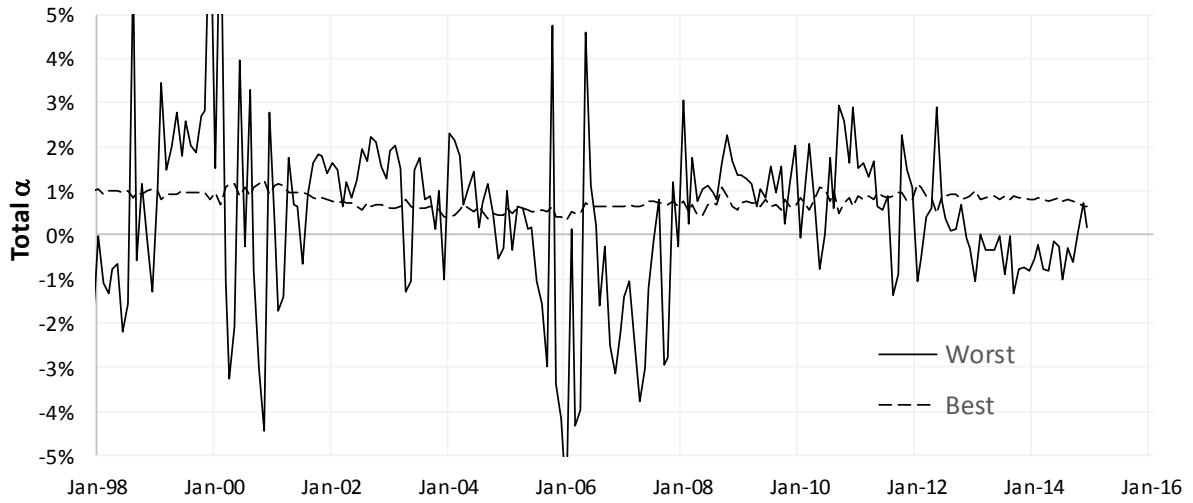


Figure 5.9: Characteristic total α s for best and worst funds.

The t -statistic of a fund, t_α , is a relevant measure. Defined as

$$t_\alpha = \frac{\bar{\alpha}}{\sigma_\alpha}$$

where $\bar{\alpha}$ is the average α measured over a suitably long period (usually > 36 months) and σ_α is the volatility of α measured over the same period. This statistic provides important information because the magnitude of t_α measures the significance of the manager's α . A value $> +1.65$ suggests that the manager could generate persistent $\alpha > 0$ over time at the 5% significance level, whereas a value below this threshold suggests that the manager's actual α may be ≤ 0 . At the 1% significance level, $t_\alpha > +2.33$. Figure 5.10 shows the results for a rolling t_α at both significance levels for the extremes of fund styles (best and worst). Neither strategy generates $\alpha > 0$ at either significance level at *any* time during the observation period (Pedersen, 2013), so both approaches generate $\alpha \approx 0$.

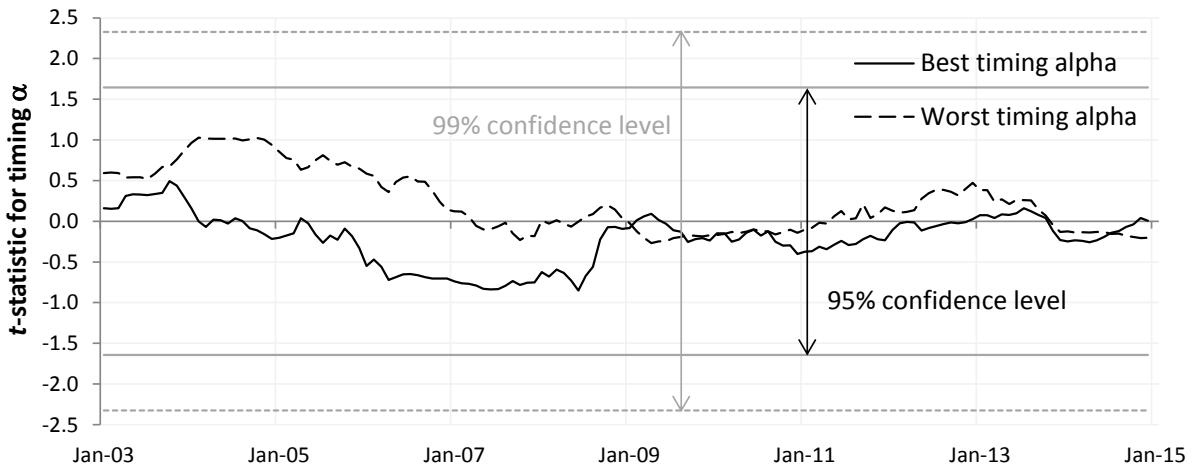


Figure 5.10: t_α for best and worst funds. Neither fund style generates $\alpha > 0$ at the 5% or 1% significance level.

Although the best performing funds' returns are low, they are consistently positive: this combined with these funds' $\beta \approx 1$ (Figure 5.11) means they take advantage of dependable market outperformance while reliably tracking it. This gives these funds the advantage over the worst performing funds which lack strategic focus and alter investment plans as often as they change stock selection whilst maintaining a high gearing to the market. This combination increases portfolio volatility and neutralises profits.

A summary of the results is provided in Table 5.4.

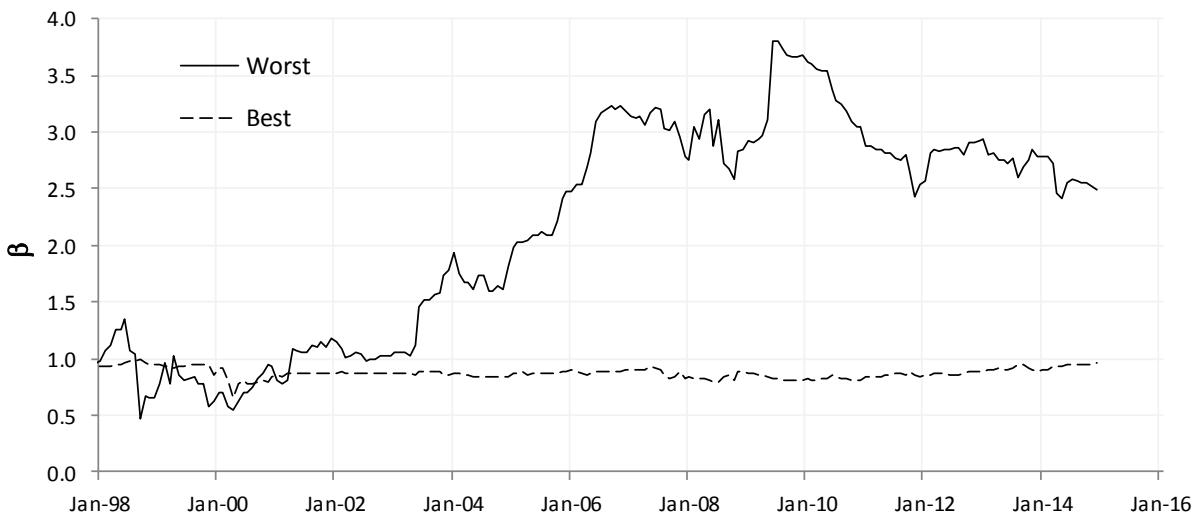


Figure 5.11: Characteristic β s for best and worst funds. Managers of the best hedge funds maintain a consistent $\beta \approx 1$ while those from the worst aim for high leverage, i.e. $\beta \gg 1$.

Table 5.4: Summary of empirical observations for best performing and worst performing hedge funds. Note from Figure 5.10 that the best funds comprise predominantly global macro style, while the worst are mainly made up of managed futures.

| Components | Best funds (mainly global macro) | Worst funds (mainly managed futures) |
|--|--|---|
| Market timing α | Low variability ($\sigma \sim 0.15\%$) Average $\sim 0\%$ | High variability ($\sigma \sim 1.50\%$) Average $\sim 0\%$ |
| Stock selection α $(\beta - \bar{\beta})(r^M - r^f)$ | Low variability ($\sigma \sim 0.15\%$) Average $> 0.0\%$ | High variability ($\sigma \sim 1.00\%$) Average $\sim 0.0\%$ |
| Total α | $> 0\%$ Low variability ($\sigma \sim 0.10\%$) | $\sim 0\%$ High variability ($\sigma \sim 2.50\%$) |
| β | ~ 1.00 stable, constant Low variability ($\sigma \sim 0.10\%$) | $\gg 1$ – high (~ 3) during 08 crisis High variability $\sigma \sim 2.00\%$ |

A comparison between the regression plots between global macro and managed future excess returns and the excess market returns is shown in Figure 5.12. There is almost no relationship ($R^2 = 0.02$) between the two regressor variables for the managed future hedge fund style, while for the global macros style, there a strong relationship holds ($R^2 = 0.58$).

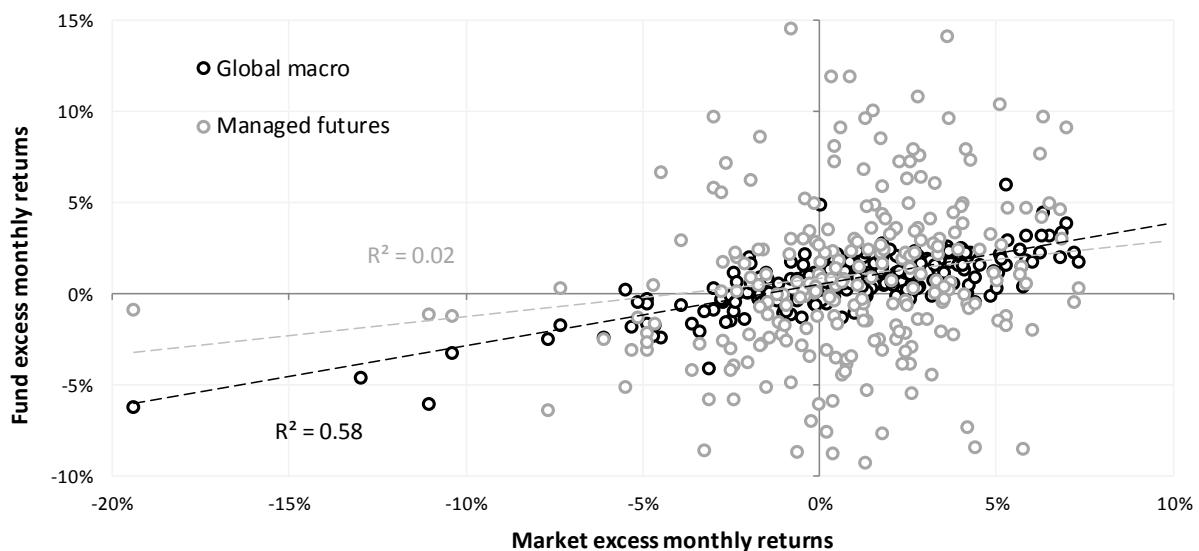


Figure 5.12: Regression plot of global macro and managed futures funds.

