Extensions and applications of Merton's model of capital structure

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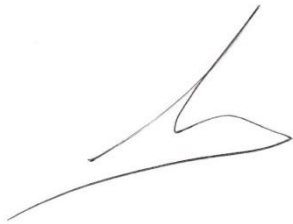YUNIBESITI YA BOKONE-BOPHIRIMA
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This thesis is dedicated to my family. To Carla, Nate and Chris who have lived through the LONG gestation and to my mom, Yvonne, who would have been so very proud.

I think in probabilities

DECLARATION

I, Leon Benoit Sanderson, declare that

1. The contents of this thesis, except where otherwise indicated, are the result of my own work.

2. This thesis has not been previously submitted, in part or in whole, to any university or institution for any degree, diploma, or other qualification.

3. This thesis does not contain data, graphs or other information from other people or resources unless specifically acknowledged as such. Where other written sources have been quoted the text has been rewritten and the information attributed to the original author.

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ABSTRACT

Merton's structural model offers a powerful and intuitively appealing approach to the evaluation of a firm's capital structure choices and market behaviour across a firm's issued debt and equity securities. Historical empirical evaluations of the efficacy of the model to replicate observed behaviour in the context of firm choices and market prices have produced mixed results. We show by way of two distinct examples that adopting different metrics for the evaluation of the model's performance exposes very positive results. We evaluate the performance of the debt and equity markets in Anglo American Plc and BHP Billiton Plc in the period 2006 to 2015 using Merton's structural model. We consider the failure of African Bank using Merton's structural model. Separately, we construct a robust, extended and expanded interpretation of the Merton conceptual framework that incorporates many real world features of firm behaviour and market activity. Notably, we introduce the notion of liquidity that captures the requirement for firms to settle all payments and receipts through a cash account. We show that this model captures observed market behaviour. We apply the model to the two examples introduced previously.

KEYWORDS

ACKNOWLEDGEMENTS

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1 INTRODUCTION

Firms finance their activities by accessing a broad spectrum of instruments encompassing debt, equity and much in between. The resulting combination of these instruments defines the firm's capital structure at a point in time. The range of instruments available to a firm is wide and the characteristics of each is complex. The choices made in respect of financing influence how the firm operates in future. The study of these choices and their effect on firm behaviour has occupied academia for many years and many theories of firm behaviour and instrument impact have been postulated. In this thesis we explore one such theory.

Merton (1974) provided an elegant solution to the capital structure puzzle, linking the behaviour of a firm's debt and equity to the variability of its underlying business activity. He expressed the instruments used to finance a firm's activity as options on the underlying assets of the firm. This approach allowed for analysis and valuation in a risk neutral context. The approach has been extensively debated, tested and extended in the academic literature in the years since and is the genesis of the structural model approach to capital structure theory. Structural models are in essence an evaluation of the trade-off between the leverage benefits of debt and the costs of debt when considering value for equity holders as well as total firm value.

In the chapters that follow we consider the application of the Merton model in finance as well as offering a novel expression of the model that is significantly enhanced to cater for real world characteristics of financial instruments and the general behaviour of firms regarding capital structure choices.

We begin, in Chapter 2, with a detailed literature review. We consider broad capital structure theory before focussing on structural models. We pay particular attention to the empirical performance of these models across a variety of applications. We note that while structural models are intuitively appealing, the empirical performance is mixed, as is the case with other theories of capital structure.

In Chapter 3 and Chapter 4 we consider two applications of the Merton model in its most basic form, highlighting the value of the approach. We deviate from the literature which tends to focus on prediction and look to evaluate the model in an applied sense. Firstly, we look at two diversified resources companies, Anglo American Plc and BHP Billiton Plc, through the lens of the Merton model. We do not consider whether Merton is a "good" predictor of the prices of debt and equity, rather we devise an investment strategy that uses the output of the model to make investment decisions. Secondly, we look at African Bank Limited, a consumer lender in the South African market that went into curatorship in 2014. We target their performance in the period leading up to the curatorship and evaluate whether the traded prices of their various securities, when interpreted through the Merton model, provided any particular insight or forewarning of the problems to come. In both instances we are using the Merton model to provide guidance on changes in prices in instruments, rather than the absolute level of the prices.

In Chapter 5 we change tack completely. We construct the S-model, which is an extended and expanded expression of the Merton model catering for a wide range of real world elements. Coupons, dividends, relative tax benefits, bankruptcy charges, equity issuance, subordinated debt and deviation from absolute priority of claims on default are introduced. In addition, and perhaps most importantly, we present liquidity as a key consideration in firm behaviour.

In Chapter 6 we provide a detailed analysis of S-model behaviour across a wide range of parameter choices and compare and contrast our results with the literature.

In Chapter 7 we conclude. We draw the work undertaken across the earlier chapters together by briefly evaluating the companies considered in Chapter 3 and Chapter 4 in the context of the S-model, highlighting how the extensions and expansions of the S-model relate to some of the results derived in the earlier chapters. We summarise the findings of the thesis and suggest areas for further research.

2 LITERATURE REVIEW

Firms choose to finance their activities from a range of financing sources including equity, debt and a plethora of variations in between. The study of capital structure is an effort to explain these choices. A number of theoretical models of capital structure have been proposed and numerous empirical studies have been undertaken.

Modigliani and Miller (1958) are seen to offer the genesis of modern capital structure theory. Their work provides a robust frame of reference that shows capital structure choices to be irrelevant under certain conditions. In the absence of these ideal conditions we have a variety of theories and avenues for study. In practise taxes, information asymmetry and agency costs (amongst others) make capital structure choices relevant. Myers (2001) summarises the dominant theories of capital structure that guide the mix of debt and equity issued by firms.

- The **trade-off theory** emphasises the balance between the costs and benefits of debt finance. Potential tax benefits are offset by the dead weight costs associated with bankruptcy.
- The **pecking order theory** emphasises differences in information held by insiders (managers) and investors. Managers with superior knowledge about the firm's financial position will only issue equity if it is overvalued or all other financing sources have been exhausted.
- The **agency costs theory** emphasises the divergence between firm management and firm funders (debt holders and owners). Managers may pursue risky activities that benefits owners at the expense of debt holders. Managers may ignore favourable investment opportunities that may benefit debt holders at the expense of owners. Managers may limit payments to owners and destroy firm value through increased management incentives and perquisites.

Merton (1973) provides a robust conceptual framework for evaluating capital structure in the presence of risk. His work gave rise to a class of structural models that cater primarily for the trade-off theory but have been expanded to include elements of the pecking order theory and the agency costs theory.

The body of work covering empirical analysis includes the key contributions of Titman and Wessels (1988), Frank and Goyal (2003) and Graham and Leary (2011).

This chapter provides a summary of the fundamentals of capital structure theory. In addition we consider structural models and conclude with an overview of the empirical work undertaken.

2.1 Capital Structure Theory

Modigliani and Miller (1958) formulated their approach in a simplified expression of the economy. Their metaphor, as described by Miller (1977), considered an environment where firms were divided into distinct classes. Within each class the expected rate of return offered by each firm is the same as other members of the class. In effect, each class is a grouping of entities with similar risk characteristics. The result of this approach is that the capitalisation rate applicable to returns within a given class is constant. In addition for the purposes of their analysis in the original paper (although relaxed in subsequent work) the bond market is represented by a single rate and is considered to be risk free, namely free from default. The three propositions are proven by way of arbitrage arguments, whereby market participants would take advantage of mispricing of market securities and either add to or undo the implied leverage in a given firm.

PROPOSITION 1

The market value of a firm is independent of the capital structure used to finance the firm,

or equivalently

the average cost of capital of a firm is independent of the capital structure used to finance the firm and is equal to the capitalisation rate of a pure equity stream in the class relevant to the firm,

$$V = (S + D) = \frac{X}{P} \qquad (2.1.1)$$

$$\frac{X}{(S+D)} = \frac{S}{V} = p \qquad (2.1.2)$$

where V is the value of the firm, S is the market value of the common stock of the firm, D is the market value of the debt issued by the firm, X is the expected return of the firm and p is the capitalisation rate appropriate for the firm's class.

PROPOSITION 2

The expected yield of a share of common stock is equal to the appropriate capitalisation rate for a pure equity stream in its class, p, plus a premium related to the financial risk of the firm concerned,

$$I = p + (p - r)\frac{D}{S} \qquad (2.1.3)$$

where I is the expected yield of the stock, p is the capitalisation rate appropriate for the firm's class, r is the risk free rate, S is the market value of the common stock of the firm and D is the market value of the debt issued by the firm.

PROPOSITION 3

A firm will exploit an investment opportunity if and only if the rate of return of investment I is greater than the appropriate capitalisation rate for a pure equity stream in its class p.

In effect the cut off point for investment is a minimum return of p and is independent of the type of security package used to finance such investment.

Propositions 1 and 2 provide a theory of valuation of firms and common stock in the presence of uncertainty. This leads to a cost of capital framework, effectively proposition 3, and provides a rational investment decision making process within the firm. In their original paper Modigliani and Miller (1958) reflect on some of the potential issues with their ideal environment and the results of their analysis.

- They recognise that default is a possibility in the bond market regardless of how positive the expected return is for a given firm but they argue that in the aggregate their arbitrage proof remains as the market will move to offset opportunities as participants adjust their holdings of debt and equity.
- They recognise that differential tax treatment across debt and equity may impact on their analysis but they argue that the results remain and that all that one must change is to make use of the after tax return for a given firm. In addition they briefly discussed the notion that taxation must be considered in the context of both corporates and individuals and that any gain to a firm as a result of debt issuance may be offset by the cost to the individual holder of such debt.

The Modigliani and Miller (1958) propositions provided a frame of reference that was fundamentally at odds with the market norms at the time and generated significant debate. Their work came under significant scrutiny and criticism, with value invariance and the impact of taxation broadly debated (see Durand (1959)), which led to the publication of a correction. Modigliani and Miller (1963) recognised that the presence of debt on a firm's balance sheet provided incremental value,

$$V_L = V_U + tD_L \qquad (2.1.4)$$

where V_L is the value of the leveraged firm in a given class, V_U is the value of an unleveraged firm in the aforementioned class, t is the applicable tax rate on income at a firm and D_L is the value of the debt of the leveraged firm. They argued however that the tax gain afforded to debt was the only permanent difference between the levered and unlevered firm.

Miller (1977) subsequently revisited the debate regarding the tax value of debt. He defined the gain from leverage as the net improvement in market value after taking account of taxation at all levels,

$$GL = \left(1 - (1 - T_c)\frac{(1 - T_{ps})}{(1 - T_{pd})}\right)D_L \qquad (2.1.5)$$

where G_L is the gain from leverage, T_c is the corporation tax rate, T_{ps} is the individual tax rate on gains on stock holdings, T_{pd} is the individual tax rate on gains on debt holdings and D_L is the debt issued by the levered firm.

In essence the advantage of deductibility for a given market participant on the issue of a specific instrument must be offset by the disadvantage of the tax levied on another market participant on the return offered by the instrument. The desire at an aggregate level to hold bonds will be impacted by the tax consequences of doing so. He argued that there may be advantages to debt issuance but he questioned the quantum of such value. He concluded by stating that the supply / demand function at a corporate sector level will guide the market towards an optimum leverage but that such optimal level may not be applicable for individual firms within the sector.

Miller (1988) again revisits the debate and provides some commentary on the progress of the discussion as well as some perspective on the thought process.

On the value invariance construct as per proposition 1 he argues that the resources for the aggregate economic investment by the business sector ultimately comes from the savings in the household sector. In this instance the T-accounts below reflecting the basic balance sheets of the business sector and the household sector are instructive. We have the assets and liabilities of Businesses and Households,

Businesses

Assets	Liabilities
Productive Capital	Debt owed to Households
	Equity owed to Households

Households

Assets	Liabilities
Debt of Firms	Household net worth
Equity in Firms	

which simplifies on consolidation to

Assets	Liabilities
Productive Capital	Household net worth

In effect the mix between debt and equity is not relevant.

He recognises that individuals may not be able to effect the arbitrage proof as outlined in the original proposition 1 but suggests that the result is guided by the law of one price, namely

that a given outcome should command the same price regardless of how the outcome is arrived at, and that the result remains.

In dealing with specific criticism of the Modigliani and Miller model he recognises that dividends and dividend policy do provide information about firm performance but that rather than this being a refutation of the original proposition, it is a recognition that the base assumption that all market participants have the same level of information about the underlying firm is at fault.

In considering the tax impact of debt issuance Miller looks to two market developments that support both the tax benefit to firms of debt issuance and the importance of considering tax at firm and individual level. He argues that the development of the leveraged buyout (LBO) market at relatively high leverage levels provides ample support for the thesis that debt is good. He further argues that share buyback programs undertaken by firms, which are primarily motivated by the differential tax treatment for individuals on the return of capital by way of sale as opposed to dividends, highlights the importance of the interaction between corporate tax rates and personal tax rates.

The impact of taxation on Capital Structure has been widely studied. Graham (2000) focusses on whether the tax benefits of debt impact capital structure choices and how much value they add to the firm. He highlights the difficulty in answering these questions given the complexity of the tax code which makes the determination of a tax rate onerous, the impact of interest taxation at a personal level and the bankruptcy process and its associated costs.

Graham (2000) defines a tax benefit function. The tax benefit function is a series of marginal tax rates, with each tax rate associated with a specific level of interest deductions. Each marginal tax rate incorporates the impact of non-debt tax shields, tax carry forwards, tax credits as well as a measure of the probability that interest tax shields will be utilised in a given year.

He argues that having the whole tax benefit function at your disposal allows for three distinct contributions :

1. one can quantify the tax advantage of debt by integrating the area under the curve;
2. one can assess how aggressively firms use debt, notably whether any incremental coupon interest on new debt is fully shielded from tax. Graham defines the *KINK* in the tax benefit function as that point where the benefit on incremental interest deductions declines (i.e. where incremental interest will not be fully deductible); and,
3. one can estimate how much incremental value a conservative, low debt firm could add were it to increase leverage. Graham argues that firms should issue debt up to the point where the *KINK* in the tax benefit function is reached.

He finds that amongst conservative debt users there are large, profitable and liquid companies in stable industries that face low costs of financial distress. These firms enjoy growth options and have limited intangible assets on their balance sheets. Debt conservatism is found to be persistent and is positively related to excess cash holdings.

He argues that firms could add up to 15 % (7%) of value through additional leverage if we ignore (consider) the personal tax penalty.

Graham (2000) explores a number of non-tax explanations of debt policy including :

1. the expected costs of financial distress, noting that firms will issue less debt when such costs are high,
2. cash flow and liquidity,
3. management entrenchments and perquisites, described more fully as the agency problem,
4. the need for financial flexibility,
5. information asymmetry and the pecking order theory, and
6. the so-called "Peso Problem" whereby a very low probability but exceedingly large impact event constrains the use of leverage.

The pecking order theory as originally detailed by Myers and Majluf (1984) and Myers (1984) analysed an existing firm with existing assets considering a growth opportunity that requires external financing. They assume perfect capital markets except that investors do not know the true value of a firm's existing assets or the true value of the growth opportunity. This makes it difficult for investors to properly value the securities to be issued to finance the growth opportunity. On the assumption that managers act in the best interest of the existing shareholders, the firm will only issue equity if the shares are overvalued or the value of the growth opportunity exceeds the cost to existing shareholders through dilution of issuing the stock at a discount to the true value. They argue that on the announcement of an equity issue

stock prices will fall to cater for this uncertainty. If debt issuance is an option the impact on share prices is muted as debt is a prior claim on the firm assets (and as such is less exposed to the vagaries in asset valuation) and it would reflect manager optimism as a firm with undervalued equity would always issue debt as an alternative. Equity will only be issued if debt capacity is exhausted. This thought process is reflected in the investor's decision making process. Given the need for external financing by a firm, coupled with the fact that debt is the security whose value changes least with respect to managers inside information, investors, knowing that managers will only issue equity if it is overvalued will insist on debt issuance up to capacity exhaustion. This leads to the pecking order theory where internal resources for financing are preferred to external resources and debt issuance is preferred to equity issuance.

Uncertainty about firm value implies that equity would only be issued if it were overvalued. Uncertainty about firm volatility, however, suggests otherwise. If firm volatility were to be underestimated by the market then debt issuance would be done at a premium level as the bonds are in fact less secure than assumed. In this instance there is an incentive for investors to demand equity.

In the context of capital structure each firm's debt ratio effectively represents its cumulative requirement for external financing. In addition it is important to note that certain positive net present value projects will not be undertaken in the absence of internal resources and debt capacity because of the impact of equity issuance. This highlights the value of financial flexibility, namely having access to financial resources to meet unexpected demands.

Jenson and Meckling (1976) approach capital structure in a fashion that is fundamentally at odds with the work of Modigliani and Miller (1958). They argue that agency costs dominate decisions and develop a theory of ownership structure of the firm that draws from property rights, agency and finance. Their key definition is that of an agency relationship as a contract where a principal engages an agent to perform some service on their behalf which involves delegation of some decision making authority from the principal to the agent. If both parties to this contract act to maximise their individual utility then the agent will not always act in the best interests of the principal. The principal can limit this divergence by way of appropriate incentives for the agent and by incurring monitoring costs. The agent may expend resources by way of bonding expenditure to guarantee that he will not take certain actions and provide

some recourse for the principal where he to do so. Agency costs are defined as the sum of the monitoring costs, the bonding costs and a residual cost.

They argue that corporations are "legal fictions" that provide the nexus for a set of contracting relationships amongst individuals. This contracting environment covers parties that are both internal and external to the firm. Corporate structure is primarily motivated by the limited liability on offer to both debt and equity providers. The separation of ownership and control pits owners against managers where owners incur monitoring costs in an effort to limit the abuse of perquisites by managers.

They further argue that leverage is limited by the incentive effects of highly levered firms and the associated monitoring and bankruptcy costs.

In an interesting aside they suggest that security analysts deliver "social good" by providing a valuable monitoring function, and as such effectively shift the burden of this activity away from the firm which enjoys reduced agency costs as a result.

Jenson and Meckling (1976) vigorously argue that capital structure choices have a direct and significant impact on firm value.

Black and Scholes (1973) deal briefly with the issue of capital structure in their seminal paper. They characterise a basic corporate structure as a combination of options, albeit in a simplified environment without coupons and dividends. In effect they argue that the bond holder owns the firm assets but has issued call options to the shareholders. This approach introduces the risk of default to holders of the company's debt – corporate debt is effectively risk free debt coupled with a short position in a put option on the underlying firm assets struck at the face value of the obligation.

The failure of the original Modigliani and Miller model to cater for debt default is dealt with elegantly by considering the Modigliani and Miller proposition for value invariance in the context of an option framework, namely put – call parity (see Merton (1973) for a discussion),

$$S = C(K) + Ke^{-rt} - P(K) \qquad (2.1.6)$$

where S is the value of the firms cash flow, $C(K)$ is the market value of the levered shares of the firm, Ke^{-rt} is the market value of the debt issued by the firm if it were riskless and $P(K)$ is the value of the shareholders put to the debt holders that ensures limited liability of equity.

In this way, default is introduced but value invariance is maintained.

Merton (1974) expands on the work of Black and Scholes (1973) as he explores Capital Structure in a contingent claims environment in detail, focussing on the valuation of corporate debt. He argues that the value of a specific issue of corporate debt is a function of 3 elements, namely the required rate of return on risk free debt, the provisions and restrictions of the specific issue (including coupon, maturity, seniority, limitations on further borrowings) and the probability of default of the issuing firm. He derives the Black and Scholes solution for the valuation of simple corporate debt with the following conditions and assumptions :-

- zero coupon debt (where the obligation to repay is limited to nominal plus interest payable at maturity),
- on default of the debt (i.e. non-payment) the bondholders take control of the firm assets and shareholders receive nothing,
- no new debt issuance ,
- no dividends, and
- no share buybacks.

He deals explicitly with the risk associated with corporate debt and he solves for the price of risk as a yield spread over the risk free rate.

As discussed in Miller (1988) above, Merton provides an effective proof, in the absence of corporate income taxes and any bankruptcy charges, of the 1st proposition of Modigliani and Miller, namely firm value invariance to capital structure choice in the presence of default.

The introduction of the possibility of default to the analysis does require an adjustment to the 2nd proposition of Modigliani and Miller that relates the required return on equity to the amount of leverage undertaken by a firm. The weighted average cost of capital of a firm is unchanged and remains the cap rate for a pure equity stream in a given class but the relationship between required returns on debt and equity is no longer linear.

Merton's structural approach, extended and expanded by a number of authors to cater for amongst other elements, the value of tax shields and the costs of bankruptcy, effectively encompasses the trade-off theory of capital structure. These extensions and expansions are discussed below.

2.2 Structural Models of Capital Structure

The original work of Merton (1974) has been expanded and enhanced by a number of researchers who have made use of the contingent claims conceptual framework proposed. These expansions and enhancements have sought to model actual security definitions as well as observed market behaviour. Coupons, dividends, variable bankruptcy conditions and a dynamic environment for capital structure choice encompass some of the features explored. In addition elements of the pecking order theory and the agency theory of capital structure are introduced into the modelling framework. In essence these are all structural models, however some have static trade-off features (i.e. debt is constant with limited corporate flexibility, consistent with the original formulation of Merton (1974)) whilst others have dynamic trade-off features with enhanced corporate flexibility. The highlights of this lengthy exercise undertaken by numerous parties is chronicled below.

Black and Cox (1976) extend the earlier work of Black and Scholes (1973) and Merton (1974) to cater for a number of specific security indentures. They consider safety covenants that limit losses to bondholders by way of a reorganisation of a firm's assets and liabilities prior to the maturity of the debt (as distinct from Merton's model which permits no such change). In addition they evaluate the impact of subordination on value and the outcome of restrictions on how coupon payments and dividend payments might be framed (e.g. limiting asset sales for this purpose). They found that the imposition of the specific security indentures detailed above act to increase the value of the bond.

Geske (1977) developed a compound option formula for the valuation of risky securities with sequential payments (in this case coupon bonds). At each coupon payment date the equity holders can effectively purchase an option that extends to the next coupon date (or the maturity date of the bond) by making the coupon payment or they can default on the coupon payment and forfeit the firm to bondholders. He considers the application of his model to a variety of common security indenture features including sinking fund provisions, safety covenants and subordinated debt issuance.

Brennan and Schwartz (1978) extend Modigliani and Miller and Merton to consider capital structure choices in an environment with coupon bonds, corporate tax rates and bankruptcy costs. In their correction, Modigliani and Miller (1963), the value of the tax deductibility of coupon payments was introduced as a certain stream. Brennan and Schwartz introduce the notion that on bankruptcy the interest tax saving will cease. They note that the incremental debt issuance will impact on firm valuation by increasing tax savings on the assumption of survival, but simultaneously decrease the probability of survival. Their analysis is premised on the Modigliani and Miller risk class assumption, namely that similar risk demands the same return. They construct a differential equation relating the levered firm to the unlevered firm. In this construct any cash flows required to service debt are assumed to be financed by a fresh equity issue. The resulting differential equation is identical to that considered by Black and Scholes however the boundary conditions are different. They introduce a bankruptcy cost and assign a tax saving to each coupon payment. Bankruptcy is triggered on a positive net worth basis whereby asset value dropping below the face value of the bonds outstanding triggers default. They apply numerical methods in solving their differential equation and consider the relative impact of tax rates, bankruptcy costs and leverage. These parameters are optimised in pursuit of maximising firm value.

Shimko, Tejima and van Deventer (1993) extend Merton's model by allowing for stochastic interest rates. They note that the correlation between interest rates and the underlying assets of a firm is an important determinant of the firm's credit spread.

Leland (1994) revisits the work of Modigliani and Miller (1958), Merton (1973) and Brennan and Schwartz (1978). He produces closed form results for risky debt value, yield spreads and optimal capital structure. He delivers an analytical solution by considering corporate securities that depend on underlying firm value but are time independent, namely perpetual debt alongside equity (which is by definition perpetual). The model follows Modigliani and Miller, Brennan and Schwartz and Merton in that the activities of the firm are unchanged by capital structure and that capital structure decisions, once made, remain fixed. Coupons, as with Brennan and Schwartz, will be financed by fresh equity issuance. Leland evaluates two models for triggering bankruptcy. In addition to the positive net worth default consideration (considered protected debt), default is only triggered when the company is unable to issue

sufficient fresh equity to finance the net cash flow obligations (considered unprotected debt). Leland argues that protected debt offers some defence against the cost to bondholders of the agency problem, namely that equity holders are incentivised to increase borrowings and increase the volatility of the firm's assets as they are effectively holders of a call option that benefits from increased leverage and increased uncertainty.

Leland and Toft (1995) extend the results of Leland (1994) and cater for a firm that has the capacity to choose both the quantity and the maturity of its debt. They argue that short term debt does not exploit the tax benefits available to the same extent as long term debt. Short term debt does however limit the risks associated with asset substitution and the related agency costs thereof as it balances the incentives between debt holders (who wish to earn a high yet secure coupon for as long as possible) and equity holders (who would be incentivised to switch the firm into riskier assets if access to capital was certain for longer). A key observation is that the optimal debt ratio depends on the debt maturity and is significantly lower when firms are financed by short term debt.

Longstaff and Schwartz (1995) develop an approach for valuing risky debt by extending the earlier work of Black and Cox (1976) to incorporate default risk and interest rate risk as well as allowing for explicit deviation from absolute payment priority on default. An important implication of their model is that two firms with similar default risk could have very different credit spreads depending on the relative correlation between underlying firm assets and interest rates. A key result is that in general the credit spread will be negatively related to the level of interest rates. This somewhat counterintuitive result is a function of higher interest rates resulting in a higher drift rate for firm assets which in turn implies a lower theoretical default probability. To assess the performance of their model, they consider monthly data for a number of Moody's corporate bond yield averages for the period 1977 to 1992. The results of their evaluation suggest that the model output is consistent with observed market credit spreads.

Anderson and Sundaresan (1996) produce a dynamic framework for valuing risky debt. Their approach allows for strategic debt service (namely deviations from absolute priority and

contractual obligations of the firm) as the cost of bankruptcy encourages creditors to accept terms other than those contracted originally. This flexibility allows them to fold in elements of the costs of agency and the pecking order theory of capital structure to a structural model.

Mello and Parsons (1992) adapt a contingent claims framework of the firm to reflect the impact of the capital structure of the firm on manager incentives. This approach allows them to measure the impact of agency costs of debt in addition to the traditional incorporation of tax shield benefits balanced against bankruptcy and reorganisation costs.

Leland (1998) produces a dynamic capital structure model that explores a unified framework that incorporates elements of the Modigliani and Miller valuation invariance theory of capital structure and the Jensen and Meckling agency problem approach and its associated asset substitution concerns. In his model managers are able to make investment decisions after the firm has raised debt (i.e. they have risk flexibility which allows for asset substitution). He studies the impact on leverage, debt maturity and yield spreads of this uncertainty and measures the scale of the distortion in a firm's choices with respect to risk given the presence of debt.

Zhou (2001) extends the static structural model of Merton by incorporating a jump diffusion process for the underlying firm asset. The framework allows for a flexible term structure of credit spreads and can be parameterised to cater for a number of observed empirical patterns that have been found to be inconsistent with the traditional structural model. An example of this is that the possibility of a jump to default can explain the relatively wide credit spreads observed (as compared to the theoretical spreads generated by the Merton model) for very short dated investment grade and near investment grade bonds.

Sundaram (2001) discussed the KMV approach to pricing credit risk, which is a commercial variant of Merton's structural model of default that has found application in practise in a number of areas. In the KMV model (described in detail by Bohn and Crosbie (2003)) the original Merton formulation is extended in the following ways :

1. they introduce a degree of uncertainty relating to the default boundary for the firm's assets,
2. they extract estimates for firm value and firm volatility in the context of an uncertain default boundary using equity values and equity volatility, and
3. they map the distance from default for a given firm (standardised in terms of standard deviation) to a proprietary database of historical defaults over a 12 month time horizon.

Goldstein, Ju and Leland (2001) propose a model of dynamic capital structure. In their framework a firm has the option to increase future debt levels. A number of striking observations follow from this flexibility in the hands of the firm's managers.

1. Initial debt issuance will be lower than it would have been without the option to issue debt at a later stage (i.e. in a static capital structure model), and
2. the price at which the initial debt is issued will reflect the potential for further issuance and the associated increased risk of default and will therefor carry a higher credit spread than would be the case without the option to issue debt at a later stage.

These results go some way towards explaining the observation of lower than optimal debt ratios and higher than theoretically expected credit spreads as generated by the Merton model and a number of the relatively simple variants discussed above. This work highlights the potential benefits of applying dynamic trade-off models relative to static trade-off models, where dynamic models allow for changes to the absolute levels of debt through time.

In a key distinction from previous work, whilst they recognise the differential tax treatment for debt and equity, they highlight the fact that any tax is a cost to the firm. Increases in the tax rate charged might increase the share of firm value attributable to equity but they will certainly reduce overall firm value. As a consequence they consider an appropriate after tax risk free rate in their framework. In addition they take cognisance of the importance of actual earnings (and by implication actual cash flow) as opposed to asset revaluations by evaluating the firm's EBIT (earnings before interest and tax) as the stochastic variable as opposed to firm asset value.

Delianidis and Geske (2001) analysed the components of the credit spread. They conduct their analysis by way of a structural model as it provides a framework for the decomposition of the credit spread. They make use of a modified Brennan and Schwartz and Merton structural model.

The modified structural model caters for payments by the firm, namely coupons on debt and dividends on stock. The approach used effectively accrues all dividends and coupons up to the maturity of the assumed debt. As these payments are made during the life of the debt instrument they are considered less risky and given priority over the debt principal and any residual equity value on default. Valuation of debt and equity will include the accrued payments. In addition the modified structural model allows for a fractional recovery on default (albeit after accounting for accrued payments).

Leland (2002) examines differences in expected default frequencies (EDFs) that are generated by alternative structural models of risky corporate debt. Exogenous default boundary models are those where default is triggered when the asset price process breaches a fixed level (often set as the face value of the outstanding debt, implying a positive net worth constraint). Endogenous default boundary models are those where the decision to default is made by managers who look to maximise equity value and constantly test whether it is optimal to continue to service debt or not. Endogenous default boundary models provide a more flexible means for describing default and Leland argues that these are superior to exogenous default boundary models.

Giesecke and Golding (2004) develop a structural credit model premised on incomplete information. This approach implies that investors cannot observe a firm's default boundary. This uncertainty allows their model to match a number of observed empirical nuances including positive short term credit spreads.

Chen (2010) extends the earlier dynamic capital structure work of Leland (1998). He introduces a cost adjustment, which is applied when debt levels are changed, to the model and evaluates its impact on asset volatility and the cost of capital. The incentive for equity holders to shift downside risk to debt holders in times of distress and the associated risk

premium demanded by debt investors are impacted by a costly risk adjustment. Risk shifting is deferred and equity risk is higher as a result.

Hurd and Zhou (2011) consider two factor capital structure models for equity and debt. They model firm value and debt value as correlated stochastic variables. They argue that the added complexity as compared to one factor structural models that have firm value as a stochastic variable but debt value as a deterministic function is more than compensated for by an ability to better match observed empirical levels and the variability of equity and credit markets.

Anderson and Carverhill (2012) extend the conceptual framework of Merton by considering the impact of liquid asset holdings – be they positive cash balances or short term borrowings – on capital structure choices. They model operating revenue as a stochastic variable and consider debt and equity as claims on this variable cash flow stream constrained by limits to short term borrowings, reduced returns on positive cash holdings and expensive equity issuance. The model allows for excess cash flow to be paid out in good times, short term borrowings to be used to cover cash flow needs when required and equity to be issued when short term borrowing capacity is exhausted. The model is solved by way of numerical techniques, namely finite differences.

Anderson and Carverhill (2012) argue that the model allows for consistent mapping to observed market behaviours. The flexible model structure provides an environment, where depending on initial cash holdings an improvement in firm cash flow could result in savings or increased dividends. They observe that there is a relatively low sensitivity of firm value to the quantum of long term debt and as such it is not a key decision driver in the context of capital structure choice. They argue that increases in long term debt and the associated increase in potential tax value are offset by the increased cost of bankruptcy and the need to hold larger inefficient cash reserves to cater for potential liquidity needs.

The pecking order theory is not dealt with explicitly by the model but they argue that the high costs of bankruptcy and equity issuance versus internal cash flow use provide a reduced form representation of the information asymmetries that underpin the theory.

Anderson and Carverhill (2012) make a case against the merits of asset substitution as they argue that increased volatility will lead to a larger liquid cash balance requirement which will in turn reduce equity value. Hedging activity by firms is considered in a favourable light as it

should reduce the risk of financial distress which implies a lower liquid cash holding requirement which in turn should improve equity value.

Structural models of credit and capital choice allow market practitioners the capacity to model corporate decision making as well as an ability to calibrate theory to observed market levels. A popular alternative to structural models when mapping to observed market levels is the reduced form hazard rate models detailed briefly below.

Jarrow and Turnbull (1995) provided a new framework for pricing and hedging derivative instruments in the presence of credit risk. They make use of a foreign exchange spot market analogy to introduce a Poisson bankruptcy process. They assume that default is an independent event not related to any other market variables. This approach allows them to calibrate to observed market prices and to produce consistent pricing and hedging parameters and was an early expression of a reduced form hazard rate model of credit risk.

Duffie and Singleton (1999) presented a reduced form hazard rate model that extended earlier work in the area to allow for specific parameterisation of losses at default. They were able to relax the assumption of independence of default from underlying value that was a feature of earlier reduced form models.

2.3 Empirical Studies of Capital Structure

The extended history of theoretical capital structure research detailed above is matched by a voluminous body of empirical study that has been undertaken to attempt to match theory to market experience. The area covered encompasses studies undertaken to assess the performance of competing models of capital structure as well as relative predictive performance within the trade-off theory or structural model space.

Titman and Wessels (1988) undertook a factor analysis for estimating the impact of a number of attributes (asset structure, non-debt tax shields, growth, uniqueness, industry classification, size, earnings volatility and profitability) on capital structure choice. They evaluated 469 US

firms for the period 1974 to 1982. Their results were mixed but of particular interest was the finding that showed that there was no impact on debt ratios arising from non-debt tax shields and earnings volatility.

Harris and Raviv (1991) undertook a survey of capital structure theories, highlighting the main implications of each and compared them to the available empirical evidence. They cover models based on agency, models based on asymmetrical information, models based on product and market interactions and models based on corporate control considerations. They do not deal with the trade-off theory as they exclude models driven by tax considerations.

They argue that the models evaluated share similar outcomes and are generally supported by empirical evidence that shows that stock prices increase with added leverage and that impending changes to equity (be it via new issuance, share buybacks etc.) impacts on market prices so as to support models premised on signalling.

Delianidis and Geske (2001) define the residual spread as the observed market credit spread on a given instrument minus the theoretical option based default spread. The option based default spread takes account of the probability of default as well as recovery rates on default. They conducted an empirical analysis of the residual spread on industrial corporation bonds over the period 1991 through 1998. Their analysis considered only non-callable coupon bonds. The company debt profiles were mapped to a single duration adjusted bond that reflected the sum of long term and short term debt obligations. Firm value and firm volatility was recovered by fitting to observed levels for equity price and equity volatility. They observed that the market credit spread was not well described by the theoretical option based default spread and they considered various factors that could determine the observed market credit spread. They found that liquidity (as measured by stock volume) was positively related to the residual spread. High stock volatility was shown to reduce the residual spread as it narrowed the gap between observed and theoretical levels. Stock returns were positively related to the level of the residual spread as increased stock levels implied lower default probabilities.

