THE INFLUENCE OF AN INDUCTIVE TEACHING APPROACH ON THE LEARNING OF THE CONCEPT FUNCTIONS IN GRADE 11

BY

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MINI – DISSERTATION

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EVERYTHING IS POSSIBLE THROUGH GOD IN JESUS CHRIST WHO GIVES ABUNDANTLY. IT IS THROUGH HIS MERCY, LOVE AND GRACE THAT I CAME TO REALISE MYSELF.

Dedication

I would like to dedicate this work to my late mother **Dolly, Daisy Masebe** who has always been my pillar of strength and who always encouraged me to work hard in my studies. May she find favour, joy and love in the face of the **Almighty God**, and may her soul rest in peace.

Declaration

I **Tshidiso Phanuel Masebe** declares that "The Influence of an Inductive teaching approach on the learning of the concept Functions in Grade 11" is my own work and all the sources cited herein have been duly acknowledged in full by means of complete references.

SUMMARY

The study presents a pragmatic evaluation of the influence of inductive teaching on grade 11 learners in two high schools in Tshwane West District in the Gauteng province in a form of pseudo experiment complemented with a qualitative investigation. The study focussed on the influence of inductive teaching on the nature of conceptualisation of and the learning achievement with regard to functions in Grade 11. A model adopted by O'Callaghan that identifies and applies the four competencies of modelling a function, interpreting a function, translating and reifying a function proved to be relevant for the investigation and hence was adapted for the study.

The methodology used included data collection through pretest-posttest control group experimental design complemented with unstructured interviews. The verification of the reliability of research instruments and data analysis were done with the assistance of the Northwest University (Potchefstroom Campus) Statistical Consultation Services and through identification of common perceptions and experiences of participants. The results of the study did indicate positive influence of inductive teaching on the nature and quality of conceptual learning of the function concept.

Key words: Teaching approach; inductive/deductive teaching; mathematics learning; function concept; learning performance/achievement.

OPSOMMING

Die invloed van induktiewe onderrig op die leer van die funksiebegrip in Graad 11.

Hierdie studie is 'n pragmatiese evaluering van die invloed van induktiewe onderrig op Graad 11 leerders in twee Hoërskole in die Tshwane-Wes distrik in Gauteng provinsie in die vorm van pseudo-eksperiment, wat deur'n kwalitatiewe ondersoek aangevul is. Die studie fokus op die invloed van induktiewe onderrig op die aard van konseptualisering van die funksie-begrip in Graad 11. 'n Model wat deur O'Callaghan ontwikkel is en wat vier bevoegdhede ten opsigte van die konseptualisering van die funksiebegrip identifseer en toepas, naamlik modelering, interpretasie, vertolking ("translation") en reïfikasie van 'n funksie, is vir doel van die ondersoek geskik bevind en gebruik.

Die metodologie wat gebruik is, het data-versameling ingesluit deur middel van 'n voortoets-natoets-kontrole-groep eksperimentele ontwerp, wat aangevul is deur ongestruktureerde onderhoude. Die verifiëring van die geldigheid van die ondersoek-instrumente en data-analisie is met die hulp van Noordwes-Universiteit (Potchefstroom-kampus) Statistiese Konsultasie Dienste gedoen. Die identifisering van gemeenskaplike persepsies en ervarings van deelnemers is ook gebruik. Die studie toon positiewe invloede van induktiewe onderrig op die aard en gehalte van die konseptuele leer van die funksiebegrip.

Sleutelwoorde: Onderrigbenadering; induktiewe/deduktiewe onderrig; wiskundeleer; funksiebegrip; leerhandeling/-prestasie.

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CHAPTER 1

STATEMENT OF THE PROBLEM AND PROGRAMME OF STUDY

1.1 INTRODUCTION

The 1995 Third International Mathematics and Science Study (TIMSS), its repeat (TIMSS-R) in 1999, and TIMSS 2003 show that achievement in school mathematics in South Africa is below international average and lags considerably behind other countries (Howie, 1999:20; Kanjee, 2004:1). The poor achievement in mathematics in general, and particularly in respect of functions, is attributed to the traditional way of teaching in the form of deductive teaching which overemphasises symbolism, manipulative skills and rote memorization of facts at the expense of the development of concepts and problem solving abilities (O'Callaghan, 1998:22). In addition, under-achievement in mathematics occurs when teachers over-indulge in using deductive teaching approaches in their teaching, resulting in little regard for learners' abilities and learning styles, and disallowing them an opportunity to benefit from inductive teaching which proceeds from the particular to the general.

The traditional way of teaching is mostly deductive in nature as the teacher first states the general principle and then leads the class to particular applications of the principle (Van der Horst & McDonald, 2001:124). On the other hand, inductive teaching intends to guide learners by appropriate questions, examples and learning experiences to the apprehension of an idea or principle before it is stated as a formal idea.

The use of an inductive approach in teaching algebra concepts, of which "function" is central, is in line with the changes in the education system in South Africa. The critical outcomes in general, as outlined in the mathematics curriculum document, and the first critical outcome in particular, promote a balanced view of reality and deep conceptual understanding by requiring development of all learners' critical thinking powers and their problem solving abilities (DoE, 2003:1; Van der Horst & McDonald, 2001:4). O'Callaghan (1998:24) argues that the essential feature in the construction of mathematical knowledge is the creation of relationships, which is the hallmark of conceptual and relational understanding and problem solving. In the context of the school mathematics curriculum, functions are particularly useful tools in problem solving as they are often used to describe relationships. Fey (1984, quoted by O'Callaghan, 1998:23) argues that the function concept is operationally a relation between quantities (variables) that change as the situation changes.

The researcher seeks to investigate the relationship between an inductive teaching approach and the nature of conceptualisation of and the learning achievement in functions in Grade 11.

1.2 CONTEXT AND APPROACH TO THE RESEARCH

According to Ernest (1996:11), children first learn a mathematical concept as an algorithm (a procedure or method). Later the algorithm or procedure is transformed into an object. For example, it is not difficult to connect two points by a straight line. It is rather hard to conceptualise the straight line as an entity in itself, apart from operations. The concept function is a central idea around which mathematics grows in importance as one progresses deeper into the inner circles of understanding mathematics. The function concept is believed to be a fundamental object of algebra (Cadwell, 1997), which involves five representational systems: contextual, graphical, equations, tables and language (Van de Walle, 2004:436; DoE, 2003:13). Graphs represent earliest representations in mathematics at which learners use a symbolic system to expand and understand a function concept. Equations and graphical representations are used to jointly construct and define the mathematical concept, function. A function captures the spirit and essence of connections and interdependencies, and embraces elements of input and output, control and observation and cause and effect (Cadwell, 1997). In the school mathematics curriculum (DoE, 2003:12, 22, 48), Learning Outcome 2 (Patterns, Functions and Algebra) asserts that understanding of the function concept can also be realized when a learner is able to describe a situation by interpreting a graph of a situation, and when s/he draws a graph from a description of a situation. O'Callaghan (1998:24) distinguishes four fundamental components that learners need to acquire in order to conceptualize the idea of a function, namely modelling, translation, interpretation and reification (see 2.9). It must be emphasized that the model does not describe what learners do to understand functions. The model accounts for components of knowledge relevant to the function concept and form a basis of analyses for learners' conceptualization of functions (O'Callaghan, 1998:24).

The ways in which an individual characteristically perceives, prefers and organises his/her processing, acquisition, retention and retrieval of mathematical information are collectively termed the individual's learning style. Mismatches often occur between the learning styles of the learners and the teaching approaches of teachers with unfortunate effects on the quality of learners' learning and their attitudes towards mathematics (Felder & Henriques, 1995).

1.3 PROBLEM STATEMENT

The teaching and learning of mathematics in South African schools continue to experience grave problems leading to low quality learning and frequent failure (Kanjee, 2004:1). DoE (2008:28) indicated a drop in overall percentage in mathematics from a 55,7% pass in 2005 to a 52,2% pass in 2006. Hence, this research seeks to contribute towards finding viable and durable solutions to the problem. In particular, mismatches between traditional teaching and learning of mathematics are blamed for the problems encountered. The aim is to try to minimize teaching-learning mismatches that tend to disadvantage learners and to this end the research will look into a particular teaching approach that may offer hope towards improving the conception of mathematical concepts.

A deductive teaching approach, such as traditionally found in mathematics classes (Van der Horst & McDonald, 2001:124) , is based on the principle of a *priori* logic that proceeds from some general law or premise, the truth or validity of which is taken for granted in advance, to some particular case. In other words, principles and rules are given and applications are deduced. An inductive teaching approach is based on the principle of *a posteriori* logic that proceeds from a particular set of causes or facts to the general law or principle, i.e. facts and observations are given and underlying principles and rules are inferred (Van der Horst & McDonald, 2001:124).

The nature of an inductive teaching approach suggests that it can be best applied in problem solving situations (Van de Walle, 2004:38). Problem solving is a process of applying existing knowledge to a new or unfamiliar situation to gain new knowledge or better understanding of the situation. A problem refers to a task or an activity for which learners have no prescribed or memorised rules or methods, nor is there a perception by the learners that there is a specific solution procedure. Problem solving requires that learners exert active effort and high thinking levels, not merely recalling previously learned facts (Van de Walle, 2004:47).

Problem solving tends to engage learners actively in learning, helps them develop critical thinking skills and encourages them to make informed judgements. For obvious reasons, recent literature promotes a teaching-learning approach in mathematics that centres about problem solving as a way to resolve the mismatches associated with the traditional approach (Van de Walle, 2004:47). The choice of Grade 11 in this study is informed by a level in the

Further Education and Training (FET) band, and compared to the Grade 10s they have some exposure to the concept of functions. A Grade 11 class is a transition in the FET band that plays an important role in determining the learners' progress and how well they perform in a school mathematics curriculum in general and in particular in Grade 12. This study, therefore, is focused specifically on the learning of functions in Grade 11 mathematics through an inductive teaching approach.

In view of the above discussion, the study sought to provide answers to the following problem questions:

- What is the influence of an inductive teaching approach on the conceptualisation of functions in Grade 11?
- What is the influence of an inductive teaching approach on learning achievement with regard to functions in Grade 11?

1.4 RESEARCH AIM

The purpose of the study is to investigate:

- The relationship between inductive teaching and the conceptualisation of functions in Grade 11.
- The relationship between inductive teaching and learning achievement with regard to functions in Grade 11.

1.5 PROGRAMME OF STUDY

The programme of this research is presented as follows:

1.5.1 Literature Study

An intensive and comprehensive review of the literature on inductive and traditional deductive teaching, as well as on the relationship between teaching approach and learning style of mathematics, particularly functions, was done. Primary sources were used to gather information, while secondary sources were only used when primary sources were not available. Electronic databases and search engines ERIC, EBSCOhost and Google Scholar were consulted, using the following keywords:

Teaching; deductive/inductive teaching; learning; learning style; problem solving; functions; functional relations; achievement; mathematics; Grade 11.

1.5.2 Research Design and Methods

The research design was a pragmatic evaluation study in a form of pseudo experiment complemented with a qualitative investigation. The methods used in the research design were twofold: a quantitative investigation by means of paper and pencil tests and a qualitative investigation by means of interviews.

- Quantitative investigation: A pretest-posttest-control group experimental design (Leedy & Ormrod, 2005:225) was employed.
- Qualitative investigation: Semi-structured interviews were conducted on a teacher and on students who volunteered from the experimental group to get their views, experiences and their preferences regarding the teaching approach and learning that took place in their class.

1.5.3 Population and sampling

The population consisted of Grade 11 mathematics learners from the Gauteng Province (Tshwane West District). The researcher enlisted the services of the Subject Advisory to identify best performing schools that practised Outcomes-based Education well and the schools that taught in the traditional way. Interviews were conducted on teachers of the schools concerned to confirm the assertion by the Subject Advisory. Two schools from the original eight were randomly selected. The best OBE practising school formed the experimental group, while the one practising the traditional method formed the control group. The sample consisted of 122 Grade 11 learners from the two schools.

1.5.4 Data collection

The following procedure was followed to gather data:

- The experimental teacher was coached in the application of an inductive teaching approach with regard to functions in the first quarter of the year.
- The pre-test was set by the researcher and moderated by two mathematics teachers and a Subject Advisor. The test was piloted on four non-participating learners. After some comments and ascertaining the appropriateness for the Grade 11 classes, the test was ready to be administered to both experimental group and control group learners.
- Pre-testing: A test was administered to 122 Grade 11 learners in both the experimental group and the control group.
- Intervention: The experimental classes were subjected to inductive teaching of functions for a period of at least four weeks, while the control classes followed the traditional approach.
- The researcher set the post test that was moderated by two mathematics teachers and a Subject Advisor. A test was set based on four components of modelling, translation, interpretation and reification (see 2.9). The test was piloted on four non-participating learners. After some comments and ascertaining the appropriateness for the Grade 11 classes, the test was ready to be administered to both experimental group and control group learners.
- Post-testing: A test was administered to 122 Grade 11 learners in both the experimental group and control group.
- Interviews: Four students from the experimental group who volunteered and their teacher were interviewed after completion of the experimental phase.

1.5.5 Data Analysis

- Quantitative analysis: Inferential statistics by means of t-tests, effect sizes and analysis of variance were used to analyse the experimental data (Leedy & Ormrod, 2005:274). The assistance of the Statistical Consultation Services of the NWU was sought.
- Qualitative analysis: The researcher grouped information into segments that reflected various aspects of the experience. Divergent perspectives were identified. The researcher used various meanings identified to develop an overall description of the experience (Leedy & Ormrod, 2005:144). All data analysed and interpretations were subjected to literature control.

1.5.6 Procedure

The following procedure was followed to gather data:

- Training of an experimental teacher in the application of an inductive teaching approach with regard to functions was done.
- The pre-testing for both the control and the experimental groups was done before the teaching of the concept to test the conceptualization of functions based on the Grade 10 work.
- Intervention: the experimental classes were subjected to inductive teaching of functions for a period of at least four weeks, while the control classes followed the traditional approach.
- The post-test was piloted on four non-participating learners in a similar nearby school to determine whether questions were up to the required standard of Grade 11.
- Post-testing: A post-test was administered to both the control and the experimental groups to check improvement as well as progression.
- Interviews: Learners who volunteered and a teacher were interviewed after completion of the experimental phase.

1.5.7 Ethics

In line with ethical aspects, the following ethical procedure was followed:

- An application letter was written to the Gauteng Department of Education to conduct research in schools in the Tshwane West Region (Appendix A).
- Further arrangements were made with the Principals of the selected schools.
- Consent was sought from the learners participating in the research (Appendix B).
- The identity of participants in the research was kept anonymous.

1.6 STRUCTURE OF THE DISSERTATION

Chapter 1	This chapter focuses on the background and statement of the problem, the aims of the research, the context and approach to the research, research hypotheses and the programme of study.
Chapter 2	This chapter focuses on the modes of learning, learning as viewed by some authors, and identifies four main theories of learning. This chapter also focuses on learning and teaching styles, teaching paradigms, inductive and deductive reasoning, differentiates between inductive and traditional teaching, and looks at the perspectives of the two teaching approaches, the function concept, the conceptual framework adopted by O'Callaghan (namely modelling, interpreting, translating and reifying)(see 2.9), and a summary.
Chapter 3	This chapter focuses on research approach. Experimental design comprises quantitative and qualitative investigations. Included in the chapter are population and sample, variables, data collection, procedure and measuring instruments, ethical aspects and data analysis.

Chapter 4 This chapter deals with information extracted from the data after analysis is done. A summary of findings is given based on the supplied information and is cross-referenced to literature.

Chapter 5 The chapter includes a summary of the study, the findings and recommendations of the study, and a final conclusion.

1.7 CONCLUSION

This chapter pointed out the research problem and outlined the programme to be followed to find solutions to the research problem. The context and the approach give a precise strategy employed in attaining the objectives of the investigation. The purpose of the study was to investigate the effect inductive teaching has on the level of conceptualisation of and achievement on the concept functions. For the exercise the sample consisted of 122 Grade 11 learners from Tshwane West District.

CHAPTER 2

INDUCTIVE AND TRADITIONAL TEACHING APPROACHES AND THE LEARNING OF FUNCTIONS

2.1 INTRODUCTION

Learners learn mathematics in several ways such as in reflection and acting, reasoning logically, intuitively, memorizing, and visualizing. However, the learner's natural ability and prior preparation as well as the teacher's characteristic approach to teaching, governs much of how a learner learns in a classroom (Felder & Henriques, 1995:1 ; Huetinck & Munshin, 2004:52). Different teachers use different teaching approaches. Some teachers lecture, others demonstrate or leads learners to self-discovery, some focus on principles and others on applications, some emphasize memory and others understanding. Huetinck and Munshin (2004:52) contend that the way in which an individual acquires, retains and retrieves information is typical of his or her learning style. In the light of the preceding discussion, the researcher seeks to gain a better understanding of how an inductive teaching approach influences learners' learning styles of functions.

Often, there are mismatches in the teaching input and the learning outputs that reflect that there is dissonance between the teaching approach and learning styles (Huetinck & Munshin, 2004:52). Characteristically, an over-emphasis of one teaching approach tends to advantage some learners while it disadvantages others. The mismatches between the learning style of the learners and the teacher's teaching style often leads to a situation in which learners become bored and inattentive in class, do poorly in assignments and tests, get discouraged about the subject, and consequently conclude that they are no good in the subject and give up (Felder & Henriques, 1995:2; Huetinck & Munshin, 2004:52). As a result, this research project seeks to determine the influence of Inductive teaching approach to the learning of functions by further exploring the following questions, as posed by Felder and Henriques (1995:2):

a) Which aspects of learning styles are particularly significant in the learning of functions?

b) Which aspects of learning styles do the teaching styles of most mathematics teachers in teaching functions favour?

c) Which learning styles are promoted by most teachers in the teaching of mathematical functions?