The EWMA volatility of both styles are shown in Figure 5.13. Managed futures style returns are considerably more volatile – by a factor of between 2 and 3 – than the global macro style returns. The correlation between these returns was low (~ 0) in the early 2000s, then became negative briefly in 2003 before increasing considerably until the onset of the credit crisis in 2008. Since then, the correlation has remained at ~ 0.5 .

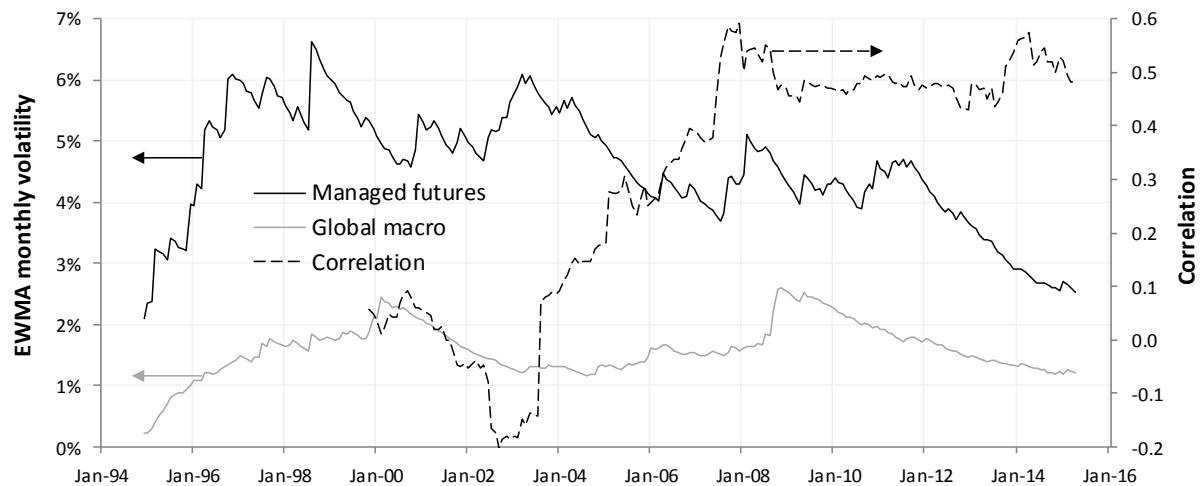


Figure 5.13: Global macro and managed futures monthly return EWMA volatilities and correlation.

Global macro and managed futures strategies are both long-term trend followers and both have a common source of returns: major trends generated by long term secular shifts in capital flows. There are many features, however, that differentiate between the two investment approaches: these are summarised in Table 5.5 below.

Table 5.5: Comparison of global macro and managed futures investment style approaches.

| Characteristic | Global macro (best) | Managed futures (worst) |
|-----------------------|--|---|
| Investment style | Relative value traders and value investors Discretionary, fundamentals-driven Ability to step in and out of markets as fundamentals change | Momentum traders Systematic Narrower market breadth, less diversification amongst assets |
| Style themes | Trade within a relatively small series of themes (Warsager, Duncan & Wilkens, 2004) | Broadly diversified across a wide range of markets (Hurst, Ooi & Pederson, 2013) |
| Investment components | Use various types of OTC derivatives Have more flexibility in terms of the instruments they employ | Use futures markets and interbank FX markets which are generally more volatile than those invested in by global macro |
| Vulnerabilities | Sometimes miss some or all the final stage of a trend | Vulnerable to givebacks during market reversals |
| Traded volatility | As discretionary traders, may be short volatility in the form of relative value positions or flat (unlike managed futures) if they have an- | Can be used offensively (by high return seekers) or defensively (to obtain a long option-like risk profile, e.g. long |

| | | |
|---|--|---|
| | ticipated market turmoil, thus avoiding exposure in potentially dangerous environments | volatility, but volatility derived from trends rather than external market shocks) |
| Historical return volatility | Lower (Figure 5.11) | Higher (Figure 5.11) Futures contracts are by nature more volatile because of leverage |
| Regression plots | Figure 5.10 Less dispersed with lower R^2 | Figure 5.10 More dispersed with a higher R^2 |
| Regression variables (α & β) | More stable Current β is an <i>average</i> value – determined using several years of monthly excess returns | More variable Current β is a contemporaneous value |
| Kalman variables (α & β) | Stable Leads to lower spread differences | Highly reactive to more volatile portfolio returns Far more reactive than regression variables, leads to a high spread difference |
| Jain, <i>et al</i> (2011): α due to market timing using $(\beta - \bar{\beta})(r^M - r^f)$ | Small Indicates minimal to no market timing attempts Suggests <i>little to no performance contribution</i> | Highly variable Large absolute values, but small negative average over time Indicates that attempts at market timing <i>do not contribute positively</i> to performance |
| Jain, <i>et al</i> (2011): Total α | Entirely derived from stock selection ($\mu_\alpha \approx 0$) | α from market timing ($\mu_\alpha \approx 0$ but σ_α high) detracts from total α |
| Measurement error (spread difference between Kalman derived values and regression variables) | Small | Large |

5.5.3 Alternate interpretation of market timing α

An alternative interpretation is posited with respect to the interpretation of the market timing α term, proposed by Jain, *et al*, (2011). It is contested that the 'timing α ' $(\beta - \bar{\beta})(r^M - r^f)$ is due to measurement uncertainty or error which arises from differences between β

obtained from the Kalman filter and $\bar{\beta}$ obtained from standard linear regression rather than the positive identification of timing α .

As demonstrated in Chapter 4 (see Figure 4.7), the instantaneous CAPM measurement variables (α and β) are more accurately measured using the Kalman Filter than either unweighted or weighted regression techniques. The product of the difference between instantaneous β , as measured by the Kalman filter, and the long run market exposure $\bar{\beta}$ as measured using regression and $(r^M - r^f)$, should be interpreted as measurement error dependent on the underlying volatilities of the constituent portfolio assets. The higher and more volatile the average absolute spread between these values $(\beta - \bar{\beta})(r^M - r^f)$, the worse (i.e. lower) the fund performance. Jain, *et al.* (2011) argued that this difference provided an estimate of market timing α and concluded that funds with low and stable “market timing α ” outperformed those which displayed large and volatile market timing α . These results should instead be interpreted as accepting that more volatile portfolios, where there is a more pronounced difference between instantaneous β and long run $\bar{\beta}$ resulting from differences in calculation methodology, have historically underperformed portfolios investing in less volatile assets.

The regression scatter plot (Figure 5.11) for a *more* volatile fund (managed futures) is more dispersed, so the regression slope will respond more slowly to incoming data points (for a rolling calculation) than Kalman. This is a direct consequence (as the calculation ‘rolls’ one month forward) of the regression slope losing the oldest data point as it incorporates the most recent. The Kalman approach incorporates all historical data and lessens noise through variance reduction, thereby generating *instantaneous* measures. For volatile underlying return data, a more pronounced difference between the measures is the result – giving the impression of timing skills.

For *less* volatile funds (global macro), measurements obtained using the two techniques are similar. Regression parameter estimates still change slowly, but because the points it gains and loses are similar as the calculation rolls forward, this flaw is disguised. Because the underlying returns are not volatile, Kalman’s instantaneous parameter estimates are similar to those obtained by regression estimates, and the difference between β and $\bar{\beta}$ is small.

This proposal is illustrated using the results obtained in this research (Figure 5.14).

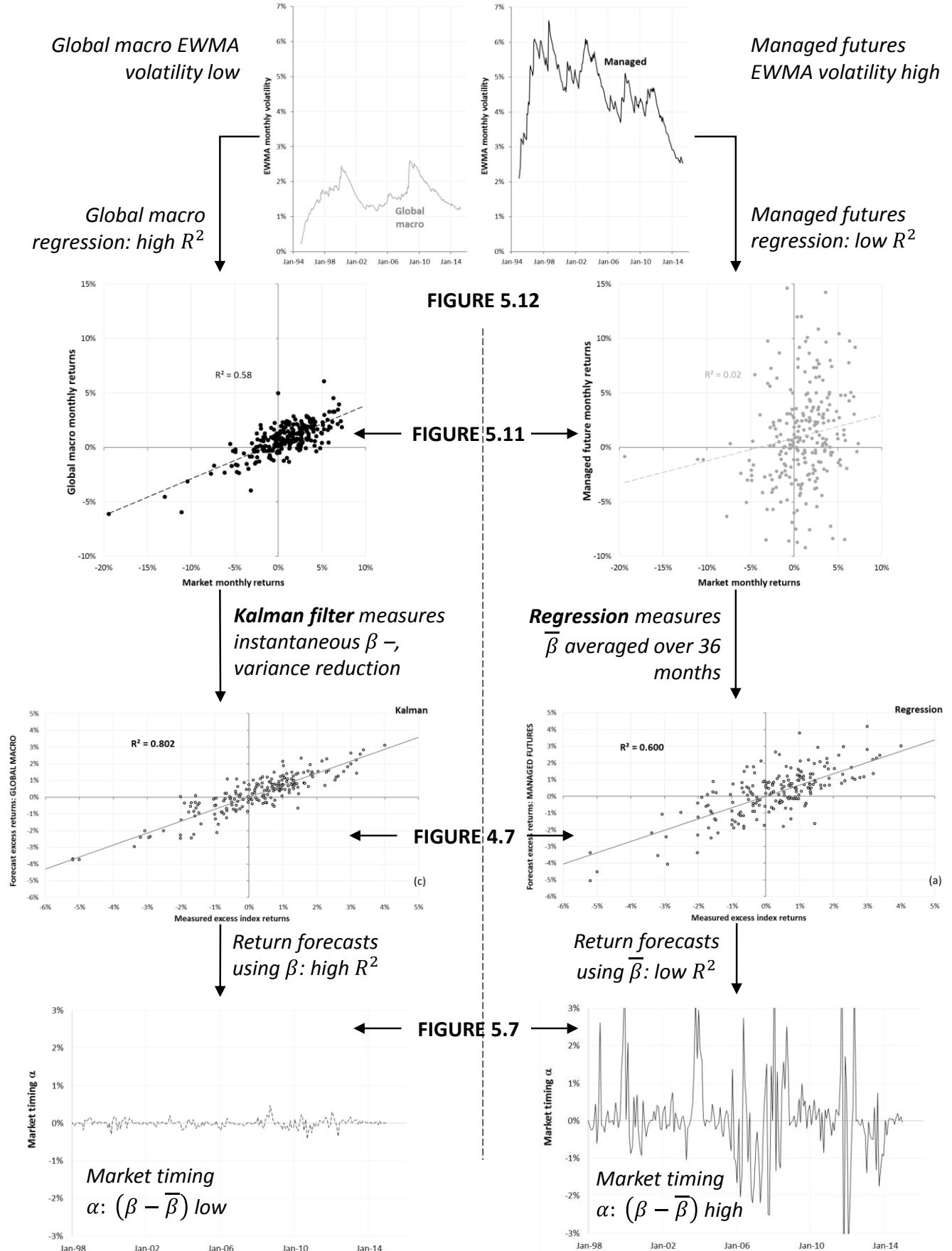


Figure 5.14: Flow chart illustrating the logic behind Table 5.5.