They postulate that the markets for credit instruments lack liquidity and as such are incomplete. This would result in high hedging costs which would imply wider spreads. The relevance of this observation through time bears consideration. Credit derivatives markets

grew rapidly through the 1990's and have been curtailed following the 2008 global financial crisis.

Frank and Goyal (2003) evaluate the relative importance of 39 factors (including industry leverage, market to book ratio, profits and dividends) in the leverage decisions of publically traded US firms. Their work follows the earlier exercises of Harris and Raviv (1991) and Titman and Wessels (1988) whose results were wholly inconsistent. Their analysis covers data for the period 1950 to 2000 on all US firms with the exception of financial firms and those firms engaged in significant merger and acquisition activity. They found that the pecking order theory is a poor descriptor for the data but that structural models that balance tax shields with bankruptcy costs performed adequately.

Frank and Goyal (2003a) conduct a detailed test of the pecking order theory on a broad cross section of publically traded US firms. They consider data for the period 1971 to 1998 excluding financial firms, utilities and those firms engaged in significant merger and acquisition activity. They find that internal resources are not sufficient to cover investment spending and that external financing is used extensively. Debt is not shown to dominate equity as is posited by the pecking order theory. Equity issuance, in contrast to what would be expected under the pecking order theory, tracks the requirement for external financing closely whilst debt financing does not. They find that the pecking order theory's descriptive performance improves when considering only large firms and data from the early part of their study period.

Welch (2004) argues that the primary determinant of changes in capital structure as described by the debt to equity ratio is stock price returns. He uses annual data for all publically traded US corporations in the period 1962 to 2000. Debt ratio dynamics are evaluated across an array of factors. He finds that corporate issuing activity is relatively high (with significant debt and equity issuance and debt and equity buybacks) but is not related to the pursuit of a fixed debt to equity ratio. The results show that stock returns are the primary determinant of debt to equity ratio changes but Welch concedes that direct and indirect costs of issuance may well dampen the response of managers to changes in the debt to equity ratio.

Gaug, Hosli and Barden (2005) evaluate capital structure choice across a sample of more than 5000 European obligors for the period 1998 to 2000. They test trade-off, pecking order and agency models by way of a panel analysis of firm specific determinants of debt or equity choice. They conclude that neither the trade-off model nor the pecking order model offer a suitable description of capital structure policies and choices. In addition they find some support for the agency model with profitable firms choosing to increase dividends rather than reducing debt.

Anderson and Sundaresan (2000) conduct a high level empirical evaluation of the capacity of firm value based structural models to describe corporate bond prices. They consider a general framework that covers the work of Merton (1973), Leland (1994) and Anderson and Sundaresan (1996). They make use of monthly data on long dated bonds with Standard and Poor's credit ratings of AAA, A and B respectively in the period 1970 to 1996. Using proxies for volatility and leverage they find that their general framework accounts for the majority of the observed movements in historical yields on generic corporate bonds.

Hull, Nelson and White (2004) develop a new approach to implementing Merton's structural model. They use implied volatilities on the firm's shares to estimate model parameters. Given that in Merton's model equity is effectively an option on the firm's assets, options on equity can be valued as compound options. They extend the analysis of Geske (1977), who provided a valuation framework for compound options, to show that the credit spread in Merton's model can be calculated from the implied volatility of two equity options. In assessing the performance of their approach they focus on credit default swaps (CDS) as they target the default element embedded in the credit spread of a corporate bond. Using a relatively short (January 2002 to December 2002) but rich data period they found that their approach outperformed the traditional method of implementing Merton's model in predicting observed CDS spreads.

Eom, Helwege and Huang (2004) conduct an empirical analysis of the performance of 5 distinct structural models of corporate bond pricing. These models encompass both 1-factor and 2-factor variations. They make use of a data set spanning the period 1986 to 1997 of 182 bonds issued by firms with simple capital structures. They excluded financial firms and

utilities and ensured that the instruments considered were non-callable, senior obligations with fixed coupons and principal paid at maturity. The models were used to predict corporate bond spreads as measured relative to constant maturity treasuries. The predicted spreads were compared to observed market data.

All the models generated substantial spread prediction errors. The sign and magnitude of these errors differed across the model set but they found that very low risk bonds produced predicted spreads that were substantially lower than those observed in the market.

Huang and Huang (2002) attempt to answer the following question : "How much of the observed corporate – treasury yield spread is due to credit risk ?". Their approach is to calibrate a range of structural models to historical default experience across both expected default frequency and actual loss given default. Whilst calibration to a consistent data set does not guarantee that the range of structural models considered will produce consistent prediction, they observe that across a large and reasonable span of economic variables the models produce similar estimates for credit risk. They conclude that credit risk accounts for a relatively small proportion (20 % to 30 %) of the observed spread in investment grade bonds and near investment grade bonds, but credit risk accounts for a far larger portion of the observed credit spread in so-called junk bonds.

Schaefer and Strebulaev (2008) argue that whilst structural models of credit risk generally overvalue corporate bonds and provide a poor prediction of bond prices and bond returns they perform well as a predictor of the sensitivity of debt to equity as the hedge ratios produced are consistent with those observed empirically. They find that both the simple structural model of Merton and the more complex structural models of Leland provide good estimates of empirical observations. In addition they highlight that corporate bond returns are significantly related to factors that do not reflect standard measures of credit exposure – namely ratings, leverage and asset volatility, and that default risk accounts for only a fraction of the observed yield spread.

Yu (2005) evaluates the risk and return of the so called "Capital Structure Arbitrage" trading strategy. He argues that the capital structure arbitrageur will make use of a structural model, generally a variant of Merton, to gauge the relative price of Credit Default Swap (CDS)

spreads. High (low) CDS spreads will be sold (bought) and hedged via the equity market where offsetting delta positions would be held in the shares of the obligor referenced by the CDS. He considers the strategy using daily spreads for 5 year CDS's on 261 North American industrial obligors for the period 2001 to 2004. He finds that the individual trades can be very risky but that when trades are aggregated and performance is evaluated on a monthly basis the strategy offers attractive risk adjusted performance and provides returns that are not correlated with equity market and fixed income market performance.

Di Cesare and Guazzarotti (2010) conduct an analysis on how the financial market turmoil post 2007 has changed the way in which credit default swaps (CDS) are priced. They consider a number of factors and determinants in their regression models including a theoretical Merton based credit spread calculation. They assessed a large data set of CDS quotes on US non-financial companies for the period January 2002 through March 2009. The data set is sourced from Bloomberg and is limited to 5 years standardised instruments. They found that the inclusion of theoretical Merton based credit spreads improved the explanatory power of their model and that the importance of equity volatility as a factor was reduced. The contribution of leverage as a factor has increased significantly post 2007.

Ghosh and Cai (2011) evaluate a data set spanning the period 1983 to 2003. They consider whether firms in a given industry adjust their debt to equity ratio over time to an industry norm. This behaviour would support the optimal capital structure argument of the trade-off theory. They find that rather than a single point representing the optimal capital structure, there is a range of debt to equity ratios that are optimal. In addition, within this range they find strong evidence for the pecking order theory, namely that internal resources are preferred to external resources and debt is preferred to equity when external resources are indeed utilised.

De Jong, Verbeek and Verwijmeren (2011) test the static trade-off theory against the pecking order theory focussing on a key difference in predicted behaviour. The static trade-off theory sees firms increase leverage until the target debt ratio is reached whilst the pecking order theory argues that debt will be issued until debt capacity is reached. Their findings show that

the pecking order theory outperforms when new debt is issued but that the static trade-off theory is a superior prediction tool when repurchase decisions are considered.

Smit, Swart and van Niekerk (2003) test the Merton model and the model of Shimko, Tejima and van Deventer (1993) in the context of risky South African debt. They find that of the two models tested the Merton model underperforms when derived credit spreads are compared with empirical data.

Venter and Styger (2008) modify Merton's structural model of default. In their model both assets and liabilities follow geometric Brownian motion where the underlying stochastic processes are correlated. Equity is evaluated as a swap or exchange option. They find that their model provides a reasonable fit to a representative set of South African banking data over the period 1996 to 2006.

Holman, van Breda and Correia (2011) make use of the Merton model to quantify default probabilities of non-financial South African firms. They find weak correlation between the derived Merton default probabilities and those of ratings agencies.

2.4 The synthesis of Theory and Empirical behaviour

As detailed above the empirical evidence offers limited support for the various models of capital structure proposed. A number of authors have revisited the approach taken in the various empirical studies with a view to exposing market features and model factors that have masked the true capacity of the theory to accurately describe behaviour.

Graham and Leary (2011) conduct a review of empirical capital structure research. They evaluate capital structure variation across three dimensions, namely, across firms, across industries and within a given firm over time. They state that much of the research undertaken has focussed on the static trade-off model and the pecking order theory. There has been limited success for both models and that rational explanations for the underperformance include mismeasurement of variables, the impact of leverage on non-financial stakeholders, supply side constraints with respect to capital, the limited value impact of capital structure

variation on firm value across a wide range of leverage assumptions and the impact of financial contracting. They argue that one could interpret the trade-off theory as balancing any number of costs and benefits, including information content, thereby folding the pecking order and trade-off theories into a single frame.

In considering the explanations for the underperformance they highlight the following in each category :

- Mismeasurement of leverage, costs of financial distress, value of tax shields and the implication off balance sheet items coupled with a limited and fragmented market for credit supply;
- Non-financial stakeholders include customers and suppliers (where leverage might be a concern where future service needs are high) and employees for whom high leverage might imply higher risk of job losses;
- Theoretical firm value has been shown to exhibit limited change across a wide variety of leverage assumptions which coupled with high execution costs could limit rapid adjustment to target debt ratios;
- Financial contracting includes the collateral impact of a firms assets as well as the split between tangible and non-tangible elements of the balance sheet.

They conclude that a dynamic trade-off theory that caters for costly adjustment of capital structure offers much promise.

Welch (2011) addresses two specific problems in capital structure research. Firstly he considers the calculation of leverage ratios. He argues for the careful measurement and accurate reflection of items on a firm's balance sheet. A company's assets will be offset by financial liabilities, non-financial liabilities and equity. A simple calculation of leverage as financial liabilities divided by total assets effectively treats non-financial liabilities as equity which can grossly understate the firm's true leverage. In addition he highlights the confusing impact that short term loans and deposits can have on a firm's leverage ratios. For example, short term borrowings funding liquid near cash assets will distort the calculated ratios. Secondly he argues that a simple interpretation of equity issuance as a deleveraging exercise is flawed as it fails to recognise the broad spectrum of activities that drive equity issuance, including some that result in increased leverage.

Chen and Gong (2012) address the observed failure of structural models of default to accurately predict leverage ratios as a function of the level of corporate tax rates. They highlight the traditional outcome that suggests that higher tax rates imply higher leverage ratios, but counter this conclusion by pointing out the reduction in firm value that accompanies the higher tax rate. The combination of higher tax shield benefits and firm value reduction associated with an increase in tax rates gives rise to a non-linear relationship between leverage ratios and tax rates.

Mirza (2011) considers optimal capital structure and default decisions in an environment where firms compete in product markets and underlying asset markets. In his model firm default results in a fire sale of the underlying firm assets (i.e. a forced sale at reduced prices). He finds that the larger the possibility of a fire sale the lower the optimal leverage and the higher the probability of default.

Davydenko (2012) studies whether default is triggered by low market values for firm assets or by liquidity shortages. He defines financial distress as the state where a firm has difficulty honouring its current financial obligations. He defines economic distress as the state where a firm's prospects deteriorate and the value of its business decreases. In general, at default most firms are insolvent both economically and financially, however he documents instances and circumstances where either economic distress or financial distress was responsible for default in isolation (i.e. default triggered by low asset values even when current cash flow needs are met, or default triggered by inability to meet current cash flow needs even when asset values exceed debt obligations). His findings suggest that the assumptions of the early trade-off models (Merton, Leland et al) that consider only asset values as triggers for default and assume frictionless access to equity issuance as a certain source of cash flow should be revisited. He argues that trade-off models that incorporate elements of economic distress and financial distress will provide superior results.

In addition to the academic expansion described above the global financial crisis of 2008 has brought renewed, focussed scrutiny to the field of quantifying and managing credit exposures (see BIS (2010)).

2.5 Conclusion

The field of capital structure research highlighted above spans a variety of theories. We have highlighted numerous and disparate empirical studies and various economic rationalisations for observed behaviour. There is no clear solution to the questions relating to capital structure. We have endeavoured to outline the problem space and to set the scene for the empirical and theoretical work to follow in subsequent chapters.

3 AGL AND BHP

3.1 Introduction

Equity and debt are both claims on the assets of a firm. Debt is generally serviced first up to the contractual obligation with equity enjoying a residual but unlimited share thereafter. We consider it reasonable to expect the performance of both equity and debt to be related to the performance of the assets of the underlying firm. The firm's assets encompass both physical assets and intangible assets. This spans plant and equipment, brand, human capital, know-how and licenses amongst others. In our study we consider the physical assets of a firm to be the ultimate driver of performance of the firm's debt and equity instruments.

The Merton structural model provides a robust and simple conceptual bridge between the theoretical values of a firm's debt and equity instruments and the characteristics of the assets underlying the operations of the firm. Numerous extensions to the conceptual framework, for example Black and Cox (1976) and Geske (1977), have been explored but at the cost of added complexity. As discussed in Chapter 2, empirical analysis of the performance of the Merton model and its successors, when applied to actual market data has been inconclusive with limited explanatory power observed and numerous issues relating to measurement highlighted. We have chosen to make use of a simple expression of capital structure that limits the number of assumptions required.

In this chapter we assess the capacity of the Merton model as a tool to describe market behaviour. We test for statistical and economic significance of the model. Statistical tests

encompass regression and co-integration, while economic significance is evaluated by applying the model to an investment process and considering the resulting returns. The investment process is a pair trading strategy where positions in the underlying equity are offset with opposite positions in the underlying asset. We consider two firms, Anglo American Plc (AGL) and BHP Billiton Plc (BHP), both of which are diversified mining companies. Their financial performance is intimately linked to the prices they receive for the commodities they produce. We expect that the fortunes of both companies are related to the performance of their underlying commodity markets. In this instance we consider the physical assets of each firm to be the basket of commodities they sell. We argue that in both cases, the underlying assets of each of the firms can be reasonably described by the construction of proxy indices that are made up of observable metrics for underlying commodity markets. We consider semi-annual balance sheet data for each of the firms and make use of market prices for equity instruments and debt instruments to generate implied underlying asset value and asset volatility for each of AGL and BHP using the Merton structural model. We evaluate the relationship between the implied measures (asset value and asset volatility) and the value and volatility derived from the proxy indices. In addition, we construct a simple pair trading strategy that will be long (short) the proxy index and short (long) the relevant equity when large relative deviations from the average (of the ratio of the proxy index and the equity) are observed. The trading strategy takes advantage of the expectation that a strong relationship between the proxy index, which represents the underlying firm assets, and the equity will be observed.

3.2 Data

We require data on the equity of the firm, the debt of the firm and underlying assets of the firm. We consider the period 6 January 2006 to 25 December 2015. The period was chosen to encompass the financial crisis experienced in 2008 and the period of significant commodity price weakness in 2015. We make use of weekly closing prices for equity markets, credit markets, US interest rate markets (Treasury bills and notes) and foreign exchange markets. We make use of weekly closing prices for the UBS Bloomberg CMCI Indices (constant maturity commodity indices that we use to construct proxy indices). We make use of balance sheet and income statement information for both AGL and BHP. AGL has a December year end and BHP has a June year end. We have interim financial statements and annual financial

statements for both in June and December each year. AGL and BHP report in US Dollars, with their product lines, namely the commodities they produce, also denominated in US Dollars. All the information is sourced from Bloomberg. Equity data is adjusted for historical splits and spin offs.

We note that AGL generally has a higher level of financial leverage (as expressed by total debt relative to the balance sheet) and that in the period leading up the financial crisis experienced in 2008, AGL held a larger proportion of their debt in shorter dated instruments. We observed relatively larger swings in the prices of the debt and equity instruments of AGL as compared to BHP.

3.2.1 Equity Data

AGL and BHP are listed on a variety of stock exchanges and have a number of lines of equity. Total equity is the sum of these distinct listings (or lines). AGL has a primary listing on the London Stock Exchange (LSE), with all other listings convertible or exchangeable into the LSE line on a one for one basis. Accordingly, AGL equity is calculated as the price of the LSE line multiplied by the total shares in issue converted into US Dollars. BHP is dual listed, with distinct lines on the LSE and the Australian stock exchange (ASX). All other lines are convertible into either the LSE line or the ASX line on a one for one basis. The ASX line and the LSE line are not interchangeable but have identical economic interests (dividends and votes). Accordingly, BHP equity is calculated as the price of the LSE line multiplied by the total shares in issue (of both the LSE line and the ASX line) converted into USD dollars.

3.2.2 Debt Data

Data on secondary market trading of debt instruments for AGL and BHP is limited. We make use of Credit Default Swap (CDS) data to provide summary information on the performance of the credit market. A CDS provides the holder of the instrument with insurance against potential losses on an investment in bonds. In the event of default, the CDS holder has the right to deliver bonds to the CDS writer against receipt of a fixed nominal. This insurance has a fixed term, commands a regular premium and provides cover on a fixed nominal. A CDS is effectively an American option (an option that can be exercised at any time up to its maturity) whose premium is paid over the life of the instrument, where further premium payments are extinguished on exercise (namely default). We source standardised data on 5 year and 10 year CDS markets. Liquidity in these instruments is limited, however daily pricing information is

available. AGL CDS is only available in Euros. We assume that AGL CDS in US Dollars is equivalent to AGL CDS in Euros. This assumption ignores the potential quanto effect in shifting currencies. As discussed in Chan-Lau (2009) this quanto effect is driven by the convertibility risk and the transfer risk between the underlying currency markets, which in this case, Euros to US Dollar, is considered negligible. In addition we note that our interest lies primarily in the changes in CDS levels rather than the absolute values.

We extract two distinct estimates for total debt for each of AGL and BHP from the available balance sheet information. We denote these estimates as D1 and D2. The first (D1) is total liabilities (Bloomberg field BS_TOT_LIAB2). The second (D2) is total liabilities reduced by current assets (Bloomberg field BS_CUR_ASSET_REPORT) and augmented by inventories (Bloomberg field BS_INVENTORIES). The motivation for the choice of both D1 and D2 relates to the measurement issues highlighted in Chapter 2. D1 is likely to exceed formal debt obligations but better reflects the obligations that must be met prior to value being available to service equity holders in the event of default. D2 is an attempt to adjust D1 to account for working capital.

3.2.3 Asset Data

Many commodity markets do not have liquid, transparent price discovery mechanisms for the spot market but enjoy deep and liquid derivative markets. The UBS Bloomberg CMCI indices provide a blended and consistent mechanism for reflecting available prices for baskets of commodities (see UBS (2011) for the detailed index methodology and calculation). The CMCI family of indices covers many sectors, time frames and return profiles. The CMCI indices reflect tradeable market levels across a variety of derivative instruments with the relevant commodities as underlyings. We make use of the total return benchmark indices that encompass Precious Metals, Industrial Metals and Energy. The Bloomberg codes for these indices are **CMPMTR Index**, **CMIMTR Index** and **CMENTR Index** respectively. The data used in constructing the proxy indices is historical information that is available as at the relevant calculation date.

We map each firm to a combination of these indices. AGL and BHP provide a breakdown of divisional assets in their financial statements. These divisions are delineated by underlying commodities. We map divisional Assets information from the AGL and BHP financial statements for the financial years 2006 to 2015 to three distinct groups. Precious Metals includes Gold, Platinum and Diamonds. Industrial Metals includes Iron Ore, Base Metals,

Aluminium, Stainless Steel, Nickel, Zinc and Manganese. Energy includes Petroleum and Coal. On the basis of this categorisation we determine proxy assets for each of AGL and BHP. The proxy weights are adjusted on an annual basis to reflect the latest information available in the financial statements. Proxy index values are calculated as a linear combination of the three CMCI benchmark indices (Precious Metals, Industrial Metals and Energy). We adjust the linear combination parameters every six months (in June and December) to reflect the relevant proxy weights (*w*), as determined from the latest annual financial statements. In the case of AGL, over the period, Precious Metals (*CMPM*) contributed 25% to 36%, Industrial Metals (CM*IM*) contributed 52% to 59% and Energy (CM*EN*) contributed 7% to 21%. In the case of BHP, over the period, Precious Metals (*CMPM*) contributed 3% to 4%, Industrial Metals (CM*IM*) contributed 60% to 66% and Energy (CM*EN*) contributed 31% to 37%. The formulae for the proxy index values for each of AGL and BHP on a given day, *t*, using the applicable weights for each of AGL and BHP in the benchmark indices, are shown below.

$$ProxyAGL_t = w_{PM,i}^{AGL} * CMPM_t + w_{IM,i}^{AGL} * CMIM_t + w_{EN,i}^{AGL} * CMEN_t \qquad (3.2.3.1)$$

$$ProxyBHP_t = w_{PM,i}^{BHP} * CMPM_t + w_{IM,i}^{BHP} * CMIM_t + w_{EN,i}^{BHP} * CMEN_t \qquad (3.2.3.2)$$

where $w_{y,z}^x$ is the weight for company *x* in benchmark index *y* applicable for period *z*.

We calculate an historical volatility series for each of the proxy index values by generating a series of annualised standard deviations on the log changes of the weekly data points using a rolling four week window. The one month historical variability is annualised by assuming that variances are additive over time and are scaled accordingly from one month to twelve months. In effect we are considering the one month historical variability of each of the proxy indices. In applying the Merton model we are evaluating debt and equity by considering the underlying assets and their associated volatility. We use the calculated historical volatility (as described in Hull (2012)) of the proxy index series as an estimate for current volatility of the proxy index series. The choice of a one month period is effectively a compromise, motivated by our requirement to capture current information without suffering the impact of large historical moves. The formulae for the proxy historical volatility for each of AGL and BHP on a given day, *t*, are shown below.

$$PrVolAGL_t = \sqrt{\frac{\sum_{j=1}^{4}\left(\ln\frac{ProxyAGL_{t-j}}{ProxyAGL_{t-j-1}} - \overline{\ln\frac{ProxyAGL_t}{ProxyAGL_{t-1}}}\right)^2}{4}} \qquad (3.2.3.3)$$

$$PrVolBHP_t = \sqrt{\frac{\sum_{j=1}^{4}\left(\ln\frac{ProxyBHP_{t-j}}{ProxyBHP_{t-j-1}} - \overline{\ln\frac{ProxyBHP_t}{ProxyBHP_{t-1}}}\right)^2}{4}} \qquad (3.2.3.4)$$

The proxy index values and the proxy index realised volatility levels are shown below in Figure 3.1 and Figure 3.2.

Figure 3.1 Proxy Index Levels

Figure 3.2 Proxy Index Volatility Levels
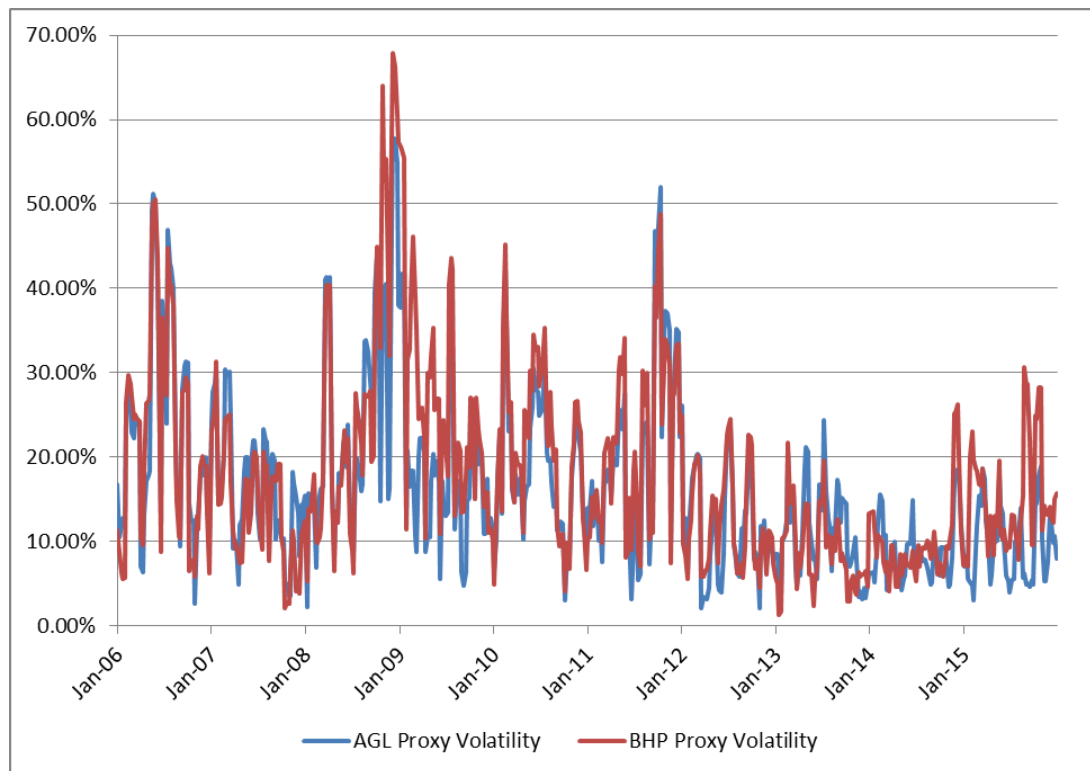
3.3 Methodology

In an efficient market we expect the market value of a firm's assets to be equal to the market value of a firm's liabilities. Let us consider a simple firm funded with non-dividend paying equity and zero coupon debt (nominal and interest payable only at maturity). Merton's (1973) insight allows one to apply a standard option pricing model to solve for the value of equity and debt, given the underlying asset price and the associated asset volatility of a firm. Equity and debt are both claims on the underlying assets. Equity is the residual, if any, after debt has been fully serviced by the underlying assets. Debt is a senior claim on the underlying assets, limited to the face value of the debt obligation. Equity can be modelled as a call option on the underlying asset struck at the future value of the debt obligations. Debt can be modelled as a risk free zero coupon bond coupled with a risky component that reflects the potential for losses, namely that the assets will not be sufficient to fully service the debt obligation. The risky component of debt can be modelled as a put option on the underlying asset struck at the future value of the debt obligations. Merton's (1973) original formulation considered equity and debt instruments in the context of European options, with no intermediate payments. In effect he evaluated default only at the maturity of the underlying debt instrument, which is assumed to be a zero coupon bond.

Expressed mathematically, we have,

$$A = E + D \qquad (3.3.1)$$

where A is the value of the assets of the firm, E is the value of the equity and D is the value of the debt

At the maturity of the debt the value of the equity is given by

$$E = Max(0, A - K) \qquad (3.3.2)$$

where K is the face value of the debt, which by definition is zero coupon and which has value today of D.

Prior to the maturity of the debt we make use of Merton's model for the value of equity, expressed as a call option on the assets of the firm,

$$E = AN(d_1) - Ke^{-rT}N(d_2) \qquad (3.3.3)$$

where

$$d_1 = \frac{ln\left(\frac{A}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \qquad (3.3.4)$$

and

$$d_2 = d_1 - \sigma\sqrt{T} \qquad (3.3.5)$$

where $N(x)$ is the cumulative probability distribution function for a standardised normal distribution and σ is the volatility of the firm assets.

The value of the debt is arrived at by way of put call parity, rearranging terms and recognising that the debt value is equal to the asset value reduced by the equity value

$$E + Ke^{-rt} = P + A \qquad (3.3.6)$$

$$A - E = D = Ke^{-rt} - P \qquad (3.3.7)$$

where P is the value of a put option on the assets A struck at the face value of the debt K.

This approach provides a mechanism for relating asset value and asset volatility to values for equity and debt. In general we have values for equity and debt, which we use to solve for the underlying asset price and the associated asset volatility.

As detailed above in the Data section, we have market data on both equity and debt for AGL and BHP in the form of historical equity values and historical CDS levels.

We consider two distinct formulations when determining the face value of the underlying debt as reflected on the balance sheet (D1 and D2). AGL and BHP have issued a variety of coupon bearing debt instruments. In our model we map these instruments to a single zero coupon debt instrument for each of AGL and BHP. We set the term of this zero coupon bond to match the tenor of the CDS instrument used (5 or 10 years). The face value of this representative zero coupon debt instrument is calculated by grossing up the face value of the coupon paying debt as reflected on the balance sheet (and calculated in D1 and D2) by the risk free interest rate augmented by an assumed credit spread. This approximates the effective coupon paid by the underlying debt instruments. The risk free interest rate used is the then prevailing relevant US Treasury rate (either 5 year or 10 year). We note that post 2008 the choice of risk free rate may differ from the relevant sovereign interest rate.. The credit spread applied is the average of the relevant CDS over the full period under consideration (6 January 2006 to 28 December 2015). The higher the credit spread used, the higher the effective face value of the representative zero coupon debt instrument. The face values of these representative debt instruments are the effective strike prices in our application of the Merton model. Expressed mathematically we have

$$K = D_{BS}(1 + r + CS)^T \qquad\qquad (3.3.8)$$

where K is the face value of the grossed up representative debt instrument, D_{BS} is the face value of the balance sheet debt (D1 or D2), r is the relevant risk free interest rate (NACA, annual effective rate) and CS is the credit spread determined as the average of the relevant CDS and T is the tenor of the relevant CDS.

We model equity as a call option on the underlying assets. The market value for this option is equal to the market value of equity as discussed in the data section above. We model debt as a risk free zero coupon bond coupled with a put option on the underlying assets. This approach, widely applied in the literature and described in Hull, Nelken and White (2004), ignores both the early exercise nature of the CDS as well as the contingent premiums and simply aligns the CDS to a European put option. It is a simplification of the mechanics of the underlying instruments which attempts to capture the essence of their behaviour whilst limiting the complexity of the calculation. We proceed by solving for the premium of the put option. The strike price of both the call option and the put option is set equal to the calculated face value of the debt. The term of the option is set equal to the tenor of the CDS (either 5 years or 10 years). We have assumed zero coupon debt, and as such over time the value of the debt outstanding will increase until the assumed maturity at which point the debt value

equals the face value. A standardised CDS assumes a fixed nominal. To cater for this discrepancy, we calculate the equivalent put option premium by multiplying the CDS level by the average of the initial value of debt outstanding and the nominal of debt outstanding (final value), multiplied by the tenor of the CDS (reflecting the number of CDS payments), present valued to today. This maps the annual payment of the CDS premium as insurance on a fixed nominal to the firm's growing debt obligation. The equivalent put option premium, P, is thus given by

$$P = \frac{(T*CDS*\frac{(D_{BS}+K)}{2})}{(1+r)^T} \qquad (3.3.9).$$

When evaluating options, we assume that the underlying asset pays no dividends, we apply the same risk free rate as that used in determining the face value of the representative debt instruments and we set the term to match the tenor of the CDS instrument used (5 or 10 years).

We solve for the implied asset price (A) by way of Put − Call parity (shown below) as we have the market values for equity (modelled as a call option) and debt (modelled as a risk free zero coupon bond coupled with a put option).

$$E + Ke^{-rt} = P + A \qquad (3.3.10)$$

$$A = E + Ke^{-rt} - P \qquad (3.3.11)$$

We use the implied asset price together with the equity value to invert the Black − Scholes equation to solve for the implied asset volatility numerically (σ).

We proceed by generating implied asset price levels and implied asset volatility levels at each time period, namely weekly for debt levels D1 and D2 and maturity 5 years and 10 years.

3.4 Results

We have four distinct series for each of AGL and BHP. We consider 5 and 10 year terms across two distinct definitions of debt (D1 and D2).

 We will consider the relationship between changes in the proxy asset levels and changes in the implied asset levels, and changes in the historical proxy asset volatility and changes in the implied asset volatility. A direct comparison is not applicable however as there are changes to the quantum of debt and equity through time that must be accounted for (e.g. new equity issuance, new debt issuance, share buybacks or the retirement of debt all impact on the balance sheet value of the company but are unrelated to changes in the underlying asset values). These changes are already reflected in the market data by way of adjustments in the total shares in issue and adjustments to the liabilities on the balance sheet. At each data point we adjust the implied asset levels to reflect the cumulative change in debt (we adjust the face value) and equity (we adjust the total number of shares) from the beginning of the assessment period. In effect we reverse the impact of changes in the structure of debt and equity in an effort to focus on changes in value related to underlying asset price variability only. We then apply log differences to these adjusted implied levels as well as the proxy levels that were generated.

We make use of linear regression models of the form

$$y = \beta_0 + \beta_1 x + \varepsilon \qquad (3.4.1)$$

where y is the dependent variable, x is the independent variable, β_0 and β_1 are equation parameters and ε is the error term.

We regress the log differences of the proxy asset levels on the log differences of the adjusted implied asset levels and consider the correlation between the two series.

We regress the log differences of the proxy asset volatility on the log differences of the implied asset volatility and consider the correlation between the two series.

The results of this correlation study are shown in Table 3.1 and Table 3.2 below.

Table 3.1 AGL correlation study

AGL Asset Correlation			AGL Volatility Correlation	
	5y	10y	5y	10y
D1	0.63	0.52	0	0
D2	0.62	0.51	0.02	0.02

Table 3.2BHP correlation study

BHP Asset Correlation			BHP Volatility Correlation	
	5y	10y	5y	10y
D1	0.69	0.68	-0.03	-0.03
D2	0.69	0.68	-0.03	-0.03

In all cases we find that asset levels are highly correlated but that asset volatility is not highly correlated. In an effort to understand the nature of the relationship between the volatility series we then consider the extent to which the proxy asset volatility and the implied asset volatility are co-integrated. In effect we wish to assess whether there is a long-run equilibrium relationship between the volatility series.

We note that if two time series x and y (both $I(1)$, namely, with variance proportional to time T and as a result the time series is non-stationary) are co-integrated, then a linear combination of them, u, must be stationary ($I(0)$, with a unit root) (more fully described in Engle and Granger ((1987)).

$$y_t - \beta x_t = u_t \qquad (3.4.2)$$

We make use of Engle and Granger's (1987) two step procedure to test for co-integration.

We test the proxy asset volatility and the implied asset volatility time series and report the Dicky – Fuller (Dickey and Fuller (1979)) test statistic in Table 3.3 below.

Table 3.3 AGL and BHP co-integration results

AGL Dicky-Fuller Test Statistic			BHP Dicky-Fuller Test Statistic		
	5y	10y		5y	10y
D1	-7.53	-7.82		-4.71	-4.45
D2	-7.24	-8.24		-6.3	-5.58

At a 95% confidence interval, in all cases we find that proxy asset volatility and implied asset volatility are co-integrated.

We consider the stability of the test results shown above by considering three distinct time periods, January 2006 to December 2009, January 2010 to December 2012 and January 2013 to December 2015. We repeat the tests outlined above (excluding the volatility correlation) on these three periods and reflect the results in Table 3.4, Table 3.5, Table 3.6 and Table 3.7 below.

Table 3.4 AGL Asset correlations across time periods

AGL Asset Correlation	5y			10y		
	2006/2009	2010/2012	2013/2015	2006/2009	2010/2012	2013/2015
D1	0.66	0.7	0.41	0.65	0.71	0.27
D2	0.66	0.68	0.4	0.66	0.69	0.28

Table 3.5 AGL co-integration results across time periods

AGL DF Test	5y			10y		
	2006/2009	2010/2012	2013/2015	2006/2009	2010/2012	2013/2015
D1	-5.68	-4.04	-2.22	-4.83	-4.57	-3.89
D2	-5.69	-3.64	-2.38	-5.3	-4.75	-2.34

Table 3.6 BHP Asset correlations across time periods

BHP Asset Correlation	5y			10y		
	2006/2009	2010/2012	2013/2015	2006/2009	2010/2012	2013/2015
D1	0.67	0.79	0.57	0.67	0.79	0.56
D2	0.67	0.78	0.57	0.67	0.78	0.56

Table 3.7 BHP co-integration results across time periods

BHP DF Test	5y			10y		
	2006/2009	2010/2012	2013/2015	2006/2009	2010/2012	2013/2015
D1	-3.81	-2.42	-1.68	-3.91	-2.27	-1.44
D2	-4.89	-3.23	-0.71	-4.76	-2.82	-1.84

In addition we consider alternate values for the credit spread applied in determining the face value of the representative zero coupon debt instrument. We evaluated credit spread levels significantly below and significantly above the calculated average and found similar results.