2.2 LEARNING

2.2.1 Introduction

Mathematical learning requires that learners need to learn with understanding, actively building new knowledge from experience and prior knowledge. This involves accumulating ideas and building successively deeper and more refined understanding (Cangelosi, 2003:144). What the statement expresses is that learners can understand a concept such as functions, using the notion of dependence that they are familiar with. They build on this notion to note the relation between quantities and extend it to the representation of relations in different forms.

Most psychology books define learning as a change in behaviour resulting from some experience. This view depicts learning as an outcome and a product of some process (Smith, 1999:1). Learning is viewed as both a product and process, and the latter takes us into the arena of competing learning theories. Learning theories are ideas about how learning may happen (Smith, 1999:1).

The schools of educational psychology have opposing views to learning.

- Some psychologists prefer to base their understanding of learning on the observable behaviour, while others prefer to focus on the working of the mind (Huetinck & Munshin, 2004:39).
- A second area in which educators differ concerns the definition of the nature of learning. The educators agree that learning is a change, but disagree on whether the change is primarily in behaviour or in mental associations (Smith, 1999:4).

Huetinck and Munshin (2004:39) identify four main theories of learning, namely behaviourism, social cognitive theory, information processing and constructivism.

- Behaviourism is the study of observable behaviour and the related environments. Behaviourism does not focus on what is happening "inside" the human mind during learning, but rather on observable overt behaviours that can be measured (Smith, 1999:3).
- The social cognitive theory was originally known as observation learning theory due to the assertion that much could be learned from observing others. The theory differs from behaviourism by defining learning as change in mental associations, and emphasizes cognitive processes rather than observable behaviour (Huetinck & Munshin, 2004:41).
- The information-Processing theory views learning as a significant change in mental processes and in considering those internal processes. Information-Processing theory differs from behaviourism and social cognitive theories in that it examines how humans attain, remember and organize information (Huetinck & Munshin, 2004:42).
- Constructivism is a cognitive learning theory. Constructivism is founded on the premise that reflecting on our experiences we construct our own understanding and make sense of the world we live in. According to the theory, learning mathematics needs active construction and not passive reception, and to know mathematics requires constructive work with mathematical objects in a mathematical environment (Huetinck & Munshin, 2004:39; Smith, 1999:4).

Of the four learning theories mentioned above, both Constructivism and Information-processing are learner–centred. The two learning theories are well suited for Inductive teaching, because learners construct their own understanding from some engagement or experience. If learners understand a concept they attain, remember and organize information (Huetinck & Munshin, 2004:42). Associated with the learning and understanding of mathematical concepts are the learning styles.

2.2.2 The learning style

Learning styles refer to different approaches or ways of learning. The three main mathematical learning styles are visual learning, abstract learning and tactile learning. Visual learning refers to learning related to sight or using sight. Learners inclined to this learning style use drawing of models and also rely on abstract drawing to represent and manipulate physical phenomena in their minds. Abstract learners prefer to think and work in more general terms. Abstract learners see a bigger mathematical picture by making connections between abstract concepts. Tactile learners learn best with physical models in their hands. Tactile learning refers to learning by touching (Dossey *et al.*, 2002:521).

2.2.3 Dimensions of the learning styles

The composition of learners in a class accounts for different learning styles due to the learners' differing social and knowledge backgrounds. The statement refers to learners' accessibility to learning materials, library etc. and the learner's own condition and environment. Teachers have to consider these factors when preparing their mathematics lessons. Felder and Henriques (1995:2) describe three learning style dimensions that do not meet the educational needs of learners when teachers apply traditional approaches to mathematics teaching. The three learning style dimensions described below refer to:

a) the type of mathematical information that the learner preferentially perceives. Do learners prefer sensory information (sights, sound or physical sensation) or intuitive information (memories, ideas or insight)?
b) the modality through which learners effectively perceive sensory mathematical information. Is the modality visual (pictures, diagrams, graphs, demonstrations) or verbal (written and spoken words or formulas)?
c) the manner in which learners prefer to process mathematical information. Do learners process mathematical information actively (through

application in solving problems) or reflectively (through introspection or giving a problem a thorough thought thinking of application and the relevance to a particular situation)?

In view of the above, the following is a discussion of some learning styles that can be associated with mathematical learning and inductive teaching.

2.2.3.1 Sensing and intuitive learners

Hjelle and Ziegler (1992:175) opine that sensing is a direct, realistic perception of the external world without judgement. According to the assertion, a sensation-oriented person is acutely aware of the taste, smell or feel of the stimuli around the world. Intuiting by contrast is characterised by subliminal, unconscious perception of daily experiences. Furthermore, Hjelle and Ziegler (1992:175) assert that an intuition-oriented person relies on hunches and guesses to grasp the meaning of life. Felder and Henriques (1995:2) view sensation and intuition as the two ways in which people perceive the world. The following table illustrates some striking differences of sensing learners and intuitive learners.

Table 2.1: Sensing and Intuitive Learners (Felder & Henriques, 1995:3)

Sensing Learners	Intuitive Learners
Sensing involves gathering information through senses.	Intuition involves indirect perception by accessing memory, speculating and imagining.
Sensors tend to be concrete and methodical, like in solving problems using the well-established methods and linking the information to the real world.	Intuitors are inclined to be abstract and imaginative, like in solving a problem they prefer to be innovative and dislike repetitions.
Sensors like facts, data and experimentation.	Intuitors deal with principles, concepts and theories.
Sensors are patient with detail, but do not like complications. In solving	Intuitors are bored with detail and welcome complications. Intuitors may

problems, sensors prefer well – established methods and resent to be tested on material that they have not covered in class.	be better at grasping new concepts in mathematics and would be comfortable with mathematical formulations to discover relationships.
Sensors are more inclined to rely on memorisation as a learning approach and they learn more comfortably by following procedures and rules.	Intuitors accommodate new concepts and exceptions to rules. Intuitors do not prefer information that involves much memorisation and routine calculations.

In view of these differences by Felder and Henriques (1995:2), traditional teaching would best suit the sensors while inductive teaching would appeal to both the sensory and intuitive learners. Moodley, Njisane and Presmeg (1992:17) argue that intuition discourages investigation because it is obvious. However, one may argue that intuition plays a vital role in the learning and teaching of mathematics because on some occasions a learner may guess the solution to the problem before actually learning the rules and procedures of solving it. One can acquire learning by intuition with experience and it should be actively encouraged and cultivated.

In the light of the preceding statements, it appears that, as the human mind is complex and dynamic, it would be arguable to confine learning experience to one specific teaching approach. For example [in a classroom] sensation would best serve the whole class when demonstrating plotting points on the board in a graph of a straight line in the two dimensional coordinate system to the Grade 11 class. The researcher is of the opinion that using a combination of inductive and traditional teaching approaches in a lesson would reduce advantaging one group of learners and disadvantaging the other. However, inductive teaching caters for all learners in that it ensures their participation and encourages them to work in their preferred style.

2.2.3.2 Visual and verbal learners

Visual learners prefer that information be presented in pictures, diagrams, flow charts, timelines, demonstrations etc. rather than in spoken or written words, while verbal learners would prefer spoken or written words. Felder and Henriques (1995:3) describe visual learning techniques as the graphical ways of working with ideas and presenting information. Diagrams, graphs and demonstrations enable learners to clarify their thinking process, and organise and prioritise new information. Significantly, diagrams reveal patterns, interrelations and interdependencies among objects under study, in order to stimulate creative thinking (Lefrançois, 1997:196). Felder and Henriques (1995:3) assert that, visual learning promote learners' mathematical learning in several ways such as in:

(i) clarifying their mathematical thinking - learners see how mathematical ideas are connected and realize how to group or organize information. Learners easily and thoroughly understand new concepts.

(ii) reinforcing their mathematical understanding – learners recreate what they have learned using their own words and try to form mathematical relationships between objects they have learned. This helps them assimilate and internalize new information, giving them ownership of their ideas.

(iii) integrating new knowledge – diagrams prompt learners to build upon their prior knowledge and internalize new information. By reviewing diagrams created previously, learners see how facts and ideas fit together.

(iv) identifying misconceptions – discovery of misdirected links and wrong connections can help learners realize what they do not understand.

Verbal learners benefit more from words, either spoken or written. Verbal learning promotes learners' mathematical learning in the following ways.

(i) When studying, write summaries and mathematical formulae of important concepts or course material in your own words.

(ii) Try to work in groups where members explain mathematical concepts and ideas.

(iii) A learner benefits more or learns more when he or she does the explaining (Moodley *et al.* 1992:19).

Felder and Henriques (1995:2) contend that effective mathematical instruction reaches out to all learners, not just those with one particular learning style. Learners taught entirely by methods that are antithetical to their learning style tend to be too uncomfortable to learn effectively. However, the argument is that learners should at least have some exposure to different teaching approaches and methods to develop a full range of learning skills and approaches. It stands to reason that mathematical instruction should contain elements that appeal to sensors and other elements that appeal to intuitors. To this effect, the material presented in every class should be a blend of concrete information (definitions, mathematical rules) and concepts. In addition, the material chosen should fit and be appropriate to the level of the course, age of learners, as well as level of sophistication of the learners' learning styles (Felder & Henriques, 1995:3).

2.2.3.3 Active and reflective learners

Most psychologists, including Lefrançois (1997:18), identify experimentation and observation as mental processes that convert information to knowledge. Active learners retain and understand information best by discussing it, applying it, or even explaining it to others, whereas reflective learners prefer to think quietly about it first. Active learners prefer working in groups, while a reflective learner finds it very comfortable working alone (Lefrançois, 1997:18). Felder (1993) recommends that reflective learners could enhance their learning by

- compensating for the lack of discussion or problem solving activities in a classroom when they are studying;
- studying in groups, with members taking turns to explain different topics; and
- finding ways to apply information for better retention.

To improve the quality of learning for active learners, Felder (1993) suggests that learners could do well if they

- compensate for the lack of thinking time about new information when they are studying;
- stop periodically to review the information they read and to think of possible questions or application, and not just memorise the material; and
- write short summaries of reading notes in their own words.

Human learning entails strategies for thinking, understanding, remembering and producing language. As an active learner, one can learn effectively from actively discussing problem solving and finding applications for new information. Learners who have a strong preference for active learning need to be aware of the potential dangers of jumping to conclusions prematurely about things without thinking them through. Reflective learners, on the other hand, learn best when they allocate time for thinking about and digesting new information. It would be very helpful for one to review new work periodically, write summaries and think of possible ways to apply new information. The danger of reflective learning, however, is that one can spend too much time thinking about something rather than getting it done.

From discussion of the learning style dimensions, mathematics teachers who apply inductive teaching need to plan learning activities that will cater for these differing styles of knowledge acquisition. Teachers need to select learning activities in inductive teaching that promote communication of mathematical ideas among learners. Learners should be able to write about, describe, explain and share mathematical ideas (Van de Walle, 2007:4). Teachers need to select learning activities in inductive teaching that develop connections among mathematical ideas, mathematical ideas and the real world and to other disciplines and enhance problem solving skills (Van de Walle, 2007:4).

2.3 TEACHING

A primary objective of teaching is to enable learners to perform tasks expected of them. Effective mathematical teaching requires understanding what learners know and need to learn and challenging them to learn it well (Cangelosi, 2003:145). The researcher concurs that learning has to start where the learner is. The learner's pre-knowledge can help shape his or her assimilation and understanding of new matter that he or she has to learn. A learner's penchant for a particular learning style informs how he or she learns.

Huetinck and Munshin (2004:282) view teaching as the weaving of the learners' knowledge and understanding of concepts to be able to initiate, guide and respond to dynamic situations, so that the learners' ability may grow. The essence here is that learners should be able to demonstrate that they have learned the concepts by being able to apply skills and knowledge to solve unprecedented problems in unfamiliar situations. Mathematics teaching involves having specialised knowledge, competent teaching strategies and behaviours and appropriate professional dispositions. Teaching does command an expertise unique to the profession and teachers should never cease to develop and refine their own expertise (Berry & King, 1998: 408).

Flinders (1989), as quoted by Huetinck and Munshin (2004:282), suggests four modes of viewing teaching, namely:

- Communication: Oral, written and nonverbal clues such as eye contact, nods, smiles, leaning forward etc. are all coordinated to enhance communication.
- Perception: Teachers should strive to see, hear and understand their learners. Intuitive receptivity requires flexibility and imagination to move with learners in order to enhance their understanding.
- Cooperation: Some of the teachers' strategies for negotiating cooperative relationship with the learners are:
    a) using humour and self-disclosure to promote learner solidarity;
    b) allowing learners to choose activities;
    c) bending classroom and even school rules in the learner's favour;

d) providing opportunities for individual recognition; and
e) creating time to allow interaction with the learners on a one-on-one basis.

- Appreciation: Teachers appreciate their own work by describing how their efforts prevailed over a difficult endeavour.

In teaching, there is no set of rules on how to become an outstanding teacher, but only guidelines. This lack of prescription demands that the teacher be creative and be dedicated to the profession (Huetinck & Munshin, 2004:283).

2.4 TEACHING PARADIGM

Education paradigms influence the Inductive and traditional teaching approaches. Before the year 1994, the South African education system perpetuated race, class, gender and ethnic divisions, and it was based on values and principles that emphasised separateness rather than common citizenship and nationhood (DoE, 1997:1). Subsequently, the system gave rise to the crisis of lack of equal access to schools, irrelevant curricula, shortages of educational materials and inadequately qualified teaching personnel (Van der Horst & McDonald, 1997:5). Typically, educational reform became inevitable in order to transform the curriculum that was content driven, compartmentalised, and arranged into traditional subjects, and replaced it with an outcomes based education. Notably, curriculum changes in South Africa evolved from a situation where there were limited learning opportunities and the curriculum was characterised by contest learning and competitions (Van der Horst & McDonald, 1997:5). What was also crucial for the curriculum change was that the curriculum prepared individuals for limited career and job opportunities as the emphasis was on content reception without application and not on skills development (DoE, 2003:2).

In view of this country's history, it has been imperative to restructure the curriculum to reflect values and principles of the new democratic society (DoE, 1997:1). Van der Horst and McDonald (1997:5) allude to the fact that it was also important to move away from rote learning to understanding and doing. To generate the required knowledge, skills and habits as well as to achieve the ideal of a prosperous and democratic country require a paradigm shift in classroom teaching. As a result, Outcomes-based Education was introduced in the post 1994 era to drive educational reform for the implementation of the new curriculum that encourages integrated learning. It is noted that Outcomes-based Education promotes integration of inductive and traditional teaching approaches but places more emphasis on inductive teaching approach as it shows commitment to learning for all learners and success orientation for all (DoE, 2003:3).

In recent years, the National Curriculum Statement that enlists as its precursor Curriculum 2005 endorses a concept of lifelong learning (DoE, 2003:3). This provides all people (adults, youths who left school and scholars) who need to learn the opportunity to do so. It is common knowledge that learning occurs mostly inductively at the workplaces, for the simple reason that most of the mathematics applied there, is different from the one taught in classrooms. The researcher noted that the new educational system encourages that all people should be granted the opportunity to develop their potential to the full whether by means of formal or non-formal schooling. Just as education and training in the post 1994 era are both people centred and success oriented (Van der Horst & McDonald, 1997:4), applying inductive teaching approach leads to an inclusive and integrated approach in the teaching praxis.

One of the mechanisms used to drive the process of attaining the goals of the new education and training system is to come up with the most appropriate teaching approaches. Van der Horst and McDonald (1997:133) define a teaching approach as a broad plan of action for teaching activities with an aim of achieving an outcome. A learner acquires new knowledge if it links with the knowledge and experience he or she already has.

Teachers should strive to prepare sequences of activities where new knowledge builds on the old. Notably, a learner lives in the concrete world, and hence learns by observing concrete things. Teachers would enhance the learners' concept learning if they sequence learning activities so that learners can visualize the information and think in abstract terms (DoE, 2003b:34). The introduction of the theme to be learned by clarifying concepts related to it before learning its different aspects, tend to give learners clear ideas of the whole theme they are about to learn. Relevant teaching methods support teaching approaches to attain a desired outcome, and one such approach used to attain the learning outcomes is the inductive approach. We start first by discussing the teaching approach before the introduction of Outcomes-based Education herein referred to as Traditional or Deductive form of teaching.

2.5 TRADITIONAL TEACHING

Traditionally mathematical teaching means that a teacher imparts knowledge and learners are expected to apply these ideas or skills to solve problems. It is mostly listen, copy, practice and drill. The learning theories associated with the traditional form of teaching from the four mentioned above are Behaviourism and Social cognitive theory (see 2.2.1). The approach encompasses an explicit teaching strategy, exposition strategy, drill strategy amongst others. Traditional teaching (also called direct instruction) is much less constructivist in approach. Traditional teaching approach emphasizes the idea that a highly structured presentation of content creates optimal learning for learners (Huetinck & Munshin, 2004:4).The teacher using a traditional approach typically presents a general concept through explanation, questioning and then providing examples or illustrations that demonstrate the idea. Learners are given opportunities to practice, with the teacher's guidance and feedback, applying and finding examples of the concept at hand, until mastery is achieved (Moodley *et al*,1992:56).