5.6. Conclusions

A simple expansion of the familiar CAPM equation was shown to allow the apportioning of hedge fund returns into components derived from stock selection, market timing and market exposure. Since the expanded CAPM required 'instantaneous' values of α and β , a different technique from regression (whether time-weighted or unweighted) was required. The Kalman filter, using Bayesian variance reduction, provided the relevant parameters. These were used to extract instantaneous β s (a dynamic β exposure which is different from the long-run market exposure, $\bar{\beta}$, that originates from regression techniques and is far more commonly employed) in the expanded CAPM equation (2.2).

The characteristics of the best performing and worst performing hedge funds were compared and top performing funds' α s were found to originate almost entirely from consistent stock selection whilst market timing α was kept as low as possible. Top-performing funds exhibited considerably lower return volatility. Worst performing funds displayed substantially higher volatilities and market timing α s that fluctuate (both positive and negative). This led to the conclusion that attempts at market timing are difficult without introducing market volatility – and that this detracts from returns over time.

The data were examined from a hedge fund style perspective and global macro and event driven funds were found to outperform managed futures and long/short equity strategies. An alternate interpretation of the 'active β ' component of the expanded CAPM equation postulated by Jain, *et al*, (2011) was posited and a statistical argument was used (using the return histories of global macro and managed futures funds) to show that active β can also be viewed as measurement error between the instantaneous Kalman filter and unweighted (or time-weighted, EWMA) regression. The return volatility of these funds determines the magnitude of the difference between these two measurement techniques, as opposed to pure market timing α .

Investors should seek a combination of low historic volatility and consistent positive returns. This work found that, contrary to conventional wisdom, high risk was *not* compensated with high return and consistently superior returns can be achieved with lower risk and smaller – but consistently positive – returns.

Finally, refinements to the Kalman filter's assumptions were suggested to enhance its predictive capability and argue that a traditional long only equity database may provide more meaningful decompositions between stock selection and timing as market β s are of more relevance to managers in these funds.

Due to the mathematical rigour of the Kalman filter, some simplifying assumptions were made in the filter's specification. These include:

1. taking into account the correlation between α and β in the observation equation. Future studies could investigate the use of the measures, empirical correlation,
2. time-varying α s and β s of the transition equation follow a random walk process. Investigation into the use of other, possibly more suitable, processes may yield even more accurate predictions and
3. the use of a traditional asset management database. With the emphasis on managers' precise market/ β exposure, further study – using data gathered from traditional asset managers who, it could be argued, manage their β exposures more closely in relation to more specific benchmarks than broad market indexes – could provide greater insight into the α from market timing.

Chapter 6

Conclusions and suggestions for future research

6.1. Summary and conclusions

The complex task of forecasting economic and financial variables is rendered particularly difficult for practitioners because of sparse data, lags in the arrival of these data, gaps in the data and other data-related issues. Establishing whether patterns are present in time series requires some knowledge of the noisiness accompanying the underlying signal. Eliminating that noise and then extracting the 'true' signal is a non-trivial task, requiring the original data to be filtered in some way, complex mathematics to determine the signal frequency and amplitude (if cyclical) and true trajectory (if not), and advanced statistics to ascertain the significance of the results. Despite these obstructions, the value of accurate forecasts to the industry, to economic policy and to finance is immeasurable.

This dissertation explored two applications of the Kalman filter – both involving forecasting.

6.1.1. Hedge fund index performance forecasting using the Kalman filter

The first application examines the robustness of time-varying parameter estimates (α and β) from the CAPM equation for hedge fund indices. These parameters are usually determined using linear regression, but using monthly fund and market returns that span a period of several years, so their validity is often questioned. The Kalman filter provides estimates of almost instantaneous parameters using a Bayesian variance reduction framework.

Because manager compensation for hedge funds is strongly linked to execution, accurate and timely measurement of fund performance has become critically important to hedge fund managers, investors and regulators. Accurate performance measurements (ex post) as well as accurate return forecasts (ex ante) are important for manager rewards, and they thus feature prominently when determining fees and charges. The time-varying parameters as determined by the Kalman filter provide better return forecasts than those determined using the standard regression-based values, as evidenced by the higher R^2 values for the former. The Kalman filter reacts faster to market changes and filters out noise to leave behind the "pure" signal, so its estimates of CAPM coefficients are demonstrably superior.

6.1.2. Hedge fund returns attribution using the Kalman filter

An expansion of the familiar CAPM equation provided insight into the apportioning of hedge fund returns into two constituents: stock selection and market timing components. This expanded CAPM uses 'instantaneous' values of α and β , values which are not obtainable from standard linear regression. The Kalman filter – which estimates time-varying instantaneous values – was used for this purpose. Instantaneous β s are different from long-run market exposure, $\bar{\beta}$, which originate from linear regression, and it has been asserted that their difference measures the α from market timing while the instantaneous (Kalman-filtered) α provides the return component due to stock selection.

The characteristics of the best performing and worst performing hedge funds were compared and top performing funds' α s were found to originate almost entirely from consistent stock selection whilst market timing α was kept as low as possible. Top-performing funds exhibited considerably lower return volatility. Worst performing funds displayed substantially higher volatilities and market timing α s that fluctuate (both positive and negative). This led to the conclusion that attempts at market timing are difficult without introducing market volatility – and that this detracts from returns over time.

The data were examined from a hedge fund style perspective and Global Macro and event driven funds were found to outperform Managed Futures and Long/Short Equity strategies. An alternate interpretation of the 'active β ' component of the expanded CAPM equation postulated by Jain, *et al*, (2011) was posited and a statistical argument was used (using the return histories of global macro and managed futures funds) to show that active β could also be viewed as measurement error between the instantaneous Kalman filter and unweighted (or time-weighted, EWMA) regression. The return volatility of these funds determines the magnitude of the difference between the measurement techniques, as opposed to pure market timing α .

6.2. Suggestions for future research

6.2.1. Forecasting returns

The CAPM, with time-varying coefficients estimated using the Kalman filter, could be connected to a formulation of a stochastic receding horizon control framework.

Standard fund returns (rather than pure hedge fund returns) could be used to evaluate and compare the accuracy and robustness of the α s and β s estimated obtained from the Kalman filter with those obtained from standard regression.

Additional further potential work could develop a Kalman filter-based model for asset prices that could be used in conjunction with a control theory oriented state space model for portfolio optimisation.

6.2.2. Return apportioning

The disparity between the distribution moments of the top performing hedge fund returns versus those of the worst performing funds should be investigated further. The extremes of skewness, kurtosis and variance deserve further work – including the drivers of these moments and the reasons for the large disparity.

The mathematical rigour required of the Kalman filter, necessarily involves some simplifying assumptions to be made in the filter's specification. Future research could relax these conditions. An example of this is the assumption of zero correlation between α and β in the observation equation. Future studies could investigate the use of realised correlation between these two variables. This correlation has been found to be non-negative in certain circumstances (Stivers, 2010): including a non-zero correlation could affect the results obtained considerably.

Time-varying α s and β s of the transition equation follow a random walk process. Investigation into the use of other, possibly more suitable, processes (e.g. see Nieto, Orbe & Zarraga, 2014) may yield even more accurate predictions and result in even more accurate estimates of time varying α and β .

Traditional asset management returns could be used (as opposed to hedge funds). With the emphasis on managers' precise market/ β exposure, further study – using data gathered from traditional asset managers who manage their β exposures more closely in relation to more specific benchmarks than broad market indexes – could provide greater insight into α due to market timing.

Greater explanatory power from CAPM regression could be also be sought using hedge fund-specific factors e.g. short and long term momentum approaches and style tilts (see e.g. Israel & Maloney, 2014).

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Appendix

THOMSON, D. and van Vuuren, G. 2016. Forecasting the South African business cycle using Fourier Analysis. *International Business and Economics Research Journal*, 15(4): 175 – 192.

Forecasting the South African business cycle using Fourier analysis

DANIEL THOMSON, North-West University, South Africa

Gary van Vuuren, North-West University, South Africa

ABSTRACT

A Fourier transform analysis is proposed to determine the duration of the South African business cycle, measured using log changes in nominal gross domestic product (GDP). The most prominent cycle (two smaller, but significant, cycles are also present in the time series) is found to be 7.1 years, confirmed using Empirical Mode Decomposition. The three dominant cycles are used to estimate a 3.5 year forecast of log monthly nominal GDP and these forecasts compared to observed (historical) data. Promising forecast potential is found with this significantly-reduced number of cycle components than embedded in the original series. Fourier analysis is effective in estimating the length of the business cycle, as well as in determining the current position (phase) of the economy in the business cycle.

Key words: Fourier, Hodrick Prescott filter, Baxter-King filter, empirical mode decomposition

JEL classification: C22, C63

1 Introduction

In finance and economics, the predominant method of analysing time-series data is usually to view these data in the time-domain, i.e., analysing changes of a series as it progresses through time. The problem in using only this approach to study financial datasets is that all realisations are recorded at a predetermined frequency. This frequency corresponds to whichever period the realisations are recorded at and the implicit assumption is made that the relevant frequency to study the behaviour of the variable matches with its sampling frequency (Masset, 2008). This can be construed as analysing inflation figures with a one year time frame and presuming that the cycle will repeat itself the following year as the cycle is presumed to be one year long. The realisations of financial and economic variables often depend on a number of frequency components rather than just one so the time-domain ap-

proach alone will not be able to process the information in the time-series precisely (Masset, 2008).