In summary we find strong, statistically significant relationships between our proxy asset levels and proxy asset volatility and the implied asset levels and implied asset volatility. However these relationships are not particularly stable when considering sub-periods within the data. The disparity between the proxy asset volatility which is an historical backward looking measure and the implied asset volatility which is a market generated estimate of future variation is noted.

3.5 Trading Strategy

We now consider a simple Pair Trading strategy that makes use of the insights gained in the evaluation above to test whether the observed relationships are of economic significance. Given the fundamental relationship between equity and the underlying assets of a company we expect to observe a strong link between the behaviour of the underlying assets and the behaviour of the equity. An increase (decrease) in underlying asset value should be accompanied by an increase (decrease) in equity value. A control for our experiment would be akin to a simple coin toss exercise which would drive an investment process which would have an expected return of zero. We include a naïve alternate trading strategy as a more relevant comparison. The naïve alternate trading strategy considers the proxy asset levels and the equity levels but ignores any implied asset values. We expect there to be a relationship between equity prices and underlying asset levels. We are evaluating whether applying the Merton model, which incorporates the concepts of leverage and asset volatility, enhances this relationship. We apply the trading strategy for the period 6 January 2006 to 25 December 2015.

In all instances of our trading strategy evaluation we consider transactions in the equity and proxy assets of AGL and BHP, as these are tradable instruments (as distinct from the implied asset levels which are constructs). Recall that the proxy assets are linear combinations of benchmark indices that are made up of tradable derivative instruments. The trading strategy formulation draws heavily on the work of Gatev, Goetzmann and Rouwenhorst (2006) who evaluated a pair trading, relative value investment strategy.

We construct our trading strategy as follows.

We generate a trade ratio series (TRS) such that at each point in time its value is the implied asset level (IAL) divided by the proxy asset level (PAL). We generate a trade average series (TAS) that is the 12 period (approximately three months, given weekly data) mean of the ratio series. We generate a trade variability series (TVS) that is the 12 period standard deviation of the ratio series.

We generate a comparison ratio series (CRS) such that at each point in time its value is the equity level (EL) divided by the proxy asset level (PAL). We generate a comparison average series (CAS) that is the 12 period (approximately three months, given weekly data) mean on the ratio series. We generate a comparison variability series (CVS) that is the 12 period standard deviation of the ratio series.

$$TRS_i = \frac{IAL_i}{PAL_i} \qquad (3.5.1)$$

$$TAS_i = \frac{\sum_{j=1}^{12} TRS_{i-j}}{12} \qquad (3.5.2)$$

$$TVS_i = \sqrt{\frac{\sum_{j=1}^{12}(TRS_{i-j} - TAS_i)^2}{12}} \qquad (3.5.3)$$

$$CRS_i = \frac{EL_i}{PAL_i} \qquad (3.5.4)$$

$$CAS_i = \frac{\sum_{j=1}^{12} CRS_{i-j}}{12} \qquad (3.5.5)$$

$$CVS_i = \sqrt{\frac{\sum_{j=1}^{12}(CRS_{i-j} - CAS_i)^2}{12}} \qquad (3.5.6)$$

The base assumption is that the ratio series (TRS, CRS) are mean reverting and as such over time the ratio series will drift back towards the average series (TAS, CAS). This assumption is premised on the strong correlation and co-integration results noted in section 3.4 above. Given this assumption, we have a buy signal on the ratio when the series is a defined distance below the average series (e.g. 1 standard deviation) and a sell signal on the ratio when the series is a defined distance above the average series (e.g. 1 standard deviation). In both cases we will close positions when the series breaches the average series.

We consider three distinct strategies. In the comparison strategy, trade signals are generated by the control series. In the trading strategy and the adjusted trading strategy, trade signals are generated by the trading series. In the comparison strategy and in the trading strategy the nominal of both the long position and the short position on trade entry are set equal to $ 1 million. In the adjusted strategy the nominal of the equity position is set equal to $ 1 million, however the nominal of the proxy asset position is determined by the calculated sensitivity of the equity to the underlying asset value.

The delta of an option is defined as the change in option value for a given change in the underlying value (see Hull (2012)). In our application of Merton's model we generate a delta for the equity of the firm with reference to the underlying firm assets, where for a given move in the implied asset level we observe some quantifiable but variable move in the equity level. In the adjusted trading strategy we make use of the relevant delta to reflect a larger nominal exposure in the proxy asset. In all cases we assume that trades are undertaken at the closing

prices of the relevant instruments. The impact of the equity delta on the nominal used in the adjusted trading strategy is outlined below.

$$N_{PA} = \frac{\Delta_C * A}{E} \qquad (3.5.7)$$

$$\Delta_C = N(d_1) \qquad (3.5.8)$$

$$d_1 = \frac{\ln\left(\frac{A}{K}\right) + \frac{(r + \sigma^2)}{2}T}{\sigma\sqrt{T}} \qquad (3.5.9)$$

where N_{PA} is the nominal exposure of the proxy asset and Δ_c is the equity delta.

The results of the 3 trading strategies for both AGL and BHP are reflected in Table 3.8 and Table 3.9 below. In each instance we consider the total number of trades undertaken over the period, the number of winning trades, the number of losing trades and the winning trade percentage. In addition we calculate the total revenue generated, the average trade return, the maximum trade return, the minimum trade return and the standard deviation of trade returns. Finally we reflect a Sharpe Ratio (Sharpe (1994)), defined as the average return divided by the standard deviation of returns over the full period. We ignore any dividends paid or received on short or long positions in the underlying equity.

We show a sample for each of AGL and BIL (10 year period and D1 as debt definition) of the detailed summary, where we consider the control strategy, the trading strategy and the adjusted strategy. The adjusted strategy shows the impact of the adjustment in isolation and the combination of the trading strategy and the adjustment.

Table 3.8 AGL 10 year D1 debt definition trading strategy performance

AGL 10y D1

Comparison			Trading			Adjusted			
# Trades	50		# Trades	58		# Trades	58	58	
Win	36	WinLoss%	Win	45	WinLoss%	Win	31	46	WinLoss%
Loss	14	72%	Loss	13	78%	Loss	27	12	79%
Total PnL	1 502 913		Total PnL	2 277 016		Total PnL	250 244	2 527 259	
Max	205 967		Max	320 782		Max	116 224	289 493	
Min	-396 452		Min	-283 097		Min	-52 329	-194 773	
StDev	30 997		StDev	34 402		StDev	10 649	30 315	
Ave	30 058		Ave	39 259		Ave	4 315	43 573	
Sharpe R	0.97		Sharpe R	1.14		Sharpe R	0.41	1.44	

Table 3.9 BHP 10 year D1 debt definition trading strategy performance

BHP 10y D1

Comparison			Trading			Adjusted			
# Trades	62		# Trades	57		# Trades	57	57	
Win	52	WinLoss%	Win	45	WinLoss%	Win	32	46	WinLoss%
Loss	10	84%	Loss	12	79%	Loss	25	11	81%
Total PnL	2 509 176		Total PnL	2 114 381		Total PnL	266 318	2 380 699	
Max	163 050		Max	499 384		Max	80 106	493 281	
Min	-95 080		Min	-139 148		Min	-35 042	-174 190	
StDev	22 919		StDev	30 565		StDev	6 264	30 679	
Ave	40 471		Ave	37 094		Ave	4 672	41 767	
Sharpe R	1.77		Sharpe R	1.21		Sharpe R	0.75	1.36	

We show summary data, encompassing total revenue generated (expressed as PnL) and the Sharpe Ratio across debt definitions D1 and D2 and 5 year and 10 year terms in Table 3.10 and Table 3.11 below.

Table 3.10 AGL Summary trading strategy performance

AGL

	5y						10y						
	Comparison		Trading		Adjusted		Comparison		Trading		Adjusted		
	PnL	Sharpe R	PnL	Sharpe R	PnL	Sharpe R	PnL	Sharpe R	PnL	Sharpe R	PnL	Sharpe R
D1	1 502 913	0.97	2 124 208	1.14	2 566 370	1.6	1 502 913	0.97	2 277 016	1.14	2 527 259	1.44
D2	1 502 913	0.97	1 702 147	1	1 855 051	1.27	1 502 913	0.97	2 306 067	1.21	2 406 328	1.39

Table 3.11 BHP Summary trading strategy performance

BHP

	5y						10y						
	Comparison		Trading		Adjusted		Comparison		Trading		Adjusted		
	PnL	Sharpe R	PnL	Sharpe R	PnL	Sharpe R	PnL	Sharpe R	PnL	Sharpe R	PnL	Sharpe R
D1	2 509 176	1.77	2 133 972	1.18	2 449 080	1.36	2 509 176	1.77	2 114 381	1.21	2 380 699	1.36
D2	2 509 176	1.77	2 401 042	1.36	2 589 800	1.46	2 509 176	1.77	2 315 697	1.59	2 502 599	1.68

In almost all cases, the strategies – comparison, trading and adjusted trading - showed a positive return of at least 200% of nominal over the period. As shown in the summary data above, when considering AGL the trading strategy outperformed the comparison on all measures. However, when considering BHP the trading strategy underperformed the comparison by some measures but outperformed the comparison in some cases in total revenue generated. In all cases the adjusted trading strategy outperformed the trading strategy in total revenue generated, winning trade percentage and Sharpe Ratio.

We considered the sub-period performance across the three trading strategies. In Table 3.12 we show a sample for BHP (5 year period, both D1 and D2 for debt definitions). In general there was limited variability in summary statistics for the three time periods across the inputs for period and debt definition, although the trading strategy and adjusted trading strategy

performed particularly well in the case of AGL in the period 2013 to 2015, which coincides with extreme levels of commodity price volatility and high leverage in the company. Table 3.13 reflects AGL (5 year and 10 year period, D2 for debt definition) and illustrates this point.

Table 3.12 BHP Summary trading strategy performance across sub-periods

BHP 5y, D1						5y, D2					
Comparison		Trading		Adjusted		Comparison		Trading		Adjusted	
PnL	Sharpe R	PnL	Sharpe R	PnL	Sharpe R	PnL	Sharpe R	PnL	Sharpe R	PnL	Sharpe R
2006/2009 729 799	1.27	595 994	0.69	757 079	0.86	729 799	1.27	688 410	0.84	781 223	0.92
2010/2012 763 936	1.94	740 937	1.77	875 456	2.13	763 936	1.94	784 321	2.01	840 446	2.18
2013/2015 1 015 441	2.18	797 041	2.04	816 544	2.25	1 015 441	2.18	928 310	1.98	968 131	2.33

Table 3.13 AGL trading strategy performance - 2013 to 2015

AGL 5y, D2						10y, D2					
Comparison		Trading		Adjusted		Comparison		Trading		Adjusted	
PnL	Sharpe R	PnL	Sharpe R	PnL	Sharpe R	PnL	Sharpe R	PnL	Sharpe R	PnL	Sharpe R
239 295	1.13	417 173	1.41	464 691	1.82	239 295	1.13	674 769	1.96	703 160	2.21

The results of the trading strategy are uniformly positive. One criticism of the approach may be the lack of an out of sample evaluation, however the only input that is dependent on the data in the full period is the credit spread applied in determining the face value of the representative zero coupon instrument. This is a constant throughout the evaluation. We evaluated credit spread levels significantly below and significantly above the calculated average and found similar results. This is to be expected as the trading strategy is not dependent on the calculation of specific descriptive statistics for the data period considered.

3.6 Conclusion

In this chapter we considered the capacity of the Merton structural model to describe market behaviour. We evaluated the Merton model using both statistical and economic measures. We chose two firms whose underlying asset behaviour can be described by tradable market indices. We found strong but unstable statistical support for the Merton model as a descriptor of market behaviour. These results, whilst favourable, were arrived at when considering only two firms and as such the broad application remains untested. We generated superior economic returns when applying the results of our analysis to a trading strategy, with particularly good performance in times of enhanced stress in market and firm conditions. The application of a specific trading strategy provided an alternative measure to that applied traditionally when considering the efficacy of the model.

4 Banking regulation as an application of Merton's structural model : an examination of the failure of African Bank

4.1 Introduction

The business of banking involves the taking of deposits and the making of loans. Banking balance sheets are highly leveraged, with equity capital generally dwarfed by debt capital. Banks are regulated entities but the nature of regulation does not generally consider the market prices of the securities issued by a particular bank. In this chapter we evaluate the capacity of the Merton structural model to provide insight into a bank's financial health in the context of the failure of African Bank in 2014.

4.2 African Bank

African Bank Ltd. was placed in curatorship on 10 August 2014 (SENS (2014 August)). This took place against a backdrop of a significant capital raise in 2013 (SENS (2013 November)) with detailed remedial action undertaken by management (SENS (2013 October))). In March

(SENS (2016 March)) and April (SENS (2016 April)) of 2016 a restructured African Bank emerged from curatorship, with significant losses imposed on creditors and shareholders.

The failure of African Bank was a shock to the market and introduced significant systemic risk in the South African financial system. Could this risk have been highlighted at an earlier stage? Was there a way to avoid this outcome?

4.3 Banking regulation

Banking is a highly regulated industry globally. This level of oversight is justified as these are institutions that take deposits from the public and generally have highly leveraged balance sheets given their capacity to create money via the fractional reserving requirement. Regulation encompasses both supervision of the activities undertaken and monitoring of exposures against prudential guidelines and limits.

Global best practice for supervision, with the United States banking industry as a specific example (see Gilbert, Meyer and Vaughn (2000) and Prescott (2008)), encompasses both on-site examinations and off-site surveillance. Off-site surveillance provides an ongoing impression of bank performance. On-site examinations are the primary supervisory tool during which banks are assessed across various risk and operational factors. These factors are captured in a CAMELS assessment, namely Capital protection (C), Asset quality (A), Management competence (M), Earnings strength (E), Liquidity (L) and more recently the overall financial market system (S). The combination of a given bank's performance across these factors results in a single score or rating being generated for the given bank. Remedial action, where necessary, across the factors will be communicated to the bank concerned and progress on improvements monitored thereafter. CAMELS type assessments are generally infrequent (certainly no more than once per year), with ongoing off-site surveillance incorporating the modelling of likely changes to these ratings.

The Basel Committee on Banking Supervision (BCBS) provide the global regulatory capital framework for the banking industry. BCBS set the standards for prudential regulation of banks via risk based capital measures (BIS (2010), BIS (2013)). The application of these standards is dependent on information provided by the banks, including the application of the bank's internal models and evaluation of risk. These standards are the primary tool of off-site surveillance.

In practice neither the supervision nor the exposure monitoring undertaken was sufficient to prevent African Bank's woes.

4.4 Market prices and bank financial health

An alternative approach to the evaluation of a bank's financial health is to consider information embedded in the market prices of a company's securities, including both debt and equity instruments. This approach draws heavily on the structural model of Merton (1974). In essence, we consider the credit risk (i.e. the likelihood of default) of the bank concerned as a financial put option (i.e. the right, but not the obligation to sell an asset at a fixed price within a predetermined period) on the underlying assets of the firm, where the put is struck at the face value of the obligations of the firm. The equity of the bank is seen as a financial call option (the right, but not the obligation to buy an asset at a fixed price within a predetermined period) on the underlying assets of the firm, struck at the face value of the obligations of the firm.

The measure commonly used is the so-called Distance to Default (DTD), which incorporates the relative leverage of the firm as well as the volatility of its underlying assets. We define default with reference to a fixed time horizon (generally one year in the literature, and in this paper) as taking place when the assets of the firm are insufficient to meet the liabilities of the firm. We follow the approach described in Allen and Powell (2010) and model the underlying assets as a stochastic variable and set DTD equal to the difference between the value of a firm's assets and the value of a firm's liabilities, divided by the standard deviation of the firm's assets (over our given time horizon). In effect, given risky assets, we are quantifying how many standard deviations the firm's assets are above (or in some cases) below the firm's liabilities. Given this measure we can infer a probabilistic expectation of firm failure (defined as liabilities exceeding assets) over our chosen time horizon.

Merton (1977) relates equity value and firm asset value as follows:

$$E = VN(d_1) - e^{-rT}FN(d_2) \qquad (4.4.1)$$

where E is the market value of firm equity, V is the value of the firm's assets, F is the face value of the firm's debt (zero coupon), r is the risk free rate, N is the cumulative standard normal distribution function and T is the time horizon. Furthermore,

$$d_1 = \frac{\ln\left(\frac{V}{F}\right) + (r + \sigma^2/2)T}{\sigma\sqrt{T}} \qquad (4.4.2)$$

and

$$d_2 = d_1 - \sigma\sqrt{T} \qquad (4.4.3)$$

where σ is the standard deviation of asset returns.

Market prices are observable for the securities issued by the firm. The asset value and asset volatility of the firm are not directly observable, however, they can be estimated (as discussed in Milne (2014)).

Asset value is generally modelled as the sum of debt and equity, and asset volatility, σ, can be derived given a value for underlying equity volatility (Milne (2014)).

$$\sigma = \left(\frac{V}{E}\right) N(d_1)\sigma_E \qquad (4.4.4)$$

where σ_E is the standard deviation of equity returns, reflecting underlying equity volatility.

A more complex estimate of asset value and asset volatility can be derived given prices for two securities issued by the underlying firm that can be valued as options on the underlying assets of the firm (e.g. equity as a call option and debt as a combination of a put option and a risk free asset).

Distance to Default (DTD) is defined as follows (Allen and Powell (2010)):

$$DTD = \frac{\ln(V/F) + (r - \sigma^2/2)T}{\sigma\sqrt{T}} \qquad (4.4.5)$$

4.5 Data

African Bank's activities were funded by a combination of debt instruments (both domestic and foreign), preference shares and ordinary shares. All the data we use in this analysis is sourced from Bloomberg, and the raw, unadjusted prices are used for equity.

We consider African Bank Ltd. equity trading data and debt trading data from March 2005, up to August 2014 when the bank was placed into curatorship. We consider so-called price return data only. We consider the curatorship to be a default event.

Our analysis includes the ordinary shares and preference shares listed on the JSE Securities Exchange and the debt instruments issued by African Bank denominated in South African Rand and other currencies. We incorporate published balance sheet information in the form of total liabilities in our estimate of total debt. We make use of closing levels for the one year swap rate; this term is consistent with the one year time horizon applied in the calculation of Distance to Default (DTD).

It is important at this stage to highlight significant weaknesses in the underlying data. Whilst the historical data for equity trading activity and preference share trading activity is representative of market activity on a daily basis, with two-way pricing and significant volumes executed, the data available on listed debt instruments, both domestic and foreign is deeply suspect. No measure of actual trading activity is recorded and any price movements observed appears to be with reference to a change in an underlying benchmark. In the case of domestic debt instruments the mark to market process incorporates a spread to a chosen underlying benchmark (or companion bond). In effect the credit spread reflected in the historical prices is more akin to that at the time of issue rather than the prevailing level. We include this data in parts of our analysis below, however, the results are not satisfactory, in that they suggest that either the debt market was oblivious to the events unfolding at African Bank, or that the prices did not reflect market reality. Neither conclusion casts a favourable light on market practice.

4.6 Methodology

We calculate a Distance to Default on a daily basis, given levels for asset value and asset volatility. We consider data for the period March 2005 to August 2014, where available. We have data for equity value (both ordinary shares and preference shares), realised equity volatility and debt value. We solve for asset value and asset volatility in three distinct ways using the equations above as detailed below.

In assessing the output we consider a DTD value of one or below to be a significant warning signal as it indicates a high probability of distress in future - in effect there is a one third chance that the asset value will drop below the value of the debt obligations at the one year time horizon.

In the first instance (DTD1) we follow the literature (Allen and Powell (2010)) and solve for equity as a call option on the assets struck at a level that takes account of total liabilities and the total quantum of preference shares issued. Balance sheet liabilities are grossed up for the one year term at a rate equivalent to the one year swap rate plus a spread commensurate with African Bank's funding costs over the period. This spread is set to 250 basis points which is consistent with the average spread paid by African Bank on its DMTN programme (SENS (2013 June), SENS (2013 March)). The grossed up balance sheet liabilities are denoted as X. Preference share nominals are grossed up for the one year term at a rate equivalent to the cash yield on the nominal of the instrument prevailing at the time. The grossed up preference share nominal is denoted by Y. Asset volatility is calculated as per Milne (2014), namely realised equity volatility adjusted for firm leverage (as per equation 4.4.4 above)

In the second instance (DTD2) we solve for asset value and asset volatility as two simultaneous equations in two unknowns, where we are given values for equity and preference shares. Equity is valued as above (in DTD1), however, the preference shares are valued as a call spread (i.e. simultaneous purchase and sale of an equal number of call options that differ only in their strike price) on the assets of the company, where the two strikes are [X] and [X+Y], in effect the preference share's claim to the assets of the company are limited by the prior claim of the balance sheet liabilities and the nominal outstanding of the preference shares.

In the third instance (DTD3) we solve for asset value and asset volatility as two simultaneous equations in two unknowns, where we are given values for equity and debt. We make use of a single debt instrument and imply a Credit Spread from the traded price of the debt

instrument. This credit spread is then interpreted as a simple one year put option on the assets of the company struck at [X]. We make use of the listed ABL10A bond. It was issued in March 2010, carried a coupon of 11.5% and was due to mature in March 2015. The ABL10A bond was benchmarked on issue against the R201 bond issued by the South African government (coupon of 8.75% maturing in December 2014). The choice of this bond is motivated by the tenor of the instrument during the period under review. We recognise that the positive yield spread of the ABL10A bond over the relevant government issued benchmark instrument incorporates more than a premium for default risk, including instrument liquidity and investor duration preferences. However, our focus is on the changes in value of equity and debt, and as such we do not adjust the prices of the bond to reflect any other information.

4.7 Results

The results of applying the three distinct methods are shown below. In assessing these results we wish to understand whether the application of Merton's model provided any forewarning of the future failure of African Bank. In effect we ask ourselves if the changes in the prices of debt and equity provide insight into the changes in the values of the underlying assets and their volatility.

To illustrate the interaction between Asset Value, Equity Value, Asset Volatility, Equity Volatility and Distance to Default, we graph Equity Value against Asset Value and Distance to Default against Asset Volatility and Equity Volatility. The general behaviour of Equity Value and Asset Value is consistent across DTD1, DTD2 and DTD3 and as such we show only the output for DTD1 in Figure 4.1 below (where asset value is reflected on the left hand axis and equity value is reflected on the right hand axis). We note that equity value changes are generally an amplification of changes in asset value. We show the graph of Distance to Default against Asset Volatility and Equity Volatility for each of the three approaches as the differences in applying these approaches are evident in these individual charts. We briefly describe the behaviour for each of DTD1, DTD2 and DTD3 above each chart and we show the key differences in the three approaches in the table below, highlighting values as at the date of the capital raised by African Bank in 2013 and on the days leading up to and including curatorship.

Figure 4.1 DTD1 Values
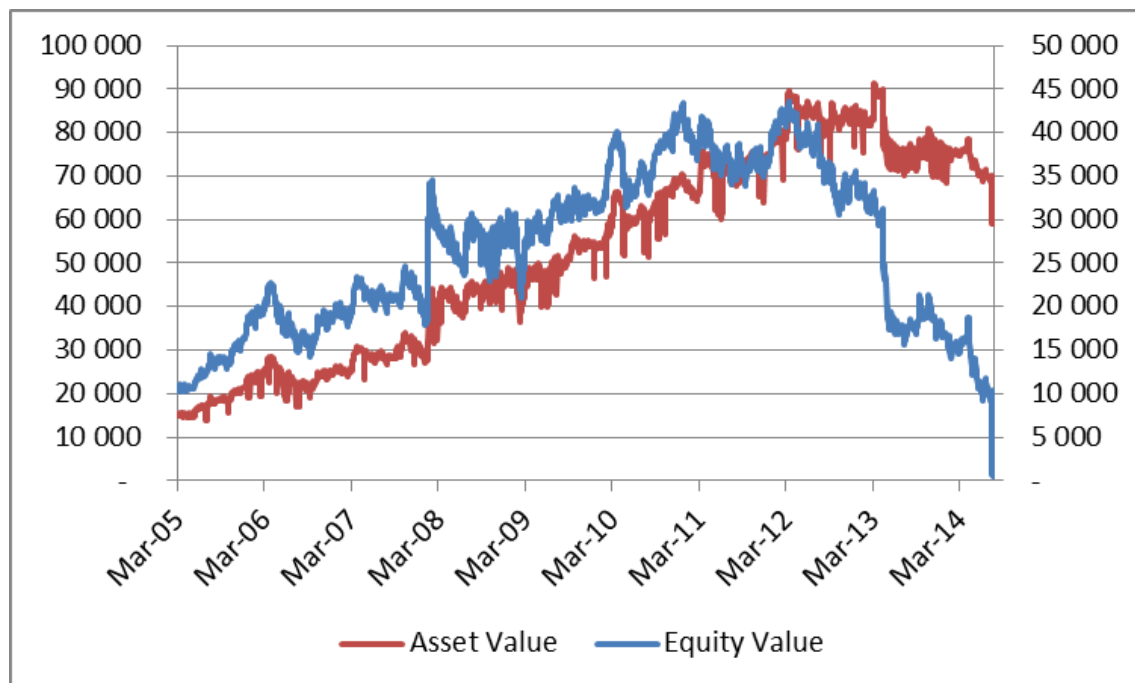
In Figure 4.2 we show the case of DTD1. DTD is reflected on the left hand axis and asset volatility and equity volatility is reflected on the right hand axis. We note that the DTD peaks above eight and declines to zero when the bank is placed in curatorship. DTD is fairly volatile, dropping to two at the peak of the global financial crisis in 2008 but recovering dramatically thereafter. DTD drops below one only in the days prior to default. In the case of DTD1, of particular interest is the behaviour of asset volatility which declines to a level of approximately 10% by the end of July 2014, in spite of the obvious uncertainty prevailing at the time. Asset volatility drops further as African Bank moves into curatorship. This is a direct result of using realised equity volatility to determine asset volatility, where asset volatility is linked to equity volatility by a leverage ratio. In this case the increase in equity volatility is more than offset by the observed increase in leverage. To be clear, we would expect an increase in asset volatility as a result of the very large observed changes in the underlying asset price.

Figure 4.2 DTD1 Volatilities
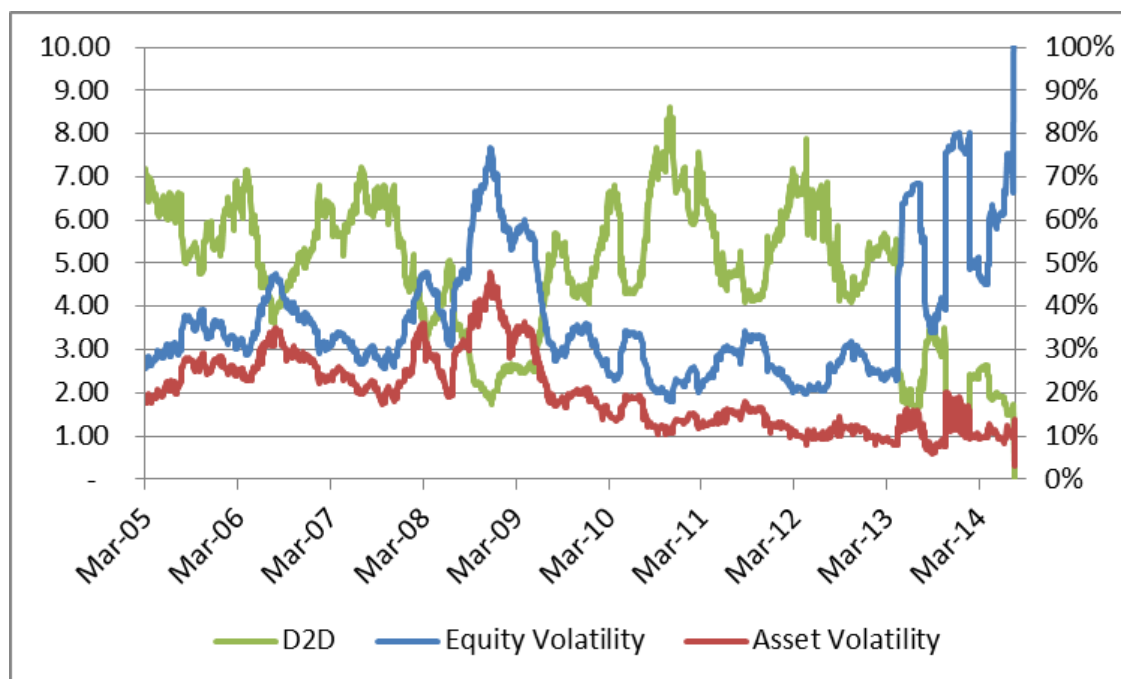
In Figure 4.3 we show the case of DTD2. DTD is reflected on the left hand axis and asset volatility and equity volatility is reflected on the right hand axis. We note that the DTD declines from a peak of approximately six to close to zero when the bank is placed in curatorship. DTD first drops below one in 2008 in the grips of the global financial crisis, recovers somewhat in 2010, only to drop below one again in 2012 and remain there until default. The calculated DTD suggests financial stress from the initiation of the global financial crisis all the way through to the bank being placed in curatorship.

Figure 4.3 DTD2 Volatilities
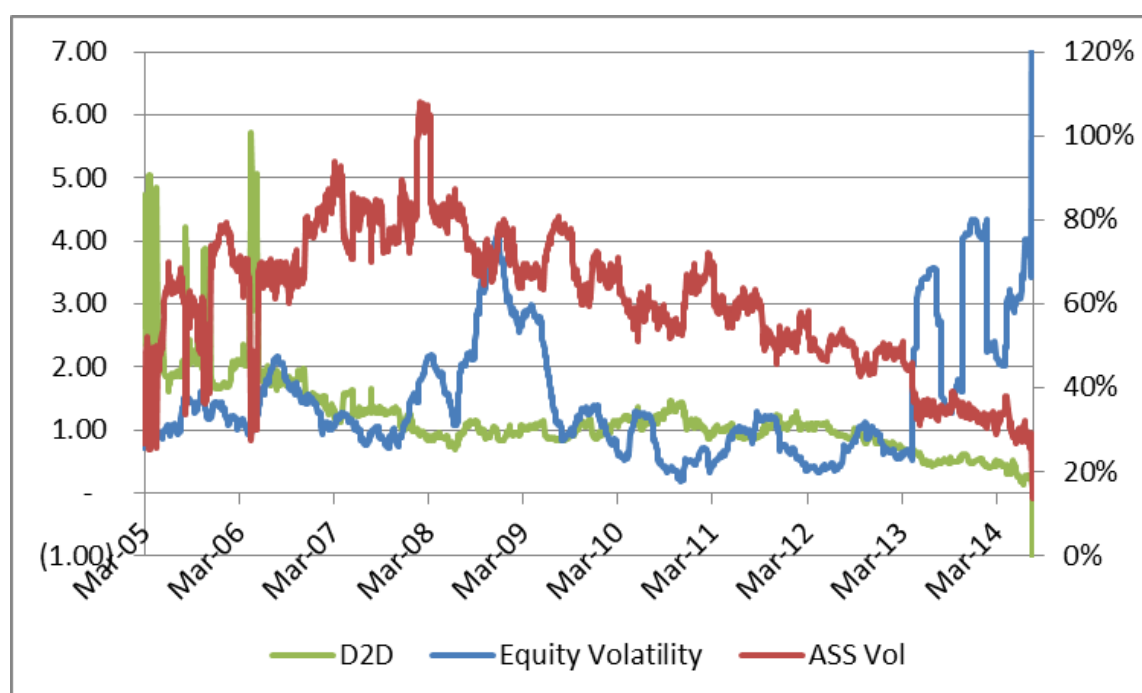
The general behaviour of DTD in DTD2 suggest financial weakness much earlier than in DTD1, however, the results are marred by the erratic behaviour of the calculated variables, namely asset volatility and DTD. Calculated asset volatility is in excess of 50% for much of the period analysed. DTD does not follow the significant improvement in equity value after 2008 and tracks close to one all the way to default. This can be ascribed to the somewhat volatile price history of the associated preference shares and our assumption that the price of the preference share is driven primarily by changes in the underlying asset value and asset volatility. We note that so-called perpetual non-cumulative non-participating preference shares as a Tier 1 capital raising exercise for banks in South Africa enjoyed much popularity prior to 2006. These instruments suffered a significant price decline as an asset class (i.e. across all issuing banks) in the period up to 2011 (see Figure 4.4 below). This decline coincided with the introduction of a dividend tax by the South African Revenue Services. In an effort to adjust for this behaviour we consider a shorter period for DTD2, namely January 2012 to August 2014, and we normalise the price series for the preference shares such that the initial value in January 2012 is set to par. In Figure 4.5 we show the case of adjusted DTD2. DTD is reflected on the left hand axis and asset volatility and equity volatility is reflected on the right hand axis. In this instance we note that the DTD declines from a peak above five in January 2012 to below zero just prior to the bank being placed in curatorship. DTD first drops

below one in June 2013, around the announcement of the rights issue in August 2013 (finalised in November 2013 (SENS (2013 November)) and remains there until default. We note that asset volatility in both iterations of DTD2 are higher than those of DTD1, with significant increases observed by the end of 2012, however, they too show a decline in asset volatility as we move towards August 2014 and curatorship.