2.5.1 An example of Traditional teaching of a functions lesson

In the function lesson, the teacher presents the concept of a slope of a graph. The slope is then defined as follows: The slope m of a graph of a non-vertical line is given by the equation, $m = \dfrac{vertical\ rise}{horizontal\ displacement}$ (Stewart *et al*, 1998:103).

The teacher would draw a graph of the slope m as follows:

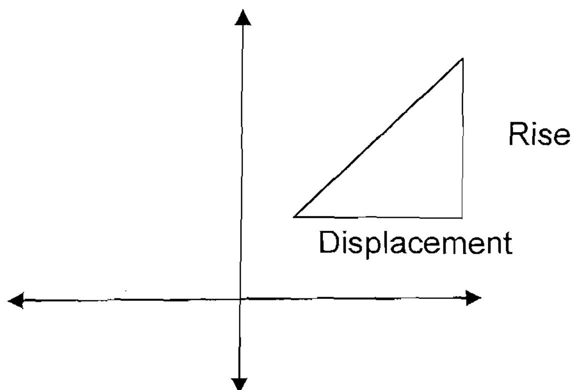

Figure 2.1: The slope m

If the given two points $(x_1; y_1)$ and $(x_2; y_2)$ lie on the non-vertical straight line $y = mx + c$, then the slope m is determined by using the equation $m = \dfrac{y_2 - y_1}{x_2 - x_1}$. The slope of a vertical line is not defined (Stewart *et al*, 1998:103). The teacher guides the learners to determine whether the following points are collinear (lie on the line) (Stewart et al, 1998:103).

$$A: \quad (1;1), (3;9), (6;21)$$
$$B: \quad (-1;3), (1;7), (4;15)$$

The calculation of the slope $m = \dfrac{y_2 - y_1}{x_2 - x_1}$, using points in A, taking two points at a time, yields the following results:

Using the points $(1;1)$ and $(3;9)$, the slope $m = \dfrac{9-1}{3-1} = \dfrac{8}{2} = 4$.

Using the points $(3;9)$ and $(6;21)$, the slope $m = \dfrac{21-9}{6-3} = \dfrac{12}{3} = 4$.

Using the points $(1;1)$ and $(6;21)$, the slope $m = \dfrac{21-1}{6-1} = \dfrac{20}{5} = 4$.

Three equal values are obtained from the calculations of the slope. The learners deduce that since the slopes are equal, then the points lie on the line (Stewart *et al*, 1998:107).

The calculation of the slope $m = \frac{y_2 - y_1}{x_2 - x_1}$, using points in B, taking two points at a time yields the following results:

Using the points $(-1;3)$and$(1;7)$, the slope $m = \frac{7-3}{1-(-1)} = \frac{4}{2} = 2.$

Using the points $(1;7)$and$(4;15)$, the slope $m = \frac{15-7}{4-1} = \frac{8}{3}.$

Using the points $(-1;3)$and$(4;15)$, the slope $m = \frac{15-3}{4-(-1)} = \frac{12}{5}.$

The calculation of the slope yields three different values. The learners deduce from the theory that since the slopes are different, then the points do no lie on the line (Stewart *et al*, 1998:107).

In the above examples the teacher led learners in the calculation of the slope and based on theory taught to them before, learners deduced desired results. Their role in this exercise is that of verification and not of construction of understanding.

2.6 INDUCTIVE TEACHING APPROACH

Inductive teaching approach or inquiry-based learning enhances the learner to be able to do things on her or his own. This is precisely the primary purpose of teaching. Inductive teaching encompasses strategies such as problem solving, investigative learning, learning by discovery and cooperative learning. Inductive teaching approach expends inductive reasoning as a process skill. Inductive reasoning is based on the principle of *a posteriori* logic, which proceeds from a particular set of causes or facts of experience to the general law, or principle (Van der Horst & McDonald, 1997:133). An *a posteriori* logic refers to using facts or observations you know now; to form a judgement about what must have happened before. Felder and Henriques (1995:7) contend that Inductive reasoning is a reasoning progression that proceeds from particulars (observations, measurements, data) to generalities (rules, laws, theories). An example of an inductive reasoning is the observation that when one adds two odd numbers the sum is an even number (Moodley *et al.*1992:48).

Example 2.6.1: Consider the following addition of odd numbers:

$$1+3=4, \quad 3+5=8, \quad 7+9=16.$$

We note that although the numbers 1 and 3, 3 and 5, 7 and 9 are odd, their sums 4, 8 and 16 respectively, are even numbers. Repeating the exercise with larger odd numbers, this is still true. For example, consider the following additions:

$$115+117=232, \; 253+345=588.$$

In all the cases, these particular examples suggest the conjecture: "*The sum of any two odd numbers is even*" (Moodley *et al.*1992:48).

In the inductive reasoning individual examples are used to arrive at the general principles underlying them. Examples that do not fit the idea (non-examples) are helpful in confirming the idea (Huetinck & Munshin, 2004:283). Conjectures are arrived at after a number of steps or procedures (Felder & Henriques, 1995:7). Inductive reasoning is extensively used in mathematics as a method of proof where primarily it serves the roles of discovery and verification (Felder & Henriques, 1995:10).

Example 2.6.2: Let us consider an example taken from Learning Outcome 1, Number and Number Relationships (DoE, 2003:19), where induction is used twofold, for confirmation and as a method of proof. The example first requires the use of reasoning to confirm the formula $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$, then we apply mathematical induction as a method of proof to prove the statement true for all natural numbers n. The proof using mathematical induction as a method of proof requires three steps that involve proving the statement or proposition true for one natural number, assuming the proposition true for a number of terms or natural numbers, and proving the proposition true for numbers beyond for those assumed true. In this approach induction is used as a method of proof.

To confirm the formula $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$, we perform a number of steps and inductively arrive at the desired result. Consider the following table where n represents the number of terms (numbers):

n	$\sum_{i=1}^{n} i = 1+2+3+....+n.$
1	$1 = \frac{1\times 2}{2}$
2	$1+2=3 = \frac{2\times 3}{2}$
3	$1+2+3 = 6 = \frac{3\times 4}{2}$
4	$1+2+3+4 = 10 = \frac{4\times 5}{2}$
5 . . n	$1+2+3+4+5 = 15 = \frac{5\times 6}{2}$. $1+2+3+4+...+n = \frac{n(n+1)}{2}$

Table 2.2 Verification of the formula $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

In the above table we conclude that after five steps we infer inductively that $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.

In the next illustration we show steps needed to prove the statement or proposition true for all natural numbers n using mathematical induction we proceed as follows:

The expanded form of $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ is $\sum_{i=1}^{n} i = 1+2+3+....+n.$ We write the mathematical statement without the sigma sign as:

$$1+2+3+4+....+n = \frac{n(n+1)}{2} \qquad (1).$$

The above equation (1) is the proposition that has to be verified by the inductive reasoning.

Step1: We prove that the proposition is true for $n=1$.

LHS = 1.

$$\text{RHS} = \frac{1(1+1)}{2} = \frac{2}{2} = 1.$$

Since both the Left hand side (LHS) and the Right hand side (RHS) are equal to 1, we conclude that the proposition is true for $n=1$.

Step2: We assume that the proposition is true for $n=k$. That is:

$$1+2+3+4+....+k = \frac{k(k+1)}{2}.$$

Step3: We prove that the proposition is true for $n=k+1$.

$$\text{RHS} = \frac{(k+1)(k+2)}{2}.$$

$$\begin{aligned} LHS &= 1+2+3+.....+k+k+1. \\ &= \frac{k(k+1)}{2} + k + 1. \\ &= \frac{k(k+1)+2(k+1)}{2}. \\ &= \frac{(k+1)(k+2)}{2} = RHS. \end{aligned}$$

From step 3 both sides are equal and are equal to $\frac{(k+1)(k+2)}{2}$. We conclude inductively that the proposition is true for all natural numbers n. Inductive reasoning, as Moodley *et al* (1992:46) contends, proceeds from a true premise to a probable conclusion, and thus, its role is that of invention or discovery. The example exposes us to the view that we use induction to discover the trends in natural numbers in each of the three steps, and finally make a conclusion about the proposition. We also use induction intuitively to confirm statements.

The National Curriculum Statement (DoE, 2003:2) emphasises the importance of prior knowledge in learning. The introduction of new material linking prior knowledge to observed or previously known material is essentially inductive in approach. Another view of inductive reasoning is the assertion by Moodley *et al* (1992:46) that it is a principle proceeding from true premises (particular) to probable conclusion (general).

Inductive teaching (also called discovery teaching or inquiry teaching) is based on the claim that knowledge is built primarily from a learner's experiences on interactions with phenomena. A teacher using an inductive approach begins by exposing learners to a concrete instance or instances, of a concept. The learners follow by observing patterns, raise questions, or generalize from their observations (Huetinck & Munshin, 2004:220). The role of the teacher is to create opportunities and the context in which learners can successfully make appropriate generalizations and to guide the learners as is necessary (Huetinck & Munshin, 2004:220).

Inductive teaching has close ties with the instructional approach of guided discovery where the teacher sets a problem and helps the learners investigate it. Phenomena are explored before concepts are named. The learners are encouraged to discover patterns and draw conclusions that are shared in whole class discussion (Huetinck & Munshin, 2004:220). Inquiry-based teaching, in which learners are asked to continually develop and test hypotheses in order to generalize a principle, is also closely related to inductive teaching (Huetinck & Munshin, 2004:220).

2.6.1 An example of Inductive teaching functions lesson

In the lesson on functions (see 2.8), the teacher gives the learners a visual depiction of the scenarios where the slope of the graph is to be induced from interaction with the equations. The teacher asks the learners to sketch the graphs of the linear functions $y=3x+2, y=-3x+2$ and $y=-\frac{x}{3}+2$ using point-by-point plotting. The teacher asks the learners to use any two points from each line to calculate the slope. The learners are instructed to observe their calculations and compare their values to those from the equations. After calculations, group discussions, and reflections the teacher consolidates learning that has taken place by providing the name of the concept and the learners become convinced that m indicates the slope in the graph of a linear function $y=mx+c, x, y \in R$ and c is the $y-$intercept (Stewart, Redlin & Watson, 1998:105). In this way, general rules are often accepted inductively from experience and interaction with the phenomenon.

2.7 PERSPECTIVES ON TRADITIONAL AND INDUCTIVE TEACHING APPROACHES.

Inductive learners prefer making observations and poring over the data looking for patterns so that they can infer larger principles. Traditional learners like to have general principles identified and prefer to deduce the consequences and examples from them (Huetinck & Munshin, 2004:220).

In the Inductive teaching approach, the learners may draw other meanings from the examples and data provided, than what the teacher intended. It is important that clear guidelines are set beforehand to avoid this from happening. The inductive teaching approach may also take more time and be less efficient than a traditional teaching approach if the learners do not have an idea of what the teacher expect them to uncover. One clear disadvantage of traditional teaching is that it can be too rigid. Traditional teaching sometimes does not allow for divergent learner thinking nor emphasise reasoning and problem solving (Huetinck & Munshin, 2004:220).

Traditional teaching is a one way form of instruction. Learners are passive receivers of information. Drilling information can be aimless and boring. Learners chant matter in a parrot-like manner with little understanding and vitality (Stewart *et al*, 1998:103,). Inductive teaching offers learners good opportunity to develop mathematical communication skills. Traditional teaching offers little involvement in that learning outcomes are marginal at best (Berry & King, 1998:183).

Van de Walle (2007:39) suggests that open-ended inductive exercises posed as problem solving activities may present severe challenges to learners with learning disabilities. Learners with learning disabilities have difficulty getting started, understanding their roles in the exercise, and staying focussed on the activity. For these learners to succeed when engaged inductively, it is essential that the teacher creates clear guidelines for the behaviour, provides explicit directions of the activity, and be prepared to offer extra guidance as needed.

Inductive teaching is very advantageous in that learners taught in this approach can display a better long-term retention of concepts than those taught with traditional approach. Felder and Henriques (1995:26) contend that the benefits claimed by inductive teaching include increased academic achievement and enhanced abstract reasoning skills, longer retention of information and improved ability to apply principles. The reason is that inductive thinking demands deeper processing (Huetinck & Munshin, 2004:220; Van de Walle, 2007:39).

We now apply inductive teaching specifically to the concept of a function. A concept of a function is vital in the construction of mathematical knowledge in that it typifies a creation of relationships, which is the hallmark of conceptual and relational understanding and problem solving.

2.8 THE FUNCTION CONCEPT

The definition of a function as a relationship between variables stresses a stronger operational base (a relation between quantities (variables) that change as the situation changes) than the structural conception of a function.

Furthermore, a function is defined as a rule that uniquely defines how the first or the independent variable affects the second or the independent variable. The object is to determine how the change in one variable affects the change in the other (Van de Walle, 2004:436). O'Callaghan, (1998:22) depicts variables as quantities that change as the situation changes. A practical example is the ever-changing petrol price. The change in the petrol price is governed by amongst others (a) the volatility of the South African currency (rand) against major world currencies and (b) the conflicts among countries, in particular Middle East countries.

The modern set theory views a function as a binary relation f, with the property that for an element x in the set A, there is no more than one element y or $f(x)$ in another set B for that relation. The definition of a binary relation is that it is a relation between ordered pairs $(x;y)$, and subsequently a relation is any relationship between sets of information (Stewart, Redlin & Watson, 1998:132). This definition presents the structural view of the function. The view asserts that if one thinks of a function as a machine, then if you input x, then the output is $f(x)$.

The figure below illustrates a function as an input and output machine, that is, if one inputs x, under the function f then your output is $f(x)$.

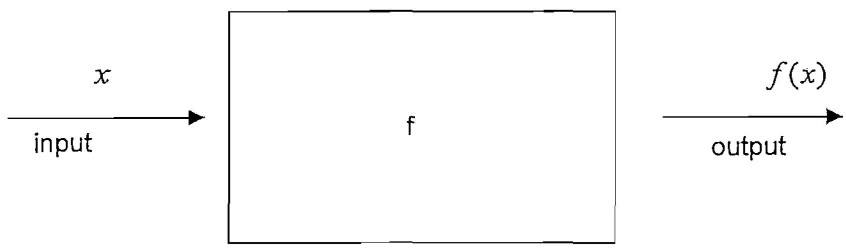

Figure 2.2: The structural view of a function

Another way to picture a function is by using an arrow diagram. The arrows connect an element in the set A to an element in the set B.

The figure below illustrates a function f from a set A to a set B by an arrow connection of an element in the set A to an element in the set B (Stewart *et al*, 1998:132).

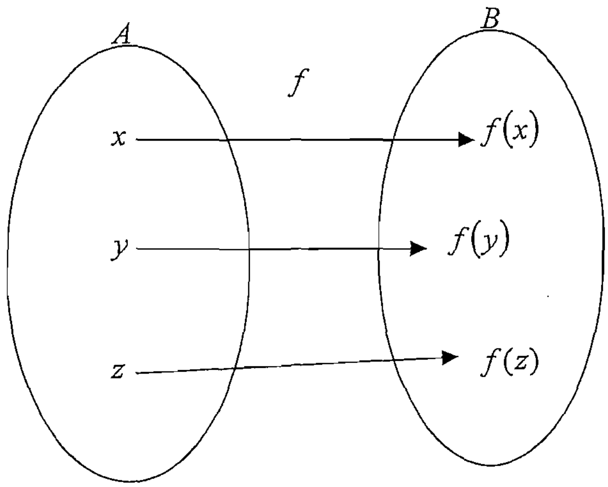

Figure 2.3: Function from a set A to a set B

The figure 2.4 below does not represent a function from a set A to a set B (Stewart *et al*, 1998:132).

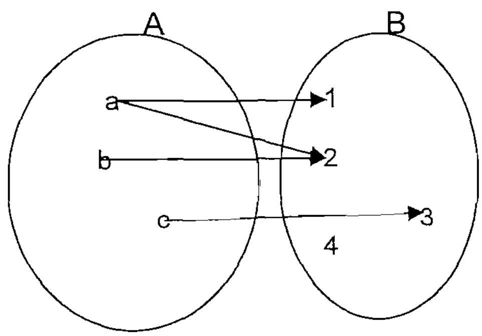

Figure 2.4: Not a function

The figure 2.5 below does represent a function from a set A to a set B (Stewart *et al*, 1998:132).

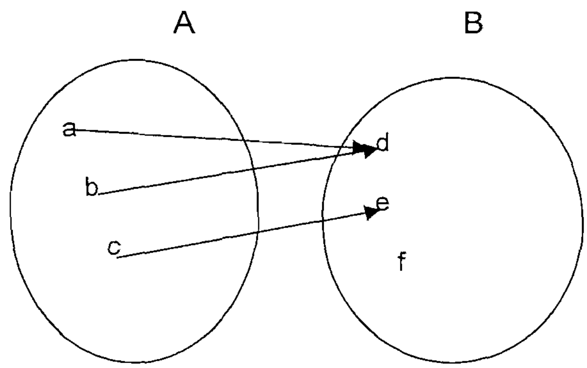

Figure 2.5: A function

O'Callaghan (1998:23) asserts that not all functions have real world context, but it is important at the General Education and Training phase of schooling to place functions in contexts that make sense to the learners. The object is to represent a function in a model that one can operate on and assess.

Kieran (1992:411) differentiates three representations of functions namely algebraic, graphical and tabular. Algebraic refers to the equation defining a function. Tabular is the solution of a function in a table for different values of variables. An interplay of tabular, graphical and symbolic experiences offer meaning oriented towards conception of functions (Kieran, 2007: 712).