Spectral analysis methods that enable a frequency-domain representation of the data, such as Fourier series and wavelet methods, are able to identify at which frequencies the time series variable is active. The strength of the activity may be measured using Fourier analysis to construct a frequency spectrum (or periodogram)¹¹ – a graphic representation of the intensity of a frequency component plotted against the frequency at which it occurs. This method is particularly attractive for the use of economic variables that exhibit cyclical behaviour as the cycle length may be identified using the Fourier transform (Baxter & King, 1999).

Understanding the business cycle of a region and having an idea of its current position (or *phase* in the cycle) enables participants in the economy to make informed decisions. Because business cycle information is so valuable, much research has been done to identify its behaviour and the South African business cycle is no exception (see Venter (2005), Bosch & Ruch (2012) and Du Plessis *et. al* (2014)). In fact, owing to South Africa's volatile political and economic history, modelling its behaviour provides a robust test to structural breaks and regime shifts of any technique (Aaron & Muellbauer, 2002 and Chevillon, 2009).

This article examines and attempts to forecast South African Gross Domestic Product (GDP) time-series data using Fourier series analysis. Significant cycles are detected and the length of these cycles quantified by examining the data in the frequency-domain rather than the time-domain, thus providing an alternative method of business cycle analysis to most in the literature. Using single frequency components or a combination thereof also provides a novel perspective of South African GDP forecasting. A further aim of this work is that it paves the way for further, alternative methods of analysis of economic and financial variables, wavelets in particular, through highlighting potential limitations of pure frequency-domain analysis.

The remainder of this paper is structured as follows. Section 2 provides a brief literature review providing an overview of spectral methods applied to finance and previous attempts at modelling South African GDP. A description of the data used in the analysis as well as detail

¹¹ The terms frequency spectrum and periodogram are used interchangeably.

of methodologies employed is given in Section 3. The analysis, results and discussion of the problem are presented in Section 4. Section 5 concludes and provides recommendations for further study.

2 Literature review

Spectral analysis methods have a broad range of applications in the real world. The Fourier integral formula – which enjoys widespread physics and engineering applications – is regarded as one of the most fundamental results of modern mathematical analysis (Debnath, 2012), including spectroscopy, crystallography, imaging, signal processing and communications. Fourier series have more gained traction as a tool in finance and econometrics, starting with the early works of Cunningham (1963), Nerlove (1964) and Granger (1966), and others on simple economic time series in the 1960s to modern day applications in derivative pricing and wavelet analysis.

This literature review focuses on the application to finance and econometrics. In particular, literature that describes practical uses of Fourier analysis as applied to modelling and forecasting economic data is presented.

Hamilton (1994) discussed spectral analysis and introduced the concepts of the population spectrum, the sample periodogram and estimation based on the population spectrum.¹² An analysis of US manufacturing data demonstrated the use of spectral methods on real time series. Hamilton (1994) explains why adjustment (in the form of taking natural logs) of the data must be performed, owing to the assumptions of a covariance-stationary process implicit in the transform (for a mathematical definition of a covariance-stationary process, see Lindgren, Rootzén & Sandsten (2013)). To omit seasonal effects which appeared in the periodogram, (Hamilton, 1994) suggested using year-on-year growth rates.

General and theoretical work on spectral analysis in financial time series was introduced by Cunningham (1963), Granger & Hatanaka (1964) and Granger (1966). Shumway & Stoffer (2000) provide a wide range of time series analysis techniques and applications, covering spectral analysis and filtering. Granger & Morgenstern (1964) used Fourier analysis to examine stock market prices on New York Stock Exchange price series.

¹² Spectral analysis meaning the study of a variable over the frequency spectrum or frequency-domain.

Granger & Morgenstern (1964) found that stock prices followed the random-walk hypothesis in the short term, but long run components exerted greater influence than the random-walk hypothesis suggested (a non-parametric test of this was demonstrated by a flat spectrum of share price changes). Seasonal variation and the business-cycle components were found to be largely irrelevant in explaining the evolution of stock market prices.

Praetz (1973) studied Australian share prices and share price indices in the frequency domain using spectral analysis methods. In contrast to the findings of Granger & Morgenstern (1964), they found small departures from the random-walk hypothesis in their share price series from 1947 to 1968, although not large enough to abandon the hypothesis as a crude first approximation. Some clearly-defined seasonal patterns were present. Praetz's (1973) study of share price indices and other market sectors were shown to lead or lag the market as a whole. This evidence contrasted with Granger & Morgenstern's (1964) and Godfrey, Granger & Morgenstern's (1964) studies on the New York Stock Exchange and the London Stock Exchange, where such patterns and lags were far less significant. Praetz (1973) offered the tentative conclusion that Australian share markets were less efficient than their overseas counterparts.

Iacobucci (2003) elaborated on the issues of cross-spectral analysis and filtering, and explored the concepts of coherency and phase spectra. This analysis was applied to US inflation and unemployment data and showed that a Phillips relation existed at typical business cycle components of 14 and six years. Cross-spectral analysis and filtering was used to measure the correlation between the two factors through the Phillips curve. Unemployment was found to lead inflation by a full 12 months.

Masset (2008) provided an introduction to spectral and wavelet methods of analysis with many practical examples using real economic data. A spectral analysis on residential property prices in New York covering the period January 1987 to May 2008 was performed using both parametric and non-parametric methods to show the subsequent differences in the frequency spectrum. Spectra from the non-parametric methods (using Welch method (Welch, 1967) contained more noise than the spectra obtained from the parametric methods (Tjostheim & Paulsen, 1983). Fourier analysis of the data confirmed that strong seasonalities affected New York home prices with a cycle of 12 months. A detailed exposition on

wavelet analysis was proffered as a way to overcome many of the shortfalls of Fourier transform and filtering methods (Masset, 2008).

Liu *et al.*, (2012) investigated business and growth cycles in the frequency domain by performing Fourier analyses on several data sources including electricity demand, foreign currency data, monthly retail sales, quarterly GDP, labour market and productivity statistics from Statistics New Zealand. In their analysis of the GDP data, the data were transformed using natural logarithms and detrended using the Hodrick-Prescott filter before conducting Fourier analysis of the detrended transformed data. Using a periodogram, definitive cycles corresponding to eight years and four-and-a-half years were found. Because of the distance between energy spikes in the periodogram, it was proposed that the cycle length varied between 4.5 to 8 years. Liu *et al.*, (2012) argued that Fourier analysis could be used to detect cyclical behaviour in any type of time series data, although no cyclical behaviour was detected in the majority of the time series data tested and proposed that a natural extension to Fourier analysis was the use of wavelets.

Omekara, Ekpenyong & Ekrete (2013) used Fourier series analysis to identify cycles in the Nigerian all-items monthly inflation rates from 2003 to 2011. A square root transformation was used to increase stability and normality of the inflation rate data. Periodogram analysis showed a short term and a long term cycle of 20 months and 51 months respectively with the long cycle corresponding to the length of two different government administrations that existed during the sample period. A general Fourier series model was then fitted to the data to make accurate short term monthly inflation rate forecasts from an out-of-sample period from September 2011 to September 2012.

More recent academic research (Masset, 2008 and Liu *et al.*, 2012) of Fourier series often leads to a recommendation of wavelet analysis as a natural extension to the limited, frequency-domain only methods such as Fourier transforms. Masset (2008) presents two principal drawbacks to the use of spectral analysis and standard filtering methods:

- (i) they impose strong restrictions regarding the possible processes underlying the dynamics of the series (e.g. stationarity), and,
- (ii) they lead to a pure frequency-domain representation of the data, i.e. all information from the time-domain representation is lost in the operation.

Much literature governing spectral analysis now embraces wavelets, with Fourier analysis forming part of the development process (see, for example, Daubechies, 1988; Vetterli & Herley, 1992; Scargle, 1993 & Cody, 1994).

Aron & Muellbauer (2002) developed a GDP forecasting model for South Africa to measure interest rate effects on output. Multistep forecasting models were found to be preferable to recursive forecasting with vector autoregressive (VAR) models because of the structural breaks present in the South African economy. The multistep model comprised a factor model which was evolved to a single equation equilibrium correction model with a built in term for the stochastic trend. The model made forecasts for up to four quarters and was tested for stability using sample breaks. Tests for normality and heteroscedasticity yielded satisfactory, robust results (Aron & Muellbauer, 2002).

Venter (2005) discusses the methodology used by the South African Reserve Bank (SARB) to identify business cycle turning points. This methodology included the use of three composite business cycle indicators and two diffusion indexes. Leading, lagging and coincident indicators make up the composites while movement in historic and current diffusion indexes helped to confirm whether changes in the economy were localised or all-encompassing.

Chevillon (2009) drew on the research of Aron & Muellbauer (2002) and established whether direct multi-step estimation improved the accuracy of forecasts. Chevillon (2009) assembled 779 different models and applied them to South African GDP data to ascertain which provided the most accurate forecasts and coped best with the large number of regime changes and structural breaks. Aron & Muellbauer's (2002) direct multi-step model was found to perform best within short time horizons whilst multivariate and univariate models performed better with intermediate to long term time horizons.

Bosch & Ruch (2012) provided an alternative methodology to dating business cycle turning points in South Africa by using a Markov switching model and the Bry-Boschan method to date the turning points and found that the model estimates generally coincided with the business cycle turning points determined by the SARB. The model was applied to GDP data, but also to 114 of the 186 stationary variables the SARB uses to date the business cycle (Bosch and Ruch, 2012). Using Principle Component Analysis (PCA) on these variables a more accurate measure of the business cycle turning points than using GDP data alone was obtained.

Du Plessis, Smit & Steinbach (2014) developed a dynamic stochastic general equilibrium (DSGE) model for the South African economy. The model used Bayesian techniques to incorporate prior information about the economy into the parameter estimates. Its forecasting capability extends up to seven quarters and was tested against a panel of professional forecasters and a random walk. It was found to outperform the professional forecasters and the random walk, especially over longer time horizons, when used to predict CPI inflation and GDP growth.

Other filters exist – such as the Kalman filter – which may be used to extract underlying patterns (including periodic ones) from signal data by reducing or eliminating noise. To calibrate the Kalman filter, however, requires many data and most GDP data are only reported and recorded quarterly. Considerably more data are required for the filter to 'learn' the underlying pattern (i.e. distinguish the coherent pattern from the noisy signal) as well as test out of sample data than are needed for use in, e.g. the Fourier transform technique. For this reason, the Kalman filter was not used in this work.

3 Data and methodology

3.1 Data

This study seeks a simple and readily available proxy for South African economic activity from which to identify potentially meaningful cycles. GDP, although not a perfect measure of the business cycle (see Boehm & Summers, 1999), provides a reasonable measure of economic activity and the business cycle, over a satisfactory sample period.