Figure 4.4 ABL preference share vs. average of other bank issues
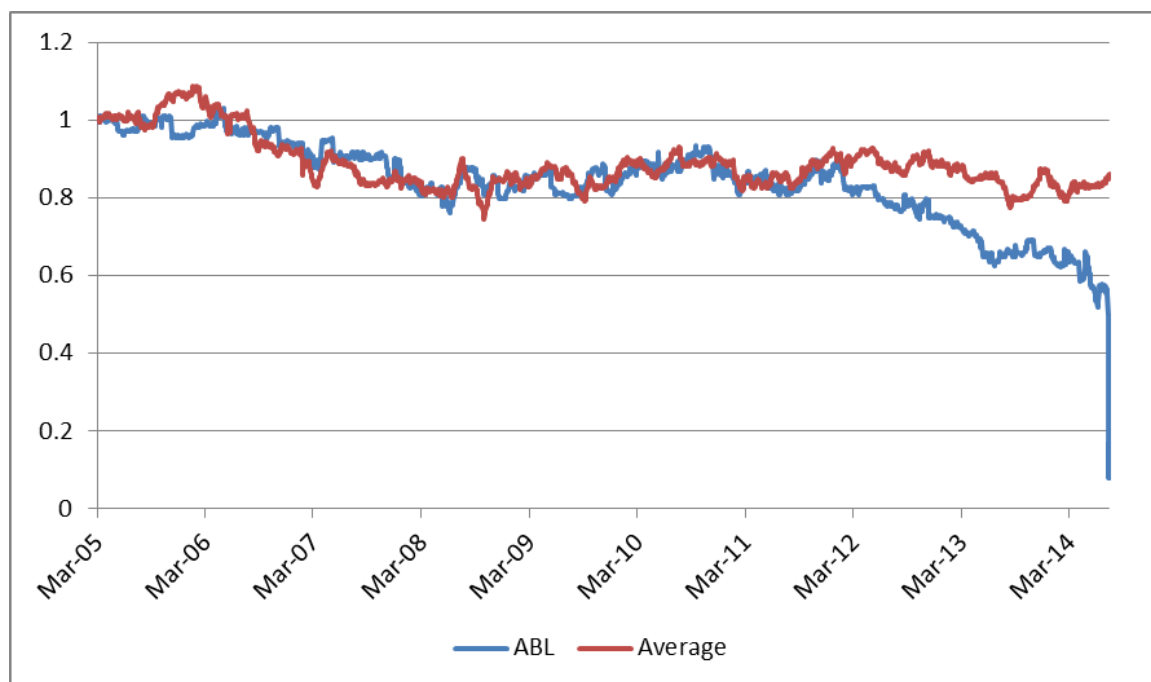
Figure 4.5 DTD2 Volatilities (adjusted)
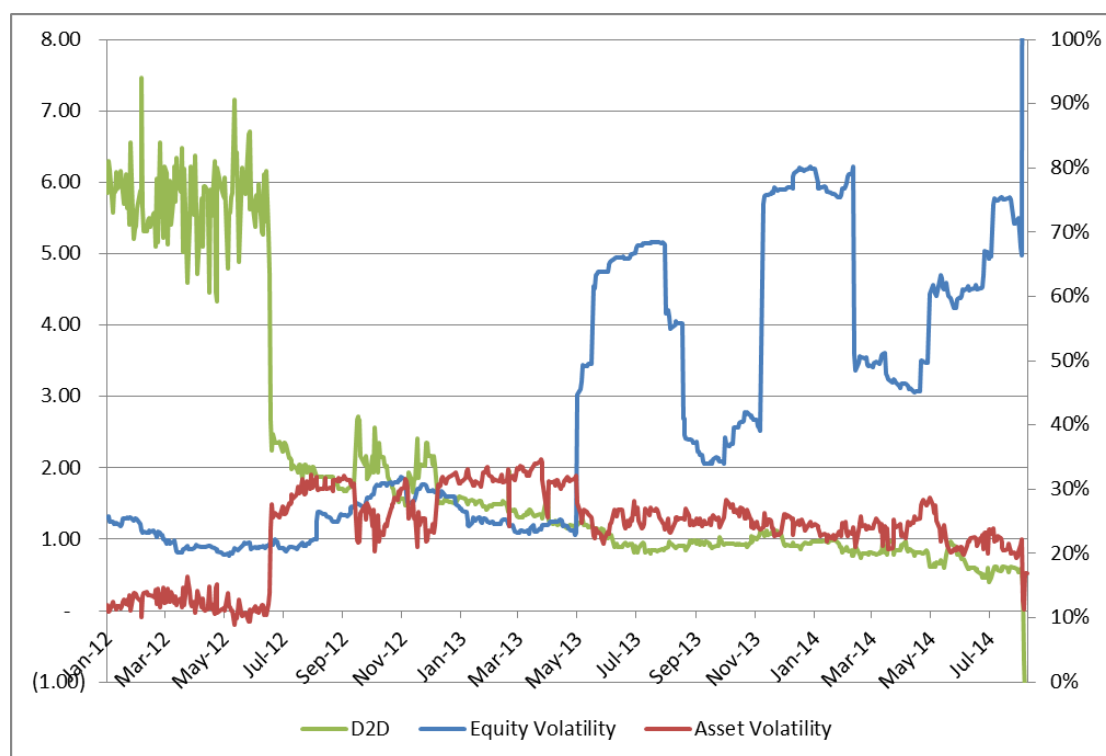
In the case of DTD3 we consider the period March 2010 to August 2014. With reference to Figure 4.6 (where bond spread is reflected on the left hand axis and equity value is reflected on the right hand axis) we note that the mark-to-market spread over the reference bond declines from a level of approximately 320 basis points at issue to 200 basis point in August 2014 when African Bank is placed in curatorship. In effect the marked credit spread is lower at default than at issue. There appears to be limited actual trading that results in adjustments to the effective mark-to-market as the spread to the benchmark bond is broadly constant for extended periods of time. In Figure 4.7 we show the case of DTD3. DTD is reflected on the left hand axis and asset volatility and equity volatility is reflected on the right hand axis. As a result, we note that the DTD is just above one in March 2010 and drifts down from May 2013 to be just below zero on default. The observed credit spread and the resultant calculated values for DTD are incongruous with the events unfolding at the time. In the case of DTD3, almost perversely the asset volatility is seen to move dramatically lower over the period. This

is a direct result of the effective credit spread declining whilst leverage increased and the realised risk in the underlying asset ballooned.

Figure 4.6 Bond spreads vs. Equity value
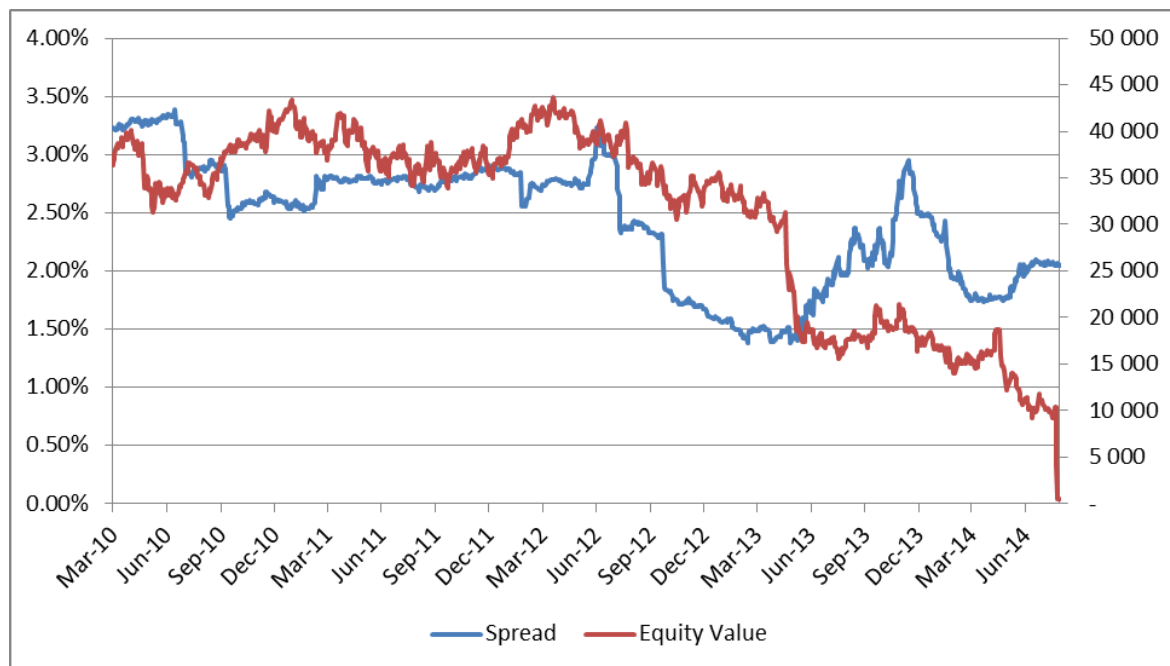
Figure 4.7 DTD3 Volatilities
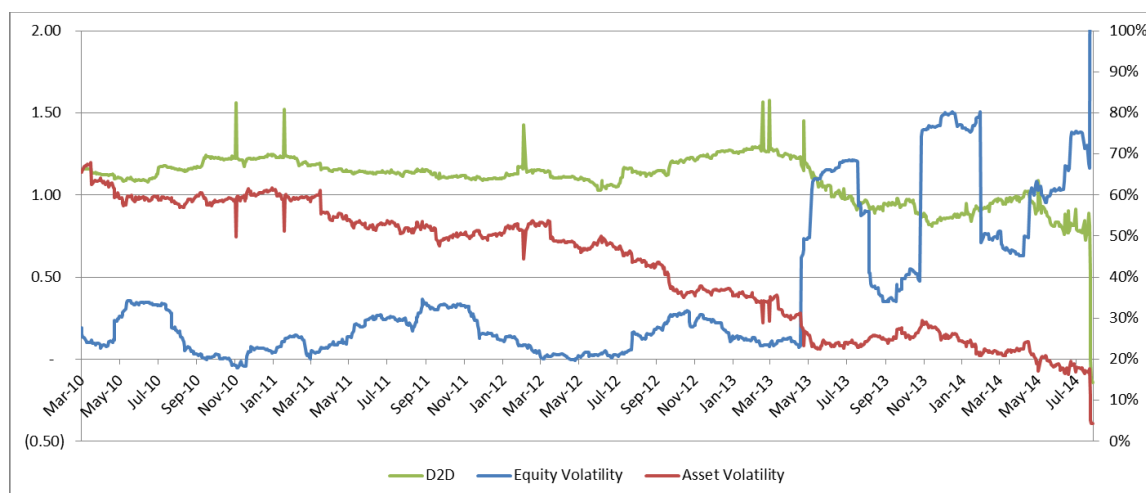
Table 4.1 Equity volatility, Asset volatility and DTD

Date	30-Dec-05	31-Dec-07	31-Dec-08	31-Dec-10	31-Dec-12	03-May-13	31-Jul-13	31-Dec-13	31-Jul-14	07-Aug-14	08-Aug-14
DTD1											
Equity Value	36%	32%	70%	23%	27%	45%	68%	80%	72%	387%	397%
Asset Value	28%	22%	43%	14%	11%	12%	12%	19%	10%	5%	3%
Distance to Default	5.38	5.48	2.03	6.83	4.85	2.7	1.69	1.38	1.59	0.04	0.26
DTD2											
Equity Value	36%	32%	70%	23%	27%	45%	68%	80%	72%	387%	397%
Asset Value	74%	76%	77%	64%	49%	40%	34%	34%	26%	14%	20%
Distance to Default	1.7	1.25	0.83	1.13	0.81	0.65	0.45	0.52	0.26	-0.84	-1.32
DTD2 (Adjusted)											
Equity Value					27%	45%	68%	80%	72%	387%	397%
Asset Value					31%	28%	25%	24%	19%	11%	17%
Distance to Default					1.6	1.2	0.85	0.97	0.58	-0.7	-1.22
DTD3											
Equity Value				23%	27%	45%	68%	80%	72%	387%	397%
Asset Value				61%	37%	28%	23%	26%	17%	5%	4%
Distance to Default				1.23	1.27	1.18	0.91	0.87	0.76	0	-0.12

In Table 4.1 we show equity volatility, asset volatility and DTD for each of the methods applied at regular intervals across a wide term with particular focus on periods of significant change in the capital structure of African Bank.

In all of the approaches detailed above the market data shows a wide variation in the financial health of African Bank over the period, covering periods of no stress (with Distance to Default in excess of four) to periods of potential stress (with Distance to Default less than two). The introduction of the African Bank preference share into the analysis shows a wider variation of output. The recovery of financial health from the stresses associated with the Global Financial Crisis in 2008 proved to be short lived. The adjusted DTD2 approach shows that the market was highlighting significant risks of financial stress as early as December 2012. The DTD3 approach shows very little variability in the financial health of African Bank over a period of excessive volatility and significant corporate activity. This can be attributed to the static spreads applied to the closing prices of listed African Bank debt which did not reflect the fundamental changes in the financial health of African Bank.

We believe that the analysis above suggests that the consideration of market data provides significant explanatory and predictive output regarding the potential for future bank failures. As is the case with all analysis, the quality of the data is key. Closing prices in the absence of arms-length transactions between market participants do not reflect market prices and should be treated with suspicion.

4.8 Conclusion

The Merton structural model and the Distance to Default measure provide valuable insight into the financial position of a bank. These measures should be incorporated in the regulatory process undertaken on banks and should be an essential component of the toolkit for any financial market professional evaluating the performance and standing of a bank.

Traditional regulation and supervision was not able to prevent the failure of African Bank. A considered evaluation of market prices offered an early warning signal for impending distress; however, not all market prices are of equal quality and some are categorically suspect. The equity and preference share markets for securities issued by African Bank showed significant weakness and stress in the period leading up to the effective default of the institution. This feature was not mirrored by the closing price data available for the debt instruments issued by African Bank. It is our belief that the closing prices on these instruments did not reflect market conditions at all, and as such they offered almost no value to the financial markets. The investment process encompasses a wide range of data evaluation and information assembly, which process is undermined by the quality of the data available for debt instruments. The prices available must capture the market or they should not be made available at all.

The results expressed above reflect only a single instance of default, namely the failure of African Bank, and as such the broad application of the approach remains untested. However we are firmly of the view that there is merit in making use of market prices of debt and equity instruments to assist in the evaluation of the financial health of a bank.

5 BEYOND MERTON : THE S MODEL - A DISCRETE, DYNAMIC STRUCTURAL MODEL OF FIRM CAPITAL INCORPORATING LIQUIDITY

5.1 Introduction

The purpose of this chapter is to define our structural model of firm capital (the S Model). In the following chapter we evaluate model behaviour across a wide range of inputs and assumptions. The model recognises that a firm's operations are driven by cash receipts and cash payments. These cash flows, be they dividends, coupons or revenue, impact on the cash holdings of a firm. We define the net position of a firm's cash holdings as firm liquidity. The model incorporates firm liquidity and provides for adjustments to leverage by way of subordinated debt and equity. The model is expressed in discrete time and is delivered in a binominal tree framework following the work of Cox, Ross and Rubinstein (1979).

Merton (1974) derives solutions for corporate debt using a simplified expression of firm structure and limited capital structure dynamics. We extend Merton's conceptual framework

of the firm as well as expanding on capital structure dynamics. We extend the conceptual framework by allowing for coupon payments, tax benefits between debt and equity and incorporating bankruptcy charges. These extensions span similar work undertaken by Black and Cox (1976), Brennan and Schwartz (1978) and Leland (1994). We expand on capital structure dynamics by focussing on firm liquidity as a key determinant of firm value and of firm behaviour. The introduction of firm liquidity mirrors aspects of the work of Anderson and Carverhill (2012). We include subordinated debt issuance and equity issuance as a means for the firm to access cash resources. In addition we consider costs in the context of equity issuance as well as relaxing the requirement for absolute priority on bankruptcy, namely that on bankruptcy all debt obligations may not be fully serviced before equity receives any value.

5.2 Conceptual Outline

It is instructive to set out the basic conceptual framework of our model prior to a formal presentation.

We assume a firm with a given risky asset base can be financed by a combination of equity, fixed long term debt, variable short term debt and variable subordinated debt. The asset base follows a stochastic process, is indivisible and is consistent with Modigliani and Miller (1958), namely that its value is invariant to the capital structure of the firm. We set the time horizon for evaluation equal to the term of the fixed long term debt. The asset base generates cash periodically, where the cash generated is a fixed yield on the prevailing asset price. The asset base incurs operating cash costs periodically, where the cash costs incurred may be a fixed amount or a fixed yield on the prevailing asset price or a combination of both. In addition to the risky asset generating periodic cash flows, the firm may have either a positive or a negative cash balance (but never both), in the form of short term cash investments or short term debt. We recognise that in practise firms may hold both short term debt and short term investments but for the purposes of our model we consider the net position only. Positive cash balances (short term investments) generally earn interest at a rate lower than that incurred on negative cash balances (short term debt). Short term debt is limited and is assumed to be risk free (i.e. on default the company is required to settle short term debt obligations first and in full). All payments, be they coupons, dividends or operating costs must be serviced out of the firm's cash balance (which as described above may be positive or

negative). In the event that there is insufficient cash capacity to meet payments (i.e. the short term debt exceeds the limits imposed) then the firm may source additional cash resources. Cash may be generated by issuing subordinated debt up to a defined limit and equity thereafter. Subordinated debt issuance is limited by a positive net worth constraint (i.e. at the instant of subordinated debt issuance the assets of the firm must be sufficient to settle all debt obligations, both senior and subordinated). Equity issuance is limited by bankruptcy provisions and incurs additional costs. Debt coupons, be they senior or subordinated may be determined by firm leverage or may be fixed. Bankruptcy is considered at the maturity of the fixed long term debt and at each periodic payment date prior to maturity. At the maturity of the fixed long term debt, default is triggered if the firm assets, adjusted for the cash balance, are unable to meet the requirement to settle debt obligations. At each periodic payment date, default may be triggered in one of two ways. In the first instance, if calculated equity is zero, or in the second instance, if the firm assets, adjusted for the cash balance, are unable to meet the requirement to settle debt obligations. On default the firm assets suffer a dead-weight bankruptcy charge. Default proceeds are distributed according to seniority of claims, with short term debt serviced first and thus rendered risk free. In general absolute priority of claims is observed, however this assumption can be relaxed providing equity with some value on default. Excess cash balances can be distributed by way of subordinated debt repurchases or special dividends or can be held by the company in perpetuity. Potential tax benefits, taking account of the differential tax treatment of interest and dividends (coupons being tax deductible and dividends being after tax payments), may accrue on net interest payments made.

5.3 The Model

We model total firm assets, W, as a combination of a risky asset, V, that pays a fixed net yield, k, coupled with a deterministic short term cash balance, C, that may be positive or negative.

$$W = V + C \quad (5.3.1)$$

The fixed net yield, k, reflects the gross yield paid by the firm assets reduced by any fixed operating costs calculated as a yield on the firm assets. The short term cash balance collects the fixed net yield paid by the risky asset and is impacted by debt coupon payments (senior and subordinated), dividend payments, subordinated debt issuance and buyback, special dividends and equity issuance.

Our model proceeds by first building a tree of asset prices. We then move forward through the tree by way of forward induction, taking account of all cash flows and their consequences (be they coupons, dividends, taxes etc.). We conclude by rolling back through the tree, from the leaf nodes, considering bankruptcy and calculating values for debt and equity.

Throughout the formal presentation below we provide a blueprint that illustrates the core working of the model and acts as an introduction to the mathematical presentation that follows in the various sections. The blueprint is not referred to in the text.

5.3.1 Asset Process

In this section we describe the way in which we model the behaviour of the risky asset, V, as well as how we construct the binominal tree that provides the underlying structure for our model.

In considering the behaviour of the risky asset V, we follow Hull (2007) and assume that the return of the risky asset V follows a generalised Wiener process

$$\frac{dV}{V} = \mu dt + \sigma dz, \quad (5.3.1.1)$$

where the expected return (or drift) is given by μ, the expected variance is given by σ^2 and dz is a Wiener process

We have chosen to model the firm as a combination of a risky asset that generates regular cash flows coupled with a dynamic but deterministic cash holding. This approach is distinct from that followed by Anderson and Carverhill (2012), who model firm value as a function of

risky cash flows. In essence we are concerned with both the asset value and the cash balance as opposed to only the cash balance.

We model the asset price, *V*, as a multiplicative, recombining binomial process over discrete time periods. In this approach we divide the time horizon for evaluation, *T* (where *T* is equal to the term of the senior debt), into a large number of small time steps of length *Δt*. Over each period the asset price, *V*, may move to one of two values. It may move up to *V x u* with probability *q*, or it may move down to *V x d* with probability *(1-q)*.

V x u, with probability *q*

V

V x d, with probability *(1-q)*
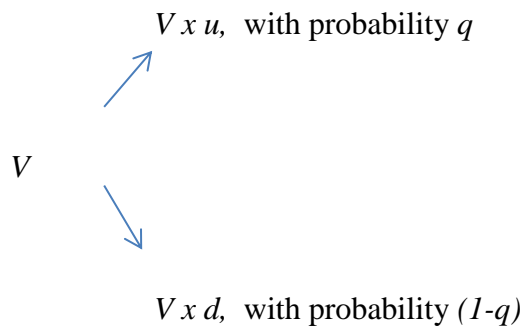
We assume a constant interest rate, *r*, over the time horizon *T* and we require that

$$d < e^{(r-k)\Delta t} < u \qquad (5.3.1.2)$$

to eliminate simple arbitrage. In the absence of this requirement the asset could generate returns (including the fixed net yield) that were either always above the risk free rate or always below the risk free rate. This could facilitate simple arbitrage, for example if both *u* and *d* were above $e^{(r-k)\Delta t}$ then the asset *V* would generate a return in excess of *r-k*.

In a risk neutral world the expected total return of *V* over each period *dt*, *μ*, would be equal to *r*. As described above, *V*, pays a fixed return, *k*, and as such the expected value of *V* should grow at *r-k*. We follow Cox, Ross and Rubinstein (1979) and consider the risk neutral probabilities *p* and *(1-p)* which are distinct from the actual probabilities *q* and *(1-q)*. The model allows for a subjective estimate of total return in any given period, however this would result in the output no longer being consistent with risk neutrality (see Chance (2010) for a discussion on risk neutral probabilities)..

Accordingly, we have the following formulae for the expected return and variance of our multiplicative binominal process.

$$pVu + (1 - p)Vd = e^{(r-k)\Delta t}V \qquad (5.3.1.3)$$

$$pu + (1 - p)d = e^{(r-k)\Delta t} \qquad (5.3.1.4)$$

$$\left(ln\left(\frac{u}{d}\right)\right)^2 p(1 - p) = \sigma^2\Delta t \qquad (5.3.1.5)$$

The choice of *u,d* and *p* is not trivial and is considered in detail in Chance (2010), where a number of alternative models are evaluated in the context of maintaining no-arbitrage conditions. We make use of Chriss's model (1996) in determining values for *u,d* and *p*. This model is chosen as it correctly preserves no-arbitrage conditions and recovers the required volatility for any number of times styeps.

$$u = \frac{2e^{(r-k)dt+2\sigma\sqrt{dt}}}{e^{2\sigma\sqrt{dt}}+1} \qquad (5.3.1.6)$$

$$d = \frac{2e^{(r-k)dt}}{e^{2\sigma\sqrt{dt}}+1} \qquad (5.3.1.7)$$

$$p = \frac{1}{2} \qquad (5.3.1.8)$$

We set the number of steps, *N*, equal to *T* divided by *Δt*. By definition we have that the leaf nodes coincide with debt maturity and the final cash flows (the sale of the asset, redemption of the debt to the extent possible with the residual, if any, available for equity).

Blueprint 5.1 : Asset Process

The asset tree recombines and as such asset prices (V) at the next level in the tree are either a move up or a move down from the current asset price.
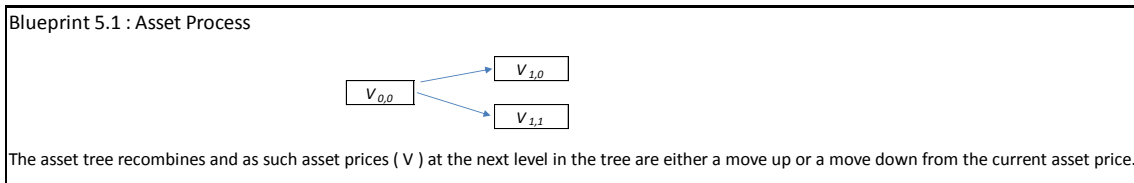
We proceed by constructing a tree of asset prices. At each level (i) of the tree we have nodes (denoted j, running from 0 to i). The value of the asset that corresponds to node j at level i on the tree is given by

$$V_{i,j} = V_0 u^j d^{i-j} \quad i = 1,2,\dots N; j = 0,1,\dots i \tag{5.3.1.9}$$

where V_0 is the initial level for the asset.

The tree of asset of prices described above provides the structure for our model, as all cash flows and capital structure decisions are tied to the nodes in the tree.

5.3.2 Cash Process

In this section we describe how we deal with cash and all its related consequences in our model. We detail how cash is described, how cash flows are incorporated, how interest on cash balances is calculated, how cash shortfalls and cash excesses are dealt with and how differences between the taxation of cash interest and cash dividends are incorporated.

We move forward through the tree, taking account of all cash flows and their consequences.

Blueprint 5.2 : Cash Process

The cash available (C) at any given node is a function of the cash levels at prior nodes.
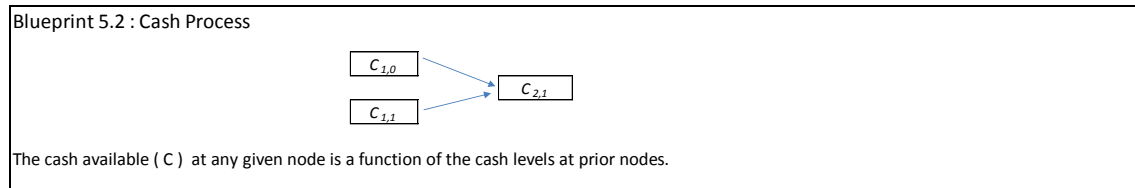
We recall from above that the total firm assets, W, are a combination of the risky asset, V, and the short term cash balance, C.

Formally, we define C as

$$C = VC + TC \tag{5.3.2.1}$$

where VC *(Asset Cash)* reflects the cumulative cash value of the fixed yield paid by the risky asset and TC *(Transmission Cash)* reflects the current total of all other cash flows to date (coupons, dividends, debt, equity), be they payments made or payments received. The split of C into VC and TC is cosmetic and facilitates easier explanation.

In addition to collecting the fixed yield paid by the risky asset, all cash flow impacts the short term cash balance C. To simplify our analysis, without any loss of generality, we assume that all cash flows other than the fixed yield from the risky asset only occur periodically on so-called coupon dates.

Figure 5.1 Basic nodal structure
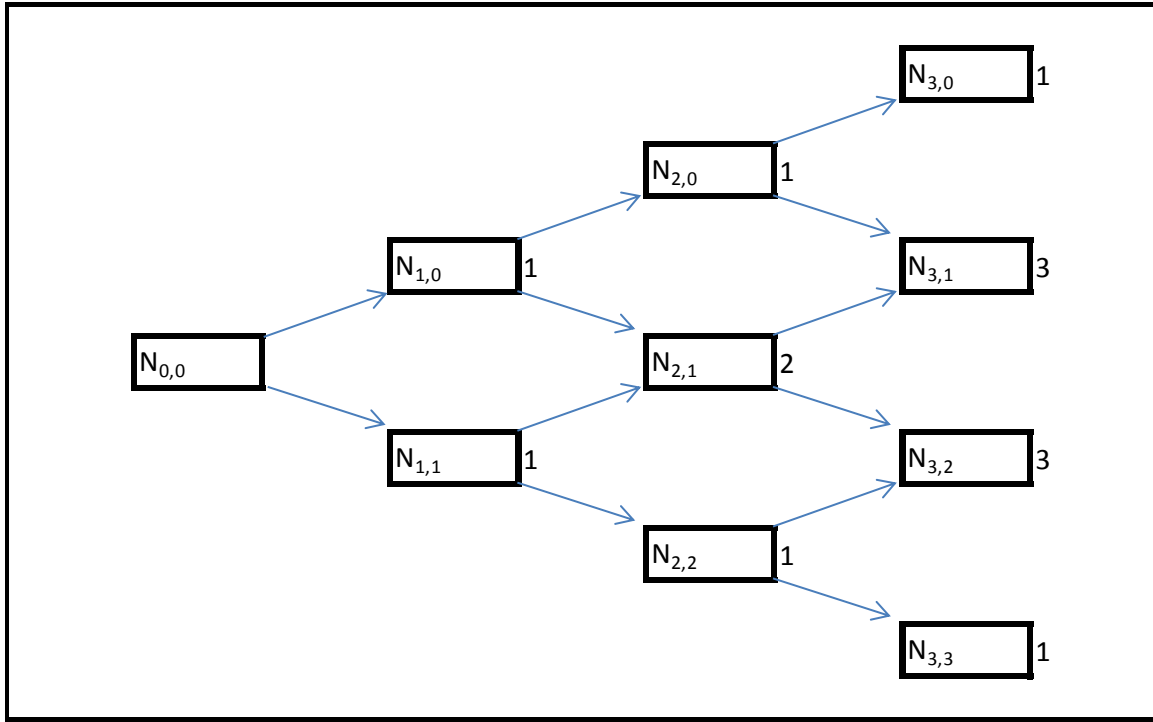
We make use of the basic nodal structure in Figure 5.1 to illustrate a number of the concepts introduced below. The number of paths to each node is shown next to the nodal label ($N_{i,j}$).

We map cash to the multiplicative binominal process for the risky asset. In doing so we must consider the many different paths that may be taken to reach any given node of our binominal tree and the cash flows that have occurred on these paths.

We define, for a given node i,j (node j of level i in the tree)

$$P_{UP} = \frac{(i-1)!}{(i-j)!(j-1)!} \qquad (5.3.2.2)$$

$$P_{DN} = \frac{(i-1)!}{(i-j-1)!(j)!} \qquad (5.3.2.3)$$

$$R_{UP} = \frac{2P_{UP}}{P_{UP}+P_{DN}} \qquad (5.3.2.4)$$

$$R_{DN} = \frac{2P_{DN}}{P_{UP}+P_{DN}} \qquad (5.3.2.5)$$

where P_{UP} and P_{DN} are the number of paths leading to a given node, where the given node is either a move up or a move down from the prior node. Where R_{UP} and R_{DN} provide a measure for a given node of the ratio of paths that reach the given node where the given node is either a move up or down from the prior node.

As an example, referring to figure 5.1 above, let us consider node $N_{3,1}$, which can be reached by 3 distinct paths, namely $N_{0,0} \rightarrow N_{1,0} \rightarrow N_{2,0} \rightarrow N_{3,1}$ and $N_{0,0} \rightarrow N_{1,0} \rightarrow N_{2,1} \rightarrow N_{3,1}$ and $N_{0,0} \rightarrow N_{1,1} \rightarrow N_{2,1} \rightarrow N_{3,1}$. Whilst the probability of any given up move or down move is set equal to 0.50 (by definition), the contribution to values at node $N_{3,1}$ from node $N_{2,1}$ is twice that of node $N_{2,0}$ given the number of paths that lead to node $N_{2.1}$.

5.3.3 Asset Cash generation and interest considerations

The overall cash balance, C, accrues or incurs interest over time depending on whether it is positive or negative. The underlying asset, V, generates cash periodically and over time this asset cash balance, VC, accrues interest. VC is by definition set to zero initially and is strictly increasing in value thereafter. The transmission cash balance, TC, can be positive, negative or zero initially. TC accrues or incurs interest over time, depending on whether it is positive or negative and taking account of the overall cash balance.

Blueprint 5.3 : Asset Cash Generation

As we roll forward through the tree, total cash available (C) is augmented by the net return (k) on the asset price (V).
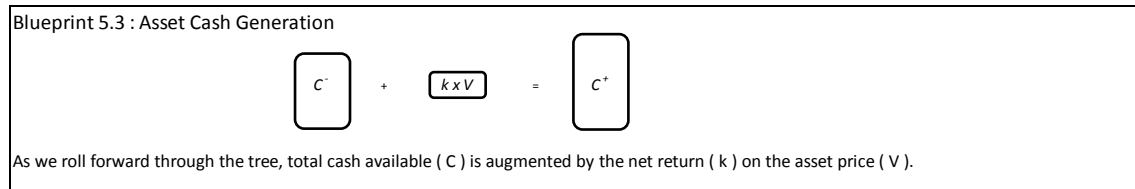
When we consider VC, we must evaluate nodes on the upper spine, nodes on the lower spine and interior nodes. The value of VC must incorporate the nodal cash flow (net yield k on the asset V) as well as the running total of prior levels of VC plus interest.

For nodes on the upper spine (where $j=0$) we have

$$VC_{i,o} = V_{i,0} \left(\frac{e^{r\Delta t}}{e^{(r-k)\Delta t}} - 1 \right) + VC_{i-1,0}e^{r\Delta t} \qquad (5.3.3.1)$$

For nodes on the lower spine (where $j=i$) we have

$$VC_{i,i} = V_{i,i} \left(\frac{e^{r\Delta t}}{e^{(r-k)\Delta t}} - 1 \right) + VC_{i-1,i-1}e^{r\Delta t} \qquad (5.3.3.2)$$

For interior nodes we have

$$VC_{i,j} = V_{i,j}\left(\frac{e^{r\Delta t}}{e^{(r-k)\Delta t}} - 1\right) + \left(R_{UP}(1-p)VC_{i-1,i-1} + R_{DN}\, p\, VC_{i-1,i}\right)e^{r\Delta t} \quad (5.3.3.3)$$

To illustrate, let us consider node $N_{3,1}$ from figure 1 above. $VC_{3,1}$ is a function (earning a net yield k on the asset V) of the asset value at node $N_{3,1}$ plus an appropriate contribution from the asset cash at nodes $N_{2,0}$ and $N_{2,1}$ adjusted for interest.

The initial transmission cash holding TC_o may be zero, positive or negative.

The transmission cash holding, TC, is subject to interest over time. The rate applied is dependent on whether the cash balance, C, is positive or negative. If C is negative then a borrowing rate, r_B is applied. If C is positive then an investing rate r_I is applied. We define r_B and r_I such that $r_B < r < r_I$. In the calculation of the asset cash balances, VC, which is always greater than or equal to zero, the risk free rate r is applied, but is adjusted below. Interest is applied to TC such that the effective rate on C, namely the combined change in TC and VC, correctly reflects whether C is positive or negative. We calculate an interest adjusted TC, namely TC^+ for use in subsequent calculations.

$$If\ TC_{i,j} + VC_{i,j} > 0\ then$$

$$TC_{i,j}^+ = TC_{i,j}e^{r_I\Delta t} - VC_{i,j}\left(e^{(r-r_I)\Delta t} - 1\right) \qquad (5.3.3.4)$$

$$else\ If\ TC_{i,j} + VC_{i,j} < 0\ then$$

$$TC_{i,j}^+ = TC_{i,j}e^{r_B\Delta t} + VC_{i,j}\left(e^{(r_B-r)\Delta t} - 1\right) \qquad (5.3.3.5)$$

As we move through the tree, the values for TC are a function of prior values of TC^+. When we consider TC, we must evaluate nodes on the upper spine, nodes on the lower spine and interior nodes.

For nodes on the upper spine (where $j=0$) we have

$$TC_{i,0} = TC^+{}_{i-1,0} \qquad (5.3.3.6)$$

For nodes on the lower spine (where $j=i$) we have

$$TC_{i,i} = TC^+{}_{i-1,i-1} \qquad (5.3.3.7)$$

For interior nodes we have

$$TC_{i,j} = R_{UP}(1-p)TC^+_{i-1,j-1} + R_{DN}\, p\, TC^+_{i-1,j} \hspace{2cm} (5.3.3.8)$$

To illustrate, let us consider node $N_{3,1}$ from figure 5.1 above. The transmission cash for the relevant node, $TC_{3,1}$ is a function of the transmission cash at nodes $N_{2,0}$ and $N_{2,1}$ adjusted for interest. The interest adjustment is a function of the levels of both the relevant transmission cash nodal value ($TC_{3,1}$) and the asset cash nodal value ($VC_{3,1}$).

The process described above facilitates the calculation of interest on cash balances as we move through the tree.

5.3.4 Cash events (Coupons, Dividends and Operating Costs)

In this section we describe how we cater for the payment of dividends, coupons and operating costs.

Blueprint 5.4 : Cash Events

As we roll forward through the tree, total cash available (C) is reduced by Coupons and Dividends.
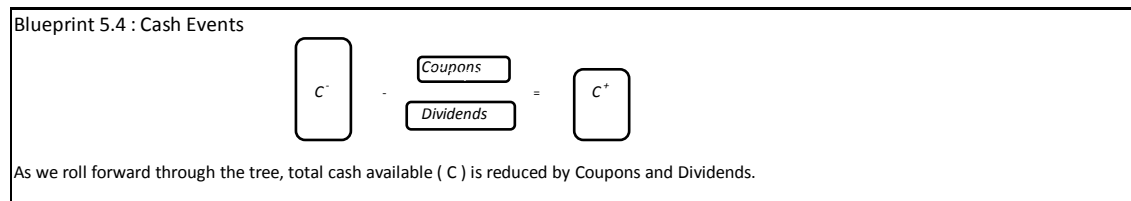
There are a number of potential cash flows that will impact *TC*. We define *D* as the quantum of total debt. *D* is constant in our model. We define *DCpn* as the coupon rate applicable on debt. *DCpn* may be fixed or linked to leverage at inception (intuitively, higher leverage will be related to higher coupons), however it is constant through the tree. We define *SD* as the total quantum of subordinated debt. *SD* is set to zero at inception but can vary in quantum through the tree. We define *SDCpn* as the coupon rate applicable on subordinated debt. *SDCpn* may be fixed or linked to leverage through the tree. We define *TG* as the potential tax gain available given a tax rate of *TR*. The differential tax treatment afforded to interest payments and dividend payments may generate a tax gain. We define *DIV* as the dividend payable. *DIV* may be a constant or a fixed yield, *DIVy*, payable on the prevailing asset value.