Teachers normally introduce the concept of functions using set-theoretical definitions (i.e. correspondence between members of different or the same set as in figure 2.2). The definition leads to the drawing of diagrams (one to one, many to one), equations and ordered pairs. These representations are extended to include table values and Cartesian graphs (Kieran,1992:408). Although the approach emphasizes structural and procedural interpretations, it detaches itself from the real life problem being solved as it does not refer to a particular context. It gives learners the impression that this is the standard definition of a function. It is precisely for this reason the researcher opted for O'Callaghan's model (see 2.9) as an approach to the conception of functions as well as the five different representations of function by Van de Walle (2004:436).

Some of the difficulties from different studies relating to the conception of functions that Kieran (1992:409) identifies are:

- Constant functions: Most learners confuse a constant and a point in the Cartesian plane.
- Functions defined by piecewise.
- Neglect of the domain and range: Learners assume that all functions are defined over real numbers.
- Translations among different representations of functions.
- Exclusive attachment to linearity and lack of dynamic conception of functions.
- Inability to view functions as mathematical objects.

The choice of O'Callaghan's model (see 2.9) as an approach to this study sought to address some of the above-stated difficulties. The purpose of the study is to investigate the relationship between inductive teaching and the nature of conceptualisation and the learning achievement of functions in Grade 11.

Van de Walle (2004:436) distinguishes five different ways of interpreting or representing functions, namely contextual representation, tabular representation, language expression of a function, graphical representation and using equations.

2.8.1 Contextual representation of Functions

To give an example of the five representations of a function in a real-life context, we will use a vendor selling tomatoes in a stall at a shopping mall. [The vendor, Molefe, is trying to make some money to support his impoverished family. The company Molefe has been working for recently retrenched him from work due to restructuring. As an unskilled labourer with no immediate future job prospects, he uses the little money he got from his past employment to sell tomatoes to put food on the table for his family.] The stall costs a monthly rental of R50. He packages the tomatoes by putting five of them in a plastic bag, and sells them for R10. The tomatoes cost the vendor R50 a crate, and there are about 50 in a crate. The profit gained from a single crate is R50, which amounts to R1 per tomato (Van de Walle, 2004:436).

The representation begins with the clear context: selling tomatoes and the resulting profit. The interest is in Molefe's profit in terms of the number of tomatoes sold (Van de Walle, 2004:437). The more packets of tomatoes he sells the more profit he will make. His profit is dependent (is a function of) on the number of packets he sells (Van de Walle, 2004:437).

2.8.2 Table representation of Functions

Molefe might hypothesise on his sales and calculate some possible income figures. This will give him some idea of how many packets he has to sell to break even, and what his profit might be for the month (Van de Walle, 2004:437). If he sells no tomatoes for the month, his profit would be below by R50, as he has to pay for the stall. If he sold 10 packets per month then his profit would be $10 \times R10 - R100 = 0$, which is determined by taking the difference between the Selling Price, Cost Price and the monthly rental of the stall. A table of similar values might look like the following table:

Packets of Tomatoes	Profit
0	- R50
10	R0
20	R50
30	R100
40	R150
60	R250

Table 2.3: The packets of tomatoes sold and the profit made from the sales of tomatoes (Van de Walle, 2004:437)

The packets of tomatoes sold is purely a matter of speculation, Molefe could sell far more than the number of packets indicated here depending on the market.

2.8.3 Language Expressions for Functions

Van de Walle (2004:437) views functional relationships as dependent relationships or rules of correspondence. In the tomatoes vendor situation, Molefe's profit depends on the number of tomatoes sold. The phrase "is a function of" according to Van de Walle (2004:437) expresses the dependent relationship. We conclude using the verbal expression that the profit depends on or is a function of the tomatoes sales.

2.8.4 Graphical Representations of functions

The comprehension of the function by learners becomes complex because of their inability to translate among the different representations of the concept. The learners need to be able to apply their knowledge to real-problem solving situations (O'Callaghan, 1998:22). There is a saying that "a picture is worth a thousand words" (Van de Walle, 2004:437). The picture in the case of functions is a graph. The information in Table 2.2 is represented graphically if we consider the horizontal axis to represent the number of packets of tomatoes sold and the vertical axis to represent the profit.

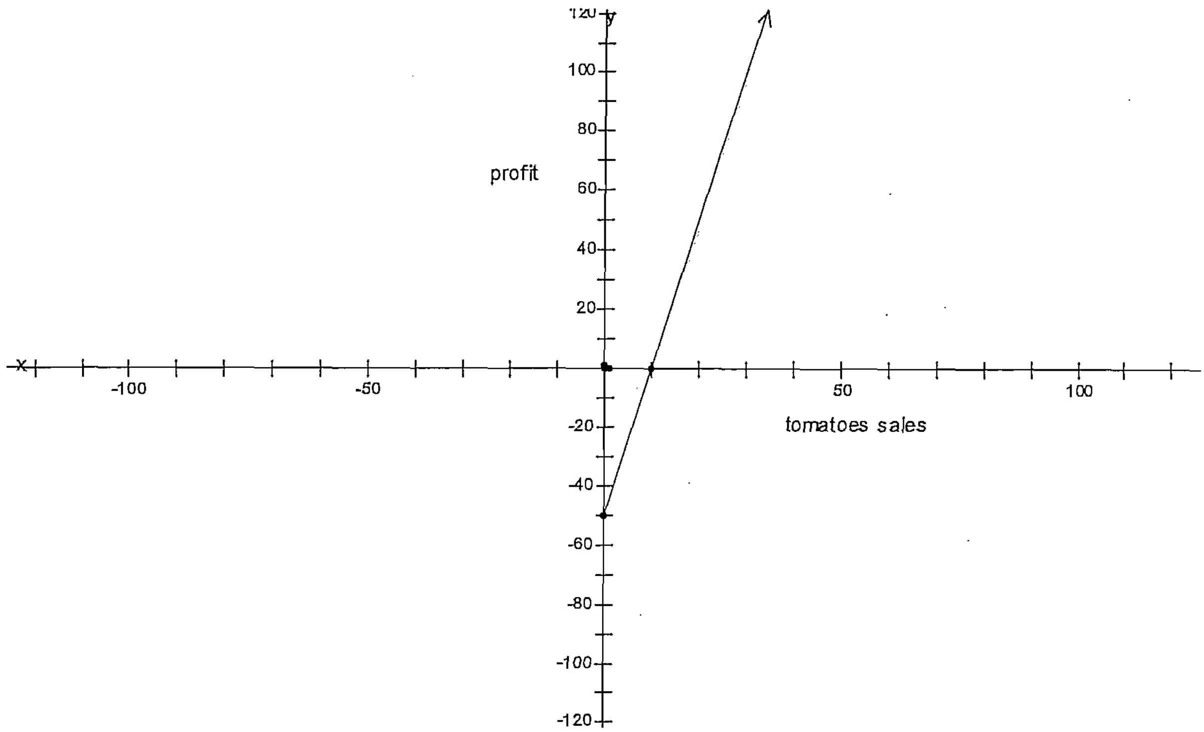

Figure 2.6: Graph of the profit of tomatoes sales (Van de Walle, 2004:438)

The graph shows that the relationship between the tomatoes sales and the profit is linear (a straight line) and is increasing. The graph allows one to determine Molefe's profit. It tells us how many tomatoes Molefe must sell to break even (the point where the line crosses the horizontal axis). Although the graph is increasing without bounds, it would not make sense to say that Molefe would sell an infinite number of tomatoes. The interpretation would be that it is a signal that indicates that if the vendor sells more tomatoes the profit would increase (Van de Walle, 2004:438).

2.8.5 Equation representation of Functions

Suppose that we pick the letter p to represent the packets of tomatoes Molefe sells. For each packet he sells, his income is 10p (Van de Walle, 2004:438). In order to determine the profit, we have to subtract from his income the rental cost of the stall per month and the cost price of each box of tomatoes. The profit on each packet is R5, which translates to R50 per crate. The equation representing the profit as a function of p per month is given by Profit$=5p-50$, where p denotes the number of packets of tomatoes sold (Van de Walle, 2004:439).

2.9 CONCEPTUAL FRAMEWORK FOR THE CONCEPT OF A FUNCTION

The conceptual framework is essentially the attempt by the researcher based on the ideas of Thompson (1985), Kaput (1989) and Fey (1992) as quoted by O'Callaghan (1998:24) to impose a structure and organisation onto the meanings associated with the function concept. The model chosen for this study is one adopted by O'Callaghan (1998:24). This model identifies and applies four competencies of modelling a function, interpreting a function, translating and reifying a function. This model best describes the conception of a function concept. It must be emphasized that the model accounts for components of knowledge relevant to the function concept and forms a basis of analyses for learners' conceptualization of functions, and is not a prescription of how functions can be learned (O'Callaghan, 1998:24).

2.9.1 Modelling

Modelling involves the transition from a real-life experience to a mathematical representation of that experience. We recall the situation of the tomatoes vendor (see 2.8.1). The problem was to identify the best possible way to maximise the profit from the sales of tomatoes to make a decent living. The process entailed the use of variables and a function to form an abstract representation of the quantitative relationships in the vendor's situation (O'Callaghan, 1998:25). This component further divides into a number of subcomponents depending on the representation system used to model the situation. Modelling real-world situations to help organize the physical world is one of the most common uses of functions. O'Callaghan (1998:25) concedes that the perception of functions as an appropriate tool in this regard is a *sine qua non* condition for making any sense of the function concept.

Example 2.9.1.1: Molefe (see 2.8.1) buys a crate of tomatoes from which he gets 10 packets of five (5) in a packet. Find the following:

a) The total number of tomatoes in five (5) crates;
b) The number of crates Molefe buys if he obtains 150 packets.
c) If a crate costs R50 and the monthly rental of the stall is also R50, write the profit per crate of tomatoes as a function of x, where x represents the number of packets of tomatoes sold if each packet sells for R10.

The solutions of the above questions are:

a) The number of tomatoes in five crates $= 5$ crates $\times$ 10packets / crate $\times$ 5 tomatoes / packet = 250 tomatoes.
b) To obtain 150 packets, Molefe will need to buy three (3) crates of tomatoes.
c) The profit per crate is defined as the difference of the selling price per crate and the cost price.

   $\therefore$ Profit $= 10x - 50$.

2.9.2 Interpretation

Interpreting is the reverse process to modelling. It gives functions in different representations in terms of real-life applications. Interpreting is the second component of the function model (O'Callaghan, 1998:25). This component can be partitioned into subcomponents, which would correspond to the three most frequently used representations for functions namely equations, tables, and graphs (O'Callaghan, 1998:25). O'Callaghan (1998:25) contends that learners could be confronted with problems that require them to make different types of interpretations of functions or to focus on different aspects of the graph, for example individual points versus global features.

O'Callaghan (1998:25) argues that learners have trouble with the way they operate with and conceptualise functions. The sources of these difficulties evolve through their developing and integrating structures on both the levels of specific values and overall patterns of behaviour.

Example 2.9.2.1: The following graph describes the motion of a train pulling into a station and off-loading its passengers.

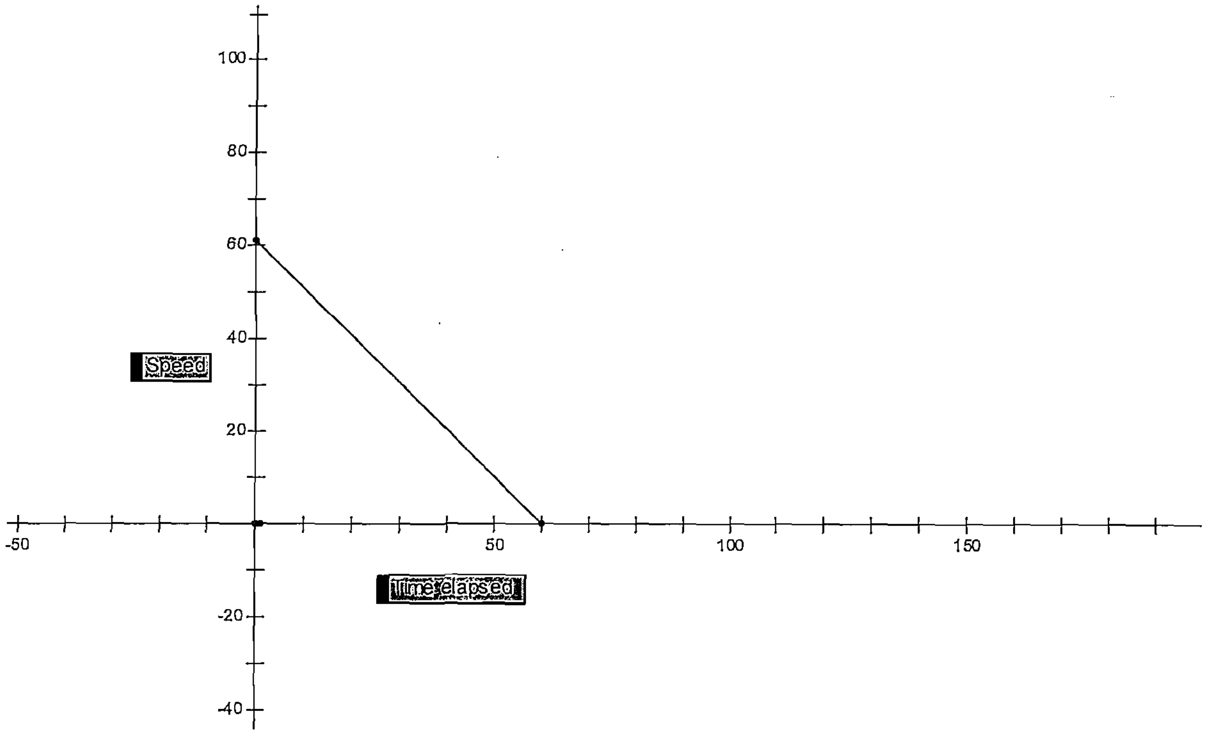

Figure2.7: Speed versus elapsed time

The graph is used to describe the reduction and calculation of the train's speed until it finally comes to a stop.

2.9.3 Translation

Translation refers to the ability to move from one representation of a function to another. From 2.8 the mathematical model may be represented by various systems. The five core representational systems that a function can translate among are contextual, graphical, equations, tabular and language use (O'Callaghan, 1998:25; Van de Walle, 2004:436).

Example 2.9.3.1: Let us suppose that the table below gives the value (V), in rands, of a Hyundai Getz 1.4 GL car for the different number of years (t) after it is purchased.

Number of years (t)	Value (V) in rands
0	117600
2	95200
4	72800
6	50400
10	?

Table 2.4: Number of years and the value in rands

We use the information in the table to write a symbolic rule expressing V as a function of t, and use the rule to determine what the price of the car will be ten years after its initial purchase.

2.9.4 Reification

Reification defines the creation of a mental object from what was initially perceived as a process or procedure (O'Callaghan, 1998:25). In simple terms, reification refers to the treatment of something abstract as if it existed as a real and tangible object. This mathematical object possesses the property that one can operate on by other processes such as transformation or composition. Arguably, reification is a truly difficult process that entails a conceptualisation of functions achieved by few learners (O'Callaghan, 1998:26).

Example 2.9.4.1: Suppose that Molefe works forty hours a week at a furniture store, earning a R220 weekly salary plus a 3% commission on sales over R5000. Given that Molefe's sales is represented by x, then his commission as a multiple of his sales will be represented by the function $f(x) = 0.03x$. The other condition placed on Molefe's commission is that the sales should be more than R5000. Represented as a function we have $g(x) = x - 5000$. Molefe's commission as function of his sales (x) is represented by the composition of functions $C = (f \circ g)(x) = 0.03x - 150$. In this example the commission could be attained once the sales have exceeded R5000. The amount of R5000 is the break-even.

2.10 CONCLUSION

The teaching and learning of mathematics in South African schools continue to experience grave problems leading to low quality learning and frequent failure. Mismatches often occur between the learning styles of the learners and the teaching approaches of teachers with unfortunate effects on the quality of learners' learning and their attitudes towards mathematics.

The curriculum requires that the learners should be able to identify and solve problems using critical and creative thinking (DoE, 2003a:2). The inductive strategy fosters such creative thinking skills, as the learners would have to think the problems through before attempting to give their perspectives and suggest solutions. The inductive teaching approach helps lead learners to transfer skills from familiar to unfamiliar situations. Although both traditional and inductive teaching approaches are used frequently in teaching mathematical concepts to learners, mathematics teachers are urged to apply inductive teaching more often as it encourages self-discovery and better understanding of concepts among learners.

Inductive teaching intends to guide learners by appropriate questions, examples and learning experiences to the apprehension of an idea or principle before it is stated as a formal idea. It is hoped that by its constructivist nature, inductive teaching which encourages learner involvement and participation, can help ease the problem of low quality learning and frequent failure in mathematics.

CHAPTER 3

DESIGN AND METHOD OF RESEARCH

3.1. INTRODUCTION

The study was conducted through primary data collection at two schools in the Gauteng Province. The schools were selected with the assistance of the mathematics Subject Advisory in the area.

The purpose of the study was to investigate the relationship between inductive teaching and the nature of conceptualisation of functions, and the learning achievement with regard to functions in Grade 11. To achieve this, the study sought to provide answers to the following problem questions:

1. What is the influence of inductive teaching on the conceptualisation of functions in Grade 11?
2. What is the influence of inductive teaching on the learning achievement with regard to functions in Grade 11?

The results of the study may not necessarily be generalised to include all schools. The aim of the research was not to draw generalisations but to elicit rich information regarding the research questions posed in the first chapter (see 1.2).

3.2. RESEARCH APPROACH

3.2.1 Experimental design

The methods that were used in the research design are twofold: a pragmatic evaluation study in a form of a pseudo experiment complemented with a qualitative investigation by means of semi-structured interviews.