The data employed are seasonally-adjusted, nominal GDP in South African rand (ZAR) measured monthly from January 1969 to July 2015. The in-sample period for the establishment of dominant cycles using Fourier analysis was January 1969 to December 2011 and the out-of-sample forecast period was January 2012 to July 2015 (SARB, 2015).

Fourier analysis uses a Fast Fourier Transform (FFT) which enables much faster computing, but restricts the number of data points to 2^n , where $n \in \mathbb{Z}$ (Cooley & Tukey, 1965).

3.2 Methodology

The Hodrick-Prescott (HP) and Baxter-King (BK) filters are time-domain techniques to decompose time-series data into trend and cycle components. The cycle component of the data may then be analysed using methodologies which require stationarity, such as the FFT.

3.2.1 Time series filtering methods

Hodrick-Prescott filter

Hodrick & Prescott (1997) presented a procedure for representing a time series X_t as the sum of a smoothly-varying trend component τ_t , and a cyclical component c_t , where,

$$X_t = \tau_t + c_t \quad t = 1, 2, \dots T \quad (1)$$

The trend component τ_t is found by choosing a positive value of λ and solving for

$$\min \left\{ \sum_{t=1}^T (y_t - \tau_t)^2 + \lambda \sum_{t=2}^T [(\tau_{t+1} - \tau_t) - (\tau_t - \tau_{t-1})]^2 \right\}.$$

The parameter λ is a smoothing parameter which penalises variability in the growth (trend) component series (Hodrick & Prescott, 1997). The larger the value of λ , the smoother is the output series.

The HP filter has been criticised for a number of limitations and undesirable properties. Canova (1994 and 1998) used the HP filter to extract business cycles from macroeconomic data of average length of four to six years, but was sceptical of the methodology used to determine key parameter inputs after inconsistent results were obtained. Spurious cycles and distorted estimates of the cyclical component when using the HP filter were obtained by Harvey and Jaeger (1993). Cogley & Nason (1995) also found spurious cycles when using the HP filter on difference-stationary input data. Application of the HP filter to US time series data was found to alter measures of persistence, variability and co-movement dramatically (King & Rebelo, 1993). Many of these critiques do not provide sufficient compelling evidence to discourage use of the HP filter (van Vuuren, 2012). As a result, it remains widely-used among macroeconomists for detrending data which exhibit short term fluctuations superimposed on business cycle-like trends (Ravn & Uhlig, 2002).

BK Filter

Baxter and King (1999) argued that a time series function X_t comprised three components: a trend τ , a cyclical component c , and a 'noise' (random) component, ϵ , such that:

$$X_t = \tau_t + c_t + \epsilon_t \quad t = 1, 2, \dots, T \quad (2)$$

The BK filter removes the trend and noise components, leaving only the cycle component:

$$c_t = X_t - \tau_t - \epsilon_t \quad t = 1, 2, \dots, T.$$

Guay & St-Amant (2005) provided a measure of the ability of the HP and BK filters to extract the business cycle component of macroeconomic time series using two different business cycle definitions. The first definition was that the duration of a business cycle was between six and 32 quarters (as argued by Baxter & King (1999)). The second definition made a distinction between permanent and transitory components. Guay & St-Amant (2005) concluded that in both cases the filters performed adequately if the spectrum of the original series peaked at business-cycle frequencies. Low frequencies dominant in the spectrum were found to distort the business cycle. Guay & St-Amant's (2005) results suggest that the use of HP and BK filters on series resembling the Granger shape of an economic variable may be problematic.

In this work, Fourier analysis was performed on the cyclical components of the HP filter (1) and the BK filter (2), for comparison.

3.2.2 Data stationarity

A practical difficulty arises when the deterministic time-varying component (drift) of the underlying data is time-varying. How may this deterministic component be removed and the random process component be isolated? The drift may be thought of as the local mean value at each point in time, while the variation around this means represents the signal. Removing this drift may be accomplished through several techniques – two are considered below.

Log of time series

Masset (2009) emphasises that spectral methods such as Fourier transforms require that data under investigation must be stationary (for a comprehensive definition of strict stationarity and weak stationarity, see Pelagatti (2013) who takes stationarity to mean *weak* stationarity (covariance-stationary), unless otherwise specified). An augmented Dickey-Fuller (ADF) test was used to check for this condition. The results obtained from the ADF test on nominal GDP data used failed to reject the null hypothesis that the index levels series is non-stationary. Natural logarithms of the time-series were thus taken ($\ln(x_t)$) –

$\ln(x_{t-1})$), where x_t and x_{t-1} are consecutive months in the GDP time series. This transformation converts the monthly nominal GDP data into monthly returns, a stationary series.

Empirical mode decomposition (EMD)

The EMD is specifically designed to expand a function into a trend plus a number of intrinsic mode functions (IMFs), whose defining features are a) that all local minima and maxima are negative and positive, respectively, and b) that their local mean is zero. To apply the EMD, the envelope of a signal $x(t)$ may be generated by first determining the sequences $H = \{h_i\}_{i \in \mathbb{Z}}$ and $L = \{l_i\}_{i \in \mathbb{Z}}$ i.e. the local maxima and minima of x , respectively. Let $\text{Lin}_{A,B}(t)$ be the continuous, linearly interpolated function assuming the values $B = \{b_i\}_{i \in \mathbb{Z}}$ on $A = \{a_i\}_{i \in \mathbb{Z}}$ and changing slope on A only. Let x^\uparrow be the pointwise maximum between x and $\text{Lin}_{H,x(H)}(t)$ and x^\downarrow the pointwise minimum between x and $\text{Lin}_{L,x(L)}(t)$. The pair x^\uparrow, x^\downarrow forms the envelope of x ($x^\downarrow(t) \leq x(t) \leq x^\uparrow(t)$) and the drift is:

$$\bar{x}(t) = \frac{(x^\uparrow(t) + x^\downarrow(t))}{2}$$

$x_1 = x - \bar{x}$ is the oscillating component, and this method may be applied multiple times to obtain $x_1 = \bar{x}_1 + x_2$, etc. Higher order splines (such as cubic splines) are applied at this point to generate smoother envelopes than those generated from piecewise linear splines (Huang, Shen, Long, et al, 1998).

3.2.3 Fourier analysis

The central idea of spectral analysis is to re-express the original time-series $x(t)$ as a new sequence $X(f)$, which evaluates the significance of each frequency component, f , in the dynamics of the original series (Masset, 2008). This is achieved by using the discrete version of the Fourier transform, which decomposes a periodic signal into its constituent frequencies. Time series data that comprise periodic components can be written as a sum of simple waves (that is, oscillations of a *single* frequency) represented by sine and cosine functions (Brown & Churchill, 1993). A Fourier *series* is an expansion of a periodic function in terms of an infinite sum of sines and cosines by making use of the orthogonality relationships of the sine and cosine functions (Askey & Haimo, 1996). The *generalised* Fourier series, obtained using the functions $f_1(x) = \cos x$ and $f_2(x) = \sin x$ (which form a complete orthogonal system over $[-\pi, \pi]$) gives the Fourier series of a function $f(x)$:

$$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos(nx) + \sum_{n=1}^{\infty} b_n \sin(nx) \quad (3)$$

where

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) dx$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos(nx) dx \text{ and}$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin(nx) dx.$$

For a function $f(x)$ periodic on an interval $[0, 2L]$ instead of $[-\pi, \pi]$, a simple change of variables may be used to transform the interval of integration from $[-\pi, \pi]$ to $[0, 2L]$ by letting

$$x = \frac{\pi x'}{L}$$

Solving for x' and substituting into (3) gives (Krantz, 1999):

$$f(x') = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x'}{L}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x'}{L}\right)$$

where

$$a_0 = \frac{1}{L} \int_0^{2L} f(x') dx$$

$$a_n = \frac{1}{L} \int_0^{2L} f(x') \cos\left(\frac{n\pi x'}{L}\right) dx \text{ and}$$

$$b_n = \frac{1}{L} \int_0^{2L} f(x') \sin\left(\frac{n\pi x'}{L}\right) dx.$$

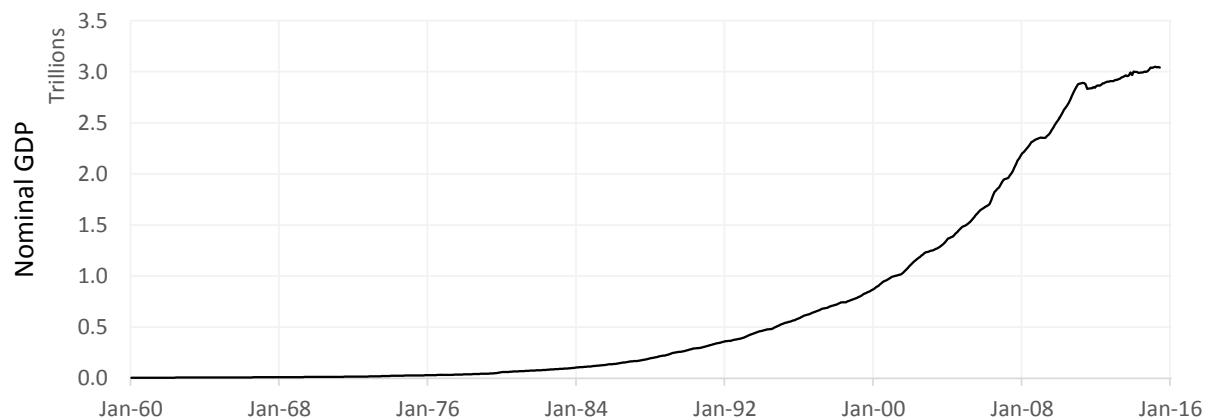
A periodogram which plots those frequency components with the greatest *intensity* or amplitude against the period shows which components bear significant meaning and which components are random 'noise'. In cyclical data, it may be found that a few frequencies are able to model the behaviour of the series relatively accurately. The low amplitude, noise frequencies may be discarded and a new, 'cleaner' time-series – free of noise and comprising only time-series signals characterised by the dominant frequencies – may thus be constructed.

4 Results and Discussion

4.1 Results

Cycle frequency identification

The discrete Fourier transform assumes that the input signal (in this case, the nominal GDP) is statistically stationary, i.e. it has a constant mean through time. If the data were taken as is (due to the convex growth curve), considerably more weight would be given to more recent fluctuations as the scale has increased substantially in later years (2000s onwards), relative to the initial years (pre 2000s) – as shown in Figure 1. This would not present an accurate representation of the time series and the Fourier analysis would not identify cycles effectively.



Source: SARB (2015).

Figure 1: Nominal GDP in ZAR trillions, seasonally adjusted.

An ADF test was performed shown in Table 1(a) and (b) to establish whether the data were non-stationary.