If *DCpn* is linked to leverage at inception then

$$DCpn = r + \sigma \left(\frac{D}{V_0 + TC_0}\right) SR \qquad (5.3.4.1)$$

where *SR* is a constant that determines the sensitivity of the calculated debt coupon (applied to both debt and subordinated debt) to asset volatility. In effect the higher the leverage and the higher the asset volatility the higher the coupon.

If *DIV* is not constant then on coupon dates

$$DIV_{i,j} = V_{i,j} \, DIVy \qquad (5.3.4.2).$$

As we move through the tree, the values for *SD, SDCpn* and *TG* are a function of prior values of *SD, SDCpn* and *TG*. When we consider *SD, SDCpn* and *TG,* we must evaluate nodes on the upper spine, nodes on the lower spine and interior nodes.

For nodes on the upper spine (where *j=0*) we have

$$SD_{i,0} = SD_{i-1,0} \qquad (5.3.4.3)$$

$$SDCpn_{i,0} = SDCpn_{i-1,0} \qquad (5.3.4.4)$$

$$TG_{i,0} = TG_{i-1,0} \qquad (5.3.4.5)$$

For nodes on the lower spine (where *j=i*) we have

$$SD_{i,i} = SD_{i-1,i-1} \qquad (5.3.4.6)$$

$$SDCpn_{i,i} = SDCpn_{i-1,i-1} \qquad (5.3.4.7)$$

$$TG_{i,i} = TG_{i-1,i-1} \qquad (5.3.4.8)$$

For interior nodes we have

$$SD_{i,j} = R_{UP}(1-p)SD_{i-1,j-1} + R_{DN} \, p \, SD_{i-1,j} \qquad (5.3.4.9)$$

$$SDCpn_{i,j} = R_{UP}(1-p)SDCpn_{i-1,j-1} + R_{DN} \, p \, SDCpn_{i-1,j} \qquad (5.3.4.10)$$

$$TG_{i,j} = R_{UP}(1-p)TG_{i-1,j-1} + R_{DN} \, p \, TG_{i-1,j} \qquad (5.3.4.11)$$

As we move forwards through the tree, on coupon dates we must account for the relevant cash flows and the consequences thereof. There are a number of steps to this process.

We make payment of coupons, ordinary dividends and fixed contracted operating costs, COP, out of the transmission cash balance, TC, such that $TC_{i,j}$ is reduced by

$$DDCpn - SD_{i,j}SDCpn_{i,j} - COP - DIV_{i,j} \qquad (5.3.4.12)$$

5.3.5 Tax Benefits

In many countries there is a difference in the treatment of interest and dividends for tax purposes. Interest paid is tax deductible, interest earned is taxable and dividends are considered to be after tax payments. This disparity has been explored extensively in the literature (see Durand (1959), Modigliani and Miller (1963), Miller (1977), Miller (1988) and Graham (2000)).

We apply a simple tax regime whereby coupons and interest are tax deductible but where dividends are paid after tax. We follow the work of Brennan and Schwartz (1978) and Leland (1994) and incorporate this differential tax treatment into our model by adding / subtracting the tax saved / paid on coupons and interest to the assets available to meet the claims of equity and debt holders. This tax benefit is extinguished on default. We quantify the potential tax gain generated by considering the total interest payments made on debt and subordinated debt as well as the net cash interest, NCI, earned or paid on the cash balance, C, since the last coupon date. $TG_{i,j}$ is augmented by

$$\left(D\ DCpn + SD_{i,j}SDCpn_{i,j} - NCI_{i,j}\right) TR \quad (5.3.5.1),$$

where

$$NCI_{i,j} = \left(VC_{i,j} + TC_{i,j}\right)\left(e^{-r} - 1\right) \qquad (5.3.5.2)$$

In the event that the firm has no debt and earns net interest there will be a tax cost adjustment applied as per above.

5.3.6 Capturing Cash Adjustments

The model explicitly recognises that all payments made by a firm must take place out of a cash balance. The limits imposed on the firm's cash holdings are a key determinant in both valuation and capital structure behaviour. In this section we describe the conditions under which the model will seek additional cash resources or return cash resources to the holders of the firm's securities, as well as how these adjustments are made.

When we consider cash shortfalls and cash excesses we may make cash adjustments to the cash holdings that are specific to a given node, as opposed to coupons and ordinary dividends that take place consistently through the tree. As is the case with coupons and ordinary dividends, these adjustments only take place at those levels in the tree that coincide with coupon dates. However when we consider cash shortfalls and cash excesses, cash adjustments at a given node in the tree generates cash that is available for use at all, or removed for use at all, subsequent nodes reachable from the given node as we move forward through the tree. Unlike coupons and ordinary dividends, these cash adjustments do not take place at every node within a given level of the tree, and as such we must keep track of the impact of these specific cash adjustments. When we roll back through the tree we will need to adjust cash values to account for any cash adjustments at a given level within the tree. Cash introduced at a given level in the tree is only available for inclusion in the calculation of debt and equity values at levels in the tree above that level where the cash was introduced. Cash removed at a given level in the tree is only unavailable for inclusion in the calculation of debt and equity values at levels in the tree above that level where the cash was removed. We wish to highlight that cash adjustments propagate forwards through the tree in a fashion that is distinctly different to how those resulting adjustments are incorporated in the model when we roll back through the tree. This difference is detailed later in the presentation.

To illustrate the impact of cash adjustments as we move forward through the tree let us consider figure 5.2, where the number of paths to a given node is shown next to the nodal label. In addition we show (below the nodal label) the propagation of the introduction of 1 unit of cash at node $N_{1,0}$. We highlight node $N_{2,1}$, noting that it is reachable from two nodes, namely $N_{1,0}$ and $N_{1,1}$, and focus on the impact of the cash adjustment of 1 at node $N_{1,0}$, namely a probability weighted value of 0.5.

Figure 5.2 Cash adjustments and nodal structure
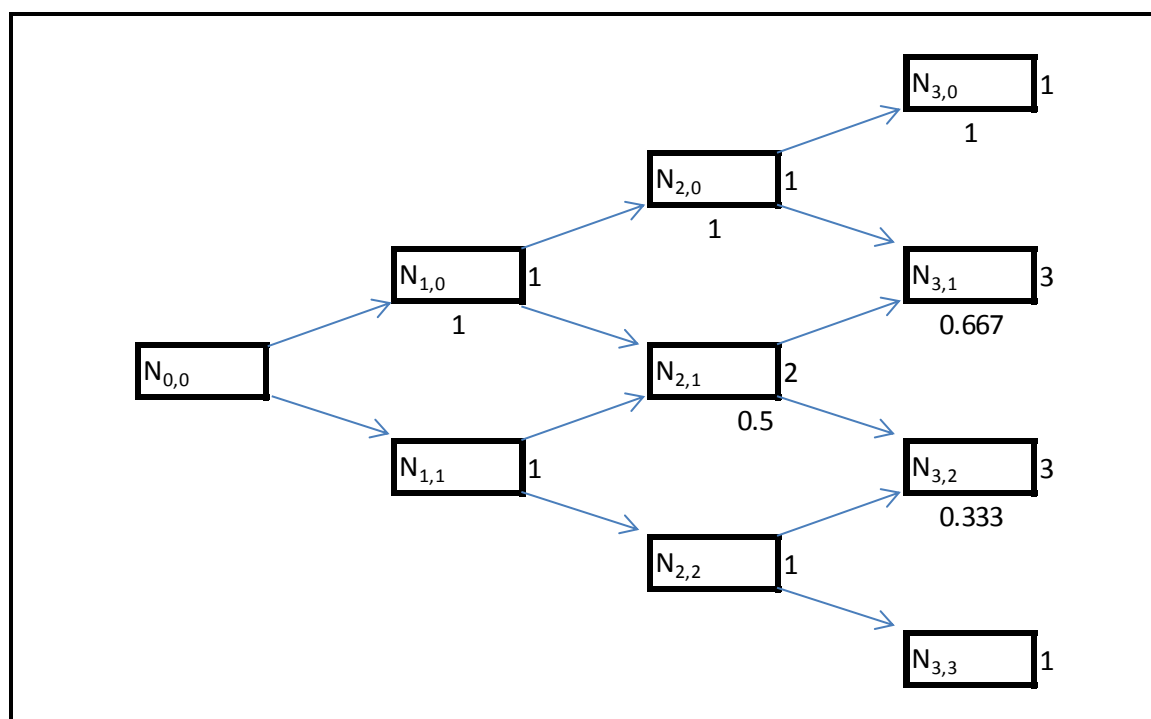
5.3.7 Cash Shortfall

In this section we describe how we assess whether additional financing is required, and if so which form of additional financing (equity or subordinated debt) is to be used and how this additional financing is reflected in the model.

When additional financing is raised we increment the total cash balance, C, by adding the new financing to TC.

Blueprint 5.5 : Cash Shortfall

As we roll forward through the tree we check if we need additional cash resources. If we do, then the incremental cash is raised by way of Equity or Subdebt or both.
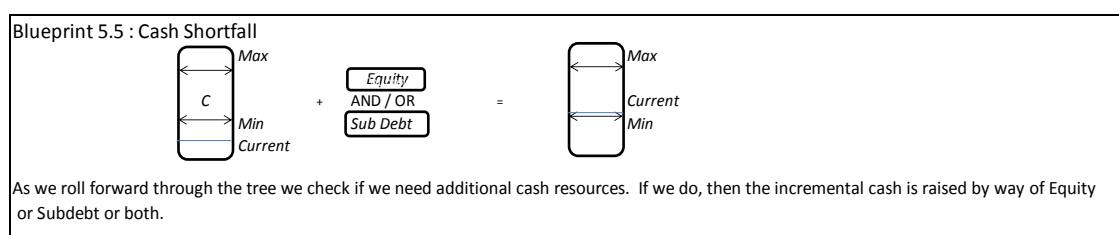
We seek additional financing in the event that our cash balance, C, is negative and the asset value reduced by a fixed dead weight charge on bankruptcy, BC, is insufficient to cover the outstanding debt and the negative cash balance, namely

$$V_{i,j}(1 - BC) < D - C_{i,j} \qquad (5.3.7.1)$$

If additional financing is required, then the total amount raised ensures that on bankruptcy the asset value reduced by *BC* is sufficient to cover the outstanding debt and the negative cash balance.   We raise an amount equal to

$$D - C_{i,j} - V_{i,j}(1 - BC) \qquad (5.3.7.2)$$

We will issue subordinated debt up to a limit and equity thereafter.

Blueprint 5.6 : Subordinated Debt Issue

If the additional cash is raised by the fresh issue of subordinated debt, then the total subordinated debt outstanding is increased accordingly.
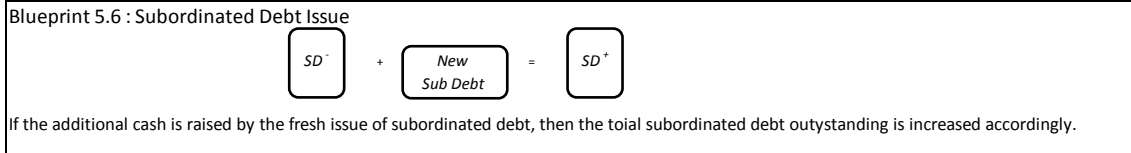
The total quantum of subordinated debt that may be issued is either set to zero (i.e. excluded from the model) or limited by a net worth consideration. In effect, at issue, the assets of the firm together with the short term cash balance must at least equal the total of ordinary debt and subordinated debt (including the new issue) outstanding, namely

$$V_{i,j} + C_{i,j} \geq D + SD_{i,j} \qquad (5.3.7.3)$$

This evaluation is distinct from that undertaken to determine if there is any requirement to raise cash, as the assets are not adjusted by the bankruptcy charge.

If *SDCpn* is linked to leverage through the tree then when new subordinated debt is issued we have

$$SDCpn_{i,j} = r + \sigma \left(\frac{(D+ESD+NSD)DR - D\,SDR}{ESD+NSD} \right) SR \qquad (5.3.7.4)$$

where *ESD* is the existing quantum of subordinated debt in issue, *NSD* is the new subordinated debt to be issued, and *DR* and *SDR* are the total debt ratio and senior debt ratio as defined below, and *SR* is as defined above.

$$DR = \frac{D+ESD+NSD}{V_{i,j}+C_{i,j}+NSD} \qquad (5.3.7.5)$$

$$SDR = \frac{D}{V_{i,j}+C_{i,j}+NSD} \qquad (5.3.7.6)$$

As was the case with calculated debt coupons, the higher the leverage and the higher the asset volatility the higher the subordinated debt coupon.

Subordinated debt issued at a given node in the tree generates cash that is available for use at all subsequent nodes reachable from the given node as we move forward through the tree. When we roll back through the tree we will need to adjust subordinated debt and cash values

to account for any subordinated debt issuance at a given level within the tree. The proceeds of subordinated debt issued at a given level in the tree are only available for inclusion in the calculation of debt and equity values at levels in the tree above that level where the subordinated debt was issued. To facilitate this adjustment we quantify the value of cash raised by way of subordinated debt issuance, SCR, from any given level of the tree (at node i,j where the issuing level is denoted by h).

Therefor on subordinated debt issuance we have

$$SCR_{h,i,j} = NSD \qquad \text{(5.3.7.7)}$$

As we move through the tree, the values for SCR are a function of prior values of SCR. When we consider SCR we must evaluate nodes on the upper spine, nodes on the lower spine and interior nodes.

For nodes on the upper spine (where $j=0$) we have

$$SCR_{h,i,0} = SCR_{h,i-1,0} \qquad \text{(5.3.7.8)}$$

For nodes on the lower spine (where $j=i$) we have

$$SCR_{h,i,i} = SCR_{h,i-1,i-1} \qquad \text{(5.3.7.9)}$$

For interior nodes we have

$$SCR_{h,i,j} = R_{UP}(1-p)SCR_{h,i-1,j-1} + R_{DN}\, p\, SCR_{h,i-1,j} \qquad \text{(5.3.7.10)},$$

where h determines the level of the tree that corresponds to that of the original subordinated debt issuance.

Equity, E, will be issued to the extent required, namely

$$E_{i,j} = D - C_{i,j} - V_{i,j}(1 - BC) - NSD \qquad \text{(5.3.7.11)}.$$

We incorporate an equity issue cost, EIC, that is used when we roll back through the tree considering bankruptcy. Equity will not be issued at leaf nodes.

Equity issued at a given node in the tree generates cash that is available for use at all subsequent nodes reachable from the given node as we move forward through the tree. When we roll back through the tree we will need to adjust cash values and equity values to account for any equity issuance at a given level within the tree. Equity issued at a given level in the tree is only available for inclusion in the calculation of debt and equity value at levels in the

tree above that level where the equity was issued. To facilitate this adjustment we quantify the value of cash raised by way of equity issuance, *ECR*, from any given level of the tree (at node *i,j* where the issuing level is denoted by *h*).

Therefor on equity issuance we have

$$ECR_{h,i,j} = E_{i,j} \qquad (5.3.7.12)$$

As we move through the tree, the values for *ECR* are a function of prior values of *ECR*. When we consider *ECR* we must evaluate nodes on the upper spine, nodes on the lower spine and interior nodes.

For nodes on the upper spine (where *j=0*) we have

$$ECR_{h,i,0} = ECR_{h,i-1,0} \qquad (5.3.7.13)$$

For nodes on the lower spine (where *j=i*) we have

$$ECR_{h,i,i} = ECR_{h,i-1,i-1} \qquad (5.3.7.14)$$

For interior nodes we have

$$ECR_{h,i,j} = R_{UP}(1-p)ECR_{h,i-1,j-1} + R_{DN}\, p\, ECR_{h,i-1,j} \quad (5.3.7.15)$$

Where *h* determines the level of the tree that corresponds to that of the original equity issuance.

The motivation for the approach outlined above is twofold. Firstly, we consider any short term cash borrowings to be risk free in nature, as is the case with Anderson and Carverhill (2012), and as such will only seek additional cash when these borrowings are at risk. Secondly, we incorporate a charge on bankruptcy that recognises the impact of the direct dead weight cost to a firm that moves from a going concern to default as well as all the indirect costs to a firm related to bankruptcy. These costs are discussed extensively in the literature, notably in Miller (1977) and Miller (1988). There remains much debate as to the absolute level of these costs, with empirical studies suggesting a wide range of potential outcomes from no impact on the asset value to a complete loss (see Bris, Welch and Zhu (2006)).

5.3.8 Cash Excess

In this section we describe how we assess whether we hold too much cash, and if so how this cash is to be distributed to holders of the firm's securities (subordinated debt and equity) and how this cash distribution is reflected in the model. We impose a maximum cover ratio as a means to limit the holding of positive cash balances in the firm.

Blueprint 5.7 : Cash Excess

As we roll forward through the tree we check if we have excess cash resources. If we do, then the excess cash is distributed by way of a Special Dividend or a repurchase of Subdebt or both.

We consider whether or not excess liquidity (too much cash) may be paid out in the form of subordinated debt repurchases and special dividends. To the extent that excess liquidity is reduced, we reduce *TC*.

Excess liquidity may only be paid out if the risky assets, *V*, exceed the sum of outstanding debt, *D*, and subordinated debt, *SD*.

Excess liquidity, *EL*, is defined as the excess over a maximum cover ratio, *MCR*, on interest service costs on debt and subordinated debt.

$$EL = Max\left(0, C_{i,j} - MCR\left(DCpn\ D + SDCpn_{i,j}\ SD_{i,j}\right)\right) \qquad (5.3.8.1)$$

If we have parameterised the model to allow for subordinated debt repurchases, then to the extent that there is subordinated debt outstanding the excess liquidity can be applied towards the repurchase of subordinated debt, *SDRep* at a cost, *SDCst*, and the excess liquidity available, *EL*, will be reduced by an amount equal to

$$\frac{SDRep}{(1+SDCst)} \qquad (5.3.8.2)$$

The coupon paid on the subordinated debt outstanding is unchanged by the repurchase of subordinated debt.

Blueprint 5.8 : Subordinated Debt Repurchase

If the excess cash is distributed by way of a repurchase of subordinated debt, then the total subordinated debt outstanding is decreased accordingly.
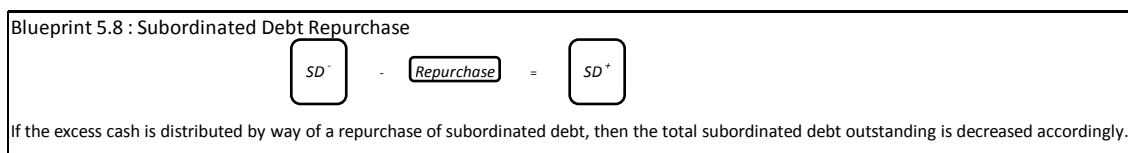
The repurchase of subordinated debt will impact on the value of subordinated debt cash proceeds available at future levels of the tree. We therefor reduce the relevant value of cash raised by way of subordinated debt issuance, $SCR_{h,i,j}$, by *SDRep*.

If we have parameterised the model to allow for special dividend payments, then to the extent that there is excess liquidity after the repurchase of subordinated debt then a special dividend equal to *EL* may be paid.

Special dividends paid at a given node in the tree removes cash that is then no longer available for use at all subsequent nodes reachable from the given node as we move forward through the tree. When we roll back through the tree we will need to adjust cash values and equity values to account for any special dividends at a given level within the tree. Special dividend payments at a given level in the tree only impact the calculation of debt and equity value at levels in the tree above that level where the special dividend was paid. To facilitate this adjustment we quantify the value of cash removed by way of special dividend, *DCR*, from any given level of the tree (at node *i,j* where the level is denoted by *h*).

Therefor when a special dividend is paid, we have

$$DCR_{h,i,j} = EL \qquad (5.3.8.3)$$

As we move through the tree, the values for *DCR* are a function of prior values of *DCR*. When we consider *DCR* we must evaluate nodes on the upper spine, nodes on the lower spine and interior nodes.

For nodes on the upper spine (where *j=0*) we have

$$DCR_{h,i,0} = DCR_{h,i-1,0} \qquad (5.3.8.4)$$

For nodes on the lower spine (where *j=i*) we have

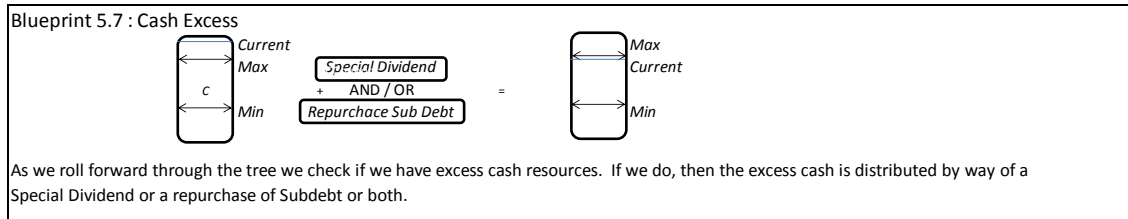$$DCR_{h,i,i} = DCR_{h,i-1,i-1} \qquad (5.3.8.5)$$

For interior nodes we have

$$DCR_{h,i,j} = R_{UP}(1-p)DCR_{h,i-1,j-1} + R_{DN} \, p \, DCR_{h,i-1,j} \quad (5.3.8.6)$$

where *h* determines the level of the tree that corresponds to that of the original equity issuance.

We move forward through the tree as described above until the leaf nodes are reached.

5.3.9 Tree Dynamics, Bankruptcy, Equity Value and Debt Value

In this section, starting from the leaf nodes of the tree we calculate values for debt and equity. These values are effectively a function of the assets of the firm being shared between the holders of the firm's debt and equity securities. These values are adjusted by events of bankruptcy, which are determined with reference to either asset value and / or equity value and generally follow the notion of absolute priority of claims. Cash balances are an integral component of this evaluation, however we must reconstruct cash holdings as we roll back through the tree as cash adjustments propagate differently when moving back from the leaf nodes.

5.3.10 Tree Dynamics

We roll back through the tree, from the leaf nodes, considering bankruptcy and calculating values for debt and equity. This process considers asset values, cash values and the current values for debt and equity. As discussed above, with reference to Figure 5.2, cash shortfalls and cash excesses generate cash adjustments to the cash holdings that are specific to a given node and propagate through the tree to the leaf nodes. When we roll back through the tree these adjustments are reflected at the level of the tree where the original cash adjustment took place, however the adjustment is no longer specific to a single node, rather it is spread out across the nodal level. When we moved forward through the tree we calculate R_{UP} and R_{DN} which provide a measure for a given node of the ratio of paths that reach the given node where the given node is either a move up or down from the prior node. When we move backwards through the tree the relative contribution to a given node from the node that is a move up from the given node and the node that is a move down from the given node is equal. As a result the cash holdings available at each node are not the same when we roll back through the tree as those that were generated when we rolled forward through the tree, and must be recalculated.

Blueprint 5.9 : Cash Adjustments : Forwards

As we roll forward through the tree we need to keep track of additional cash raised and excess cash distributed. These irregular changes to the total cash balance propagate through the tree, with the cash adjustment balance at a given node a function of cash adjustments that take place at that node coupled with the cash adjustment balances from the relevant prior nodes.
In the illustration above we show how a cash adjustment of 1 unit is propogated through the tree.

Blueprint 5.10 : Cash Adjustments : Backwards

As we roll backward through the tree we need to keep track of additional cash raised and excess cash distributed. These irregular changes to the total cash balance propogate backwards through the tree, with the cash adjustment balance at a given node a function of cash adjustment balances from the relevant future nodes.

In the illustration above we show how the cash adjustment illustration of earlier is propogated backwards through the tree.

We introduce VC' to reflect the cumulative cash value of the fixed yield paid by the risky asset and TC' to reflect the total of all other cash flows to date as we roll back through the tree. We define C' as the sum of VC' and TC'.

As we roll backwards through the tree VC' and TC' will be adjusted to reflect the relevant changes at each node. These changes include the net return on the asset, coupons and dividends and the impact of cash adjustments.

Blueprint 5.11 : Cash : Backwards

As we roll backwards through the tree we reconstruct the relevant cash available.

This cash balance (C') at any given node is a function of the cash levels at future nodes.
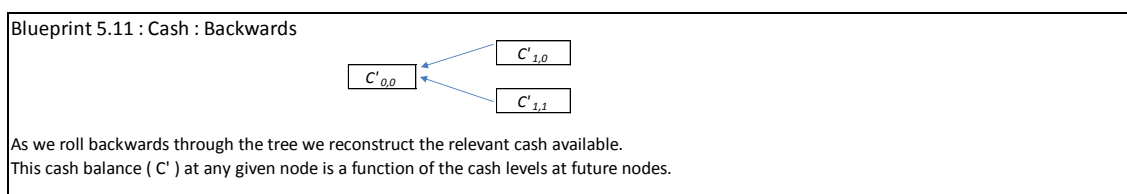
To illustrate the propagation of cash adjustments when we roll back through the tree we consider the same example as that used for Figure 5.2, namely the introduction of 1 unit of cash at node $N_{1,0}$. In Figure 5.3 we have the same leaf node impact as was evidenced in Figure 5.2 but we highlight how the original issue of 1 unit of equity at node $N_{1,0}$ is spread across nodes $N_{1,0}$ and $N_{1,1}$ as we roll back through the tree.

Figure 5.3 Cash adjustment impact on rollback
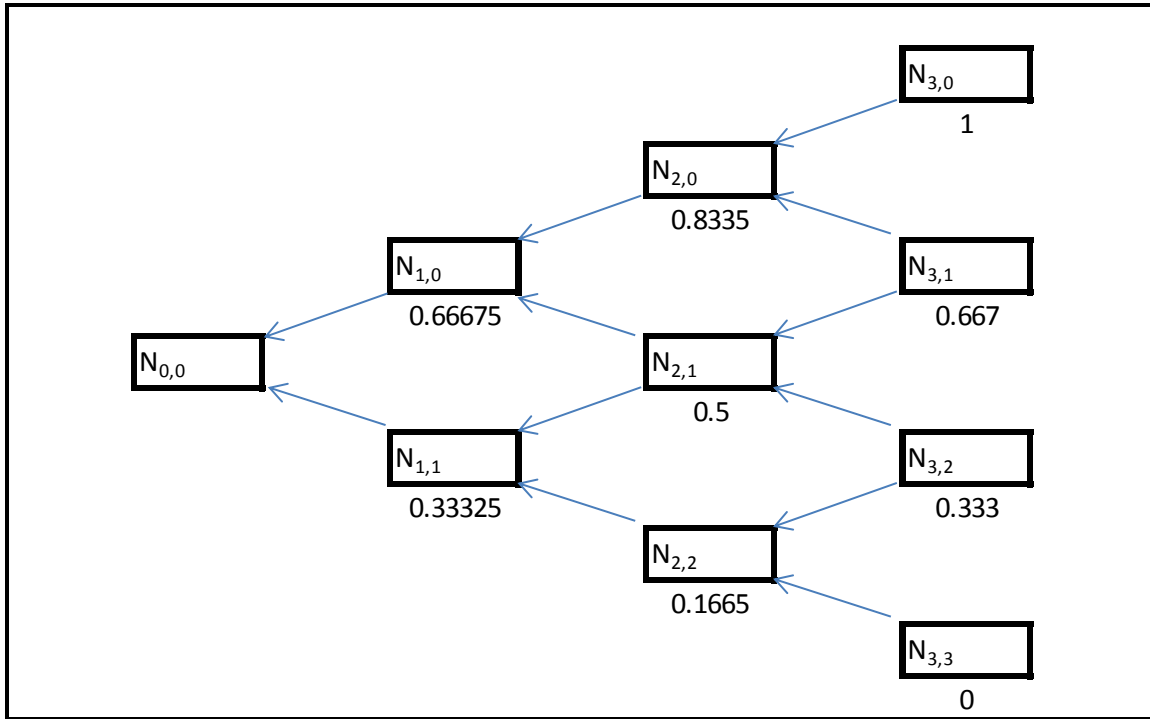
As we rolled forward through the tree we quantified the impact of the cash generated or consumed as a consequence of cash excesses and cash shortfalls. As we roll backwards from the leaf nodes through the tree we quantify the impact of these adjustments for equity issuance (*EVI*), subordinated debt issuance and buyback (*SVI*) and special dividend payments (*DVI*) at a given level within the tree. The leaf node values of *EVI, SVI* and *DVI* correspond to the leaf node values of *ECR, SCR* and *DCR*. The interior nodal values for *EVI, SVI* and *DVI* are the expected value of the nodal values reachable in the next level. The values for *EVI, SVI* and *DVI* (both at leaf nodes and at interior nodes) are defined by the equations below.

$$EVI_{h,N,j} = ECR_{h,N,j} \quad (5.3.10.1),$$

$$SVI_{h,N,j} = SCR_{h,N,j} \quad (5.3.10.2),$$

$$DVI_{h,N,j} = DCR_{h,N,j} \quad (5.3.10.3),$$

$$EVI_{h,i,j} = EVI_{h,i+1,j+1}(1-p) + EVI_{h,i+1,j}p \qquad (5.3.10.4),$$

$$SVI_{h,i,j} = SVI_{h,i+1,j+1}(1-p) + SVI_{h,i+1,j}p \qquad (5.3.10.5),$$

$$DVI_{h,i,j} = DVI_{h,i+1,j+1}(1-p) + DVI_{h,i+1,j}p \qquad (5.3.10.6),$$

where h is an indicator of the issuing level or special dividend paying level of the tree.

5.3.11 Bankruptcy, Equity Value and Debt Value

We calculate terminal values for equity and debt at the leaf nodes. We consider bankruptcy at the leaf nodes and at interior nodes corresponding with coupon dates.

Blueprint 5.12 : Equity Value : Leaf Nodes

At the leaf nodes, we assume that the assets are liquidated and all obligations are settled, with equity receiving the residual.

Blueprint 5.13 : Bankruptcy : Leaf Nodes

If the assets are insufficient to settle all obligations then bankruptcy takes place and the assets suffer a dead weight cost.
On default, obligations are settled with the reduced asset balance, with equity generaly getting none.

5.3.12 Leaf Nodes

In the absence of bankruptcy, debt and subordinated debt are assumed to be paid in full and equity receives the residual value if any. In the absence of default equity benefits from the accumulated potential tax gain, if any.

$$DV_{N,j} = D \quad (5.3.12.1)$$

$$EV_{N,j} = ma\,x\big(0, V_{N,j} + C_{N,j} - D - SD_{N,j}\big) + TG_{N,j} \qquad (5.3.12.2)$$

where DV and EV are the nodal values for debt and equity.

At the leaf nodes of the tree, bankruptcy is triggered when the assets together with the cash holdings of the firm are less than the notional outstanding of debt and subordinated debt, namely

$$V_{N,j} + C_{N,j} \leq D + SD_{N,j} \qquad (5.3.12.3)$$

In the event of bankruptcy the assets of the firm, after suffering the dead weight bankruptcy charge, *BC*, must be divided up amongst the holders of the firm's debt, subordinated debt and equity. If strict priority of claims is observed then equity holders will receive nothing, whilst debt would be serviced first up to the nominal outstanding and subordinated debt serviced, to the extent possible, thereafter. In practise equity seldom receives zero as equity holders hold valuable voting rights that impact bankruptcy proceedings (see Longhofer and Carlstrom (1995)). In our model we allow for a fraction of asset value, *EQDf*, to be apportioned to equity holders in recognition of strict priority not being observed, namely an Equity Value on Default. In the event of bankruptcy, equity does not benefit from the accumulated potential tax gain, if any. The nodal values of *DV* and *EV* in the event of bankruptcy are

$$DV_{N,j} = min\left(D, \big(V_{N,j}(1 - BC) + C_{N,j}\big)(1 - EQDf)\right) \qquad (5.3.12.4)$$

$$EV_{N,j} = \big(V_{N,j}(1 - BC) + C_{N,j}\big)\,EQDf \qquad (5.3.12.5)$$

We add coupons paid and dividends paid to the nodal value for debt and equity. Coupons and dividends are added to the value of equity and debt as their impact on default has already been accounted for by way of the cash balance. Specifically, the nodal values of *DV* and *EV*, namely $DV_{N,j}$ and $EV_{N,j}$ are incremented by *DCpnD* and $DIV_{N,j}$ respectively.

The last step prior to considering the interior nodes is to set the leaf node levels for *VC'* and *TC'*.

At both leaf nodes and interior nodes (coinciding with coupon payments) we proceed by reducing *VC'* by the asset return and increasing *TC'* by dividends and coupons.

Blueprint 5.14 : Asset Return : Backwards

$$C' \quad - \quad \boxed{k \times V} \quad = \quad C'^+$$

As we roll backwards through the tree, total cash available (C') is reduced by the net return (k) on the asset price (V).

Blueprint 5.15 : Coupons and Dividends : Backwards

$$C' \quad + \quad \boxed{\textit{Coupons} \atop \textit{Dividends}} \quad = \quad C'^+$$

As we roll backwards through the tree, total cash available (C') is augmented by Coupons and Dividends.
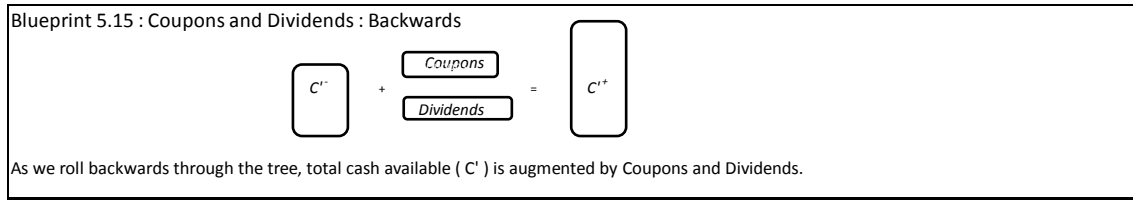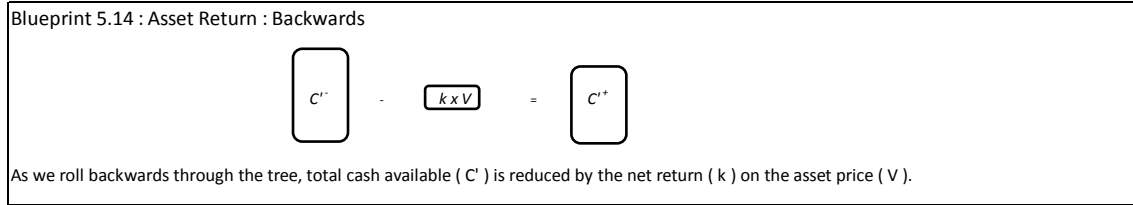
$$VC'_{N,j} = VC_{N,j} - V_{i,N}\left(\frac{e^{r\Delta t}}{e^{(r-k)\Delta t}} - 1\right) \qquad (5.3.12.6)$$

$$TC'_{N,j} = TC_{N,j} + (DDCpn - SD_{N,j}SDCpn_{N,j} - COP - DIV_{N,j}) + DVI_{T,i,j}$$

$$(5.3.12.7)$$

We effectively add back the relevant changes to the cash holdings of the firm.