- Quantitative investigation: A pretest-posttest-control group experimental design (Leedy & Ormrod, 2005:225) to provide in depth evidence on the effect Inductive teaching approach had on the learning of the concept function was employed.

Group	Numbers	Pre Test	Intervention	Post Test
Experimental Group:	Teachers: 1 Learners: 61	Math Test 1	Inductive Teaching	Math Test 2
Control Group:	Teachers: 1 Learners: 61		Traditional Teaching	

Table 3.1: Experimental design (Leedy & Ormrod, 2005:236)

- Qualitative investigation: After completion of the intervention, learners who were willing to be interviewed and a teacher of the participating classes were interviewed about their views, experiences and preferences regarding the teaching approach and learning that took place in the classes. Semi-structured interviews were used for this purpose. (Leedy & Ormrod, 2005:100).

3.2.2 Population and sample

The population consisted of Grade 11 learners from the Gauteng Province, Tshwane West Region. The sample consisted of 122 Grade 11 learners from two high schools in the region. The researcher enlisted the services of the Subject Advisory to identify the best performing schools that practise Outcomes-based Education well. Teachers from the identified schools were interviewed to ascertain the assertion that theirs schools are best Outcomes-based Education practising schools, and one out of five schools was randomly selected. From the selected school two Grade 11 classes, one with 32 learners and the other with 29 learners, were assigned as experimental group. Furthermore, with the assistance of Subject Advisory, the researcher identified a school that taught in the traditional way, and again teachers from those schools were interviewed to establish the veracity of the assertion. From this school, two Grade 11 classes, one with 35 learners and the other with 26 learners, were assigned as the control group. For qualitative investigation, four learners who voluntarily accepted to be interviewed from the experimental group, together with their mathematic teacher were interviewed.

3.2.3 Variables

The variables under quantitative investigation are as follows:

- Independent: Teaching approach (inductive teaching versus traditional deductive teaching).
- Dependent: Learning achievement in functions in Grade 11; nature of conceptualization. The test scores served as a measure for the achievement with regard to the learning of functions. Items in the tests were set and assessed according to O'Callaghan's four components of conceptualization of the function idea (see 2.9).

3.2.4 Data collection and measuring instruments

- Quantitative investigation: Self-constructed paper and pencil tests for learners on functions were used. The pre-test (Math Test 1) was based on the required entry competence of Grade 11 learners with regard to the function concept. The post-test (Math Test 2) was based on the work done during the intervention. Items in the post-test were set and assessed according to O'Callaghan's four components of conceptualization of the function idea, namely modelling, translation, interpretation and reification (see 2.9). O'Callaghan's model was chosen because it provided an appropriate description of a function in terms of these four components.

Details of the questions for each of the four categories are summarized as follows:

(i) Modelling (Question 1): Notions of Hire Purchase plan and vending were used to show that word problems on these ideas could be expressed in mathematical statements of linear equations.

(ii) Interpretation (Question 2): A train pulling into a station does not come to an abrupt standstill. It decelerates over time until the final speed is zero. A linear graph with negative slope is used to show this decrease in speed over time.

(iii) Translation (Question 3): Depreciation in the value of a car is assumed linear in this component. A table of depreciation values over time in years are calculated in this question.

(iv) Reification (Question 4): Composition of functions concept is transformed into profit and sales in this component.

- For validity purposes both tests were moderated by three Grade 11 mathematics teachers and a Subject Advisor, and were also piloted on four non-participating learners in a similar nearby school. The p-value, effect size and reliability using Cronbach's Coefficient Alpha were calculated with the assistance of the Statistical Consultation Services of the NWU.

- Qualitative investigation: Semi-structured interviews were held with four learners and the teacher focusing on their views, experiences and preferences regarding the teaching approach and learning that took place in their respective classes. Learners were interviewed with the aim of reflecting on their experiences and perceptions of the teaching during the intervention period. Interviews were audio taped and typed verbatim. An independent observer was invited to ensure that the process has been successfully completed. Recorded tapes are in the possession of the researcher.

3.2.5 Procedure for data collection

The following procedure was followed to gather data:

- Training of the experimental teacher in the application of an inductive teaching approach with regard to functions was done. The teacher was advised to expose learners to instances that relate to relationships, for example to relate the shoe sizes of the learners to their gender. Another typical real life example was to consult with the Biology teacher on the experiment of observing how fruit flies multiply after a few days. The main idea was to encouraged learners to observe patterns, raise questions and make generalizations from their observations. The teacher's role was to create opportunities and the context in which learners make appropriate generalizations and to guide them where necessary.

  It should be noted that although the experimental teacher was monitored during the course of the intervention, she was. however, given free reign to conform to the prescribed curriculum as well as to apply an inductive approach, without disturbing her schedule.

- The pre-testing for both the control and the experimental groups was done before the teaching of the concept to test the nature of conceptualization of functions based on the Grade 10 work.
- Intervention: the experimental classes were subjected to inductive teaching of functions for a period of at least four weeks, while the control classes followed the traditional approach.
- The post-test was piloted on four non-participating learners in a similar nearby school to determine whether questions were up to the required standard of Grade 11.
- Post-testing: A post-test was administered to both the control and the experimental groups to check the improvement as well as progression.

- Interviews: Learners who volunteered and a teacher were interviewed after completion of the experimental phase.

3.2.6 Data analysis

- Quantitative analysis: Inferential statistics by means of t-tests, effect sizes and analysis of variance was used to analyse the experimental data (Leedy & Ormrod, 2005:274). The assistance of the Statistical Consultation Services of the NWU was sought.
- Qualitative analysis: The researcher grouped information into segments that reflect various aspects of the experience. Divergent perspectives were identified. The researcher used various meanings identified to develop an overall description of the experience (Leedy & Ormrod, 2005:144). All data analysed and interpretations were subjected to literature control.

3.2.7 Ethical aspects

In line with ethical aspects, the following ethical procedure was followed:

- An application letter to obtain permission was written to the Gauteng Department of Education to conduct research in schools under the Tshwane West Region (Appendix A).
- Further arrangements were made with the Principals of the selected schools:
- Consent was sought from the learners participating in the research, who were assured that the research will in no way harm or hurt the participants. They were informed that the participation in the exercise is voluntary (Appendix B)
- Consent was sought from the Grade 11 mathematics teachers of the selected schools, who were informed that the participation is voluntary.

- The identity of participants in the research was kept anonymous. All interactions of the researcher and the participants, as well as all data were treated with strict confidentiality and used for research purposes only.

3.3 CONCLUSION

The purpose of the exercise was to secure participation of schools where OBE is practised well and where the traditional form of teaching is still being followed. This feat was achieved with the assistance of Subject Advisory. Consent was sought from principals to use their schools as research laboratories. The letter of permission to conduct research in schools was received only in November, long after data had been collected. Data collection was possible due to the cooperation of principals and teachers of schools that participated in the research exercise.

CHAPTER 4

DATA ANALYSIS AND INTERPRETATION

4.1 INTRODUCTION

Analysis in this chapter is in two parts. The first part is the analysis of the quantitative data carried out with the assistance of the Statistical Consultation Services of the Potchefstroom Campus of Northwest University. The second part is the analysis of qualitative data from the interviews held with both the learners and their teacher.

4.2 ANALYSIS OF QUANTITATIVE DATA

4.2.1 Reliability of measuring instruments and significance of effect

The measure of reliability of the measuring instrument was done through the calculation of Cronbach's Coefficient Alpha (Reynaldo & Santos, 1999) for the four variables in the post test. For reliability the standardised alpha should at least be 0,5. The inter-item correlation of between 0,15 and 0,55 is also an indication of reliability. The effect size [effect size is the difference the intervention makes in terms of standard deviation units in each study (Leedy & Omrod, 2005: 274)] of about 0.2 is regarded as small and provides a low practically significant difference; that of about 0.4 is regarded as moderate and gives a medium/moderate (possibly) practically significant difference. An effect size of 0.8 gives a large and practically significant effect. Information regarding minimum p-value is provided in the different tables (Schumacher & Macmillan, 2001:369).

The following table gives a summary of the results for reliability of the measuring instrument.

Table 4.1: Cronbach's Coefficient Alpha for different components in the post test

Variable	Mean = 14.8934 Std.Dv = 4.85629 Cronbach alpha: 0.400481 Standardised alpha: 0,502017 Average inter-item correlation: 0,207267				
	Mean if deleted	Var. if deleted	St Dv. If deleted	Inter-item correlation	Alpha if deleted
Modelling	11.81	8.43	2.90	0.40	0.05
Translate	12.95	21.27	4.61	0.004	0.49
Interpret	5.28	12.99	3.60	0.27	0.27
Reify	14.63	20.44	4.52	0.47	0.32

The values of inter-item correlation for modelling, interpretation and reification components of approximately 0,4 , 0,28 and 0,48 respectively are between 0,15 and 0,55 and this indicates reliability. The standardised alpha coefficient of approximately 0,5 for the modelling component as well as for the measuring instrument as a whole indicates that the instrument is reliable. Similarly, the average inter-item correlation of approximately 0,2 for the measuring instrument as a whole indicates that the instrument was reliable (Reynaldo & Santos, 1999). Generally the measuring instrument was reliable.

Attention is now drawn to the analysis of the quantitative data, taking each component at a time within the groups and then between groups. Analysis between groups was carried out for post test results only while within groups the comparison was between pre test and post test results. We start by looking at the overall performance between groups in the pre-test.

4.2.2 Pre-test performance between groups

4.2.2.1 Analysis of quantitative data between groups for pre-test

The following table 4.2 outlines information on the means of both the experimental and the control groups, p-value for both the experimental and the control groups, t statistic and the effect size amongst others for the modelling component for both the experimental and the control groups in a pre-test as provided below.

Table 4.2: Analysis of Pre-test results

	Group1 = C (Control group) Group2 = E (Experimental group) T-test for Group									
	Mean C	Mean E	t-value	df	p	Valid N C	Valid N E	Std Dv C	Std Dv E	F-ratio
Mark	15	14.7	0.24	120	0.8	61	61	5.29	4.41	1.43

d = 0.04

The values for the t-value and d are low, and p>0.05. This indicates that there are neither statistical nor practically significant differences between the experimental group and the control group. The two groups are comparable and on level terms in terms of the knowledge and performance in the concept function at the beginning of the experiment.

4.2.2.2 Analysis of quantitative data in modelling component within groups

We start first by providing analysis for the modelling component within the control group. The following table 4.3 outlines information on the mean, p-value, t statistic and the effect size amongst others for the modelling component within the control group for pre-testing and post-testing as provided below.

Table 4.3: T-test for modelling component (control group)

Variable	Group = C T-test for Dependent Samples Marked differences are significant at p < 0.05000							
	Mean	Std. Dv.	N	Diff	Std. Dv Diff	t	df	p
Modelling Post-testing	2.57	2.30	61	-0.36	3.09	-0.91	60	0.37
Modelling Pre-testing	2.93	2.90						

d = 0.12

The p-value of 0.37 and the effect size of 0.12 indicate that there is no statistical difference as well as practical significant difference in the modelling component from pre-testing to post-testing for the control group.

We proceed to provide analysis for the modelling component within the experimental group. The following table 4.4 outlines information on the mean, p-value, t statistic and the effect size amongst others for the modelling component within the experimental group for pre-testing and post-testing as provided.

Table 4.4: T-test for modelling component (Experimental group)

Variable	Group = E T-test for Dependent Samples Marked differences are significant at p < 0.05000							
	Mean	Std. Dev.	N	Diff	Std. Dev Diff	t	df	p
Modelling Post-testing	4.25	2.81	61	1.01	3.36	2.36	60	0.02
Modelling Pre-testing	3.22	2.877461						

d = 0.35

The p-value of approximately 0.02 as given in table 4.4 is below 0.05, and this indicates that there is a statistical significant difference. The effect size of 0.35 indicates that there is almost moderate practical significant difference in the modelling from pre-testing to post-testing for the experimental group. We conclude that the differences are significance.

4.2.2.3 Analysis of quantitative data in modelling component between groups

The following table 4.5 outlines information on the means of the experimental group and the control group, and the effect size for the modelling component for post-test between the experimental group and the control groups as provided.

Table 4.5: Univariate Tests of significance for modelling component (Post test)

Group;LS Means Current effect: F(1,119)= 12.735, p = 0.00052 (Adjusted means)					
Group	Modelling Post: Mean	Modelling Post Std.Err	Modelling Post -95%	Modelling Post +95%	N
C	2.61	0.32	1.99	3.24	61
E	4.21	0.32	3.58	4.83	61

d = 0.65

We conclude that the differences are (almost) practically significant from the effect size of 0.65, the high F = 12.75 and $p < 0.05$.

4.2.3 Translation Component

4.2.3.1 Analysis of quantitative data in translation component between groups for pre-test results

The following table 4.6 outlines information on the means of both the experimental and the control groups, p-value for both the experimental and the control groups, t statistic and the effect size amongst others for the translation component for both the experimental and the control groups in a pre-test as provided below.

Table 4.6: T-test for groups for translation component (Pre-test)

Variable	Group1 = C (Control group) Group2 = E (Experimental group) T-test for Group							
	Mean:C	Std. Dv. C	N:C	t-value: C & E	P C & E	F-ratio Variances C & E	df C & E	P Variances C & E
Translation Pre-testing	2.19	1.11	61	1.97	0.05	1.90	120	0.01
	Mean:E	Std. Dv. E	N:E					
	1.69	1.63	61					

d = 0.31

The p-value of approximately 0.05 and the effect size of 0.31 indicate a statistical significant difference and a moderately practically significant difference towards the experimental group. We conclude that the differences are significant between the experimental group and the control with regard to translation component.

4.2.3.2 Analysis of quantitative data in translation component within groups

We start first by providing analysis for the translation component within the control group. The following table 4.7 outlines information on the mean, p-value, t statistic and the effect size amongst others for the translation component within the control group for pre-testing and post-testing as provided.

Table 4.7 T-test for dependent samples for translation component (control group)

Variable	Group = C T-test for Dependent Samples Marked differences are significant at $p < 0.05000$							
	Mean	Std. Dev.	N	Diff	Std. Dv		df	p
Translation Post-testing	2.66	1.29	61	0.46	Diff 1.323805	t 2.71	60	0.008
Translation Pre-testing	2.19	1.18						

d = 0.36

The p-value of 0.008 and the effect size of 0.36 indicate that the differences are of moderate practical significance.

We now look at the analysis of results of translation component within the experimental group. The following table 4.8 outlines information on the mean, p-value, t statistic and the effect size amongst others for the translation component within the experimental group for pre-testing and post-testing as provided below.

Table 4.8: T-test for dependent samples for translation component (Experimental group)

Variable	Group = E T-test for Dependent Samples Marked differences are significant at p < 0.05000							
	Mean	Std. Dev.	N	Diff	Std. Dev Diff	t	df	p
Translation Post-testing	3.09	2.20	61	1.41	2.74	4.01	60	0.0001
Translation Pre-testing	1.68	1.63						

d = 0.64

The p-value of 0.0001 and the effect size of 0.64 indicate that there is a statistical significant difference and possibly practically significant difference. We can safely conclude that the differences are significant for translation component within the experimental group.

4.2.3.3 Analysis of quantitative data in translation component between groups

The following table 4.9 outlines information on the means of the experimental group and the control group, and the effect size for the modelling component for post-test between the experimental group and the control groups as provided.

Table 4.9: Univariate Tests of significance for translation component (Post test)

Group;LS Means Current effect: F(1,119)= 16.212, p = 0.00010 (Adjusted means)					
Group	Translation Post: Mean	Translation Post Std.Err	Translation Post -95%	Translation Post +95%	N
C	9.13	0.40	8.33	9.924725	61
E	6.84	0.40	6.04	7.634558	61

d = 0.74

From the effect size of 0.74, F = 12.21 and $p < 0.05$ we conclude that the differences are (almost) practically significant.

4.2.4 Interpretation Component

4.2.4.1 Analysis of quantitative data in interpretation component between groups for pre-test results

The following table 4.10 outlines information on the means of both the experimental and the control groups, p-value for both the experimental and the control groups, t statistic and the effect size and values for other variables for the translation component for both the experimental and the control groups in a pre-test as provided.

Table 4.10: T-test for groups for interpretation component (Pre-test)

Variable	Group1 = C (Control group) Group2 = E (Experimental group) T-test for Group							
	Mean:C	Std. Dv. C	N:C	t-value: C & E	P C & E	F-ratio Variances C & E	df C & E	P Variances C & E
Interpret Pre-testing	9.50	2.57	61	-0.45	0.27	1.33	120	0.01
	Mean:E	Std. Dv. E	N:E					
	9.70	2.23	61					

d = 0.08

The p-value of approximately 0.01 and the effect size of 0.08 indicate that there is a statistical significant difference but no practically significant difference. We conclude that the differences are not significant between the experimental group and the control with regard to interpret component.

4.2.4.2 Analysis of quantitative data in interpretation component within groups

We start first by providing analysis for the translation component within the control group. The following table 4.11 outlines information on the mean, p-value, t statistic and the effect size amongst others for the interpretation component within the control group for pre-testing and post-testing as provided in table 4.11

Table 4.11: T-test for interpretation component (control group)

Variable	Group = C T-test for Dependent Samples Marked differences are significant at $p < 0.05000$							
	Mean	Std. Dev.	N	Diff	Std. Dv	t	df	p
Interpret Post-testing	2.66	1.29	61	0.46	Diff 1.32	2.718	60	0.008
Interpret Pre-testing	2.19	1.18						

d = 0.36

The p-value is less than 0.05 and the effect size is 0.36. This indicates that the differences are significant.