Table 1 (a) Augmented Dickey-Fuller test for stationarity: monthly GDP levels.

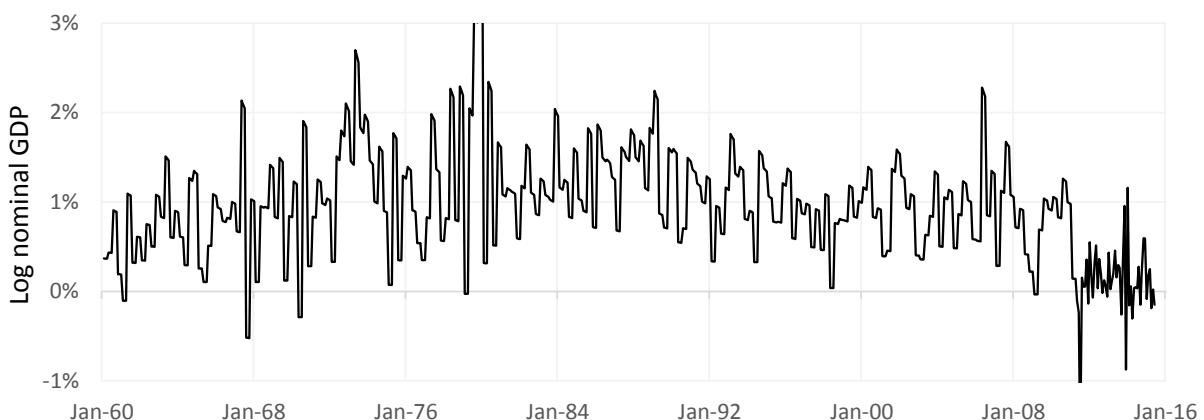
| Stationary test | | 5.0% | | |
|----------------------------|--|------|---------|----------------|
| Test | | Stat | p-value | Critical value |
| <i>No Const</i> | | -2.2 | 2.5% | -1.9 |
| <i>Const-Only</i> | | -5.8 | 0.1% | -2.9 |
| <i>Const + Trend</i> | | -7.1 | 0.0% | -1.6 |
| <i>Const+Trend+Trend^2</i> | | -7.4 | 0.0% | -1.6 |
| | | | | Stationary? |
| | | | | FALSE |

Table 1 (b) Augmented Dickey-Fuller test for stationarity: monthly GDP returns.

| Stationary test | | 5.0% | | |
|--------------------------------|------|---------|----------------|-------------|
| Test | Stat | p-value | Critical value | Stationary? |
| No Const | -2.2 | 2.5% | -1.9 | TRUE |
| Const-Only | -5.8 | 0.1% | -2.9 | TRUE |
| Const + Trend | -7.1 | 0.0% | -1.6 | TRUE |
| Const+Trend+Trend ² | -7.4 | 0.0% | -1.6 | TRUE |

The ADF test examines the inputs for the existence of a unit-root in the context of a hypothesis test. If a unit-root exists, we reject the null hypothesis and accept the alternative hypothesis that this root exists. The results showed that our assumption was correct and that we needed to perform a transformation to stationarise the data.

To stationarise the data, the natural logarithm difference from month to month was calculated to produce the percentage returns series shown in Figure 2. These returns do not scale with time and have a non-trending mean, so these are suitable for use in the Fourier analysis framework.



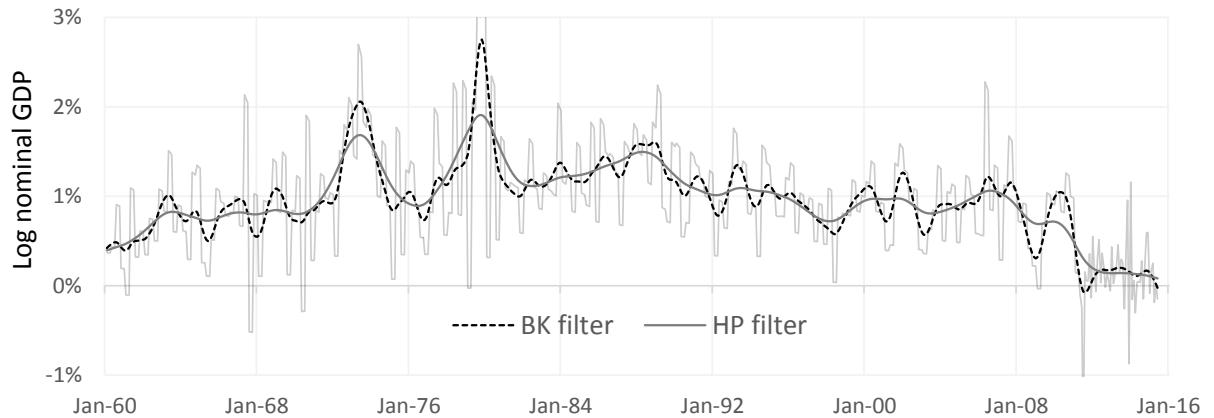
Source: Author calculations.

Figure 2: De-trended GDP returns series using first differences.

In the case of returns over the sample period, the mean monthly return is 1.05%. This positive average produces the upward ‘trend’ observed in the monthly GDP level in Figure 1. We are interested in identifying the cyclical changes around this trend.

Because of the volatility in the returns series, we apply two filtering methods which extract the trend and cycle components from the series and produce a smoother returns series. These are the HP and BK filters. The log returns series is plotted alongside the filtered series

in Figure 3. Summary statistics illustrating the effectiveness of the filters in capturing the trend and filtering through the noise appears in Table 2.



Source: Author calculations.

Figure 3: HP and BK-filtered log monthly nominal GDP (grey line).

Table 2: Summary statistics illustrating the effects of filtering.

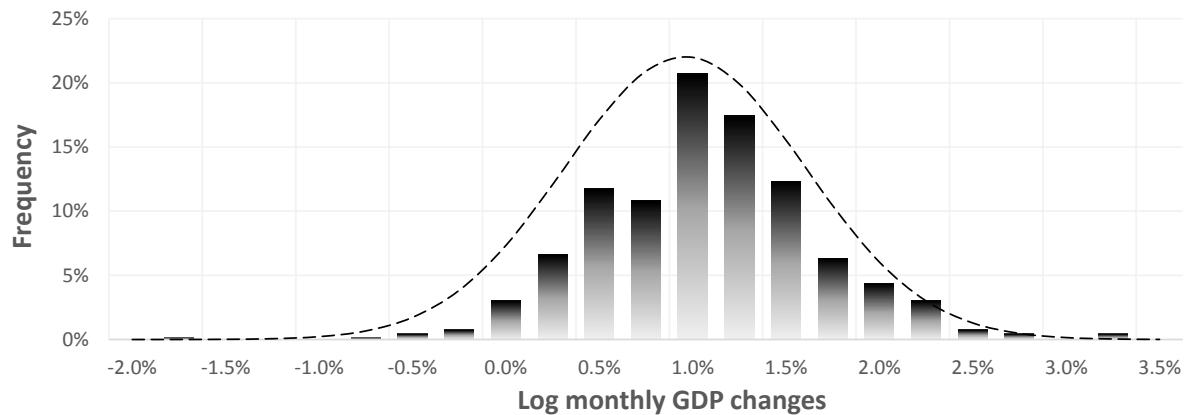
| | Log return | HP filter | BK filter |
|--------------------|------------|-----------|-----------|
| Mean | 1.05% | | |
| Standard deviation | 0.64% | 0.33% | 0.36% |
| Skew | 1.21 | -0.62 | -0.13 |
| Excess kurtosis | 6.44 | 0.92 | 0.94 |

The standard deviation of the log returns series is 0.64%, whilst the HP and BK filtered series produce less variation with standard deviations of 0.33% and 0.36% respectively. The filtered data also contain less excess kurtosis than the unfiltered returns series while there is a positive skew in the unfiltered series and negative skew of less magnitude in the filtered series. A Jarque-Bera normality test (Table 3) confirmed that the data were not normally distributed.

Table 3: Jarque-Bera test of normality on monthly returns at a significance level of 5%.

| Normality test | Z-Score | Critical Value | p-Value | Pass? |
|----------------|---------|----------------|---------|-------|
| Jarque-Bera | 988.90 | 5.99 | 0.0% | FALSE |

Severe excess-kurtosis and a positive skew, shown in a histogram plot with a normal distribution curve for comparison in Figure 4 below, characterise the monthly GDP returns series.

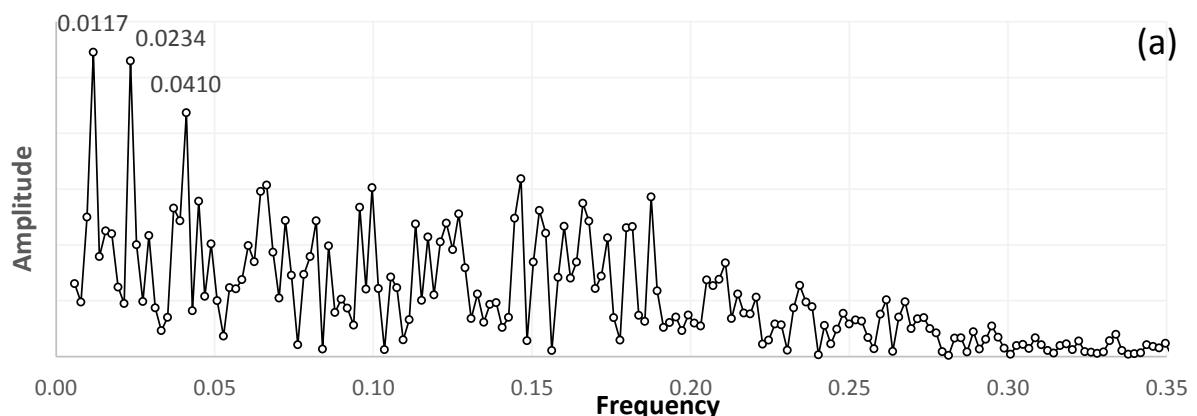


Source: Author calculations.