5.3.13 Interior Nodes

Equity and debt values at interior nodes are a function of the values of equity and debt reachable from the interior nodes.

Blueprint 5.16 : Equity and Debt : Interior Nodes

Values for Equity (E) and Debt (D) are calculated as we roll backwards through the tree.
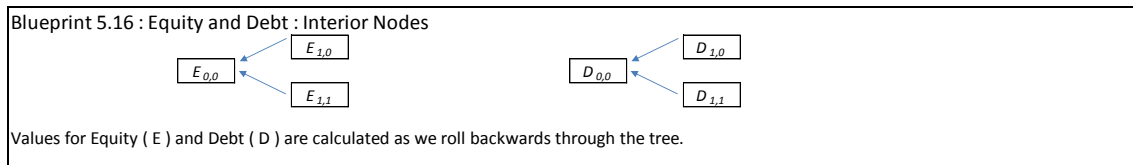
The initial Interior nodal values for debt and equity are the discounted expected value of the nodal values reachable in the next level.

$$DV_{i,j} = \frac{DV_{i+1,j+1}(1-p) + DV_{i+1,j}p}{e^{r\Delta t}} \qquad (5.3.13.1)$$

$$EV_{i,j} = \frac{EV_{i+1,j+1}(1-p) + EV_{i+1,j}p}{e^{r\Delta t}} \qquad (5.3.13.2)$$

The cash value at interior nodes is a function of the cash values reachable from the interior nodes.

In addition we account for the interior nodal values for *VC'* and *TC'*,

$$VC'_{i,j} = \frac{VC'_{i+1,j+1}(1-p) + VC'_{i+1,j}p}{e^{r\Delta t}} \qquad (5.3.13.3)$$

$$TC'_{i,j} = \frac{TC'_{i+1,j+1}(1-p) + TC'_{i+1,j}p}{e^{r\Delta t}} \qquad (5.3.13.4)$$

As we roll backwards through the tree we consider bankruptcy on coupon dates only and construct effective levels for *VC'* and *TC'*.

On coupon dates, we first adjust nodal equity value by the impact of equity issuance, if any, at that level. In essence, equity is reduced by the impact of any equity issuance that took place at that point in time (namely the level of the tree), specifically the nodal equity value is set equal to the maximum of the existing nodal equity value ($EV_{i,j}$) reduced by the relevant equity issuance ($EVI_{i,i,j}$) and zero.

In addition we reduce the nodal value of *TC'* by the impact of any cash adjustments that took place at that point in time (namely the level of the tree), specifically the relevant equity issuance combined with the relevant subordinated debt issuance less the relevant special dividend payment.

Blueprint 5.17 : Cash Adjustments (Equity, Subordinated Debt and Special Dividends)

As we roll backwards through the tree, total cash available is adjusted for any relevant cash adjustments (equity issuance, subdebt issuance, special dividends and subdebt buybacks)
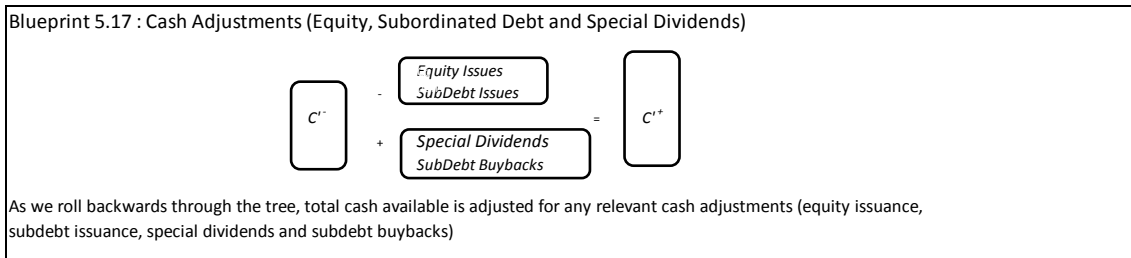
TC' is reduced by

$$EVI_{i,i,j} + SVI_{i,i,j} - DVI_{i,i,j} \qquad (5.3.13.5)$$

We highlight that the issuing level and the nodal level are the same.

The last step prior to considering bankruptcy is to adjust the nodal levels for *VC'* and *TC'*.

VC' is reduced by the relevant cash proceeds from the asset, namely

$$V_{i,j}\left(\frac{e^{r\Delta t}}{e^{(r-k)\Delta t}} - 1\right) \qquad (5.3.13.6)$$

And *TC'* is augmented by any regular cash payments made, namely

$$DDCpn + SD_{i,j}SDCpn_{i,j} + COP + DIV_{i,j} \quad (5.3.13.7)$$

When evaluating interior nodes of the tree, bankruptcy is triggered in one of two distinct ways. In the first instance bankruptcy is triggered when the assets together with the cash holdings of the firm are less than the notional outstanding of debt and subordinated debt. This is a positive net worth bankruptcy trigger (without the application of a bankruptcy charge, *BC*), namely

$$V_{i,j} + C'_{i,j} \le D + SD_{i,j} \qquad (5.3.13.8)$$

In the second instance bankruptcy is triggered when the equity value, adjusted, to the extent that equity issuance took place, for the equity issuance cost, *EIC*, is less than or equal to the Residual Equity Value on Default, *REQDf*,

$$EQ_{i,j} - EIC * EVI_{i,i,j} \le REQDf \qquad (5.3.13.9)$$

where *REQDf* is the fraction of bankruptcy value apportioned to equity (*EQDf*) multiplied by the value of the firm's assets after the application of the dead weight bankruptcy charge (*BC*).

This takes place when fresh equity issuance has no incremental value. In effect, when rolling forward through the tree, equity would have been issued at this level and we check whether there was any value in this issuance.

As above, in the event of bankruptcy the assets of the firm, after suffering the dead weight bankruptcy charge, *BC*, must be divided up amongst the holders of the firm's debt, subordinated debt and equity. The nodal values of *DV* and *EV* in the event of bankruptcy are

$$DV_{i,j} = min\left(D, \left(V_{i,j}(1 - BC) + C'_{i,j}\right)(1 - EQDf)\right) \qquad (5.3.13.10)$$

$$EV_{i,j} = \left(V_{i,j}(1 - BC) + C'_{i,j}\right) EQDf \qquad (5.3.13.11)$$

We add coupons paid and dividends paid to the nodal value for debt and equity. Specifically, the nodal values of *DV* and *EV*, namely $DV_{i,j}$ and $EV_{i,j}$ are incremented by *DCpnD* and $DIV_{i,j}$ respectively.

At the conclusion of this process, firstly having rolled forward through the tree to determine asset values, cash levels and the requirement for additional funding and secondly having rolled back through the tree quantifying value for debt and equity and triggering bankruptcy as required, we have model values for our Debt position ($DV_{o,o}$) and Equity ($EV_{0,0}$). These values reflect the parameter choices made, which drive the decisions available to firm managers as well as taking cognisance of the impact of liquidity.

6 BEYOND MERTON : S – MODEL BEHAVIOUR

The purpose of this chapter is to illustrate the behaviour of the S-model, introduced in the previous chapter, across a wide range of parameter choices. The S-model permits a range of actions, both by the managers of the firm regarding their funding choices and the market in general in the manner in which it ascribes value to instruments and choices. The model is primarily one of capital structure, focussing on firm value and the distribution of this value between equity instruments and debt instruments. The primary tools for the evaluation of the model are the values of equity and debt in isolation and the sum of the values of equity and debt.

We model a firm that holds a risky asset that pays a regular yield on asset value. The firm is funded by a combination of debt and equity and all cash flows to service these instruments as well as incremental funding when required are reflected in the firm's cash balances. The firm's capacity to be short of cash (i.e. short term borrowings) and to be long of cash (i.e. short term investments) is constrained. Default can be a function of asset value or equity value and may deviate from absolute priority of claims. The tax benefit associated with the differential treatment of interest and dividends may be reflected in the model. We make use of a binominal tree to capture the dynamics of the asset process and the accompanying cash process. We roll back through the tree, taking account of default and incremental funding to determine values for debt and equity.

We will show how the S-model broadly replicates results obtained in the literature on structural models of capital structure. In this regard we will briefly summarise the results obtained in the literature and detail how the S-model is able to replicate these outcomes. To the extent that the S-model results differ from the literature, these differences are discussed. To this end we consider the impact of coupons, dividends, default provisions and tax benefits on valuation. We extend the literature by evaluating priority of claims, use of excess cash, subordinated debt issuance, debt costs, funding rates and equity issuance costs. The major contribution of the S-model is the introduction of the concept of liquidity when evaluating capital structure. All cash flows into the firm and out of the firm take place via the firm's cash holdings. The timing, scale and variability of these flows, both positive and negative, ultimately drive capital structure choices, total firm value and the split of total firm value between debt and equity. We highlight the impact on the firm's cash holdings of the various parameter choices available. The S-model allows a wide range of parameters that result in a multitude of firm structure and firm management options. The model has been run across a large parameter set and the resulting data is available at [https://www.dropbox.com/sh/nu7eg2g2auygeam/AACWBGthef8DniLduy8Fj8eUa?dl=0]. In our discussion of the model results presented here we consider certain aspects of model behaviour across a wide range of parameter choices and highlight specific behaviour with individual parameter sets. The S-model parameters are shown below.

We begin our model evaluation by considering model convergence. We proceed by parameterising our model to closely match Merton's original formulation and in doing so we note that under specific parameterisation the S-model reduces to the continuous Black-Scholes model (Black and Scholes (1973).. We then consider basic variations to the parameters to highlight the impact of coupons and dividends. These initial variations do not permit external impacts on total firm value such as tax benefits and bankruptcy charges, and we illustrate the maintenance of initial firm value across a range of parameter choices. In essence, we highlight the split between debt and equity of the initial asset value. We introduce bankruptcy, taxation and capital issuance (both in the form of subordinated debt and equity) and we show that we are able to broadly replicate the results of earlier work (Brennan and Schwartz (1978) and Leland (1994)) that sought to extend Merton's conceptual framework. We then focus on the impact of liquidity on model values, highlighting similarities with the literature (Anderson and Carverhill (2012)) and detailing where the model differs from the literature. We conclude the evaluation by considering parameter sets

that we believe approximate real market conditions. We end the chapter by discussing the model's results in the context of the relevant literature.

In the presentation that follows we consider a wide range of parameter choices. The definitions of the subset of the S-model parameter set that we vary are repeated below.

Parameters

T	-	Time horizon for evaluation
N	-	Number of binomial steps between cash flow dates
r	-	Risk free interest rate
k	-	Asset return rate
r_I	-	Cash investment rate
r_B	-	Cash borrowing rate
V	-	Asset value
C	-	Cash balance
TC	-	Transmission cash
D	-	Debt outstanding
$DCpn$	-	Coupon paid on debt
σ	-	Asset volatility
$DIVy$	-	Dividend yield
DIV	-	Dividend, constant or yield
BC	-	bankruptcy cost (as a % of assets)
TR	-	tax rate
$EQDf$	-	Fraction of assets accruing to equity on default
EIC	-	equity issuance cost
DV	-	Debt value
EV	-	Equity value

6.1 Convergence

Valuation of financial derivatives using binominal trees exhibits a high degree of variability w.r.t the number of steps in the lattice. These models generally converge to the correct result as the number of steps increases. This issue is illustrated on page 433 in Hull (2012), where model values oscillate around the correct value ultimately converging as the number of steps increases. Prior to any application of the S-model we must confirm that the model values converge. We proceed by evaluating model output across a range of number of steps in the binominal tree. We find that the model converges rapidly as we have the number of steps between cash flows (N) exceeding 10. Rapid convergence is in part a result of the choice of the Chriss (1996) model which preserves no-arbitrage conditions and recovers the volatility for any number of time steps. In figure 6.1 below, we show the calculated equity value for a specific parameter set across a range of steps between cash flows as the model converges. To illustrate convergence across multiple parameter sets, in figure 6.2, we show the average, maximum and minimum values for calculated equity across a range of asset volatilities, debt coupon levels and bankruptcy type and a 5 year term. We show a range of steps between cash flows. In total 240 distinct iterations of the model are considered below.

Figure 6.1 Convergence - equity value behaviour
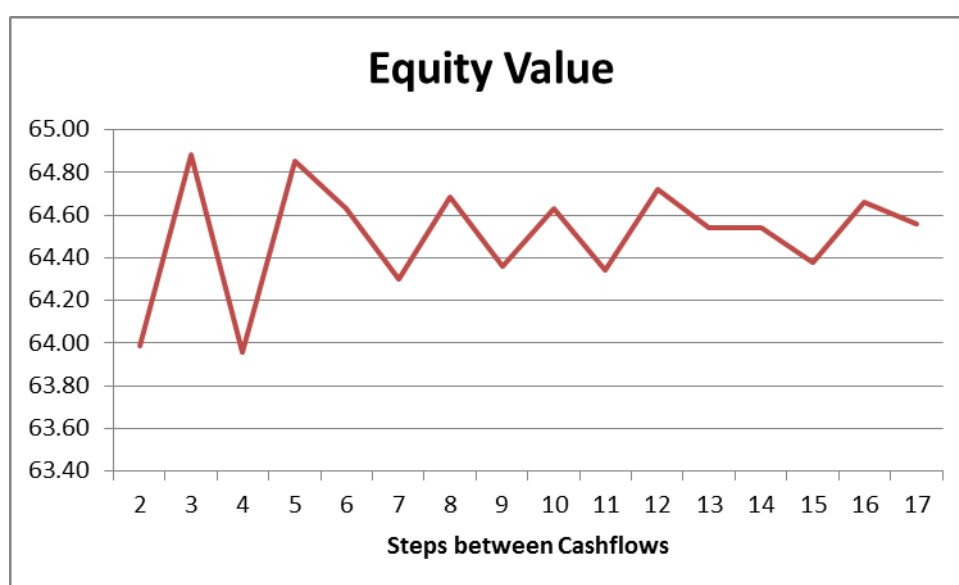
Figure 6.2 Convergence - equity value summary
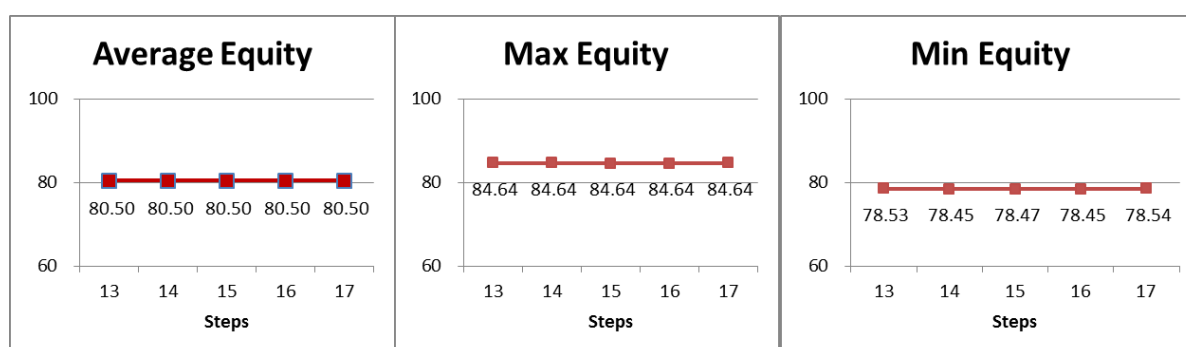
6.2 Merton

Merton (1974) detailed the first structural model of capital structure. He considered zero coupon debt in the context of a variable underlying asset value. He effectively apportioned asset value between debt and equity and highlighted the positive impact on equity of higher volatility and the negative impact on debt of higher volatility. To approximate Merton's structural model, we parameterise the S-model as follows. We specify zero coupon debt ($DCpn = 0$), no bankruptcy costs ($BC = 0$), no tax benefits ($TR=0$), no initial cash ($TC_0 = 0$), no dividends ($DIV = 0$), an equity issue cost that eliminates all equity issuance ($EIC = 1$), bankruptcy only triggered when equity has no value and subordinated debt issuance and special dividends not permitted. In addition we assume that investing and borrowing rates are the same ($r_I = r_B = 5\%$), asset value (V) is 100, term (T) is 5 years, steps between cash flows (N) is 17, asset return rate (k) is set to 5%, the risk free rate (r) is set to 5% and absolute priority of claims is observed ($EQDf = 0$). The results of using this particular combination of parameters are detailed in the table below, titled Parameter Set 1.

We consider equity value, debt value and the sum across a range of debt value (D) and asset volatilities (σ).

Equity (EV) and debt (DV) sum to 100 and the split between the two is a function of asset volatility. As asset volatility increases, so equity is afforded a larger share of total value at the expense of debt. Our results are wholly consistent with Merton (1974).

Table 6.1 Parameter set 1

Parameter Set 1	Debt (D)								
	0			**20**			**40**		
Asset Volatility (σ)	DV	EV	DV+EV	DV	EV	DV+EV	DV	EV	DV+EV
10%	-	100.00	100.00	15.58	84.42	100.00	31.15	68.85	100.00
30%	-	100.00	100.00	15.58	84.42	100.00	30.97	69.03	100.00
50%	-	100.00	100.00	15.43	84.57	100.00	28.94	71.06	100.00

The split between debt and equity, given initial debt of 40, across a range of asset volatility assumptions is shown graphically in figure 6.3.

Figure 6.3 Merton split of value across debt and equity
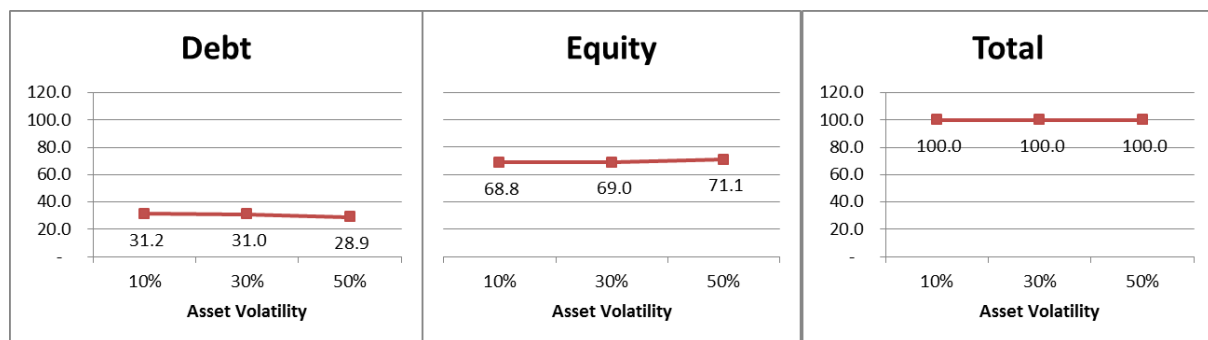
6.3 Coupons and Dividends

The original Merton (1974) model is very limiting as it fails to capture many standard features of equity instruments and debt instruments. We extend the original conceptual framework by firstly permitting coupon bearing debt and then the payment of regular dividends.

We introduce a fixed coupon on debt ($Dcpn = 6\%$). Other than the introduction of a coupon, we make use of the same combination of parameters as parameter set 1. This parameterisation differs significantly from our Merton approximation as we now make interim payments which can only be settled out of the firm's cash balance. The initial cash balance is set to zero but accumulates the asset return over time. To the extent that cash is insufficient to meet the payment obligations, equity will be raised on the assumption that such equity has value. We have set the equity issuance cost to 100% and as such the equity raised has no value and bankruptcy will take place. Bankruptcy costs are set to zero and as such, bankruptcy has no impact on the then asset value. As with parameter set 1 we note the split of

total value between equity and debt. Equity is afforded a larger share of total value at the expense of debt as asset volatility increases. The introduction of a coupon payment on the debt instrument increases the debt instrument's relative share of total value. In addition we highlight the impact on total value of the introduction of interim cash flows, namely that initial asset value is maintained. The results of using this particular combination of parameters are detailed in the table below, titled Parameter Set 2.

Table 6.2 Parameter set 2

Parameter Set 2	Debt (D)								
	0			20			40		
Asset Volatility (σ)	DV	EV	DV+EV	DV	EV	DV+EV	DV	EV	DV+EV
10%	-	100.00	100.00	20.75	79.25	100.00	41.51	58.49	100.00
30%	-	100.00	100.00	20.74	79.26	100.00	40.62	59.38	100.00
50%	-	100.00	100.00	20.22	79.78	100.00	36.79	63.21	100.00

We now link the coupon on debt to the asset volatility and leverage. As detailed in equation 5.3.4.1 in Chapter 5, this implies higher coupons at higher asset volatilities and higher debt levels which generally results in higher values for debt when compared with the results in Parameter Set 2, most notably at elevated levels of asset volatility. The results of using this particular combination of parameters are detailed in the table below, titled Parameter Set 3.

Table 6.3 Parameter set 3

Parameter Set 3	Debt (D)								
	0			20			40		
Asset Volatility (σ)	DV	EV	DV+EV	DV	EV	DV+EV	DV	EV	DV+EV
10%	-	100.00	100.00	20.32	79.68	100.00	41.51	58.49	100.00
30%	-	100.00	100.00	21.17	78.83	100.00	43.85	56.15	100.00
50%	-	100.00	100.00	21.39	78.61	100.00	42.87	57.13	100.00

We introduce a regular payment to equity investors of our firm. We change Parameter Set 2 to include the payment of a dividend (*DIVy*) of 1% of asset value. Equity benefits from this certain cash flow, however this relative benefit is only noted at model valuations where default is a possibility and as such equity will have received an incremental share of the spoils by way of payments prior to bankruptcy (consider parameter set 4 relative to parameter set 2 when asset volatility (*σ*) is 50% and debt (*D*) is 40). The results of using this particular combination of parameters are detailed in the table below, titled Parameter Set 4.

Table 6.4 Parameter set 4

Parameter Set 4	Debt (*D*)								
		0			20			40	
Asset Volatility (*σ*)	DV	EV	DV+EV	DV	EV	DV+EV	DV	EV	DV+EV
10%	-	100.00	100.00	20.75	79.25	100.00	41.51	58.49	100.00
30%	-	100.00	100.00	20.74	79.26	100.00	40.43	59.57	100.00
50%	-	100.00	100.00	20.07	79.93	100.00	36.11	63.89	100.00

Special dividends are reflected as irregular payments to the firms equity shareholders, and if permitted we see similar results. The lower the cover ratio the larger the impact of special dividend payments. A lower cover ratio will increase the size and frequency of special dividends.

We relax the equity issuance constraint by allowing the issue of equity to meet cash flow shortfalls. We change Parameter Set 2 such that the equity issue cost (*EIC*) is reduced to zero. This change allows equity to meet cash flow shortfalls where equity has value. This can delay default to later in the tree and as such is seen to be positive for equity value. The results of using this particular combination of parameters are detailed in table below, titled Parameter Set 5 (consider parameter set 5 relative to parameter set 2 when asset volatility (*σ*) is 50% and debt (*D*) is 40).

Table 6.5 Parameter set 5

Parameter Set 5	Debt (*D*)								
		0			20			40	
Asset Volatility (*σ*)	*DV*	*EV*	*DV+EV*	*DV*	*EV*	*DV+EV*	*DV*	*EV*	*DV+EV*
10%	-	100.00	100.00	20.75	79.25	100.00	41.51	58.49	100.00
30%	-	100.00	100.00	20.74	79.26	100.00	40.62	59.38	100.00
50%	-	100.00	100.00	20.22	79.78	100.00	36.71	63.29	100.00

In the results above we note that as was the case before, equity and debt sum to 100 and the split between the two is a function of asset volatility. As asset volatility increases, so equity is afforded a larger share of total value at the expense of debt. We highlight the fact the introduction of interim cash flows, be they coupons or dividends, does not impact materially on the nature of the results.

6.4 Maintenance of Asset Value

The S-model is given an initial value for the assets of the firm. The model produces values of debt and equity for the firm, which ultimately reflect the firm's asset value. In the absence of an external impact that would change the asset value, we would expect the sum of equity and debt to equal the initial asset value. This result is consistent with Proposition 1 of Modigliani and Miller (1957). We find that the model generates a consistent calculated asset value, namely where calculated debt plus calculated equity sum to 100 across a wide range of parameters. The parameter sets encompass a range of choices for debt value, asset volatility, dividends, debt coupon and term. We find this to be the case in all instances where external impacts to valuation are excluded. External impacts include tax benefits, bankruptcy charges and different rates for borrowing and investing. In total 10,080 distinct iterations of the model are considered.

6.5 Bankruptcy, Taxation and Capital issuance

We now consider external impacts to the model. These introduce changes to total asset value that are outside of the assumed volatile asset process. In addition to the direct impact on asset values and thus equity values and debt values, these external impacts result in changes to the capital structure choices made through the tree, which ultimately impacts the split of firm value between equity and debt.

We introduce the notion of a dead weight cost on bankruptcy ($BC = 50\%$) to the Merton approximation detailed in Parameter Set 1. In this case, on bankruptcy the asset value (net of any cash investments) is reduced by 50% to reflect the impact of a fire sale of assets coupled with the shift away from a going concern. As detailed in section 5.3.9 in Chapter 5, this is an external impact to the asset value and the reduction in value of the underlying asset on bankruptcy should reflect in a reduced combined value for calculated debt and equity whenever bankruptcy is considered within the model as the asset value at that particular node in the tree is reduced. The impact of this charge is noted as debt and equity no longer sum to 100 and the impact is most pronounced at higher asset volatilities which are associated with higher default frequencies. The impact is limited to debt value as the value of equity on default is zero in both parameter sets (see 5.3.9 and 5.3.10). The results of using this particular combination of parameters are detailed in the table below, titled Parameter Set 6. In particular we highlight, parameter set 6 relative to parameter set 1 when asset volatility (σ) is 50% and debt (D) is 40 – debt value is reduced whilst equity value is unchanged.

Table 6.6 Parameter set 6

Parameter Set 6	Debt (D)								
		0			20			40	
Asset Volatility (σ)	DV	EV	DV+EV	DV	EV	DV+EV	DV	EV	DV+EV
10%	-	100.00	100.00	15.58	84.42	100.00	31.15	68.85	100.00
30%	-	100.00	100.00	15.57	84.42	100.00	30.66	69.03	99.69
50%	-	100.00	100.00	15.35	84.57	99.92	27.45	71.06	98.51

We introduce a tax benefit on net interest payments ($TR = 30\%$) as a change to Parameter Set 5. In effect we add the tax saved on coupons and interest to the assets available to meet the claims of equity and debt holders (see section 5.3.5 in Chapter 5). In the absence of debt we note that we are a recipient of net interest and suffer a tax charge. The tax benefit is positively related to the quantum of debt but negatively related to asset volatility. Higher asset volatility implies higher default frequencies. In our model the tax benefit is extinguished on default. Debt values are not impacted by the tax benefit, and as such any benefit is applied to the equity value. The results of using this particular combination of parameters are detailed in the table below, titled Parameter Set 7. In particular we highlight, parameter set 5 relative to parameter set 7 when asset volatility (σ) is 50% and debt (D) is 40,

noting that debt value is maintained whilst equity value has increased, reflecting the incremental gain of the tax benefit.

Table 6.7 Parameter set 7

Parameter Set 7	Debt (D)								
		0			20			40	
Asset Volatility (σ)	DV	EV	DV+EV	DV	EV	DV+EV	DV	EV	DV+EV
10%	-	99.06	99.06	20.75	79.86	100.61	41.51	60.66	102.17
30%	-	99.06	99.06	20.74	79.87	100.61	40.62	61.31	101.93
50%	-	99.06	99.06	20.22	80.29	100.51	36.71	64.61	101.32

We now combine the impact of tax benefits ($TR = 30\%$) and bankruptcy charges ($BC = 50\%$) as changes to Parameter Set 2. The bankruptcy cost is a counterweight to the tax benefit associated with debt. At higher volatilities and high debt levels we note that the tax benefit is more than offset by the bankruptcy charge. Brennan and Schwartz (1978) produced results that were similar to those presented here, however they showed a significant gain in total value as a consequence of leverage (circa 20% of initial asset value). The source of the valuation difference is threefold. Firstly a higher tax rate assumption (50% as opposed to 30% and the resultant additional tax benefits), secondly a lower bankruptcy cost (10% as opposed to 50% and the resultant diminution of the costs of default) and thirdly a longer time horizon (25 years as opposed to 5 years, accentuating the differences). Leland (1994) produced results more in line with the S-model, with similar assumptions relating to tax rate and bankruptcy costs. In both cases it is clear that the tax benefits available on debt must be measured against the increased probability of default associated with leverage and the related charge on bankruptcy, if any. The results of using this particular combination of parameters are detailed in the table below, titled Parameter Set 8.

Table 6.8 Parameter set 8

Parameter Set 8	Debt (D)								
		0			20			40	
Asset Volatility (σ)	DV	EV	DV+EV	DV	EV	DV+EV	DV	EV	DV+EV
10%	-	99.06	99.06	20.75	79.86	100.61	41.51	60.66	102.17
30%	-	99.06	99.06	20.72	79.87	100.59	39.66	61.30	100.97
50%	-	99.06	99.06	19.92	80.29	100.21	34.08	64.47	98.55

Up to this point, the analysis has focussed on bankruptcy being triggered only when equity has no value. An alternate trigger is when the assets of the firm are worth less than the debt obligations of the firm. We adjust the assumptions used in Parameter Set 8 such that bankruptcy is now triggered when the firm does not have a positive net worth. This is a more restrictive test than positive equity value and results in a higher default frequency, which is evidenced by the lower combined values when debt and asset volatility are elevated. Leland (1994) considered both protected debt (positive net worth) and unprotected debt (positive equity) triggers for bankruptcy. His results are consistent with those of the S-model, namely protected debt is a more restrictive condition and incurs a higher overall bankruptcy impact by way of an increased probability of default. The results of using this particular combination of parameters are detailed in the table below, titled Parameter Set 9. In particular we highlight, parameter set 9 relative to parameter set 8 when asset volatility (σ) is 50% and debt (D) is 40, noting that both debt value and equity value are lower reflecting the impact of a higher default frequency.

Table 6.9 Parameter set 9

Parameter Set 9	Debt (D)								
		0			20			40	
Asset Volatility (σ)	DV	EV	DV+EV	DV	EV	DV+EV	DV	EV	DV+EV
10%	-	99.06	99.06	20.75	79.86	100.61	41.51	60.66	102.17
30%	-	99.06	99.06	20.72	79.87	100.59	39.40	61.04	100.44
50%	-	99.06	99.06	19.89	80.18	100.07	33.68	63.22	96.90

In practise, equity holders continue to wield a degree of control when a firm suffers bankruptcy. This control is evidenced through positive values for equity following default. To capture this feature, we now introduce the notion of an equity residual value on default (*EQDf*) that is not zero. We adjust the assumptions used in Parameter Set 8 such that this parameter is equal to 20%. In effect we are imposing an alternate split of asset value on default and directing a percentage of the available value to equity holders. Total value is seen to be similar to that when equity residual is zero but equity receives a larger share. The results of using this particular combination of parameters are detailed in the table below, titled Parameter Set 10. In particular we highlight, parameter set 10 relative to parameter set 8 when asset volatility (σ) is 50% and debt (D) is 40, noting that while total value is maintained the split between debt and equity has shifted, with equity receiving a larger share.

Table 6.10 Parameter set 10

Parameter Set 10	Debt (D)								
	0			**20**			**40**		
Asset Volatility (σ)	DV	EV	DV+EV	DV	EV	DV+EV	DV	EV	DV+EV
10%	-	99.06	99.06	20.75	79.86	100.61	41.51	60.66	102.17
30%	-	99.06	99.06	20.72	79.87	100.59	39.36	61.55	100.91
50%	-	99.06	99.06	19.78	80.42	100.20	33.28	65.26	98.54

In practise, market levels for borrowing rates and investment rates are not the same. We recognise that a firm will borrow cash at a rate higher than the rate it will earn on positive cash balances. We adjust the assumptions used in Parameter Set 2 to reflect the use of different investment ($r_I = 4\%$) rates and borrowing ($r_B = 6\%$) rates. Total value is reduced across all variations as less interest is earned and more interest is paid. The results of using this particular combination of parameters are detailed in the table below, titled Parameter Set 11.

Table 6.11 Parameter set 11

Parameter Set 11	Debt (D)								
	0			**20**			**40**		
Asset Volatility (σ)	DV	EV	DV+EV	DV	EV	DV+EV	DV	EV	DV+EV
10%	-	99.44	99.44	20.75	78.80	99.55	41.51	58.15	99.66
30%	-	99.44	99.44	20.74	78.81	99.55	40.61	59.05	99.65
50%	-	99.44	99.44	20.20	79.34	99.55	36.72	62.92	99.64

Thus far equity issuance has been used when additional financing is needed. We now consider the impact of subordinated debt issuance. In our model subordinated debt will only be issued when additional financing is needed. In addition subordinated debt issuance capacity will be exhausted prior to any equity issuance. Subordinated debt issuance is subject to a positive net worth constraint, namely that assets exceed liabilities on issue. The positive net worth constraint for subordinated debt issuance, generally means that subordinated debt financing takes place when we have negative initial cash balances and a bankruptcy charge. When we have positive initial cash balances and no bankruptcy charges a financing need is only triggered when liabilities exceed assets, which fails the positive net worth constraint for

subordinated debt issuance.   To illustrate the impact of the introduction of subordinated debt to the S-model framework we consider a specific set of initial parameters that results in subordinated debt issuance in the model. We make use of the assumptions from parameter set 2 except that initial cash (TC_0) is set to -30 (which given that short term cash borrowing is considered risk free in the S-model, implies that there is 70 of asset value at initiation to meet the demands of debt and equity) and we introduce a dead weight cost on bankruptcy ($BC =$ 50%). For this parameter set we find that subordinated debt issuance generally (but not always) enhances value for both equity and debt. The results of using this particular combination of parameters are detailed in the tables below. Parameter Set 12 excludes the issue of subordinated debt while Parameter Set 13 includes the issue of subordinated debt. The ability to issue subordinated debt reduces the dilution suffered by equity as equity issuance happens less often, which is generally favourable for equity values. Subordinated debt issuance is limited by the net worth constraint and any such issuance increases the cash call on the firm's assets (the firm must settle both the debt coupon and the subordinated debt coupon) which will accelerate default in cases where equity issuance does not add value. Accelerated default may be positive or negative for equity and debt. Equity raised on a branch of the tree where default ultimately takes place is negative for both equity and debt. Equity raised on a branch of the tree where default is avoided is positive for both equity and debt.

Table 6.12 Parameter set 12

Parameter Set 12	Debt (D)								
	0			20			40		
Asset Volatility (σ)	DV	EV	DV+EV	DV	EV	DV+EV	DV	EV	DV+EV
10%	-	70.00	70.00	20.67	47.74	68.41	41.51	28.49	70.00
30%	-	69.29	69.29	19.46	46.91	66.38	36.87	30.09	66.96
50%	-	69.42	69.42	16.61	49.46	66.08	29.07	35.59	64.66

Table 6.13 Parameter set 13

Parameter Set 13	Debt (D)								
	0			20			40		
Asset Volatility (σ)	DV	EV	DV+EV	DV	EV	DV+EV	DV	EV	DV+EV
10%	-	70.00	70.00	20.75	49.09	69.84	41.20	26.83	68.03
30%	-	69.97	69.97	19.39	49.43	68.82	35.28	30.88	66.16
50%	-	70.50	70.50	16.92	51.91	68.83	28.07	37.00	65.07

6.6 Impact of Liquidity

A firm's operations are driven by cash receipts, cash payments and the performance of the underlying assets held by the firm. In the analysis thus far we have focussed on the impact of coupons, dividends, asset volatility and the like on the relative distribution of value across the firm's equity instruments and debt instruments. In addition we have assessed the impact of bankruptcy on firm value as well as incorporating the differential tax treatment available across interest payments and dividend payments. We have dealt superficially with the need to raise additional cash resources, either via equity at a cost or via subordinated debt. We now focus on the impact on equity value and debt value as well as capital structure choices of our approach for incorporating liquidity in the modelling of capital structure.