We now look at the analysis of results of the interpretation component within the experimental group. The following table 4.12 outlines information on the mean, p-value, t statistic and the effect size amongst others for the translation component within the experimental group for pre-testing and post-testing as provided below.

Table 4.12: T-test for interpretation component (Experimental group)

Variable	Group = E T-test for Dependent Samples Marked differences are significant at $p < 0.05000$							
	Mean	Std. Dv.	N	Diff	Std. Dv Diff	t	df	p
Interpret Post-testing	3.09	2.20						
Interpret Pre-testing	1.68	1.63	61	1.41	2.74	4.01	60	0.0002

d = 0.64

The p-value < 0.05 and the effect size of 0.64 indicate that the differences are possibly practically significant.

4.2.4.3 Analysis of quantitative data in interpretation component between groups

The following table 4.13 outlines information on the means of the experimental group and the control group, and the effect size for the modelling component for post-test between the experimental group and the control groups as provided.

Table 4.13: Univariate Tests of significance for translation component (Post test)

Group;LS Means Current effect: F(1,119)= 2.5104, p = 0.11575 (Adjusted means)					
Group	Interpret Post: Mean	Interpret Post Std.Err	Interpret Post - 95%	Interpret Post +95%	N
C	2.61	0.23	2.15	3.07	61
E	3.13	0.23	2.67	3.59	61

d = 0.29

From the effect size of 0.29, low F value and p > 0.05 we can conclude that the differences are insignificant.

4.2.5 Reification Component

4.2.5.1 Analysis of quantitative data in reification component between groups for pre-test results

The following table 4.14 outlines information on the means of both the experimental and the control groups, p-value for both the experimental and the control groups, t statistic and the effect size and values for other variables for the reification component for both the experimental and the control groups in a pre-test as provided below.

Table 4.14: T-test for groups for reification component (Pre-test)

Variable	Group1 = C (Control group) Group2 = E (Experimental group) T-test for Group							
	Mean:C	Std. Dv. C	N:C	t-value: C & E	P C & E	F-ratio Variances C & E	df C & E	P Variances C & E
Reify Pre-test	0.36	0.66	61	1.83	0.27	1.59	120	0.07
	Mean:E	Std. Dv. E	N:E					
	0.16	0.52	61					

d = 0.04

The p-value > 0.05 and the effect size d = 0.08 indicate that there is no statistical significant difference or practically significant difference. We conclude that the differences are not significant between the experimental group and the control with regard to reification component.

4.2.5.2 Analysis of quantitative data in reification component within groups

We start first by providing analysis for the reification component within the control group. The following table 4.15 outlines information on the mean, p-value, t statistic and the effect size amongst others for the reification component within the control group for pre-testing and post-testing as provided below.

Table 4.15 T-test for reification component (control group)

Variable	Group = C T-test for Dependent Samples Marked differences are significant at p < 0.05000							
	Mean	Std. Dv.	N	Diff	Std. Dv Diff	t	df	p
Reify Post-test	0.61	0.78						
Reify Pre-test	0.36	0.66	61	0.25	0.81	2.37	60	0.02

d = 0.32

The p-value < 0.05 and the effect size of 0.32 indicate that differences are statistically but not practically significant.

We now discuss the analysis of results of reification component within the experimental group. The following table 4.16 outlines information on the mean, p-value, t statistic and the effect size amongst others for the translation component within the experimental group for pre-testing and post-testing as provided below.

Table 4.16 T-test for reification component (Experimental group)

Variable	Group = E T-test for Dependent Samples Marked differences are significant at p < 0.05000							
	Mean	Std. Dv.	N	Diff	Std Dv Diff	t	df	p
			61	1.016393			60	0.00000
Reify Post-test	1.18	1.43			1.27	-0.91		
Reify Pre-test	0.16	0.52						

d = 0.71

The p-value < 0.05 and the effect size of 0.71 indicate that the differences are possibly practically significant.

4.2.5.3 Analysis of quantitative data in reification component between groups

The following table 4.17 outlines information on the means of the experimental group and the control group, and the effect size for the reification component for post-test between the experimental group and the control groups as provided.

Table 4.17: Univariate Tests of significance for reification component (Post test)

Group;LS Means Current effect: F(1,119)= 2.5104, p = 0.11575 (Adjusted means)					
Group	Reify Post: Mean	Reify Post Std.Err	Reify Post - 95%	Reify Post +95%	N
C	0.53	0.13	0.25	0.80	61
E	1.25	0.13	0.98	1.52	61

d = 0.68

The effect size of 0.68 indicates a possibly practically significant difference. However, we may not conclude that the differences are significant due to a low F-value and that $p > 0.05$.

Finally we discuss the analysis of the total mark between the experimental and the control groups.

4.2.5.4 Analysis of total mark quantitative data between groups

The following table 4.18 outlines information on the means of the experimental group and the control group, and the effect size for post-test between the experimental group and the control groups as provided.

Table 4.18: Univariate Tests of significance for total mark (Post test)

Group;LS Means Current effect: F(1,119)= 0.27425, p = 0.60147 (Adjusted means)					
Group	Mark Post: Mean	Mark Post Std.Err	Mark Post - 95%	Mark Post +95%	N
C	14.8	0.75	13.3	16.3	61
E	15.4	0.75	13.9	16.9	61

d = 0.09

From the effect size of 0.09, low F-value and p-value > 0.05, we conclude that there is no significant difference. The two groups are comparable in terms of performance in the post-test. [The two groups differ considerable in different components; however, the overall performance does not indicate a significant difference.]

4.3 ANALYSIS OF QUALITATIVE DATA

Four learners and the mathematics teacher were interviewed on their experiences, perceptions and attitudes regarding inductive teaching. Their responses to the interview questions are summarized below. The analysis is dealt with in responses to individual interview questions (appendix E).

4.3.1 Interviews held with learners

Question 1: Can you tell any change that you noticed in the way mathematics was taught in your class since you took the pre-test?

Responses to the question revealed that learners indicated that the teacher introduced the lesson on functions, gave examples to clarify the topic and gave them time and space to go and explore more on the topic.

One of the learners said "The teacher introduced the new matter by teaching us, followed by several examples to clarify what she has been teaching. Examples are good because you can understand what is going on" (Appendix E).

Two learners preferred that at the start of teaching the concept, a lot of examples be given so that they "understand better". According to them they understood much better if more examples were given, because they could not follow the lesson if they did not have ideas of what is going on (Appendix E).

Question 2: Did you notice any difference in the way the teacher was teaching or was it just the same?

Responses to the second question indicated that two learners did not pick up any difference while others noted that there was indeed a difference in the teacher's approach to the topic. One learner said, "There was some difference because we were given a lot of examples on the topic and given research to do" and the other one said "There was not much difference to what we are used to, except that she would give us something to research after giving us several examples" (Appendix E).

Question 3: Do you like the way the teacher was teaching in the past three months or so?

There were mixed feelings to the teacher's new approach in response to the third question. Two learners were in favour of it, while other learners were not. This is what one said, "If the teacher continued teaching the way she did, I would benefit, but my peers complained that they could not do a thing without first being guided with examples,"

Question 4: Do you think that the way the teacher was teaching would make you improve your understanding or not?

There were also mixed feelings to the teacher's new approach in response to the fourth question. Two learners were in favour of it, while the other two were not. This is what one said "I do not like the way she was teaching, because if you do not know what is going on, you cannot do the research we are expected to do on the topic. I like it if she teaches" (Appendix E).

Question 5: Did you like the way the teacher was teaching or would you prefer the way s/he was teaching before you took the test?

There were mixed feelings to the teacher's new approach in response to the fifth question. Two learners were in favour of it, while other learners were not. One learner said "I would like her to teach, because we do not understand that method, and there are no textbooks with answers to the projects we are supposed to do."

Question 6: What is your attitude towards mathematics from the way the teacher was teaching?

One learner said she enjoyed doing mathematics sums, especially for the lower Grade learners. She derived pleasure in doing so as she regards this as fulfilling her mathematical ability (Appendix E). The other learner felt that mathematics is a difficult learning area even with this new approach. He said, "Mathematics is a difficult subject. If you are not properly taught you will end up getting nothing" (Appendix E).

4.3.2 Interview held with the teacher

Question 1: Tell about your experiences with inductive teaching in your classroom.

The application of inductive teaching became a burden to the teacher as some learners often became frustrated and disinterested due to a lack of resources (Appendix E).

Question 2: What are the challenges you experienced in implementing inductive teaching in the concept Functions lessons?

The teacher regarded lack of formal training in Outcomes Based Education as a challenge towards implementation of the strategies related to it. The other challenge according to the teacher is the lack of time and the pressures of the departmental requirements (Appendix E).

Question 3: Do you think inductive teaching get learners more involved in the lesson?

The teacher expressed that when properly understood and applied, inductive teaching would encourage learner involvement (Appendix E).

Question 4: Do you think if teachers used inductive teaching that it can help improve learners' understanding of the concept Function?

The teacher felt that if the teachers were to implement inductive teaching, more time would be needed to complete all the outcomes. It could serve the purpose if the schools are well resourced.

4.4 DISCUSSION OF FINDINGS

Analysis of quantitative data revealed that the differences among components between the experimental group and the control group were generally significant. This was an indication of the success of the experiment, even though the total difference between the two groups in terms of performance was not significant. The performance between the experimental group and the control group was as good as equal. This may be attributed to some of the challenges highlighted by the teacher during the interview and that the experiment was carried out over a limited period of time.

Kieran (1992: 408) identified that learners perceive functions as equations only, and cannot relate solutions of equations to values of corresponding functions in graphical solutions. This explains why learners understand functions from procedural rather than structural perspectives. Learners have a better conception of a function if it is in an equation form of computing one value of a variable based on another. Two learners felt that they needed to be taught first before they can work on exercises on function. The impression is that they find it challenging to transfer a word problem to a mathematical statement in symbols, and this is consistent with literature that learners find it difficult to transfer from graphical to algebraic and vice versa (Kieran, 1992: 411).

Most algebra textbooks define a function as a relation between two sets so that one member of the domain has only one image. This emphasises the structural rather than the procedural view of functions and learners find it hard making sense of it because they are inclined to procedural conception of functions (Kieran, 1992: 411).

Two of the learners interviewed hinted that without being taught first they cannot comprehend concept functions. The impression is that the teacher requested equations of lines, and they work out ordered pairs and translate the ordered pairs to Cartesian graphs.

4.5 CONCLUSION

The general impression is that Inductive teaching did influence the performance of the learners in the experimental group, even though this was not the case in all components. The study registered some success in improving the conception of functions in terms of the nature of understanding; however, the same cannot be said about the performance in terms of improvement of marks. Some of the challenges raised by the teacher in the implementation of inductive teaching might have had an effect on the overall outcome of the study.

Much valuable information has been received from the learners that were interviewed; however, the researcher is of the opinion that had more learners volunteered one would get a clearer understanding of what transpired in the classroom. The researcher was mindful of the lack of confidence on the part of some learners and the uneasiness in talking mathematics.

CHAPTER 5

SUMMARY OF FINDINGS, RECOMMENDATIONS AND CONCLUSION

5.1 BACKGROUND

The primary purpose of the study was to investigate the influence Inductive teaching has on the nature of conceptualisation and the learning achievement of functions. The study approach used both quantitative and qualitative methods. The quantitative approach was in the experimental form of pre-test and post-test, while the qualitative approach employed unstructured interviews.

The study was conducted on a sample of 122 Grade11 learners in two schools with one school serving as a control group and the other as an experimental group. The conceptual framework for the study was the model adopted by O'Callaghan which identifies and applies four component competencies of modelling a function, interpreting a function, translating and reifying a function. The study resulted in the following findings.

5.2 FINDINGS

5.2.1 Finding 1

The inductive teaching positively influenced the conceptualisation of functions in Grade 11. The nature of conceptualisation was in the understanding of components relevant to the comprehension of function concept. The comparison of different components of O'Callaghan's model revealed that there was improvement in modelling, translating and interpretation of functions among the experimental group. The implication was that learners improved in transferring a word problem to a mathematical statement in symbols. Learners could draw graphs of functions on a two-dimensional plane if given points or an equation. Learners could also complete tables well from recognizing a pattern given the first few entries and finally their manipulations skills such as transformation or composition also improved even though it was revealed that reification required more attention. This indicates that learners benefit from visual representations

(Appendix C). Active, visual and intuitive learners benefitted from the visual representation as these learning styles relate to translation and interpretation components as advocated by O'Callaghan (see 2.9).

5.2.2 Finding 2

Based on quantitative analyses, inductive teaching improved the learning achievement of the Grade 11 in experimental group with regard to functions. The learning achievement meant being able to translate among different representations of functions. The translation among different representations of functions enhances procedural skills. The performance between the experimental group and the control group in terms of marks was comparable in terms of the overall test results. However, they differed significantly in different components. Verbal and sensing learning styles relate to the reification component, and the analysis indicated that more attention is needed in this regard. This outcome could be imputed to some of the following limitations highlighted by the study as outlined below.

5.3 LIMITATIONS OF THE STUDY

- Although it is noted that the study did register some success, the results cannot be generalised to all Grade 11 learners.
- The study was limited to the linear functions only.
- The amount of available time for the study did not afford the researcher to possibly reveal more about the research.
- Sufficient time is needed to introduce someone to a new strategy especially since they did not receive formal training. A teacher firmly rooted in traditional teaching finds it hard to switch to new approaches such as inductive teaching.

- The study success was limited to the nature of conceptualisation and did not bring forth a measure of improvement in the performance between the experimental and the control groups in part due to time constraints and the number of concepts (linear functions as opposed to including quadratic, rational functions etc.) used in the study.

5.4 RECOMMENDATIONS

Based upon the findings in this study the following is recommended.

- Learners benefited more from visual and intuitive learning styles. This is evident by their being able to complete tables well from recognizing a pattern given the first few entries. Teachers need to encourage this kind of learning in the concept functions. Examples that relate to the learners' real life experiences similar to those in the post-test question paper (Appendix C) would go a long way in enhancing their understanding of functions as they relate to their experiences in their immediate environment.
- Verbal and sensing learning styles need attention. These relate to the reification component of conceptualization of functions (see 2.9.4). Teachers need to pick examples that relate to real life experiences of the learners in the teaching of functions. This will help improve in translating from a word problem to a mathematical statement in symbols. The translation from a word problem to a mathematical statement is seen as an important stage in problem solving. Learners need to be given tasks that will enable them to practice on translating between graphs, tables and forming mental objects out of something that is perceived to be abstract.

5.5 FURTHER RESEARCH

Future projects in this area should include determining the influence of a combination of inductive and deductive strategies to the concept functions. It would also be interesting to extend the study to quadratic and rational functions for a longer duration.

5.6 FINAL CONCLUSION

A primary objective of teaching is to enable learners to perform tasks expected of them. Learners learn mathematics well if they learn it with understanding. For the learners to make sense of what they learn, they need to be able to construct their own understanding. Strategies that will actively involve them in learning such as problem solving and cooperative learning should be applied more often in varying ways by those involved with teaching and training young ones.

In teaching, there is no set of rules but only guidelines on how to become an outstanding teacher. This lack of prescription demands that the teacher be creative and be innovative to ensure that the learners understand and benefit from what they learn. In a classroom learners should be encouraged to grapple with ideas, challenge their minds and those of other learners in order to gain understanding and make sense of the world they live in.

Based upon the findings, inductive teaching when applied properly would cater for the mathematical learning needs of the visual, active and intuitive learners, because it affords each the opportunity to construct understanding in the style best suited to him or her and also enhance good study skills among learners. Abstract and tactile learning styles need support so that learners inclined to these styles of learning may benefit more in learning the function concept.

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APPENDIX A

UMnyango WezeMfundo
Department of Education

Lefapha la Thuto
Departement van Onderwys

Enquiries : Shadrack Phele MIRMSA

Tel. No. : [+2711] 355 0285

Wednesday, 19 November 2008

Mr. TP Masebe
PO Box 911-3631
ROSSLYN
0200

Dear Mr. TP Masebe

PERMISSION TO CONDUCT RESEARCH: PROJECT

The Gauteng Department of Education hereby grants permission to conduct research in its institutions as per application.

Topic of research	: "The influence of inductive teaching approach on the Learning of functions in Grade 11."
Nature of research	: M.Ed. [Education Practice]
Name of institution	: North-West University

Upon completion of the research project the researcher is obliged to furnish the Department with copy of the research report (electronic or hard copy).

The Department wishes you success in your academic pursuit.

Yours in Tirisano,

p.p. Shadrack Phele [MIRMSA]
TOM WASPE
CHIEF INFORMATION OFFICER
Gauteng Department of Education

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APPENDIX B

LETTER OF CONSENT

I,____________________________, hereby acknowledge that:

1. The aims and methods of the research have been explained to me.
2. I voluntarily and freely give consent to my participation in the research study.
3. I understand that the results will be used for research purposes.
4. I am free to withdraw my consent at any time during the study.

I hereby give consent for the following: (Circle either yes or no)

1. Participating in a pre-test

   Yes No
2. Participating in a post-test

   Yes No
3. Being interviewed after participating in a post-test

   Yes No

Signature:__________________________________ Date:_______________

APPENDIX C

EXAMINATION (PRE TEST)	:	CONCEPTION OF FUNCTIONS
TIME ALLOWED	:	3HRS
MARKS	:	50
FIRST EXAMINER	:	MR T.P. MASEBE
SECOND EXAMINERS	:	MR P. MOKGATLE MR SHALE MS E. MAPHOTO MS T. NTHWANE

INSTRUCTIONS TO CANDIDATES

Attempt all questions.