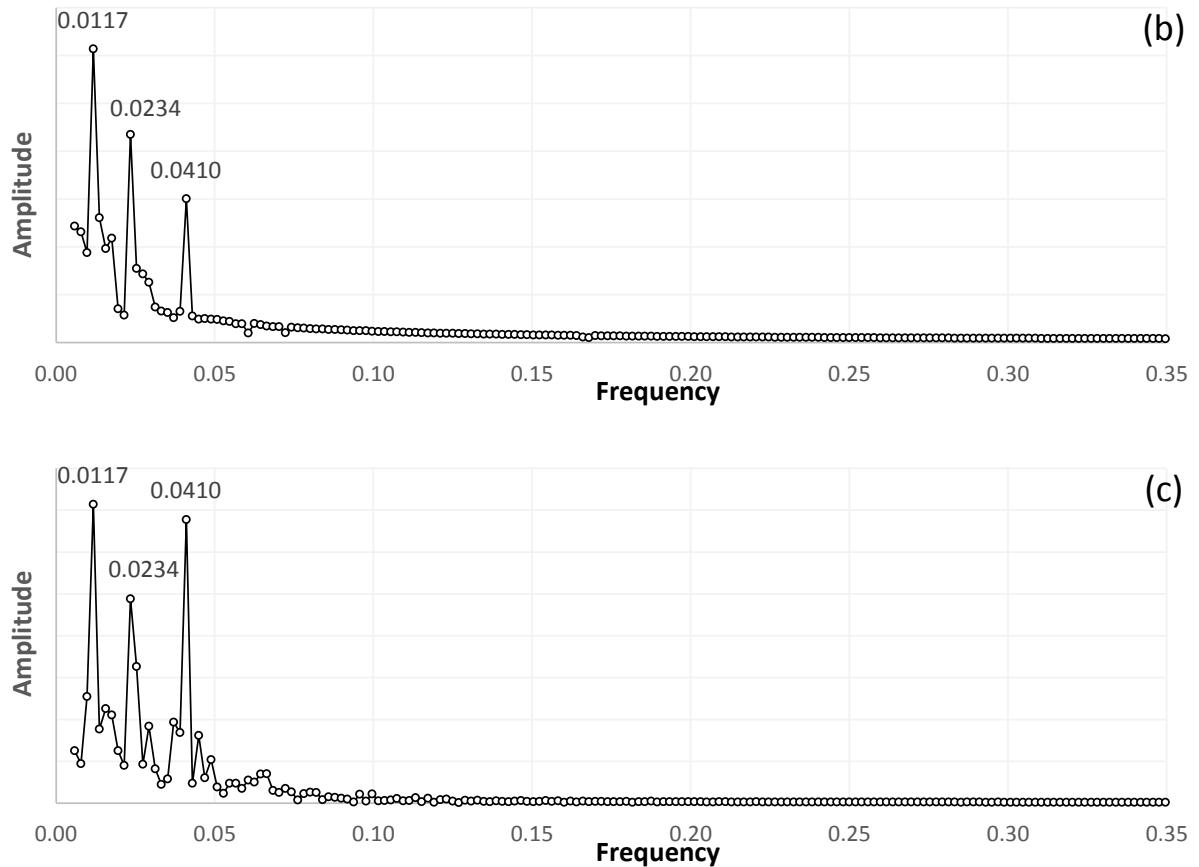
Figure 4: Frequency distribution of log returns series compared with the normal distribution curve.

Fourier analysis

Using the discrete version of the Fourier transform, the time-series of GDP returns was transformed from a time-domain representation into a frequency-domain representation. The time-series is decomposed into a series of sine and cosine waves occurring at different frequencies with different *intensities*,¹³ which in summation are able to exactly mimic the behaviour of the original signal. The *power* or amplitude of each frequency component (which explains the importance of the particular frequency in making up the original signal) is plotted against its period in Figure 5 below. The frequency is defined as $\frac{1}{\text{cycle (in months)}}$. The filtered series (Figure 5(b) and (c)) show much less noise than the unfiltered series as they rid the data of random deviations or ‘noise.’ The periodogram omits periods longer than 180 months, which distorts the analysis.



¹³ Intensity, power and amplitude are used inter-changeably.



Source: Author calculations.

Figure 5: Periodograms of (a) transformed log monthly return data, (b) the HP filtered log monthly returns and (c) BK filtered log monthly returns.

Three dominant frequencies are evident: these are at frequencies of 0.0117 (85.3 months for a full cycle or one full cycle every 7.1 years), 0.0234 (42.7 months for a full cycle or one full cycle every 3.5 years) and 0.0410 (24.4 months for a full cycle or one full cycle every 2.0 years).

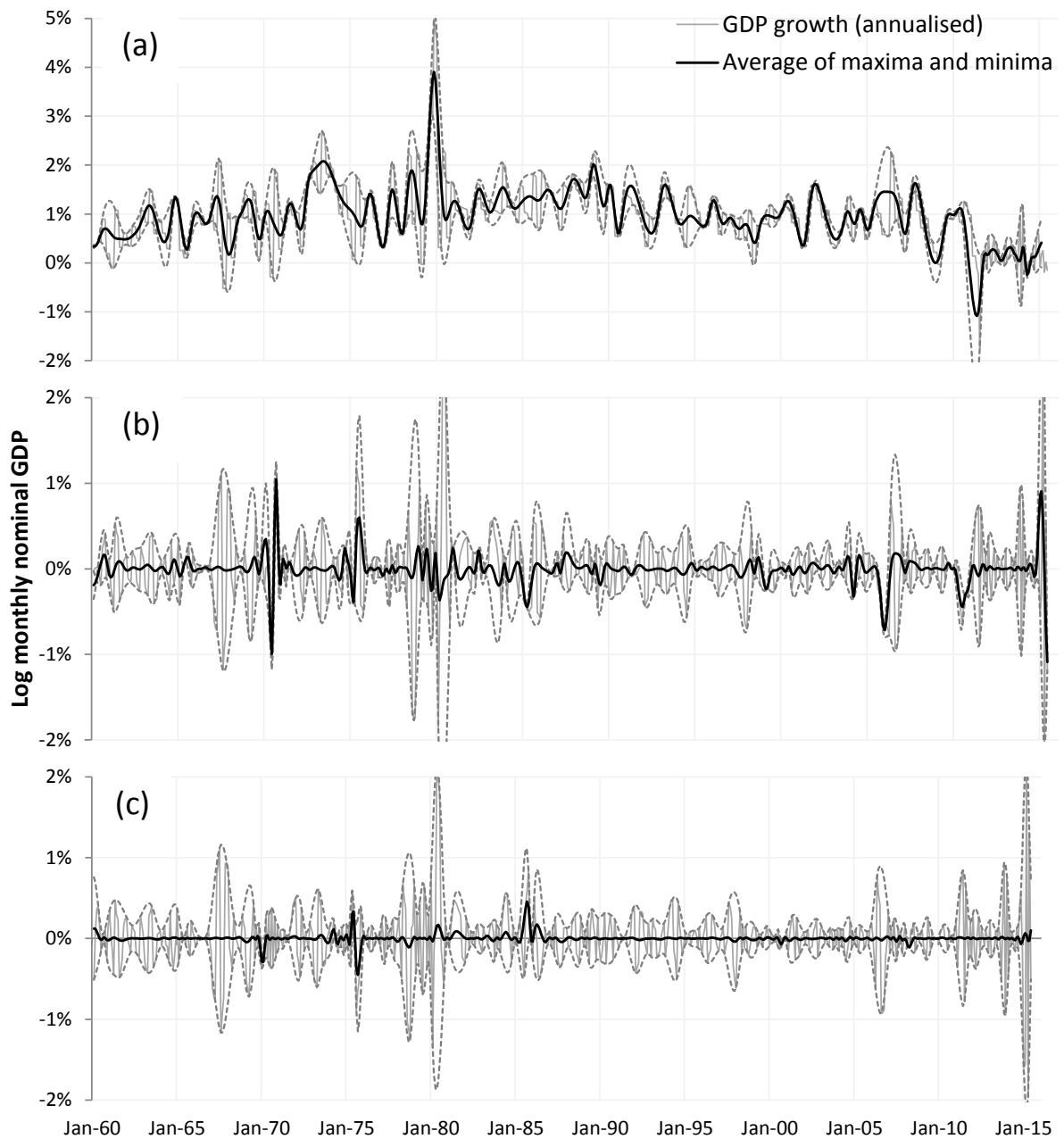
Botha (2004) found evidence of a 7.0 year cycle in South African GDP data using smooth transition autoregressive technique. This result confirms the result obtained above – that the dominant cycle present in South African GDP data using the Fourier transform is 7.1 years. The prominent cycle at 3.5 years is likely to be a harmonic of the 7.1 year cycle, so this cycle contains no economic information. The final significant cycle of 2.0 years is the South African business cycle (Gauteng Province, 2012).

EMD

The EMD was used to confirm cycle frequencies detected by the Fourier approach.

The process described in Section 3.2 (making data series stationary) is repeated until the local mean of $x(t) = 0$ everywhere (within some pre-set tolerance). The resulting function is the first intrinsic model function (IMF). This IMF is subtracted from the original function and the entire process repeated on the remainder function to obtain the second IMF. This is repeated until all IMFs have been recovered and the remainder does not have sufficient local extrema to continue: this remainder is the trend.

The EMD technique was applied to the log monthly returns of nominal GDP over several iterations, and the results are shown in Figure 6.



Source: Author calculations.

Figure 6: EMD for log monthly nominal GDP after (a) one, (b) three and (c) six iterations. Note that all graphs are on the same timescales, but different y-axis scales.

Spectral analysis of the IMFs gave the spectral decomposition as shown in Figure 7.