The effective cash position, or firm liquidity, has a material impact on the behaviour of our model.

All payments made by the firm must be made in cash. The firm maintains a cash balance that may be positive or negative. This cash balance is augmented by the return on the asset and is reduced by coupons, dividends and the like. Positive cash balances earn an investment rate. Negative cash balances incur a borrowing rate. The investment rate is generally less than the borrowing rate which in turn is less than the rate payable on debt. The cash balance is subject to maximum positive and negative limits that may prompt additional cash payments or cash financing. In the event that additional cash financing is required the firm may source funds through a combination of subordinated debt and equity. Subordinated debt is expensive and equity is dilutive and incurs additional issuance charges. We have modelled the cash balance to be risk free and as such, in the event of bankruptcy any short term debt is settled first and in full. Bankruptcy charges reduce the asset value on default and prompt earlier financing in the presence of short term debt.

We define Initial Debt (*ID*), reflecting total borrowings, as the sum of debt value (*D*) and initial cash (*TC$_0$*).

$$ID = D + TC_0 \qquad (6.6.1)$$

The components of *ID* reflect the funding mix. We can have positive or negative values of initial cash and a range values for debt (including zero). The introduction of *ID* allows us to consider behaviour within the firm for a given level of borrowings across as range of funding choices.

Thus far we have generally assumed that the firm has no initial cash holdings or borrowings when considering the impact of parameter choices on firm value and firm behaviour, we now consider values for initial cash (TC_0) other than zero. We evaluate the impact of different funding mixes for a given level of initial indebtedness. To do so, we adjust the assumptions made in Parameter set 2 to reflect a range of cash values as well as introducing a dead weight cost on bankruptcy ($BC = 50\%$). In particular we focus on instances where ID is equal to 20 and when ID is equal to 40. In the analysis below we note that negative values for initial cash (TC_0) accelerate default which negatively impacts both equity and debt values. Conversely positive values for initial cash (TC_0) delays default which positively impacts both equity and debt values. Perversely this is somewhat counterintuitive as it favours holding positive cash balances in the presence of debt. This result is consistent with our definition of short term borrowings being risk free and is reflective of negative values for cash holdings in equation 5.3.1 from Chapter 5. The results of using this particular combination of parameters are detailed in the table below, titled Parameter Set 14.

Table 6.14 Parameter set 14

Parameter Set 14	Debt (D), Initial Cash (TC0), Initial Debt (ID)											
	20, -20, 40			40,0, 40			40, 20, 20			20, 0, 20		
Asset Volatility (σ)	DV	EV	DV+EV	DV	EV	DV+EV	DV	EV	DV+EV	DV	EV	DV+EV
10%	40.74	59.09	99.83	41.51	58.49	100.00	21.51	78.49	100.00	20.75	79.25	100.00
30%	40.15	58.01	98.16	39.62	59.37	98.99	21.47	78.51	99.98	20.72	79.26	99.98
50%	37.49	59.78	97.27	34.08	63.17	97.25	20.60	79.10	99.70	19.92	79.78	99.70

We extend our analysis of liquidity by introducing a tax benefit on net interest payments ($TR = 30\%$). Interest earned on positive initial cash balances will negate the tax benefit and we highlight that in the presence of a tax benefit on interest paid, a firm earning net positive interest will suffer a reduction in overall value. The results are similar to those highlighted above, although the value in holding positive cash balances in the presence of debt is muted, as the interest earned on the positive cash balances will suffer an effective charge in the presence of a tax benefit for interest paid. The results of using this particular combination of parameters are detailed in the table below, titled Parameter Set 15.

Table 6.15 Parameter set 15

Parameter Set 15	Debt (D), Initial Cash (TC0), Initial Debt (ID)											
	20, -20, 40			40,0, 40			40, 20, 20			20, 0, 20		
Asset Volatility (σ)	DV	EV	DV+EV	DV	EV	DV+EV	DV	EV	DV+EV	DV	EV	DV+EV
10%	40.74	61.01	101.75	41.51	60.66	102.17	21.51	79.33	100.84	20.75	79.86	100.61
30%	40.19	59.58	99.77	39.66	61.30	100.97	21.47	79.34	100.82	20.72	79.87	100.59
50%	38.06	61.47	99.53	34.08	64.47	98.55	20.60	79.81	100.42	19.92	80.29	100.21

The results above highlight the interaction between bankruptcy charges, tax benefits and liquidity. We have observed that the various factors act to damp the impact of any of the costs or benefits associated with the others. The tax benefits of interest which are somewhat offset by the bankruptcy costs associated with incremental debt levels are further constrained by the limitations imposed on debt service by liquidity.

A common question posed in the literature is the pursuit of an optimal capital allocation, namely what levels of debt are appropriate for a firm, and given the value attributable to the tax benefits of interest how far does the ideal firm structure deviate from the Modigliani and Miller statement that capital structure is irrelevant. We consider the calculated asset value (i.e. the sum of the calculated debt value and the calculated equity value) as a ratio over the initial asset value across a range of values for initial debt (*ID*) and a broad range of other parameters. A ratio in excess of 1 implies a total for debt and equity in excess of the initial asset value, while a ratio below 1 implies a total for debt and equity in excess of the initial asset value. In effect, when this ratio is above 1, the benefits of leverage and tax are more than the costs of bankruptcy, while when this ratio is below 1, the benefits of leverage and tax are less than the costs of bankruptcy. Optimal capital allocation will coincide with the maximum value for this ratio. The results of using these particular combinations of parameters are detailed in the table below, titled Parameter Set 16.

Table 6.16 Parameter set 16

Paramter Set 16	Initial Debt (ID)									
Asset Volatility (σ)	-30	-20	-10	10	20	30	40	50	60	70
10%	0.978	0.982	0.986	0.996	1.003	1.011	1.015	1.008	1.011	1.009
30%	0.978	0.982	0.987	0.998	1.004	1.005	0.995	0.980	0.983	0.945
50%	0.978	0.982	0.987	0.998	0.999	0.995	0.979	0.963	0.938	0.893

We note that optimal capital structure for a given firm volatility is arrived at with some initial debt, however the variation of the optimal value is limited across a range of initial debt levels. Excessive debt and large holdings of cash in the absence of debt are seen to destroy value.

These results are broadly in line with those of Anderson and Carverhill (2012), and generally refute the findings of Brennan and Schwartz (1978) and Leland (1994). Anderson and Carverhill (2012) found optimal values for leverage across a wide swathe of parameters, noting that effectively some leverage was good but excessive leverage was bad. Brennan and Schwartz (1978) and Leland (1994) both found significant incremental value in firm leverage, however neither of them suffered liquidity as a constraint on debt service. In general large positive cash balances, in the absence of debt acts as a drag on total value. Excessive leverage, regardless of the combination of cash and debt results in lower levels of total value as bankruptcy costs dominate. Optimal capital structure suggests holding moderate levels of debt however the variation across debt levels is muted, suggesting an optimal range rather than a specific structure.

The S-model permits a wide range of parameter choices that allows for a rich description of capital structure dynamics. We now highlight the S-model's capacity to approximate normal market conditions by considering specific parameter sets. We extend the assumptions detailed in Parameter Set 15 to allow for subordinated debt, and we link coupon rates to asset volatility and debt levels and the equity issue cost (EIC) is set to 10% (in this instance the firm has access to an additional layer of debt prior to equity issuance, leverage impacts on the cost of debt capital and when equity is issued it suffers an issue cost). As above we focus on instances where ID is equal to 20 and when ID is equal to 40. We note that the presence of debt adds value to the total (DV+EV) but that the incremental gain is small. In addition we note that increases in asset volatility (as an approximation of normal market conditions) do not always coincide with an increase in equity value. The results of using this particular combination of parameters are detailed in the table below, titled Parameter Set 17.

Table 6.17 Parameter set 17

Parameter Set 17	Debt (D), Initial Cash (TC0), Initial Debt (ID)											
	20, -20, 40			40,0, 40			40, 20, 20			20, 0, 20		
Asset Volatility (σ)	DV	EV	DV+EV	DV	EV	DV+EV	DV	EV	DV+EV	DV	EV	DV+EV
10%	40.43	61.41	101.84	41.51	60.66	102.17	21.22	79.54	100.75	20.32	80.16	100.49
30%	40.70	60.06	100.76	42.39	58.68	101.07	24.02	77.54	101.56	21.15	79.57	100.72
50%	39.11	60.05	99.17	38.83	59.85	98.68	24.67	76.35	101.01	20.86	79.41	100.27

We conclude our approximation of normal market conditions by recognising that equity holders have significant say in the event of default and as such the assumption that equity receives nothing on default is not a fair reflection of reality. We adjust the parameters above such that the equity residual value on default (*EQDf*) is set equal to 20%. This effectively negates absolute priority on default and reflects equity holder's significant rights on bankruptcy. In general this is good for equity holders, bad for debt holders and has a small, but positive impact on total value. This positive impact on total value is reflective of equity holders choosing bankruptcy even when equity has value as their share of the proceeds on bankruptcy exceeds the potential value of delaying bankruptcy.

Table 6.18 Parameter set 18

Parameter Set 18	Debt (D), Initial Cash (TC0), Initial Debt (ID)											
	20, -20, 40			40,0, 40			40, 20, 20			20, 0, 20		
Asset Volatility (σ)	DV	EV	DV+EV	DV	EV	DV+EV	DV	EV	DV+EV	DV	EV	DV+EV
10%	40.43	61.41	101.84	41.51	60.66	102.17	21.22	79.54	100.75	20.32	80.16	100.49
30%	40.34	60.36	100.70	41.80	59.08	100.88	23.96	77.58	101.54	21.15	79.57	100.72
50%	38.53	60.53	99.06	37.99	60.63	98.62	23.85	77.11	100.97	20.65	79.61	100.26

6.7 S-Model Results Discussion

In the presentation above we have outlined the behaviour of the S-model across a wide range of parameter sets. The S-model replicates the initial results of Merton (1974) as well as capturing the broad dynamics of bankruptcy charges and tax benefits that were detailed in Brennan and Schwartz (1978) and Leland (1994), namely an increment to total value from tax benefits and a decrement to total value from bankruptcy charges. We explained above that the parameter choices in both implied a significantly larger increase in total firm value as a result of leverage than that evidenced in our model and we highlighted the dampening of the impact on total value as a consequence of our introduction of liquidity. Leland (1994) consideration of two distinct triggers for bankruptcy reflects the value of choice in capital structure. In an environment where equity value determines bankruptcy the choice to enter bankruptcy lies primarily with equity holders and whilst Leland does not cater for liquidity in detail, this trigger is a function of the inability to source additional cash. In an environment where asset value determines bankruptcy (by way of a positive net worth requirement) the choice to enter bankruptcy lies primarily with debt holders and this trigger is not a function of the inability to source additional cash. These distinct triggers are consistent with the work reported by Davydenko (2012) who considered empirical evidence around different causes of bankruptcy, namely a lack of liquidity or low asset values. He discusses academic research conducted in this area and finds that both triggers are relevant. Leland (1994) also considered a deviation from absolute priority on bankruptcy and noted results mirrored by the S-Model, namely that the split of total value was skewed to equity holders. Anderson and Carverhill (2012) deal explicitly with liquidity in their model but their model formulation is very different from that of the S-model. Their stochastic variable is the revenue generated by the firm, with equity, debt and asset value derived as results of their process. The S-model considers a stochastic asset value with a fixed yield on asset value as the primary driver of firm revenue, with equity and debt derived as a result of the process. Their treatment of cash balances and cash constraints is very similar to that of the S-model and a number of results are wholly consistent. The counterintuitive findings detailed above, where positive cash balances coupled with long term debt are considered optimal, is mirrored in their paper. Furthermore, as is the case with the S-model, they find increases in volatility are not always good for equity at the expense of debt and that lower volatility may in fact imply higher total values for the combination of equity and debt. This result refutes to some extent the notion of asset substitution as detailed in Myers (2001). In practise, both the S-model and Anderson

and Carverhill (2012) capture capital structure dynamics that cater for the three dominant theories, namely trade-off, pecking order and agency costs. Trade-off is the essence of Merton (1974) whilst the pecking order is explicitly captured by way of the use of funds (namely positive cash balances exhausted first, then negative cash balances up to a maximum and finally the issue of equity). The S-model and Anderson and Carverhill (2012) differ in a number of specific areas. The S-model considers both liquidity and asset value as triggers for bankruptcy, which is consistent with the empirical work highlighted above whereas Anderson and Carverhill cannot consider low asset values as these are results of their approach rather than elements of their model.

6.8 Conclusion

We defined our structural model of firm capital. The model incorporates firm liquidity and provides for adjustments to leverage by way of subordinated debt and equity. In addition the model allows for deviation from absolute priority of claims on default. The model is expressed in discrete time and is delivered in a binominal tree framework. The model provides a robust frame for expressing firm dynamics. Model behaviour is consistent with the results of Anderson and Carverhill (2012) and in particular we note that optimal leverage is not so much a single point but rather a range within which total firm value is broadly in line. Excessive leverage is seen to destroy firm value, while deviations from absolute priority are good for equity but bad for firm value. Subordinated debt issuance is seen as expensive funding but is a viable alternative to equity when absolute priority of claims on default is not observed. Positive short term cash balances in the presence of long term debt provides for the maintenance of asset value.

7 CONCLUSION

In this chapter we conclude. Firstly, we draw the work undertaken across the earlier chapters together by briefly evaluating the companies considered in Chapter 2 and Chapter 3 in the context of the S-model, highlighting how the extensions and expansions of the S-model relate to some of the results derived in the earlier chapters. Secondly, we summarise the findings of the thesis and suggest areas for further research.

7.1 Applying the S-Model to Anglo American Plc and African Bank Limited

We consider the application of the S-model, detailed in Chapter 5, to the capital structures of Anglo American Plc and African Bank Limited. In particular we explore in brief whether the S-model, with its expanded parameter set and focus on liquidity casts any additional light on the behaviour of these particular companies. In Chapter 3 we looked at two diversified resources companies, Anglo American Plc and BHP Billiton Plc, through the lens of the Merton model and devised a trading strategy that took advantage of significant deviations in traded prices from that derived from the model. In the course of this evaluation we noted differences in the capital structure of Anglo American Plc and BHP Billiton Plc and formulated a view on the impact of these differences on the relative behaviour of the securities of the two firms. It is these differences that we focus on in the application of the S-model on these two firms. In Chapter 4 we looked at the performance of the trading prices of various securities of African Bank Limited in the period leading up to it being placed in

curatorship. We highlight the relatively high level of leverage on the African Bank Limited balance sheet and the subdued level of asset volatility implied by market prices. It is these two elements that we focus on in the application of the S-model on African Bank Limited.

The analyses detailed in Chapter 3 and Chapter 4 involves a series of revaluations of theoretical levels for the prices of debt securities and equity securities issued by the firms considered. The S-model permits a given set of assumptions and delivers values for debt and equity. The repetitive evaluation of parameter sets that map to the individual firms across the period under review is beyond the scope of this chapter, however there is merit in considering whether the differences noted in relative capital structure and the apparently unreasonable assumptions w.r.t certain inputs can be incorporated in stylised iterations of the S-model. The changes in S-model output across these different assumption sets can provide guidance as to expected market behaviour which can then be considered in the light of actual market behaviour over the period. These stylised parameterisations are not meant to be exact, rather they are rough approximations meant to provide illustrative value.

In the case of Anglo American Plc and BHP Billiton Plc two issues noted in the analysis detailed in Chapter 3 are the impact in the financial crisis of 2008 of the higher leverage of Anglo American Plc relative to BHP Billiton Plc and the construction of Anglo American Plc leverage (namely that a relatively larger component of outstanding debt is short term in nature). In 2008 commodity prices suffered significant moves lower and experienced elevated volatility. This resulted in very large changes in the value of debt and equity instruments issued by Anglo American Plc and BHP Billiton Plc. We map Anglo American Plc to stylised parameterisations of the S-model. We apply changes to the assumed asset value and asset volatility of these mappings that are consistent with the changes experienced in the relevant commodity proxy value and commodity proxy volatility during the financial crisis of 2008. We consider the relative changes in the S-model's equity and debt value for Anglo American Plc. These changes are contrasted on a qualitative basis with those experienced in practise.

In the case of African Bank Limited we focus on the high level of leverage and the low asset volatility implied by market prices for securities issued by African Bank Limited. We map African Bank Limited to a stylised parameterisation of the S-model. In addition we consider an alternate firm with lower leverage. We apply changes to the assumed asset value and asset volatility that are consistent with the implied changes in asset value and asset volatility in the period leading up to the rights issue in August 2013 and in the period leading up to the bank

being placed in curatorship. We also consider the impact of short term borrowings as opposed to long term debt in the context of African Bank limited.

7.1.1 The S-Model applied to Anglo American Plc

We consider the state of Anglo American Plc and the relevant commodity markets as at April 2008 and December 2008. In this period, commodity markets suffered a significant decline and heightened volatility, with the proxy basket representing the underlying assets of Anglo American Plc losing 40% of value and realised volatility move from 20% to 55%. Over the same time period, Anglo American Plc equity suffered a 65% decline and the 10 year CDS on Anglo American Plc moved from 200 basis points to 800 basis points.

We begin by constructing a stylised parameterisation of the S-model to capture the essence of Anglo American Plc as at April 2008. We make use of leverage and volatility inputs that are consistent with our analysis in Chapter 4 and specify a number of parameters that reflect our view on normal market conditions. We specify coupon bearing debt ($DCpn = 6\%$, a 2% spread over a risk free rate of 4%), bankruptcy costs ($BC = 50\%$), tax benefits ($TR = 30\%$), no initial cash ($TC_0 = 0$), dividends ($DIVy = 1\%$), equity issue cost ($EIC = 10\%$), bankruptcy only triggered when equity has no value and subordinated debt issuance and special dividends not permitted. In addition we assume that investing and borrowing rates are not the same ($r_I = 3\%$; $r_B = 5\%$), asset value (V) is 100 (reflecting the implied asset value of approximately \$ 100 billion), asset volatility (σ) is 20%, debt value (D) is 20, term (T) is 10 years, steps between cash flows is 10, asset return rate (k) is set to 4%, the risk free rate (r) is set to 4% and absolute priority of claims is not observed ($EQDf = 10\%$). As a first step from our base assumptions for Anglo American Plc we adjust initial cash (to -10) and debt value (to 10) to better reflect the firm's mix of short term and long term debt whilst maintaining the overall leverage ratio. We apply a number of adjustments to both the base firm and adjusted firm. In the first case we incorporate changes to the asset value ($V = 60$) and the asset volatility ($\sigma = 55\%$) as well as reflecting the lower risk free rate ($r = 2.5\%$; $r_I = 1.5\%$; $r_B = 3.5\%$). In the next case we reflect the extreme change in the cost of funding experienced by Anglo American Plc in the short term ($r_B = 7.5\%$). The S-model output of applying these various parameterisations is shown in Table 7.1 below.

Table 7.1 S-Model applied to Anglo American Plc

	Base Case (BaC)	BaC adjusted for short term cash borrowing (BaCA)	BaC, Asset decline, volatility increase	BaC Asset decline, volatility increase, short term borrowing rate higher	BaCA, Asset decline, volatility increase	BaCA Asset decline, volatility increase, short term borrowing rate higher
Debt Value	23.06	11.54	17.75	17.75	8.53	8.28
Equity Value	77.41	78.72	40.91	40.91	39.33	37.94
Short Term Cash Borrowings	0.00	10.00	0.00	0.00	10.00	10.00
% Change in Debt Value			-23%	-23%	-26%	-28%
% Change in Equity Value			-47%	-47%	-50%	-52%

The S-model shows the expected steep declines in the value of both debt and equity. Of particular interest is the impact of a large portion of the debt being held in the form of short term borrowings. In the S-model, these obligations are seen as risk free and as such impose a larger burden on the remaining liabilities of the firm. This impact is noted in the steeper declines in both the debt and equity of Anglo American Plc when applying an elevated short term borrowing cost relative to the base assumption of leverage taking the form of long term debt only.

7.1.2 The S-model applied to African Bank Limited

We consider the state of the debt and equity instruments of African Bank Limited as at the end of December 2012, the end of July 2013 and the beginning August 2014. In this period the implied asset value dropped by in excess of 50% (after accounting for the growth in debt), equity value was all but extinguished and debt instruments were exposed to real losses.

We begin by constructing a stylised parameterisation of the S-model to capture the essence of African Bank Limited as at December 2012. We make use of leverage and volatility inputs that are consistent with our analysis in Chapter 4 and specify a number of parameters that reflect our view on normal market conditions. We specify coupon bearing debt ($DCpn = 7.5\%$, a 2.5% spread over a risk free rate of 5%), bankruptcy costs ($BC = 50\%$), tax benefits ($TR=30\%$), no initial cash ($TC_0 = 0$), no dividends ($DIV = 0$), equity issue cost ($EIC = 10\%$),

bankruptcy only triggered when equity has no value and subordinated debt issuance and special dividends not permitted. In addition we assume that investing and borrowing rates are not the same ($r_I = 4\%$; $r_B = 6\%$), asset value (V) is 85 (reflecting the implied asset value of approximately R 85 billion), asset volatility (σ) is 12%, debt value (D) is 20, term (T) is 10 years, steps between cash flows is 10, asset return rate (k) is set to 5%, the risk free rate (r) is set to 5% and absolute priority of claims is not observed ($EQDf = 10\%$). The asset volatility assumption above reflects that found in our analysis when applying the DTD1 approach in Chapter 4. In addition we consider an alternate firm which has debt approximately 20% lower ($D = 39$). We also consider variations to both African Bank Limited and the alternate firm where a portion of the leverage is in the form of short term borrowings ($C = -10$). As a first step from our base assumptions for African Bank Limited and the alternate firm we adjust asset volatility (to 31%) to reflect the output of applying the adjusted DTD2 approach (as detailed in Chapter 4), which we consider to be the most effective given our inputs on market data. We apply a number of adjustments to both the base firm and the alternate firm. In the first case we incorporate changes to the asset value as at July 2013 ($V = 73$, $D = 56.5$) which reflects an increase in liabilities coupled with a significant decline in total asset value. In this case the asset volatility under the two approaches, DTD1 and adjusted DTD2 is used ($\sigma = 12\%$ and $\sigma = 25\%$). Finally we incorporate changes to the asset value as at August 2014 ($V = 48$, $D = 56.5$) which coincides with the effective default. The S-model output of applying these various parameterisations is shown in Table 7.2 below.

Table 7.2 S-Model applied to African Bank Limited

		Base Case (BaC)	BaC Adjusted for higher volatility	BaC adjusted for short term cash borrowing (BaCA)	BaCA adjusted for higher volatility	Alternate firm base case (ABaC)	ABaC adjusted for higher volatility
Debt Value		50.03	45.60	31.42	29.51	39.88	39.65
Equity Value		35.84	35.61	9.81	22.44	45.87	45.91
Short Term Cash Borrowings		0.00	0.00	10.00	10.00	0.00	0.00
					(As at December 2012)		
Debt Value (Adjusted for change in total debt)		27.55	28.42	18.21	18.16	37.51	33.40
Equity Value		4.05	9.37	2.49	4.27	24.93	24.86
Short Term Cash Borrowings		0.00	0.00	10.00	10.00	0.00	0.00
% Change in Debt Value		-45%	-38%	-42%	-38%	-6%	-16%
% Change in Equity Value		-89%	-74%	-75%	-81%	-46%	-46%
					(As at July 2013)		
Debt Value (Adjusted for change in total debt)		23.05	23.05	13.91	13.91	22.98	22.98
Equity Value		2.11	2.11	1.17	1.17	2.18	2.18
Short Term Cash Borrowings		0.00	0.00	10.00	10.00	0.00	0.00
% Change in Debt Value		-54%	-49%	-56%	-53%	-42%	-42%
		-94%	-94%	-88%	-95%	-95%	-95%
% Change in Equity Value					(As at August 2014)		

We note that the impact of a higher volatility as at December 2012 is a reduction in the theoretical value of the debt of the frim. This is in contrast to the alternate firm where values for debt and equity are much the same across the two volatility assumptions. The impact of adjusting the overall leverage to include a portion held in short term debt is dramatic. Across the two volatility assumptions the values of debt and equity suffer significant declines. This is a result of the punitive treatment on negative cash balances by the S-model, coupled with the high leverage and high coupon of the firm. This combination results in higher default frequencies. When we apply the changes to market to reflect the state as at July 2013 we note that equity and debt both suffer enormous declines in value across both volatility assumptions. Equity declines in the order of 90% and debt in the order of 45%, which is consistent with default. It is important to realise that this does not coincide with the curatorship but rather a

full year earlier. Of particular interest is the impact of these changes on our alternate firm which carries less gearing. In this instance debt declines in value (6% to 16% depending on the volatility assumption) and equity suffers large losses (46% across both volatility assumptions) but remains solvent. The application of the market changes to reflect the state as at August 2014 shows default characteristics across all assumptions.

7.2 Merton's model – value added vs. empirical performance

The empirical studies discussed in Chapter 2 show mixed results. Competing studies highlighted different conclusions with the explanatory power of structural models disputed. A number of adjustments to the approaches taken have been suggested which show some promise in improving the performance of the structural models. We chose to avoid a replication of the work detailed in Chapter 2 and we focussed on two specific applications of the model and looked to evaluate the performance of the model along different vectors. We consider whether the application of the Merton model in our two examples provide incremental value. The choice of measure for the test of incremental value is key. When we consider Anglo American Plc and BHP Billiton Plc we devise a trading strategy that makes use of the Merton model to deliver a highly profitable investment performance. When we consider African Bank Limited we show that the market prices for debt and equity provided an early warning of the default of the institution. In both cases the application of Merton's structural model provided incremental value and insight.

7.3 S-Model – Theory, Behaviour and Application

In Chapter 5 we detail a robust expansion of Merton's structural model. The S-model caters for a wide range of market and instrument characteristics with a focus on liquidity. The cornerstone of the model is the recognition that all payments made to or from a firm must ultimately be reflected via cash. In Chapter 6 we outline the range of S-model behaviour across a plethora of assumptions regarding both the firm and the market in which it operates. We highlight the notion of optimal capital structure as a wide range of choices regarding leverage and we note the impact of liquidity on valuation. The application of the S-model is briefly explored earlier in this chapter where choices around liquidity are seen to impact value.

7.4 Future study

In Chapter 3 and Chapter 4 we focus on the value gained by the application of structural models. The choice of measure is key, and we believe further study is warranted in expanding the application of the models and devising the measures applied.

The S-model introduced in Chapter 5 and explored further in Chapter 6 can be applied to a wide range of firm decision making and firm valuation problems. The interaction between liquidity, debt, financing options and priority of claims on default offer a rich seam for future study.

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9 APPENDICES

APPENDIX 1 : S-MODEL SOURCE CODE

We implemented the S-model in Excel VBA. The source code is provided below with limited comments. Where processes are replicated (upper spine, interior and lower spine) they are generally only described once. The Excel worksheet used in generating all of the output described in Chapter 6, as well as the output generated is provided at [https://www.dropbox.com/sh/nu7eg2g2auygeam/AACWBGthef8DniLduy8Fj8eUa?dl=0]. This implementation differs from the source code below only to the extent that it provides a mechanism for running the model many times over across a range of parameter set choices.

```
Option Explicit
Option Base 0
Const OutputLevel = 13


Public TreeArray() As Double
Public CashFwdBckEq() As Double
Public CashFwdBckSuD() As Double
Public CashFwdBckSpD() As Double
Public CashBck() As Double


'Tree is structured with annual cashflows -- Debt, dividends etc.
'Term (Years), Steps (Total Steps in Tree), Asset (Initial Asset Value), Cash (Initial Cash value), Debt (Initial Debt value), DebtCpn (Initial Debt Coupon - can be fixed or linked to Leverage + Volatility)
Public Term As Double, Steps As Integer, Asset As Double, Cash As Double, Debt As Double, DebtCpn As Double
'DivRate (Dividend Yield), DivConst (Dividend Cash Flow), RFR (Risk Free Rate), AssRet (Asset yield), AssOpConst (Asset operating costs cash flow), AssVol (Asset volatility)
Public DivRate As Double, DivConst As Double, RFR As Double, AssRet As Double, AssOpConst As Double, AssVol As Double
'InvRate (Earned on + cash balances), BorRate (Paid on - cash balances), SubDebtCpn (Subordinated debt coupon - can be fixed or linked to leverage), SubDebt (subordinated debt)
'BancrupCost (Bancruptcy costs as % of assets), StepsCF (# steps in tree between cashflows), TaxRate (Tax rate applied to interest paid and earned)
Public InvRate As Double, BorRate As Double, SubDebtCpn As Double, SubDebt As Double, BancrupCost As Double, StepsCF As Double, TaxRate As Double


'MaxCover (maximum interest and costs cover), Flags to cater for Special Dividends, Buyback of Subordinated Debt, Calculated or fixed debt coupons (Sub + Snr)  and Subordinated debt issuance
Public MaxCover As Double, SpecDivFlag As Integer, SubDebtBBFlag As Integer, DebtCPNFlag As Integer, SubDebtFlag As Integer
```

Public ExcessCash As Double, NewSubDebt As Double, AssetPLUSCash As Double, SnrDebtRatio As Double, DebtRatio As Double

'DivChFlag (Fixed or float dividends), SpreadRatio (Impacts subordinated debt cost on issuance), SubDebtBBCost (costs applied on subordinated debt buyback), EquityIssueCost (costs applied on equity issuance)

Public DivChFlag As Double, SpreadRatio As Double, SubDebtBBCost As Double, EquityIssueCost As Double

Public UsedDiv As Double, UsedOpCost As Double, UsedSubDebtCpn As Double, MaxNewSubDebt As Double, EquityIssued As Double, EquityResidual As Double, BancrupType As Integer

Public a As Double, u As Double, d As Double, p As Double, a_df As Double, u1 As Double, d1 As Double

Public dT As Double, UsedSubDebt As Double, BancrupValue As Double, NetCashInt As Double, EqtyValDflt As Double, BaseSubDebtCpn As Double

Public SubDebtSum As Double

Public loop1 As Long, loop2 As Integer, loop3 As Integer

Sub RunInstance()

  'Procedure used to generate one iteration of the model given the input parameters and generate detailed output of the resulting tree

    Term = Range("Term")

    Steps = Range("Steps")

    Asset = Range("Asset")

    Cash = Range("Cash")

    Debt = Range("Debt")

    DebtCpn = Range("DebtCpn")

    DivRate = Range("DivRate")

    DivConst = Range("DivConst")

    AssRet = Range("AssRet")

    RFR = Range("RFR")

    AssOpConst = Range("AssOpConst")

    AssVol = Range("AssVol")

    InvRate = Range("InvRate")

    BorRate = Range("BorRate")

    SubDebtFlag = Range("SubDebtFlag")

    EquityIssueCost = Range("EquityIssueCost")

    BancrupCost = Range("BancrupCost")

    StepsCF = Range("StepsCF")

    TaxRate = Range("TaxRate")

    MaxCover = Range("MaxCover")

    SpecDivFlag = Range("SpecDivFlag")

    SubDebtBBFlag = Range("SubDebtBBFlag")

    DebtCPNFlag = Range("DebtCPNFlag")

    DivChFlag = Range("DivChFlag")

    SpreadRatio = Range("SpreadRatio")

```
    SubDebtBBCost = Range("SubDebtBBCost")

    EquityResidual = Range("EquityResidual")

    SubDebtCpn = Range("SubDebtCpn")

    BancrupType = Range("BancrupType")


    DebtCPNOld = DebtCpn


    Sheets("TreeOutput").Activate

    Cells.Select

    Selection.ClearContents


    GenerateTree


    'Output Tree

    For loop1 = 0 To Steps

        For loop2 = 0 To loop1

            For loop3 = 1 To OutputLevel

                ActiveSheet.Cells(OutputLevel * loop2 + loop3, loop1 + 1).Value = TreeArray(loop1, loop2, loop3 -
1)

            Next

        Next

    Next

  For loop1 = 0 To Steps

        For loop2 = 0 To loop1

                ActiveSheet.Cells(OutputLevel * (Steps + 1) + loop2 * 2 + 2, loop1 + 1).Value = CashBck(loop1,
loop2, 0)

                ActiveSheet.Cells(OutputLevel * (Steps + 1) + loop2 * 2 + 3, loop1 + 1).Value = CashBck(loop1,
loop2, 1)

        Next

    Next

End Sub


Sub GenerateTree()

    Dim PathsFrUp As Double, PathsFrDwn As Double, CURatio As Double, CDRatio As Double, CashRatio As
Double, IntRatio As Double, CashLvl As Integer, Trigger As Integer

    'Total steps = steps per cash flow x time (NB 1 CF per year)

    Steps = StepsCF * Term

    '13 -- Asset, Cash, SubDebt, TaxGain, Costs, Cash t+, Debt, Equity, Dividends, SubDebtCpn, EquityIssued,
RN Cash, RN Asset ... Step, Node, Data

    ReDim TreeArray(Steps, Steps, 12)

    ReDim CashBck(Steps, Steps, 2)
```

```
    ReDim CashFwdBckEq(Term, Steps, Steps)
    ReDim CashFwdBckSuD(Term, Steps, Steps)
    ReDim CashFwdBckSpD(Term, Steps, Steps)


    For loop1 = 0 To Term
        For loop2 = 0 To Steps
            For loop3 = 0 To Steps
                CashFwdBckEq(loop1, loop2, loop3) = 0
                CashFwdBckSuD(loop1, loop2, loop3) = 0
                CashFwdBckSpD(loop1, loop2, loop3) = 0
            Next
        Next
    Next


    BaseSubDebtCpn = SubDebtCpn
    UsedSubDebt = 0


    'If required, calc debt coupon.  Spread over RFR linked to leverage and volatility.
    If DebtCPNFlag = 1 Then
        DebtCpn = RFR + AssVol * (Debt / (Asset + Cash)) / SpreadRatio
    Else
        DebtCpn = DebtCPNOld
    End If


    dT = Term / Steps


'Generate parameters for the 2 asset processes - "no yield" and "yield"
  a_df = Exp(dT * RFR)
  a = Exp(dT * (RFR - AssRet))


  u = 2 * Exp((RFR - AssRet) * dT + 2 * AssVol * (dT ^ 0.5)) / (Exp(2 * AssVol * (dT ^ 0.5)) + 1)
  d = 2 * Exp((RFR - AssRet) * dT) / (Exp(2 * AssVol * (dT ^ 0.5)) + 1)


  u1 = 2 * Exp(RFR * dT + 2 * AssVol * (dT ^ 0.5)) / (Exp(2 * AssVol * (dT ^ 0.5)) + 1)
  d1 = 2 * Exp(RFR * dT) / (Exp(2 * AssVol * (dT ^ 0.5)) + 1)


  p = (a - d) / (u - d)


  'Generate Cash Ratio per Node
  CashRatio = Exp(RFR * dT) / Exp((RFR - AssRet) * dT)
```

IntRatio = Exp(RFR * dT)

'Populate Asset Nodes and generate risk neutral cash

TreeArray(0, 0, 0) = Asset

TreeArray(0, 0, 12) = Asset

TreeArray(0, 0, 11) = 0

For loop1 = 1 To Steps

   For loop2 = 0 To loop1

     If loop2 = 0 Then

       TreeArray(loop1, 0, 0) = TreeArray(loop1 - 1, 0, 0) * u

       TreeArray(loop1, 0, 12) = TreeArray(loop1 - 1, 0, 12) * u1

       TreeArray(loop1, 0, 11) = TreeArray(loop1, 0, 0) * (CashRatio - 1) + TreeArray(loop1 - 1, 0, 11) * IntRatio