Questions may be answered in any order keeping all subsections of a question together.

QUESTION 1 [16 Marks]

1.1 Anna bought a HiFi set on a Hire Purchase plan and paid R200 up front as a deposit and instalments of R120 per month for 24 months.

a) Write down the equation for the cost of the phone for x months. (2)

b) Draw the graph to illustrate the changes in the amount of payment as the number of months increases. (4)

c) Use the graph in b) to determine the amount Anna would have paid in six months. (2)

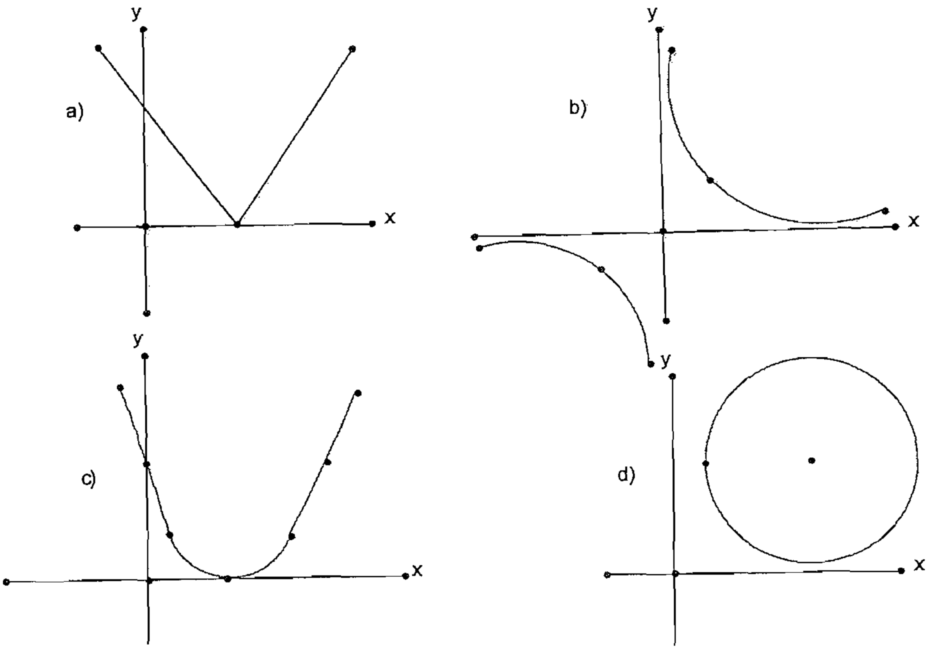
d) Use the graph to calculate the amount Anna will pay after 24 months. (2)

e) If the cash price of the HiFi set is R2000, what is the difference between the cash price and a Hire Purchase price? (2)

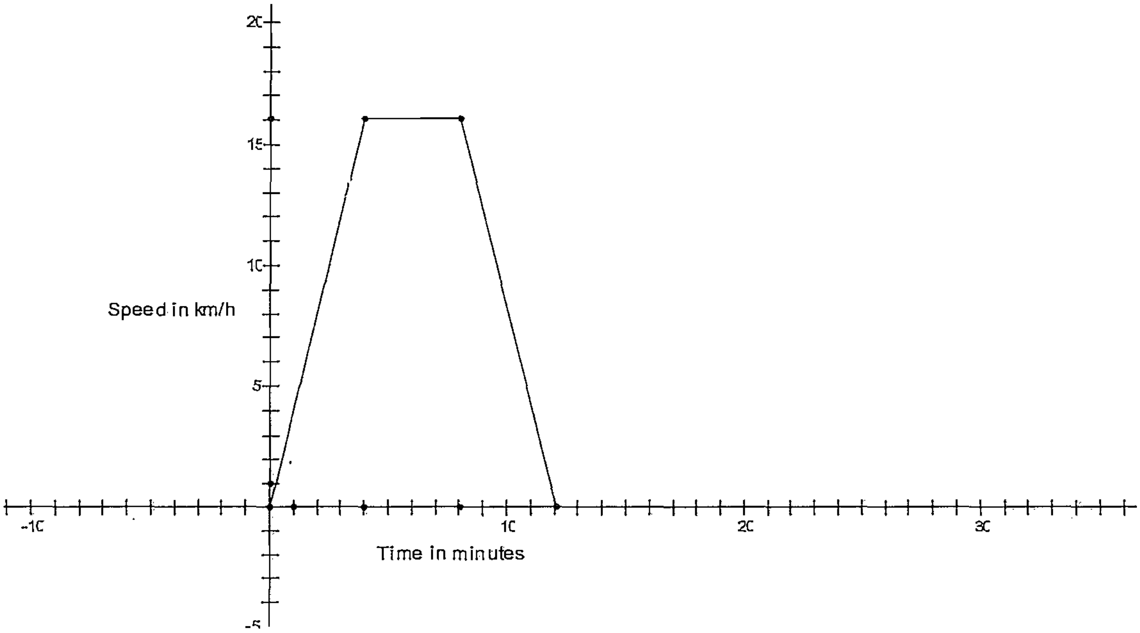
1.2 Suppose that a delivery service charge for delivery of items based on the distance in the following way: The delivery is R10 plus an additional R1, 50cents per kilometre. Sketch the graph of the delivery charges for kilometres from 0 to 10. (4)

QUESTION 2 [8 Marks]

2.1 Choose a graph that does not represent a function from the graphs below, and give a reason why you say that it does not represent a function. (2)



2. 2 The graph below shows speed in kilometres per hour of an athlete running on a treadmill for 12 minutes.



a) For how long does the athlete increase the speed (accelerate)? (1)

b) How long does the athlete maintain a constant speed, and what is the constant speed? (1)

c) How long does the athlete take to reduce the speed to 0 km/h? (1)

d) What distance does the athlete cover in 12 minutes? (3)

QUESTION 3 [19 Marks]

3.1 The following table represents the number of eggs hatched in an incubator at a chicken farm as time increases. The time is measured in hours.

Time in hours	1	2	3	4	5	6	7	8
Number of chicks hatched	1	3	5	7	9			

a) Copy and complete the table (3)

b) Is there a relationship between the time elapsed and the number of chicks hatched? If yes what kind? (2)

c) Write down the mathematical relationship between the number of chicks hatched and the elapsed time. (2)

d) How many chickens will be produced after 20 hours? (2)

e) Sketch the graph of the number of chicks hatched versus the elapsed time. (4)

3.2 The table below gives the average price of a two bed-roomed house in Akasia for every two years since 1996.

Year	Price
1996	**R140000**
1998	**R165000**
2000	**R190000**
2002	**R215000**

2004	**R240000**
2006	
2008	
2010	
2012	

(a) What would the price of a two bed-roomed house be in 2010? (2)

(b) When will the price of a two bed-roomed house be over R300 000? (1)

(c) Write the equation expressing price as a function of the number of years from 1996. (3)

QUESTION 4 [7 Marks]

4.1 The motion of a soft drink tin thrown out of a moving car can be described by the following equations: $y = -16t^2 + 24\sqrt{2}t$ and $x = 24\sqrt{2}t$.

a) Express y as a function of x. (3)

4.2 A street vendor determines that his monthly contribution (C) to the funeral scheme is dependent upon the number of items (n) sold according to the formulae: $C = 0.1(p-10)$ and $p = n^2 - 10n$

a) How much will he contribute to the society if he sold 10 items? (2)

b) Express C as a function of n. (2)

APPENDIX D

EXAMINATION (POST TEST)	:	CONCEPTION OF FUNCTIONS
TIME ALLOWED	:	3HRS
MARKS	:	50
FIRST EXAMINER	:	MR T.P. MASEBE
SECOND EXAMINERS	:	MR P. MOKGATLE MR SHALE MS E. MAPHOTO MS T. NTHWANE

INSTRUCTIONS TO CANDIDATES

Attempt all questions.

Questions may be answered in any order keeping all subsections of a question together.

QUESTION 1: [18 Marks]

1.1 Modise bought a cellular phone on a Hire Purchase plan and paid R80 up front as a deposit and instalments of R60 per month for 24 months.

a) Write down the equation for the cost of the phone for x months. (2)

b) Draw the graph to illustrate the changes in the amount of payment as the number of months increases. (4)

c) Use *the graph in* b) *to determine the amount* Modise *would have* paid in six months. (2)

d) Use the graph to calculate the amount Modise will pay after 24 months. (2)

1.2 Molefe [a tomato vendor] buys a crate of tomatoes from a vegetable market and packets them into 10 packets of five (5) per packet. Find the following.

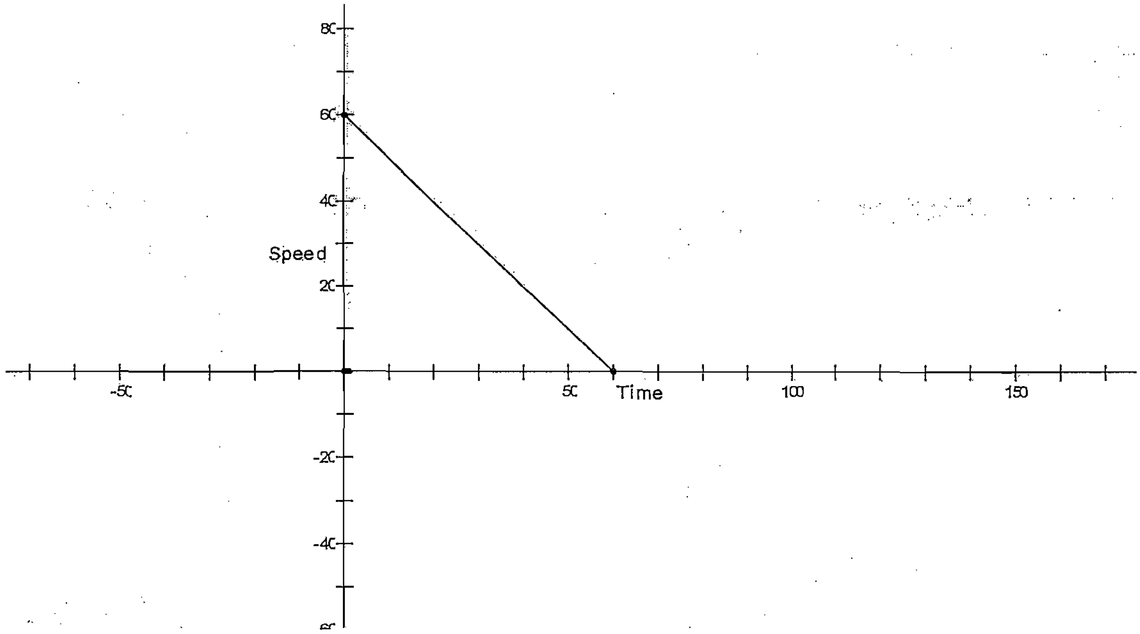
a) The total number of tomatoes in five (5) crates; (3)

b) The number of crates Molefe buys if he obtains 150 packets. (2)

c) If a crate costs R50 and the monthly rental of the stall is also R50, write the profit per crate of tomatoes as a function of x, where x represents the number of packets of tomatoes sold if each packet sells for R10. (3)

QUESTION 2: [9 Marks]

2.1 The following graph illustrates the motion of a train pulling into a station and off-loading its passengers. The speed is expressed in meters per second whereas the time is read in seconds.



Use the graph to answer the following questions.

a) How long does the train take to comes to a standstill? (1)

b) What was the train's speed when the breaks were applied? (1)

c) What is the train's breaking distance? (3)

2.2 Postage rates for an A4 size letter are determined as follows: 20 cents for the first gram plus an additional 10 cents for each gram or a fraction of a gram above 1 gram and less than 10grams. Graph a function representing postage rates for A4 letters between 0 and 10grams. (4)

QUESTION 3: [13 Marks]

3.1 The table below gives the value (V), in rand, of a Hyundai Getz 1.4 GL car for the different number of years (t) after it is purchased.

Number of years (t)	*Value (V) in rand*

0	*117600*
2	*95200*
4	*72800*
6	*50400*
10	?

Use the information in the table to answer the following questions.

a) Write a symbolic rule expressing V as a function of t. (3)

b) Use the rule to determine what the price of the car will be ten years after it is initially purchased. (2)

c) Sketch the graph of the value (V) against time (t). (4)

3.2 A turkey is taken out of a refrigerator and cooked for 2hours. The following is a table of its temperature, in degrees (D) recorded at different times during the 120minutes (m).

m	**0**	**10**	**20**	**30**	**60**	**120**
D	**50**	**100**	**140**	**170**	**200**	**220**

a) Draw a graph of the situation (4)

QUESTION 4: [10 Marks]

4.1 Molefe works forty hours a week at a furniture store, earning a R220 weekly salary plus a 3% commission on sales over R5000. Let Molefe's sales be represented by x, and answer the following questions.

a) Write a formula expressing Molefe's commission as a function of his sales. (2)

b) Write a formula expressing Molefe's total weekly earnings. (3)

4.2 A company sponsoring a charity soccer spectacular event determines its contribution to the charity organisations (C) by the profit (p) determined by the number of tickets (t) sold according to the following formulae:

$C = 0.10(p - 1000)$ and $p = t^2 - 1000t$.

a) How much will the company contribute to charity if it sold 1000 tickets?(2)

b) Express C as a function of t. (3)

APPENDIX E

INTERVIEWS HELD WITH LEARNERS

RESPONDENT NO 1

Question 1: *Can you tell any change that you noticed in the way mathematics was taught in your class since you took the pre-test?*

Response: "The teacher introduced the new content by teaching followed by several examples to clarify what she has been teaching. This is done so that we can understand better. In other cases we were given topics to research on, before the teacher would clarify the content."

Question 2: *Did you notice any difference in the way the teacher was teaching or was it just the same?*

Response: "There was no much difference to what we are used to, except that she would give us something to research on after giving us several examples."

Question 3: *Do you like the way the teacher was teaching in the past three months or so?*

Response: "I prefer that the teacher start with the content and then give several examples so that we understand better."

Question 4: *Do you think that the way the teacher was teaching would make you improve your understanding or not?*

Response: "No. I think I understand better if more examples are given. You can't follow if you do not have an idea of what is going on."

Question 5: *Did you like the way the teacher was teaching or would you prefer the way s/he was teaching before you took the test?*

Response: "I would prefer the way she was teaching before the test. She should give more examples so that one should have an idea of the content so that we are not left not knowing what is happening."

Question 6: *What is your attitude towards mathematics from the way the teacher was teaching?*

Response: "Generally I like to solve problems even before the teacher can give us examples that lead to the new matter. I would say that I generally like mathematics as a subject. This has been the case even before we took the pre-test. I normally use my existing knowledge in solving problems in lower classes."

RESPONDENT NO 2

Question 1: *Can you tell any change that you noticed in the way mathematics was taught in your class since you took the pre-test?*

Response: "The teacher gave us a topic to research on, and everyone would take part in giving feedback on their findings. This helped improve our understanding and facilitated better retention".

Question 2: *Did you notice any difference in the way the teacher was teaching or was it just the same?*

Response: "There was no much difference to what we are used to, although there was a slight difference, but not much".

Question 3: *Do you like the way the teacher was teaching in the past three months or so?*

Response: "I would prefer that she give us a research, because if she gives a research you will never forget. If for example, you learn a theorem on your own, you will never forget, unlike when the teacher teaches you. The teacher helps by giving examples of things to research on".

Question 4: *Do you think that the way the teacher was teaching would make you improve your understanding or not?*

Response: "If the teacher continued teaching the way she was teaching, I would benefit, but my peers complain that they cannot do a thing without first being guided with examples."

Question 5: *Did you like the way the teacher was teaching or would you prefer the way s/he was teaching before you took the test?*

Response: "I would prefer that the teacher give us research first and then we discuss the work later. But, then before the discussion the teacher can give guidelines. Some of my peers complain that if the teacher gives us these projects without giving guidelines, mathematics becomes very difficult, but I prefer things done that way. They have developed a negative attitude towards mathematics."

Question 6: *What is your attitude towards mathematics from the way the teacher was teaching?*

Response: "I have always liked the subject because it is an interesting subject. Every time I solve a problem I feel that I have achieved. It is always good to succeed where others feel that it is difficult. Researching on a problem before we can be taught makes me very happy. I would love that the teacher mix the approach after the pre-test and the one she is used to. The textbooks that we use do not contain most of the information that we need."

RESPONDENT NO 3

Question 1: *Can you tell any change that you noticed in the way mathematics was taught in your class since you took the pre-test?*

Response: "The teacher introduced the new matter by teaching us followed by several examples to clarify what she has been teaching. Examples are good because you can understand what is going on."

Question 2: *Did you notice any difference in the way the teacher was teaching or was it just the same?*

Response: "There was some difference because we were given a lot of examples on the topic and given research to do"

Question 3: *Do you like the way the teacher was teaching in the past three months or so?*

Response: "I do not like the way she was teaching, because if you do not know what is going on, you cannot do the research we are expected to do on the topic. I like it if she teaches."

Question 4: *Do you think that the way the teacher was teaching would make you improve your understanding or not?*

Response: "Like I said if you do not have a clue of what is being done you would be lost. I do not like it because you would be exposed that you do not know anything. The method of teaching will not help improve my understanding."

Question 5: *Did you like the way the teacher was teaching or would you prefer the way s/he was teaching before you took the test?*

Response: "I like it if she teaches us because we do not know mathematics".