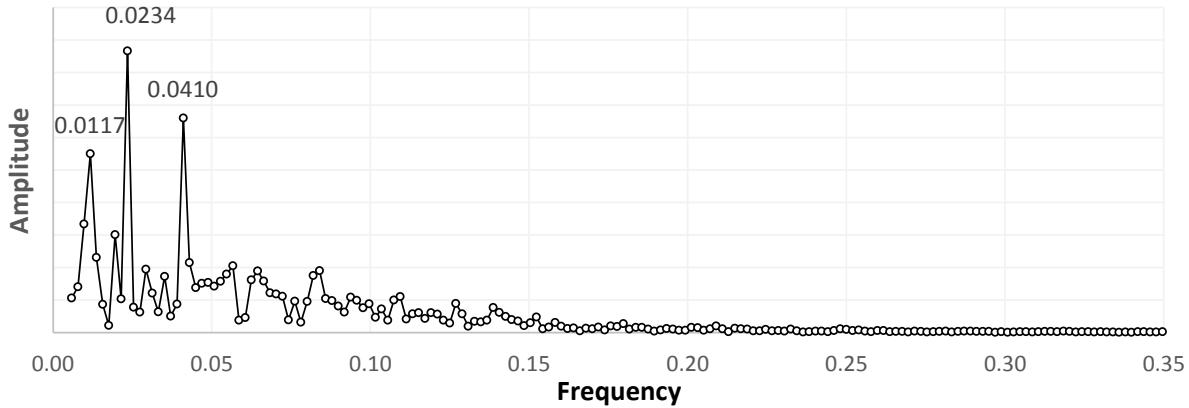


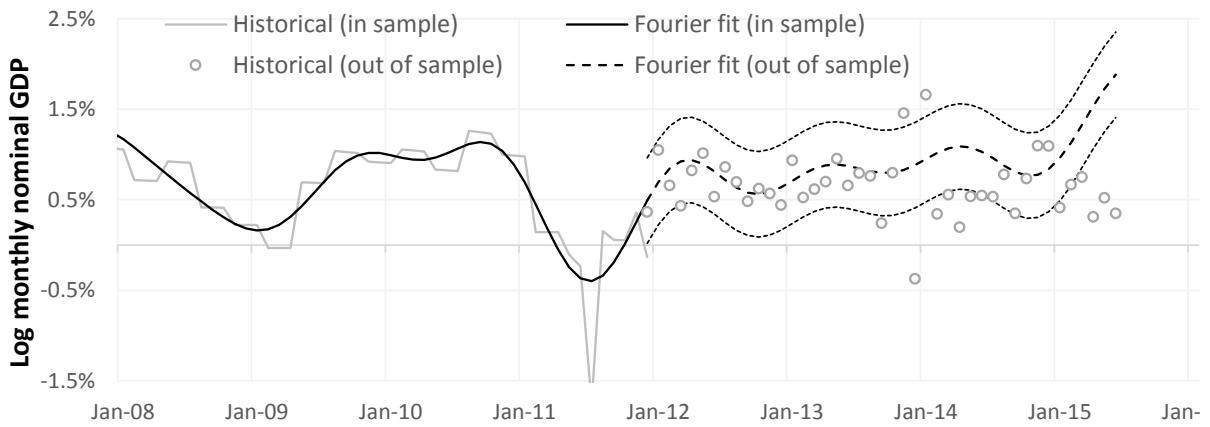
Figure 7: Spectral decomposition of EMD IMFs showing the same three dominant frequencies as those observed for the Fourier analysis.

The same frequencies as those found by Fourier analysis were again dominant, i.e. cycles of 7.1 years, 3.5 years and 2.0 years. The confirmation by another mathematical technique provides some assurance that Fourier analysis is indeed identifying the correct underlying cycles present in the data.

Forecast potential

Using the three dominant frequencies observed in the GDP dataset, a 3.5 year forecast was generated. This forecast used roughly 32 years of monthly data (i.e. from 1969 to 2012). The out-of-sample forecast of the log monthly nominal GDP was compared with historical data until mid-year 2015.

Figure 8 shows the 3.5 year log monthly GDP forecast from January 2012 to July 2015 using a combination of cycles generated from the three principal frequency components. The entire range of historical data was not plotted – only the last 16 years, since January 2005.



Source: Author calculations.

Figure 8: Historical log monthly GDP and associated (fitted) Fourier cycles using frequencies from the dominant three cycles present in the data. Also shown is the forecast (out of sample prediction) of log monthly GDP from January 2012 to July 2015 and actual [realised] log monthly GDP in circles. One standard deviation above and below the log monthly GDP forecast is also shown (dotted lines).

The forecast estimates using only three cycles forecast the future GDP reasonably accurately, although the uncertainty and negative GDP growth around January 2014 – which coincided with the Marikana unrest (Twala, 2014) – could not have been forecast and skewed the results.

4.2 Discussion

Spectral analysis provides an alternative method of, in this case, economic cycle analysis, to the traditional method used by the SARB and Venter (2005) of micro and macro-economic indicators which lead, coincide with and lag the economy. The results of this study indicate that, using 665 months of GDP data, a significant cycle exists in the log monthly South African nominal GDP with a period of 7.11 years over the period January 1969 to July 2015. Other prominent cycles are one of 3.5 years (possibly a harmonic of the 7.1 year cycle) and one at roughly 2.0 years (a possible South African business cycle).

These values were calculated using a Fourier transform methodology and these outputs were confirmed using EMD analysis.

Viewing the time series as comprising a general trend, a cyclical and an error component, the behaviour of each of these may be investigated in a similar way to Omekara (2013) who assessed each component separately before reassembling them to generate a forecast equation. Specifically, Omekara (2013) estimated the error component by evaluating the

autocorrelation function of the residual for randomness. If the residual were not random, as in Omekara's (2013) case, a first order autoregressive model may be fitted to the error values. The BK filter separates a time-series into three components, but it is critical to use the appropriate parameters in the filter to produce meaningful results. Further studies may be able to establish these parameters and use the BK-filtered data to run Fourier analysis. If serial autocorrelation is present in the error component and can be taken into account via an autoregressive model as part of the forecast equation, forecast accuracy may improve.

The combination of cycles – which constitute the out of sample prediction of log monthly nominal GDP – are reasonably accurate in determining the possible future course of South African GDP – even out to 3.5 years from the forecast date. The majority of data (at a 95% confidence level) were within one standard error of the forecast mean.

Fourier analysis does not identify the start and end points of a cycle. Liu *et al.* (2012) contend that with the *length* of a cycle identified, one can infer the start and end points. This is the reverse of the method used in Bry & Boschan (1971), in which the length of a cycle is inferred by first identifying the start and end points.

The phases of the various (three) components used to generate Figure 8 provide turning point dates and business cycle phase changes (also demonstrated by Botha, 2004). These phases – which interact and interfere via superposition – may be identified and isolated and the maxima and minima of each cycle explored. Knowledge of these points could be of importance to policy makers who may wish to be forewarned of economic slowdowns or upturns in order to manage expectations and reign in profligate lending, for example.

5 Conclusion and recommendations

5.1 Conclusion

Using Fourier and periodogram analysis to study South African GDP from January 1969 to December 2011, clear individual cycles were isolated and identified, lasting 7.1 years, 3.5 years and 2.0 year respectively. This result, though obtained using an alternative quantitative approach, compares favourably with the result of Botha (2004), who found a 7.0 year cycle in quarterly South African GDP from 1961 to 2003. A simple log transformation allows the use of Fourier analysis methods and that filtering using the HP and BK filters helps omit most random ‘noise’ whilst still preserving the integrity of explanatory cycles in the data.

Forecasting future GDP, even out to 3.5 years from the analysis period and using the three frequency components showed good agreement with realised log monthly nominal GDP values. The results using the Fourier transform are encouraging and have demonstrated that the technique has potential for forecasting GDP. Non-obvious cyclical patterns in the time domain data are considerably clearer in the frequency domain.

5.2 Recommendations for further study

Business cycle data representation

Nominal GDP was used as a proxy for the SA business cycle. GDP growth is considered the most natural indicator of an economy's aggregate business cycle by the Basel Committee for Banking Supervision (BCBS, 2010). However, several other macroeconomic indicators with useful business cycle information could potentially have been used. These include aggregate real credit growth, the credit-to-real GDP growth ratio and the leading, lagging and co-incident indicators used by the SARB to measure the business cycle. The BCBS considers real credit growth as a natural measure of supply (since boom periods leading to a peak in the business cycle are characterized by rapid credit expansion and credit contraction has typically heralded credit crunches (BCBS, 2010). For a comprehensive study of the use of Fourier series methods in detecting the length and timing of South African business cycles, the above data should also be analysed.

Data selection

Monthly GDP data showed substantial variation. Quarterly data combined with weighted moving averages and filtering would help to smooth volatility in the short term and could be considered in future studies. In South Africa's case, the recommendation is that 256 quarters (768 months) are used to ensure the reliability of the results, with the caveat that economic, environmental and political factors may have changed considerably over this period.

Return calculation

Returns were calculated by taking the natural logarithms of *month-to-month* GDP levels and not *month-on-month* returns. This approach produces monthly changes with less autocorrelation than using month-on-month data, which gives an annual change. Other methods of return calculation could be attempted.

Power spectrum estimation

Use was made of the non-parametric, periodogram method to calculate the power of a frequency component. Masset (2008) compared the parametric methods of Yule-Walker and Burg (Tjostheim & Paulsen, 1983) and the non-parametric methods of the periodogram and Welch method (Welch, 1967). The non-parametric method was accompanied by more 'noise' over the spectrum and the difference between the periodogram and Welch method was considerably greater than the difference in the parametric methods. Preceding Fourier analysis, one should ensure the most suitable method to estimate the power spectrum is used or provide a comparison of the results before further analysis.

Forecasting issues – modelling time series components

This paper made use of unfiltered returns data as the Fourier tool used was able to separate the cyclical components from the general trend internally. However, using the BK filtered returns or any method that can separate the trend, cycle and error components of a time series will allow future studies to attempt to model the error component identified if it is not random, as in Omekara (2013). This may improve the accuracy of forecasts.

Forecasting issues – determining cycles phases

The methods used to determine the length of the SA business cycle provided satisfactory results in this paper, but ascertaining the approximate current position of the economy on the business cycle was not addressed in full. For this purpose, further studies should include Fourier analysis and forecasting of various measures of the business cycle, not only the rate of GDP growth. Phase differences between leading, lagging and co-incident cycles may provide valuable information to pin-point the current position of the economy and a composite cycle of such indicators may provide a more accurate forecast.

Statistical Measurement of Forecast Accuracy

The forecasts in this paper were not tested by any statistical measure of significance and thus cannot be compared to other forecast tools. Although testing may become rigorous, at a minimum, testing for correlation and performing a simple regression to predict future GDP returns based on the path of the reconstructed signal is proposed. A multiple regression hypothesis test may yield helpful results regarding the significance of each frequency compo-

nent in predicting future values. The periodogram performed this function by ordering components according to amplitude, but a multiple regression could prove this statistically.

Wavelets

Wavelets are relatively new tools in economics and finance. These provide an attractive way of analysing financial datasets as they can represent data series from both the time and frequency perspectives simultaneously (Scargle, 1993). Hence, they permit the breakdown of market activity into different frequency components and the study of these dynamic components separately (Daubechies, 1988). Wavelets do not suffer from some of the limitations of standard frequency-domain methods, like Fourier analysis used in this paper, and can be employed to study a financial variable, whose evolution through time is dictated by the interaction of a variety of different frequency components (Vetterli & Herley, 1992). These components may behave according to non-trivial (non-cyclical) dynamics – e.g., regime shifts, jumps, long-term trends (Masset, 2008).¹⁴ Due to the complex nature and presence of such non-trivial dynamics in South African GDP data under study, using wavelet analysis on South African and other similar GDP datasets which are characterised by non-cyclical dynamics is strongly encouraged.

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¹⁴ For a non-technical introduction to wavelets and their benefits compared to pure-frequency domain analysis, see Masset (2008).

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