     ElseIf loop2 = loop1 Then

       TreeArray(loop1, loop2, 0) = TreeArray(loop1 - 1, loop2 - 1, 0) * d

       TreeArray(loop1, loop2, 12) = TreeArray(loop1 - 1, loop2 - 1, 12) * d1

       TreeArray(loop1, loop2, 11) = TreeArray(loop1, loop2, 0) * (CashRatio - 1) + TreeArray(loop1 - 1, loop2 - 1, 11) * IntRatio

     Else

       PathsFrUp = Fact(loop1 - 1) / (Fact(loop1 - 1 - loop2 + 1) * Fact(loop2 - 1))

       PathsFrDwn = Fact(loop1 - 1) / (Fact(loop1 - 1 - loop2) * Fact(loop2))

       CURatio = 2 * PathsFrUp / (PathsFrUp + PathsFrDwn)

       CDRatio = 2 * PathsFrDwn / (PathsFrUp + PathsFrDwn)

       TreeArray(loop1, loop2, 0) = TreeArray(loop1 - 1, loop2 - 1, 0) * d

       TreeArray(loop1, loop2, 12) = TreeArray(loop1 - 1, loop2 - 1, 12) * d1

       TreeArray(loop1, loop2, 11) = TreeArray(loop1, loop2, 0) * (CashRatio - 1) + (TreeArray(loop1 - 1, loop2 - 1, 11) * (1 - p) * CURatio + TreeArray(loop1 - 1, loop2, 11) * p * CDRatio) * IntRatio

     End If

   Next

 Next

'Populate Cash Nodes (1) and Populate Cost Nodes (4)

TreeArray(0, 0, 1) = Cash

TreeArray(0, 0, 4) = 0 'No Costs

TreeArray(0, 0, 2) = SubDebt 'ZERO

TreeArray(0, 0, 9) = SubDebtCpn 'ZERO

TreeArray(0, 0, 3) = 0 'Tax

'Cash + Interest (NB no Asset cash at the initial node)

If Cash > 0 Then

```
    TreeArray(0, 0, 5) = Cash * Exp(InvRate * dT)

 Else

    TreeArray(0, 0, 5) = Cash * Exp(BorRate * dT)

 End If

 'NetCashInt = TreeArray(0, 0, 5) - Cash


 For loop1 = 1 To Steps

    CashLvl = Int(loop1 / StepsCF)

    'initially we deal with the Upper spine of the Tree

    TreeArray(loop1, 0, 1) = TreeArray(loop1 - 1, 0, 5) - AssOpConst   'New Cash = Cash + Interest -
 AssOpConst [NB Asset Cash is dealt with seperately]

    For loop3 = 1 To CashLvl

       CashFwdBckEq(loop3, loop1, 0) = CashFwdBckEq(loop3, loop1 - 1, 0)

       CashFwdBckSuD(loop3, loop1, 0) = CashFwdBckEq(loop3, loop1 - 1, 0)

       CashFwdBckSpD(loop3, loop1, 0) = CashFwdBckEq(loop3, loop1 - 1, 0)

    Next

    TreeArray(loop1, 0, 2) = TreeArray(loop1 - 1, 0, 2) 'Subdebt

    TreeArray(loop1, 0, 3) = TreeArray(loop1 - 1, 0, 3) 'TaxGain

    TreeArray(loop1, 0, 9) = TreeArray(loop1 - 1, 0, 9) 'SubDebtCpn

    If loop1 Mod StepsCF = 0 Then  'if it is a Payment Date

       NetCashInt = -(TreeArray(loop1, 0, 1) + TreeArray(loop1, 0, 11)) * (1 / Exp(RFR * dT * StepsCF) - 1)
 'Calcualte the theoretical cash interest paid or earned on total cash balance

       UsedSubDebt = TreeArray(loop1, 0, 2)

       SubDebtCpn = TreeArray(loop1, 0, 9)


       'Initially we settle costs


       'Dividends -- Constant OR Yield

       If DivChFlag = 1 Then

          UsedDiv = DivConst

       Else

          UsedDiv = TreeArray(loop1, 0, 0) * DivRate

       End If

       TreeArray(loop1, 0, 4) = DebtCpn * Debt + UsedDiv + SubDebtCpn * UsedSubDebt 'Initial Costs --
 Coupons on Debt and Dividends

       TreeArray(loop1, 0, 1) = TreeArray(loop1, 0, 1) - TreeArray(loop1, 0, 4) 'Cash reduced by costs

       TreeArray(loop1, 0, 3) = TreeArray(loop1, 0, 3) + (DebtCpn * Debt + SubDebtCpn * UsedSubDebt -
 NetCashInt) * TaxRate 'Add the theoretical Tax gain

       TreeArray(loop1, 0, 8) = UsedDiv 'Store the dividend paid, it is incorporated in Equity Value as we roll
 back through the tree
```

'Check if additional financing is needed

If (TreeArray(loop1, 0, 1) + TreeArray(loop1, 0, 11) < 0) And (-1 * (TreeArray(loop1, 0, 1) + TreeArray(loop1, 0, 11)) > TreeArray(loop1, 0, 0) * (1 - BancrupCost) - Debt) Then

'Add SubDebt subject to a maximum with balance sourced from Equity

AssetPLUSCash = TreeArray(loop1, 0, 0) + TreeArray(loop1, 0, 1) + TreeArray(loop1, 0, 11)

If SubDebtFlag = 1 Then

MaxNewSubDebt = WorksheetFunction.Max(0, AssetPLUSCash - Debt - UsedSubDebt) 'Maximum sub debt is limnited by a Positive Net worth requirement for new sub debt issuance

Else

MaxNewSubDebt = 0

End If

NewSubDebt = -(TreeArray(loop1, 0, 1) + TreeArray(loop1, 0, 11) + (TreeArray(loop1, 0, 0) * (1 - BancrupCost) - Debt))

TreeArray(loop1, 0, 1) = TreeArray(loop1, 0, 1) + NewSubDebt 'Cash is augmented by new sub debt (which is sourced from sub debt and / or equity

If NewSubDebt > MaxNewSubDebt Then

EquityIssued = NewSubDebt - MaxNewSubDebt

NewSubDebt = MaxNewSubDebt

TreeArray(loop1, 0, 10) = EquityIssued

CashFwdBckEq(CashLvl, loop1, 0) = CashFwdBckEq(CashLvl, loop1, 0) + EquityIssued

End If

If NewSubDebt > 0 Then

CashFwdBckSuD(CashLvl, loop1, 0) = CashFwdBckSuD(CashLvl, loop1, 0) + NewSubDebt

UsedSubDebtCpn = 0

TreeArray(loop1, 0, 2) = UsedSubDebt + NewSubDebt

If DebtCPNFlag = 1 Then

'Calculate new SubDebt coupon rate - function of leverage and volatility

SnrDebtRatio = AssVol * Debt / (AssetPLUSCash + NewSubDebt)

DebtRatio = AssVol * (Debt + UsedSubDebt + NewSubDebt) / (AssetPLUSCash + NewSubDebt)

UsedSubDebtCpn = RFR + (((Debt + UsedSubDebt + NewSubDebt) * DebtRatio - Debt * SnrDebtRatio) / (UsedSubDebt + NewSubDebt)) / SpreadRatio

TreeArray(loop1, 0, 9) = WorksheetFunction.Max(SubDebtCpn, UsedSubDebtCpn)

Else

TreeArray(loop1, 0, 9) = BaseSubDebtCpn

End If

UsedSubDebt = TreeArray(loop1, 0, 2)

End If

End If

'If cash resources exceed maximum cover required then pay out excess to the extent allowed

'Only allow excess payment if Assets (ex Cash) exceed Debt + SubDebt

'Buy back Sub debt; Pay Special dividends

If (TreeArray(loop1, 0, 1) + TreeArray(loop1, 0, 11)) > (MaxCover * (DebtCpn * Debt + SubDebtCpn * UsedSubDebt)) And TreeArray(loop1, 0, 0) > (Debt + UsedSubDebt) Then

ExcessCash = TreeArray(loop1, 0, 1) + TreeArray(loop1, 0, 11) - MaxCover * (DebtCpn * Debt + SubDebtCpn * UsedSubDebt)

If SubDebtBBFlag = 1 Then

If UsedSubDebt > 0 Then

If UsedSubDebt > ExcessCash * (1 + SubDebtBBCost) Then

TreeArray(loop1, 0, 2) = UsedSubDebt - ExcessCash / (1 + SubDebtBBCost)

TreeArray(loop1, 0, 1) = TreeArray(loop1, 0, 1) - ExcessCash

CashFwdBckSuD(CashLvl, loop1, 0) = CashFwdBckSuD(CashLvl, loop1, 0) - ExcessCash / (1 + SubDebtBBCost)

ExcessCash = 0

Else

CashFwdBckSuD(CashLvl, loop1, 0) = CashFwdBckSuD(CashLvl, loop1, 0) - UsedSubDebt

TreeArray(loop1, 0, 2) = 0

TreeArray(loop1, 0, 9) = 0

TreeArray(loop1, 0, 1) = TreeArray(loop1, 0, 1) - UsedSubDebt * (1 + SubDebtBBCost)

ExcessCash = ExcessCash - UsedSubDebt * (1 + SubDebtBBCost)

End If

End If

End If

If SpecDivFlag = 1 Then

TreeArray(loop1, 0, 1) = TreeArray(loop1, 0, 1) - ExcessCash 'cash

'TreeArray(loop1, 0, 8) = TreeArray(loop1, 0, 8) + ExcessCash 'divs

CashFwdBckSpD(CashLvl, loop1, 0) = CashFwdBckSpD(CashLvl, loop1, 0) + ExcessCash

End If

End If

End If

'Cash + Interest (NB Asset cash considered)

If (TreeArray(loop1, 0, 1) + TreeArray(loop1, 0, 11)) > 0 Then

TreeArray(loop1, 0, 5) = TreeArray(loop1, 0, 1) * Exp(InvRate * dT) - TreeArray(loop1, 0, 11) * (Exp((RFR - InvRate) * dT) - 1)

Else

TreeArray(loop1, 0, 5) = TreeArray(loop1, 0, 1) * Exp(BorRate * dT) + TreeArray(loop1, 0, 11) * (Exp((BorRate - RFR) * dT) - 1)

End If

For loop2 = 1 To loop1 - 1

'Next we deal with the interior nodes

'Interior nodes, can be reached by 2 routes. Value used is probability weighted average (taking account of the number of paths to each of the two nodes)

PathsFrUp = Fact(loop1 - 1) / (Fact(loop1 - 1 - loop2 + 1) * Fact(loop2 - 1))

PathsFrDwn = Fact(loop1 - 1) / (Fact(loop1 - 1 - loop2) * Fact(loop2))

CURatio = 2 * PathsFrUp / (PathsFrUp + PathsFrDwn)

CDRatio = 2 * PathsFrDwn / (PathsFrUp + PathsFrDwn)

TreeArray(loop1, loop2, 1) = (1 - p) * TreeArray(loop1 - 1, loop2 - 1, 5) * CURatio + p * TreeArray(loop1 - 1, loop2, 5) * CDRatio - AssOpConst

TreeArray(loop1, loop2, 2) = (1 - p) * TreeArray(loop1 - 1, loop2 - 1, 2) * CURatio + p * TreeArray(loop1 - 1, loop2, 2) * CDRatio

TreeArray(loop1, loop2, 3) = (1 - p) * TreeArray(loop1 - 1, loop2 - 1, 3) * CURatio + p * TreeArray(loop1 - 1, loop2, 3) * CDRatio

For loop3 = 1 To CashLvl 'need to change to CURatio etc. ?

CashFwdBckEq(loop3, loop1, loop2) = (1 - p) * CashFwdBckEq(loop3, loop1 - 1, loop2 - 1) * CURatio + p * CashFwdBckEq(loop3, loop1 - 1, loop2) * CDRatio

CashFwdBckSuD(loop3, loop1, loop2) = (1 - p) * CashFwdBckSuD(loop3, loop1 - 1, loop2 - 1) * CURatio + p * CashFwdBckSuD(loop3, loop1 - 1, loop2) * CDRatio

CashFwdBckSpD(loop3, loop1, loop2) = (1 - p) * CashFwdBckSpD(loop3, loop1 - 1, loop2 - 1) * CURatio + p * CashFwdBckSpD(loop3, loop1 - 1, loop2) * CDRatio

Next

'If sub debt has been issued then calculate implied coupon

If TreeArray(loop1, loop2, 2) > 0 Then

TreeArray(loop1, loop2, 9) = ((1 - p) * TreeArray(loop1 - 1, loop2 - 1, 9) * TreeArray(loop1 - 1, loop2 - 1, 2) * CURatio + _

p * TreeArray(loop1 - 1, loop2, 9) * TreeArray(loop1 - 1, loop2, 2) * CDRatio) / TreeArray(loop1, loop2, 2)

Else

TreeArray(loop1, loop2, 9) = 0

End If

If loop1 Mod StepsCF = 0 Then

NetCashInt = -(TreeArray(loop1, loop2, 1) + TreeArray(loop1, loop2, 11)) * (1 / Exp(RFR * dT * StepsCF) - 1)

UsedSubDebt = TreeArray(loop1, loop2, 2)

SubDebtCpn = TreeArray(loop1, loop2, 9)

If DivChFlag = 1 Then

UsedDiv = DivConst

Else

UsedDiv = TreeArray(loop1, loop2, 0) * DivRate

End If

TreeArray(loop1, loop2, 4) = DebtCpn * Debt + UsedDiv + SubDebtCpn * UsedSubDebt

TreeArray(loop1, loop2, 1) = TreeArray(loop1, loop2, 1) - TreeArray(loop1, loop2, 4)

TreeArray(loop1, loop2, 3) = TreeArray(loop1, loop2, 3) + (DebtCpn * Debt + SubDebtCpn * UsedSubDebt - NetCashInt) * TaxRate

TreeArray(loop1, loop2, 8) = UsedDiv

If (TreeArray(loop1, loop2, 1) + TreeArray(loop1, loop2, 11) < 0) And (-1 * (TreeArray(loop1, loop2, 1) + TreeArray(loop1, loop2, 11)) > TreeArray(loop1, loop2, 0) * (1 - BancrupCost) - Debt) Then

AssetPLUSCash = TreeArray(loop1, loop2, 0) + TreeArray(loop1, loop2, 1) + TreeArray(loop1, loop2, 11)

If SubDebtFlag = 1 Then

MaxNewSubDebt = WorksheetFunction.Max(0, AssetPLUSCash - Debt - UsedSubDebt)

Else

MaxNewSubDebt = 0

End If

NewSubDebt = -(TreeArray(loop1, loop2, 1) + TreeArray(loop1, loop2, 11) + (TreeArray(loop1, loop2, 0) * (1 - BancrupCost) - Debt))

TreeArray(loop1, loop2, 1) = TreeArray(loop1, loop2, 1) + NewSubDebt

If NewSubDebt > MaxNewSubDebt Then

EquityIssued = NewSubDebt - MaxNewSubDebt

NewSubDebt = MaxNewSubDebt

TreeArray(loop1, loop2, 10) = EquityIssued

CashFwdBckEq(CashLvl, loop1, loop2) = CashFwdBckEq(CashLvl, loop1, loop2) + EquityIssued

End If

If NewSubDebt > 0 Then

CashFwdBckSuD(CashLvl, loop1, loop2) = CashFwdBckSuD(CashLvl, loop1, loop2) + NewSubDebt

UsedSubDebtCpn = 0

TreeArray(loop1, loop2, 2) = UsedSubDebt + NewSubDebt

If DebtCPNFlag = 1 Then

SnrDebtRatio = AssVol * Debt / (AssetPLUSCash + NewSubDebt)

DebtRatio = AssVol * (Debt + UsedSubDebt + NewSubDebt) / (AssetPLUSCash + NewSubDebt)

UsedSubDebtCpn = RFR + (((Debt + UsedSubDebt + NewSubDebt) * DebtRatio - Debt * SnrDebtRatio) / (UsedSubDebt + NewSubDebt)) / SpreadRatio

TreeArray(loop1, loop2, 9) = WorksheetFunction.Max(SubDebtCpn, UsedSubDebtCpn)

Else

TreeArray(loop1, loop2, 9) = BaseSubDebtCpn

End If

UsedSubDebt = TreeArray(loop1, loop2, 2)

End If

End If

```
        If (TreeArray(loop1, loop2, 1) + TreeArray(loop1, loop2, 11)) > (MaxCover * (DebtCpn * Debt +
SubDebtCpn * UsedSubDebt)) And TreeArray(loop1, loop2, 0) > (Debt + UsedSubDebt) Then

            ExcessCash = TreeArray(loop1, loop2, 1) + TreeArray(loop1, loop2, 11) - MaxCover * (DebtCpn *
Debt + SubDebtCpn * UsedSubDebt)

            If SubDebtBBFlag = 1 Then

                If UsedSubDebt > 0 Then

                    If UsedSubDebt > ExcessCash * (1 + SubDebtBBCost) Then

                        TreeArray(loop1, loop2, 2) = UsedSubDebt - ExcessCash / (1 + SubDebtBBCost)

                        TreeArray(loop1, loop2, 1) = TreeArray(loop1, loop2, 1) - ExcessCash

                        CashFwdBckSuD(CashLvl, loop1, loop2) = CashFwdBckSuD(CashLvl, loop1, loop2) -
ExcessCash / (1 + SubDebtBBCost)

                        ExcessCash = 0

                    Else

                        CashFwdBckSuD(CashLvl, loop1, loop2) = CashFwdBckSuD(CashLvl, loop1, loop2) -
UsedSubDebt

                        TreeArray(loop1, loop2, 2) = 0

                        TreeArray(loop1, loop2, 9) = 0

                        TreeArray(loop1, loop2, 1) = TreeArray(loop1, loop2, 1) - UsedSubDebt * (1 +
SubDebtBBCost)

                        ExcessCash = ExcessCash - UsedSubDebt * (1 + SubDebtBBCost)

                    End If

                End If

            End If

            If SpecDivFlag = 1 Then

                TreeArray(loop1, loop2, 1) = TreeArray(loop1, loop2, 1) - ExcessCash

                'TreeArray(loop1, loop2, 8) = TreeArray(loop1, loop2, 8) + ExcessCash

                CashFwdBckSpD(CashLvl, loop1, loop2) = CashFwdBckSpD(CashLvl, loop1, loop2) +
ExcessCash

            End If

        End If

    End If


    If (TreeArray(loop1, loop2, 1) + TreeArray(loop1, loop2, 11)) > 0 Then

        TreeArray(loop1, loop2, 5) = TreeArray(loop1, loop2, 1) * Exp(InvRate * dT) - TreeArray(loop1,
loop2, 11) * (Exp((RFR - InvRate) * dT) - 1)

    Else

        TreeArray(loop1, loop2, 5) = TreeArray(loop1, loop2, 1) * Exp(BorRate * dT) + TreeArray(loop1,
loop2, 11) * (Exp((BorRate - RFR) * dT) - 1)

    End If

Next


    'and we finish with the Lower spine of the Tree
```

TreeArray(loop1, loop1, 1) = TreeArray(loop1 - 1, loop1 - 1, 5) - AssOpConst

TreeArray(loop1, loop1, 2) = TreeArray(loop1 - 1, loop1 - 1, 2)

TreeArray(loop1, loop1, 3) = TreeArray(loop1 - 1, loop1 - 1, 3)

TreeArray(loop1, loop1, 9) = TreeArray(loop1 - 1, loop1 - 1, 9)

For loop3 = 1 To CashLvl

   CashFwdBckEq(loop3, loop1, loop1) = CashFwdBckEq(loop3, loop1 - 1, loop1 - 1)

   CashFwdBckSuD(loop3, loop1, loop1) = CashFwdBckSuD(loop3, loop1 - 1, loop1 - 1)

   CashFwdBckSpD(loop3, loop1, loop1) = CashFwdBckSpD(loop3, loop1 - 1, loop1 - 1)

Next

If loop1 Mod StepsCF = 0 Then

   NetCashInt = -(TreeArray(loop1, loop1, 1) + TreeArray(loop1, loop1, 11)) * (1 / Exp(RFR * dT * StepsCF) - 1)

   UsedSubDebt = TreeArray(loop1, loop1, 2)

   SubDebtCpn = TreeArray(loop1, loop1, 9)

   If DivChFlag = 1 Then

     UsedDiv = DivConst

   Else

     UsedDiv = TreeArray(loop1, loop1, 0) * DivRate

   End If

   TreeArray(loop1, loop1, 4) = DebtCpn * Debt + UsedDiv + SubDebtCpn * UsedSubDebt

   TreeArray(loop1, loop1, 1) = TreeArray(loop1, loop1, 1) - TreeArray(loop1, loop1, 4)

   TreeArray(loop1, loop1, 3) = TreeArray(loop1, loop1, 3) + (DebtCpn * Debt + SubDebtCpn * UsedSubDebt - NetCashInt) * TaxRate

   TreeArray(loop1, loop1, 8) = UsedDiv

   If (TreeArray(loop1, loop1, 1) + TreeArray(loop1, loop1, 11) < 0) And (-1 * (TreeArray(loop1, loop1, 1) + TreeArray(loop1, loop1, 11)) > TreeArray(loop1, loop1, 0) * (1 - BancrupCost) - Debt) Then

     AssetPLUSCash = TreeArray(loop1, loop1, 0) + TreeArray(loop1, loop1, 1) + TreeArray(loop1, loop1, 11)

     If SubDebtFlag = 1 Then

       MaxNewSubDebt = WorksheetFunction.Max(0, AssetPLUSCash - Debt - UsedSubDebt)

     Else

       MaxNewSubDebt = 0

     End If

     NewSubDebt = -(TreeArray(loop1, loop1, 1) + TreeArray(loop1, loop1, 11) + (TreeArray(loop1, loop1, 0) * (1 - BancrupCost) - Debt))

     TreeArray(loop1, loop1, 1) = TreeArray(loop1, loop1, 1) + NewSubDebt

     If NewSubDebt > MaxNewSubDebt Then

       EquityIssued = NewSubDebt - MaxNewSubDebt

       NewSubDebt = MaxNewSubDebt

       TreeArray(loop1, loop1, 10) = EquityIssued

       CashFwdBckEq(CashLvl, loop1, loop1) = CashFwdBckEq(CashLvl, loop1, loop1) + EquityIssued

End If

If NewSubDebt > 0 Then

CashFwdBckSuD(CashLvl, loop1, loop1) = CashFwdBckSuD(CashLvl, loop1, loop1) + NewSubDebt

UsedSubDebtCpn = 0

TreeArray(loop1, loop1, 2) = UsedSubDebt + NewSubDebt

If DebtCPNFlag = 1 Then

SnrDebtRatio = AssVol * Debt / (AssetPLUSCash + NewSubDebt)

DebtRatio = AssVol * (Debt + UsedSubDebt + NewSubDebt) / (AssetPLUSCash + NewSubDebt)

UsedSubDebtCpn = RFR + (((Debt + UsedSubDebt + NewSubDebt) * DebtRatio - Debt * SnrDebtRatio) / (UsedSubDebt + NewSubDebt)) / SpreadRatio

TreeArray(loop1, loop1, 9) = WorksheetFunction.Max(SubDebtCpn, UsedSubDebtCpn)

Else

TreeArray(loop1, loop1, 9) = BaseSubDebtCpn

End If

UsedSubDebt = TreeArray(loop1, loop1, 2)

End If

End If

If (TreeArray(loop1, loop1, 1) + TreeArray(loop1, loop1, 11)) > (MaxCover * (DebtCpn * Debt + SubDebtCpn * UsedSubDebt)) And TreeArray(loop1, loop1, 0) > (Debt + UsedSubDebt) Then

ExcessCash = TreeArray(loop1, loop1, 1) + TreeArray(loop1, loop1, 11) - MaxCover * (DebtCpn * Debt + SubDebtCpn * UsedSubDebt)

If SubDebtBBFlag = 1 Then

If UsedSubDebt > 0 Then

If UsedSubDebt > ExcessCash * (1 + SubDebtBBCost) Then

CashFwdBckSuD(CashLvl, loop1, loop1) = CashFwdBckSuD(CashLvl, loop1, loop1) - ExcessCash / (1 + SubDebtBBCost)

TreeArray(loop1, loop1, 2) = UsedSubDebt - ExcessCash / (1 + SubDebtBBCost)

TreeArray(loop1, loop1, 1) = TreeArray(loop1, loop1, 1) - ExcessCash

ExcessCash = 0

Else

CashFwdBckSuD(CashLvl, loop1, loop1) = CashFwdBckSuD(CashLvl, loop1, loop1) - UsedSubDebt

TreeArray(loop1, loop1, 2) = 0

TreeArray(loop1, loop1, 9) = 0

TreeArray(loop1, loop1, 1) = TreeArray(loop1, loop1, 1) - UsedSubDebt * (1 + SubDebtBBCost)

ExcessCash = ExcessCash - UsedSubDebt * (1 + SubDebtBBCost)

End If

End If

```
        End If
        If SpecDivFlag = 1 Then
            TreeArray(loop1, loop1, 1) = TreeArray(loop1, loop1, 1) - ExcessCash
            'TreeArray(loop1, loop1, 8) = TreeArray(loop1, loop1, 8) + ExcessCash
            CashFwdBckSpD(CashLvl, loop1, loop1) = CashFwdBckSpD(CashLvl, loop1, loop1) + ExcessCash
        End If
    End If
End If


    If (TreeArray(loop1, loop1, 1) + TreeArray(loop1, loop1, 11)) > 0 Then
        TreeArray(loop1, loop1, 5) = TreeArray(loop1, loop1, 1) * Exp(InvRate * dT) - TreeArray(loop1, loop1,
11) * (Exp((RFR - InvRate) * dT) - 1)
    Else
        TreeArray(loop1, loop1, 5) = TreeArray(loop1, loop1, 1) * Exp(BorRate * dT) + TreeArray(loop1,
loop1, 11) * (Exp((BorRate - RFR) * dT) - 1)
    End If
  Next


    'We have completed the tree construction process whereby we have rolled forward through the structure
dealing with cash flows, costs and liquidity considerations
    'We now work backwards through the tree assessing bankruptcy and assigning equity and debt value


    'We begin with the Leaf Nodes, where the Intrinsic Value of debt and equity is considered
    For loop2 = 0 To Steps
        UsedSubDebt = TreeArray(Steps, loop2, 2)


        'Check if Assets plus Cash can pay Debt and SubDebt
        'Ensure that no New equity was issued (as this would imply default at leaf nodes)
        'Allocate asset value to Equity and Debt
        If (TreeArray(Steps, loop2, 0) + TreeArray(Steps, loop2, 1) + TreeArray(Steps, loop2, 11)) > (Debt +
UsedSubDebt) And TreeArray(Steps, loop2, 10) = 0 Then
            TreeArray(Steps, loop2, 6) = Debt 'Debt paid in full (Face Value)
            'Equity Value = Asset Value + Tax Benefit + Net Cash - Debt - SubDebt
            TreeArray(Steps, loop2, 7) = WorksheetFunction.Max(0, TreeArray(Steps, loop2, 0) + TreeArray(Steps,
loop2, 3) + _
              TreeArray(Steps, loop2, 1) + TreeArray(Steps, loop2, 11) - Debt - UsedSubDebt)
        Else
            'Bancruptcy process triggered
            'Asset value reduced by dead weight costs and value allocated according to parameters
            'Short term cash borrowings are settled in Full
            'If strict priority of claims is not observed then equity will have a residual value
```

```
        TreeArray(Steps, loop2, 1) = TreeArray(Steps, loop2, 1) - TreeArray(Steps, loop2, 10)

        BancrupValue = WorksheetFunction.Max(0, TreeArray(Steps, loop2, 0) * (1 - BancrupCost) +
TreeArray(Steps, loop2, 1) + TreeArray(Steps, loop2, 11))

        EqtyValDflt = EquityResidual * BancrupValue

        BancrupValue = BancrupValue - EqtyValDflt

        If BancrupValue > Debt Then

            TreeArray(Steps, loop2, 6) = Debt

        Else

            TreeArray(Steps, loop2, 6) = BancrupValue

        End If

        TreeArray(Steps, loop2, 7) = EqtyValDflt

        TreeArray(Steps, loop2, 10) = -1 * TreeArray(Steps, loop2, 10) 'Adjust equity issued marker to reflect
bankruptcy

    End If


    'Add Dividends paid and coupon paid at maturity

    'In effect we are assuming that these payments are not subject to bankruptcy charges and are settled just
prior to the firm entering bankruptcy

    TreeArray(Steps, loop2, 7) = TreeArray(Steps, loop2, 7) + TreeArray(Steps, loop2, 8)

    TreeArray(Steps, loop2, 7) = TreeArray(Steps, loop2, 7) + CashFwdBckSpD(Term, Steps, loop2)

    TreeArray(Steps, loop2, 6) = TreeArray(Steps, loop2, 6) + Debt * DebtCpn

    CashBck(Steps, loop2, 0) = (TreeArray(Steps, loop2, 11) - TreeArray(Steps, loop2, 0) * (CashRatio - 1)) '/
IntRatio


    UsedSubDebt = TreeArray(Steps, loop2, 2)

    SubDebtCpn = TreeArray(Steps, loop2, 9)

    If DivChFlag = 1 Then

        UsedDiv = DivConst

    Else

        UsedDiv = TreeArray(Steps, loop2, 0) * DivRate

    End If

    CashBck(Steps, loop2, 1) = (TreeArray(Steps, loop2, 1) + DebtCpn * Debt + UsedDiv + SubDebtCpn *
UsedSubDebt) + AssOpConst '/ IntRatio

    CashBck(Steps, loop2, 1) = CashBck(Steps, loop2, 1) + CashFwdBckSpD(Term, Steps, loop2)

  Next


  'We now consider the interior nodes

  'Equity value and debt value are probability weighted averages

  'Bankruptcy is only considered on payment dates

  For loop1 = Steps - 1 To 0 Step -1

    CashLvl = Int(loop1 / StepsCF)
```

For loop2 = loop1 To 0 Step -1

TreeArray(loop1, loop2, 6) = (TreeArray(loop1 + 1, loop2 + 1, 6) * (1 - p) + TreeArray(loop1 + 1, loop2, 6) * p) / a_df

TreeArray(loop1, loop2, 7) = (TreeArray(loop1 + 1, loop2 + 1, 7) * (1 - p) + TreeArray(loop1 + 1, loop2, 7) * p) / a_df

CashBck(loop1, loop2, 0) = (CashBck(loop1 + 1, loop2 + 1, 0) * (1 - p) + CashBck(loop1 + 1, loop2, 0) * p) / IntRatio

CashBck(loop1, loop2, 1) = (CashBck(loop1 + 1, loop2 + 1, 1) * (1 - p) + CashBck(loop1 + 1, loop2, 1) * p) / IntRatio

For loop3 = 1 To CashLvl

CashFwdBckEq(loop3, loop1, loop2) = CashFwdBckEq(loop3, loop1 + 1, loop2 + 1) * (1 - p) + CashFwdBckEq(loop3, loop1 + 1, loop2) * p

CashFwdBckSuD(loop3, loop1, loop2) = CashFwdBckSuD(loop3, loop1 + 1, loop2 + 1) * (1 - p) + CashFwdBckSuD(loop3, loop1 + 1, loop2) * p

CashFwdBckSpD(loop3, loop1, loop2) = CashFwdBckSpD(loop3, loop1 + 1, loop2 + 1) * (1 - p) + CashFwdBckSpD(loop3, loop1 + 1, loop2) * p

Next

If loop1 > 0 And loop1 Mod StepsCF = 0 Then 'Payment Date

'Reduce Equity by contribution from Equity Issue + Issue cost, Reduce cash by contribution from equity issue, add back special dividend contribution to cash, add back subdebt contribution to cash

TreeArray(loop1, loop2, 7) = WorksheetFunction.Max(TreeArray(loop1, loop2, 7) - CashFwdBckEq(CashLvl, loop1, loop2), 0)

TreeArray(loop1, loop2, 7) = TreeArray(loop1, loop2, 7) + CashFwdBckSpD(CashLvl, loop1, loop2)

CashBck(loop1, loop2, 1) = CashBck(loop1, loop2, 1) - CashFwdBckEq(CashLvl, loop1, loop2) '* (1 + EquityIssueCost)

CashBck(loop1, loop2, 1) = CashBck(loop1, loop2, 1) + CashFwdBckSpD(CashLvl, loop1, loop2)

CashBck(loop1, loop2, 1) = CashBck(loop1, loop2, 1) - CashFwdBckSuD(CashLvl, loop1, loop2)

'Reduce Cash by EquityIssued (??)

'TreeArray(loop1, loop2, 1) = TreeArray(loop1, loop2, 1) - TreeArray(loop1, loop2, 10)

'Check for default - can be if Equity is Zero OR if Assets < Liabs

SubDebtSum = 0

For loop3 = 1 To CashLvl - 1

SubDebtSum = SubDebtSum + CashFwdBckSuD(loop3, loop1, loop2)

Next

UsedSubDebt = SubDebtSum ' sum of cashlvls up to current - 1 (already taken current away)

Trigger = 0

BancrupValue = WorksheetFunction.Max(0, TreeArray(loop1, loop2, 0) * (1 - BancrupCost) + CashBck(loop1, loop2, 1) + CashBck(loop1, loop2, 0))

EqtyValDflt = EquityResidual * BancrupValue

```
    If BancrupType = 1 Then

        If TreeArray(loop1, loop2, 7) - CashFwdBckEq(CashLvl, loop1, loop2) * EquityIssueCost <=
EqtyValDflt Then

            Trigger = 1

        End If

    ElseIf BancrupType = 2 Then

        If (TreeArray(loop1, loop2, 0) + CashBck(loop1, loop2, 1) + CashBck(loop1, loop2, 0)) < (Debt +
UsedSubDebt) Then

            Trigger = 1

        End If

    End If


    If Trigger = 1 Then

      BancrupValue = BancrupValue - EqtyValDflt

      If BancrupValue > Debt Then

         TreeArray(loop1, loop2, 6) = Debt

      Else

         TreeArray(loop1, loop2, 6) = BancrupValue

      End If

      TreeArray(loop1, loop2, 7) = EqtyValDflt

    End If


    'Add Dividend paid and coupon paid

    TreeArray(loop1, loop2, 7) = TreeArray(loop1, loop2, 7) + TreeArray(loop1, loop2, 8)

    TreeArray(loop1, loop2, 6) = TreeArray(loop1, loop2, 6) + Debt * DebtCpn


    SubDebtSum = 0

    For loop3 = 1 To CashLvl - 1

        SubDebtSum = SubDebtSum + CashFwdBckSuD(loop3, loop1, loop2)

    Next

    UsedSubDebt = SubDebtSum

    SubDebtCpn = TreeArray(loop1, loop2, 9)

    If DivChFlag = 1 Then

        UsedDiv = DivConst

    Else

        UsedDiv = TreeArray(loop1, loop2, 0) * DivRate

    End If

    CashBck(loop1, loop2, 1) = (CashBck(loop1, loop2, 1) + DebtCpn * Debt + UsedDiv + SubDebtCpn *
UsedSubDebt) + AssOpConst


    End If
```

```
    CashBck(loop1, loop2, 0) = CashBck(loop1, loop2, 0) - TreeArray(loop1, loop2, 0) * (CashRatio - 1)
  Next
 Next
End Sub


Function Fact(n As Variant) As Double
Dim loop1 As Integer, result As Double
  result = 1
  If n = 0 Then
    Fact = 1
  Else
    For loop1 = 1 To n
      result = result * loop1
    Next
    Fact = result
  End If
End Function
```