Question 6: *What is your attitude towards mathematics from the way the teacher was teaching?*

Response: "Mathematics is a difficult subject. If you are not properly taught you will end up getting nothing".

RESPONDENT NO 4

Question 1: *Can you tell any change that you noticed in the way mathematics was taught in your class since you took the pre-test?*

Response: The teacher introduced the new matter telling us followed by several examples to clarify what she has been teaching. She said that it is done so that we can understand better.

Question 2: *Did you notice any difference in the way the teacher was teaching or was it just the same?*

Response: "I noticed that most of the time she gave examples and gave us homework to find out more about the topic".

Question 3: *Do you like the way the teacher was teaching in the past three months or so?*

Response: We are using only one textbook, which is of low standard. I say it is of low standard because it has few examples and does not help us solve most of the problems. If she gives us work to research on, we end up not doing it because we do not understand. I do not like it that much".

Question 4: *Do you think that the way the teacher was teaching would make you improve your understanding or not?*

Response: "Not at all. I think you will end up confused and not knowing anything because mathematics is a very difficult subject."

Question 5: *Did you like the way the teacher was teaching or would you prefer the way s/he was teaching before you took the test?*

Response: "I would like that she teach, because we do not understand that method, and there are textbook with answers to the projects we are supposed to do."

Question 6: *What is your attitude towards mathematics from the way the teacher was teaching?*

Response: "Mathematics is difficult learning area. Learners need to be taught it. The teacher need to lead and show his/her learners relevant examples so that they follow her and understand."

INTERVIEW HELD WITH THE TEACHER

Question 1: *Tell about your experiences with inductive teaching in your classroom.*

Teacher: Inductive teaching strategy requires one to be familiar with a lot of applications of a particular topic. It poses a very difficult challenge if one has not been trained in it. A challenge I faced is that the learners are used to being taught and told the concepts, and not for them to make self-discoveries. The other challenge is that the amount of time available to implement a new strategy is very limited considering the departmental requirements. The types of learners we have do not have a very good grounding in the mathematical concepts for them infer a concept from several examples and hints. In the light of this, a teacher cannot strictly use inductive teaching alone in one's lesson. Rather a combination of both inductive and traditional teaching would cater for all our learners.

Question 2: *What are the challenges you experienced in implementing inductive teaching in the concept Functions lessons?*

Teacher: In our training we were never exposed to inductive teaching strategy, and in most of our teaching we do not apply it. So it becomes very difficult to implement something new over such a short period. Also inductive teaching strategy requires learners to discover things for themselves; it would amount to disaster if the learners do not have prior information about the specific topic. My approach was to give learners something to research on. This proved a futile exercise as the learners became frustrated and subsequently disinterested due to lack of adequate reference material.

Question 3: *Do you think inductive teaching get you more involved in the lesson?*

Teacher: If properly applied, inductive teaching can encourage learners to do some research on the mathematical topics and discover concepts themselves.

Question 4: *Do you think if teachers used inductive teaching that can help improve your understanding of the concept Function?*

Teacher: If learners are involved in an exercise, they tend to remember and understand it well. It is my belief that if they have discovered a relationship on linear functions and are able to translate it between graphical representation, equations and tables, then they will always remember it, and be able to apply it.

APPENDIX F

PRETEST RESULTS Control Group

Participants		Modelling	Translate	Interpret	Reify	Mark: 50
Participant	1	2	3	9	0	14
Participant	2	1	3	9	0	13
Participant	3	3	2	6	0	11
Participant	4	1	3	6	0	10
Participant	5	0	2	5	0	7
Participant	6	0	3	5	0	8
Participant	7	0	3	6	0	9
Participant	8	0	1	9	0	10
Participant	9	3	2	8	1	14
Participant	10	3	3	13	1	20
Participant	11	2	1	12	1	16
Participant	12	4	2	14	1	21
Participant	13	12	2	15	3	32
Participant	14	0	1	5	0	6
Participant	15	0	4	10	0	14
Participant	16	0	1	7	0	8
Participant	17	2	2	11	1	16
Participant	18	3	2	11	1	17
Participant	19	4	3	7	1	15
Participant	20	2	2	10	1	15
Participant	21	1	2	11	0	14
Participant	22	0	0	3	0	3
Participant	23	2	4	12	0	18
Participant	24	3	3	11	0	17
Participant	25	1	3	12	0	16
Participant	26	8	4	12	0	24
Participant	27	5	1	11	0	17
Participant	28	4	0	11	0	15
Participant	29	0	3	6	0	9
Participant	30	2	1	9	0	12
Participant	31	0	0	11	0	11
Participant	32	2	1	8	0	11
Participant	33	3	3	11	1	18
Participant	34	3	1	13	0	17
Participant	35	12	1	12	0	25
Participant	36	5	3	7	0	15
Participant	37	1	3	4	0	8
Participant	38	5	2	13	2	22
Participant	39	1	3	9	0	13
Participant	40	2	2	8	0	12
Participant	41	0	4	9	0	13
Participant	42	2	2	9	0	13
Participant	43	4	1	12	0	17
Participant	44	1	3	9	2	15
Participant	45	2	2	10	0	14
Participant	46	8	5	12	2	27
Participant	47	2	2	12	0	16
Participant	48	6	0	8	0	14
Participant	49	2	3	8	1	14

Participant	50	8	4	10	0	22
Participant	51	0	0	8	0	8
Participant	52	0	3	10	0	13
Participant	53	5	1	9	0	15
Participant	54	4	1	9	1	15
Participant	55	6	1	10	0	17
Participant	56	8	2	12	0	22
Participant	57	8	4	10	1	23
Participant	58	6	3	11	1	21
Participant	59	4	2	8	0	14
Participant	60	1	3	13	0	17
Participant	61	0	3	9	0	12
TOTAL MARK		179	134	580	22	915
AVERAGE MARK		2.93	2.19	9.508	0.36	15.00

PRETEST RESULTS Experimental Group

GRADE 11

Participants		Model	Translate	Interpret	Reify	Marks:50
Participant	1	2	3	6	0	11
Participant	2	4	3	9	0	16
Participant	3	3	2	6	1	12
Participant	4	4	3	8	0	15
Participant	5	5	2	9	0	16
Participant	6	8	3	12	0	23
Participant	7	3	3	8	0	14
Participant	8	0	1	11	0	12
Participant	9	0	2	8	0	10
Participant	10	2	2	13	0	17
Participant	11	2	1	12	1	16
Participant	12	4	2	11	1	18
Participant	13	4	2	10	0	16
Participant	14	6	1	11	0	18
Participant	15	0	2	13	0	15
Participant	16	0	1	10	0	11
Participant	17	4	2	11	0	17
Participant	18	3	2	11	0	16
Participant	19	4	10	3	1	18
Participant	20	2	1	10	0	13
Participant	21	2	0	7	0	9
Participant	22	4	2	9	0	15
Participant	23	2	0	8	0	10
Participant	24	3	1	11	0	15
Participant	25	1	0	12	0	13
Participant	26	2	0	12	0	14
Participant	27	2	1	7	0	10
Participant	28	5	0	11	0	16
Participant	29	4	3	13	0	20
Participant	30	1	1	9	0	11
Participant	31	10	0	13	0	23
Participant	32	2	1	8	0	11
Participant	33	6	3	11	1	21
Participant	34	0	1	9	0	10
Participant	35	3	1	12	0	16
Participant	36	5	2	7	0	14
Participant	37	2	4	10	0	16
Participant	38	5	1	13	2	21
Participant	39	0	1	9	0	10
Participant	40	2	2	10	0	14
Participant	41	0	4	9	0	13
Participant	42	2	0	7	0	9
Participant	43	5	0	6	0	11
Participant	44	0	1	9	0	10
Participant	45	4	2	10	0	16
Participant	46	1	5	6	0	12
Participant	47	9	2	12	0	23
Participant	48	6	0	12	0	18
Participant	49	2	0	8	0	10

Participant	50	2	0	10	0	12
Participant	51	8	0	9	0	17
Participant	52	0	3	10	0	13
Participant	53	5	0	11	0	16
Participant	54	1	1	9	0	11
Participant	55	6	1	12	0	19
Participant	56	0	2	12	0	14
Participant	57	16	4	11	3	34
Participant	58	1	1	11	0	13
Participant	59	4	1	6	0	11
Participant	60	2	1	7	0	10
Participant	61	2	3	12	0	17
TOTAL		197	103	592	10	902
Average Mark						14.8

APPENDIX G

POST TEST RESULTS : CONTROL GROUP

Participants		Modelling	Interprete	Translate	Reify	Mark : 50
Participant	1	4	3	10	2	19
Participant	2	1	3	9	1	14
Participant	3	3	2	10	0	15
Participant	4	1	3	14	2	20
Participant	5	2	2	7	1	12
Participant	6	0	6	11	0	17
Participant	7	0	3	13	2	18
Participant	8	6	2	9	0	17
Participant	9	5	4	13	1	23
Participant	10	7	3	12	1	23
Participant	11	2	1	12	1	16
Participant	12	4	3	12	1	20
Participant	13	7	4	15	3	29
Participant	14	2	3	10	2	17
Participant	15	5	4	10	0	19
Participant	16	0	1	7	1	9
Participant	17	2	2	7	1	12
Participant	18	3	3	11	1	18
Participant	19	5	4	7	1	17
Participant	20	5	2	10	1	18
Participant	21	0	1	2	0	3
Participant	22	1	3	5	0	9
Participant	23	4	4	12	0	20
Participant	24	3	4	12	2	21
Participant	25	4	3	12	1	20
Participant	26	12	1	12	0	25
Participant	27	5	1	12	0	18
Participant	28	4	2	11	0	17
Participant	29	2	3	7	0	12
Participant	30	5	4	9	0	18
Participant	31	1	3	11	0	15
Participant	32	4	5	10	0	19
Participant	33	3	3	9	1	16
Participant	34	3	1	13	0	17
Participant	35	6	2	12	0	20
Participant	36	5	4	12	3	24
Participant	37	1	3	4	0	8
Participant	38	3	4	10	1	18
Participant	39	1	5	9	1	16
Participant	40	3	3	12	0	18
Participant	41	1	6	9	1	17
Participant	42	2	2	12	1	17
Participant	43	4	2	6	0	12
Participant	44	1	3	8	1	13
Participant	45	2	2	8	0	12
Participant	46	0	2	7	0	9
Participant	47	2	2	3	0	7
Participant	48	1	0	8	0	9
Participant	49	2	3	5	1	11

Participant	50	0	1	5	0	6
Participant	51	1	0	8	0	9
Participant	52	1	3	10	0	14
Participant	53	0	1	9	0	10
Participant	54	0	1	7	0	8
Participant	55	3	1	6	0	10
Participant	56	1	2	6	0	9
Participant	57	0	3	8	1	12
Participant	58	0	3	10	1	14
Participant	59	1	2	6	0	9
Participant	60	1	3	2	0	6
Participant	61	0	3	7	0	10
Total Marks		157	162	555	37	911
Average Mark		2.57	2.67	9.09	0.6	14.90

POST TEST RESULTS Experimental Group
GRADE 11

Participants		Modelling	Interprete	Translate	Reify	Mark:50
Participant	1	1	2	6	2	11
Participant	2	8	0	7	1	16
Participant	3	4	0	8	2	14
Participant	4	6	3	8	0	17
Participant	5	7	1	6	0	14
Participant	6	5	6	13	1	25
Participant	7	0	2	4	0	6
Participant	8	6	6	5	2	19
Participant	9	3	0	7	1	11
Participant	10	5	4	12	2	23
Participant	11	5	4	9	2	20
Participant	12	6	5	8	0	19
Participant	13	5	2	9	2	18
Participant	14	6	7	4	0	17
Participant	15	3	2	9	2	16
Participant	16	2	6	2	0	10
Participant	17	10	6	10	2	28
Participant	18	3	6	13	0	22
Participant	19	1	1	3	0	5
Participant	20	2	5	7	0	14
Participant	21	2	3	0	0	5
Participant	22	4	7	3	0	14
Participant	23	6	3	7	2	18
Participant	24	0	2	9	1	12
Participant	25	6	8	12	4	30
Participant	26	3	0	9	3	15
Participant	27	2	3	4	0	9
Participant	28	5	2	0	0	7
Participant	29	7	5	7	2	21
Participant	30	2	3	6	1	12
Participant	31	10	3	13	2	28
Participant	32	4	2	7	0	13
Participant	33	0	2	7	0	9
Participant	34	2	2	7	1	12
Participant	35	4	1	6	0	11
Participant	36	4	2	4	0	10
Participant	37	4	1	8	0	13
Participant	38	1	2	2	1	6
Participant	39	5	2	8	0	15
Participant	40	4	2	13	1	20
Participant	41	1	3	8	2	14
Participant	42	1	0	0	0	1
Participant	43	1	2	6	1	10
Participant	44	1	1	1	0	3
Participant	45	5	3	4	2	14
Participant	46	2	5	8	3	18
Participant	47	5	5	2	0	12
Participant	48	6	0	6	1	13
Participant	49	4	3	11	1	19

Participant	50	8	1	6	1	16
Participant	51	0	6	9	1	16
Participant	52	11	2	10	2	25
Participant	53	0	2	12	2	16
Participant	54	7	6	7	0	20
Participant	55	5	2	6	1	14
Participant	56	7	2	2	2	13
Participant	57	13	9	13	9	44
Participant	58	4	5	7	3	19
Participant	59	2	1	3	2	8
Participant	60	4	6	10	1	21
Participant	61	5	2	6	1	14
TOTAL		255	189	419	72	935
Average Mark						

Variable	Group=C T-test for Dependent Samples (MasebeTP.sta) Marked differences are significant at p < .05000							
	Mean	Std.Dv.	N	Diff.	Std.Dv. Diff.	t	df	p
TranslatePost	2.655738	1.289512						
TranslatePre	2.196721	1.180673	61	0.459016	1.323805	2.708128	60	0.008803

d=0.36

Variable	Group=E T-test for Dependent Samples (MasebeTP.sta) Marked differences are significant at p < .05000							
	Mean	Std.Dv.	N	Diff.	Std.Dv. Diff.	t	df	p
TranslatePost	3.098361	2.203822						
TranslatePre	1.688525	1.628302	61	1.409836	2.740906	4.017347	60	0.000166

d=0.64

Variable	Group=C T-test for Dependent Samples (MasebeTP.sta) Marked differences are significant at p < .05000							
	Mean	Std.Dv.	N	Diff.	Std.Dv. Diff.	t	df	p
InterpretPost	9.098361	2.947908						
InterpretPre	9.508197	2.573085	61	-0.409836	3.551418	-0.901308	60	0.371028

d=0.14

Variable	Group=E T-test for Dependent Samples (MasebeTP.sta) Marked differences are significant at p < .05000							
	Mean	Std.Dv.	N	Diff.	Std.Dv. Diff.	t	df	p
InterpretPost	6.868852	3.456759						
InterpretPre	9.704918	2.231175	61	-2.83607	3.531668	-6.27193	60	0.000000

d=0.82

Post

Effect	Univariate Tests of Significance for TranslatePost (MasebeTP.sta) Sigma-restricted parameterization Effective hypothesis decomposition				
	SS	Degr. of Freedom	MS	F	p
Intercept	277.0649	1	277.0649	85.65740	0.000000
TranslatePre	6.2664	1	6.2664	1.93733	0.166554
Group	8.1200	1	8.1200	2.51037	0.115753
Error	384.9139	119	3.2346		

Variable	Means for covariates (MasebeTP.sta) LS means are computed for these values (Estimated LS means are 'adjusted' means)
	Mean
TranslatePre	1.942623

Cell No.	Group; LS Means (MasebeTP.sta) Current effect: F(1, 119)=2.5104, p=.11575 (Adjusted means)					
	Group	TranslatePost Mean	TranslatePost Std.Err.	TranslatePost -95.00%	TranslatePost +95.00%	N
1	C	2.614910	0.232134	2.155261	3.074558	61
2	E	3.139189	0.232134	2.679540	3.598837	61

d=0.29

Effect	Univariate Tests of Significance for InterpretPost (MasebeTP.sta) Sigma-restricted parameterization Effective hypothesis decomposition				
	SS	Degr. of Freedom	MS	F	p
Intercept	179.078	1	179.0779	18.17984	0.000040
InterpretPre	66.169	1	66.1685	6.71737	0.010743
Group	159.698	1	159.6975	16.21236	0.000100
Error	1172.192	119	9.8504		

Variable	Means for covariates (MasebeTP.sta) LS means are computed for these values (Estimated LS means are 'adjusted' means)
	Mean
InterpretPre	9.606557

Cell No.	Group; LS Means (MasebeTP.sta) Current effect: F(1, 119)=16.212, p=.00010 (Adjusted means)					
	Group	InterpretPost Mean	InterpretPost Std.Err.	InterpretPost -95.00%	InterpretPost +95.00%	N
1	C	9.128690	0.402018	8.332655	9.924725	61
2	E	6.838523	0.402018	6.042488	7.634558	61

d=0.7

variable	Summary for scale: Mean=14.8934 Std.Dv.=4.85629 Valid N:122 (MasebeTP.sta) Cronbach alpha: .400481 Standardized alpha: .502017 Average inter-item corr.: .207267				
	Mean if deleted	Var. if deleted	StDv. if deleted	Itm-Totl Correl.	Alpha if deleted
ModellingPre	11.81148	8.43167	2.903734	0.403091	0.054455
TranslatePre	12.95082	21.27627	4.612621	0.004533	0.491082
InterpretPre	5.28688	12.99147	3.604368	0.271867	0.270266
ReifyPre	14.63115	20.44592	4.521716	0.478472	0.325634